ARITHMETIC

WITH

NUMEROUS EXAMPLES, EXERCISES,

AND

EXAMINATION PAPERS

ARRANGED BY

A. E. LAYNG, M.A.

Late Headmaster of Stafford Grammar School
Formerly Scholar of Sidney Sussex College, Cambridge

BLACKIE & SON LIMITED
LONDON AND GLASGOW

BLACKIE & SON LIMITED
50 Old Bailey, London
17 Stanhope Street, Glasgow

BLACKIE & SON (INDIA) LIMITED
Warwick House, Fort Street, Bombay

BLACKIF & SON (CANADA) LIMITED
Toronto

Printed in Great Britain by Blackie & Son. Limited Glasgow

PREFACE.

In this work I have endeavoured to provide a convenient and systematic manual of Arithmetic suited to the requirements of the middle and upper forms in schools, of candidates for the various public examinations, and of those preparing for commercial pursuits

In the treatment of the Theory no Algebraical symbols or formulæ have been used The four fundamental rules, with which it is assumed that the reader is already familiar, are considered in less detail than the rest. Decimals are treated as independent of Vulgar Fractions, Recurring Decimals are sufficiently isolated to allow of their omission, if desired, from a first course, and considerable space is devoted to Approximation and its practical applications Questions of the kind known as "Rule of Three" are treated, in the first instance, by the Unitary method, but only as a stepping-stone to the Fractional, or Ratio, method.

The order in which the chapters are arranged is not that in which they need necessarily be read I have preferred rather to collect together, as far as possible, into groups the more closely allied portions of the subject For instance, all the varieties of Reduction will be found in one chapter, so that any one variety may easily be found when wanted.

I have ventured to introduce a few small innovations;—a new explanation of "borrowing" in subtraction, the uniform use of the word "factor" in place of "measure", the device of "moving the points" in Contracted Multiplication. I have also inserted a short chapter on the method

of Nine Multiples, and have given greater prominence to practical, than to purely theoretical, Discount

A special feature of the book is the unusually large number and variety of examples fully worked out and explained in illustration of a rule and its converse, of methods of dealing with problems of some special device for saving labour, as well as of orderly arrangement of work In the notes which accompany these examples I have endeavoured to anticipate the difficulties of the student who is without the constant help of an oral teacher

The Exercises are all collected together at the end of the volume, each set being numbered to match the corresponding chapter of "Bookwork" They have been drawn from so many sources, and so much care has been taken in selecting, adapting, and arranging them, that I trust they will, on trial, not be found wanting as regards either variety or graduation.

In Part I. there are frequent specimens of oral, or mental, questions, such as can, of course, be easily made by the teacher during oral lessons In Part II many of the questions on the commercial rules have not been constructed to yield "neat" results, but are of a strictly practical nature.

The Answers have been carefully tested, but among so many (about 8500) some errors have doubtless crept in I shall be grateful for notification of any that may be discovered, as also for other corrections, or suggestions for the improvement of the book.

A. E. L.

CONTENTS.

BOOKWORK—Part I.

BOOKWORK—PART II.

EXERCISES.—Part I.

EXERCISES.—Part II.

BOOKWORK.—PART I.

I. NUMERATION, NOTATION.

A **unit** is a *single* thing, or object,
also a *single set* of objects when regarded as a whole

For instance, *an apple, a sheep, a pair of gloves, a pack of hounds.*

If the unit is of *some particular kind*, mentioned by name, it is called **concrete.**

A **number** states *how many units* there are in some particular group

For instance, *five* apples, *seven* sheep, *six* pairs of gloves, *three* packs of hounds

When *no particular kind* of unit is mentioned, (*i e* when the unit considered is *abstract*), the number is called an **abstract number.**

For instance, *five, seven, six, three.*

Numeration is the art of expressing numbers in *words*

As all civilized nations count *by tens*, the method of numeration is, in its main features, the same in most languages Each of the numbers from one to ten has an independent name, but the names of numbers greater than ten are almost invariably compounds of these ten names

Thus, in English, "eleven" means *one and ten*, "twelve,"* *two and ten*, "thirteen," *three and ten*, and so on "Twenty" means *two tens*, "thirty," *three tens*, and so on. Also, "hundred," and "thousand," are probably compounds of words which originally meant *ten tens* and *ten hundreds* respectively The names "million," "billion," "trillion," &c, are, however, comparatively modern inventions, and are used, in England,† for a *thousand thousands*, a *million millions*, a *million billions*, &c, respectively.

---

* The use of "eleven," "twelve," in English, where we might have expected to find *one-teen, two-teen*, is probably due to their derivation from the Gothic *lif* instead of the Anglo-Saxon *ten*

† In other parts of Europe, and in America, "billion" is used for a *thousand* millions "trillion" for a *thousand* billions, &c

The method of counting *by tens* is called the **Decimal** system (Latin, *decem*, ten), and the number *ten* is called the **base**, or radix, of the system

The world-wide use of the number *ten* as base is doubtless due to the primitive custom of counting on the *fingers*

Any number *might* be used as base

For instance, if *nine* were base, and "ny" were used for *nine* as we now use "ty" for *ten*, then, just as, with base *ten*, "thir-*ty*-seven" means *three* tens *and seven*, so, with base *nine*, "thir-*ny*-seven" would mean *three* nines *and seven*

In some special cases a limited use *is* made of other bases

For instance, in speaking of "three *score* and eight" sheep, we use *twenty* as base, and in speaking of "five *gross* three *dozen* and seven" buttons, we use *twelve* as base

Notation is the art of representing numbers by means of symbols, or *figures*

Various systems of notation have been used by different nations, but, owing to its great superiority, the only one in general use now is the Arabic *

The ten figures of this system, namely, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,† are called the **digits** (Latin, *digitus*, a finger)

By means of the ten digits any number whatever can be represented, the need of fresh symbols for large numbers being avoided by making the value of any digit depend upon its *position* as well as upon its *shape*

For instance, the figure 3, by reason of its shape always represents *three* units of some kind or other, and, if it stand *alone*, these units are *single*, but, if another figure stand to the right of it, it no longer represents three *ones*, but three *sets of ten*, and so on

Thus, the value of any figure is increased *tenfold* by moving it *one place towards the left*

* It acquired the name "Arabic" from having been introduced into Europe by the Moors when they conquered Spain, but the knowledge of it did not begin to spread in Europe till the 12th century, and it was not in general use till much later The system had its origin in India

† These symbols were in use in India at least 300 B C It has been conjectured that they originated in an arrangement of as many straight strokes as each symbol represents units, thus—

The value which any figure bears *by reason of its shape* is called its *intrinsic*, or *digit* value.

The value which it gains *by position* is called its **local** value (Latin, *locus*, place).

Any number *might* be used as base in a system of notation embodying this principle of local value.

Thus, for instance, just as in the decimal system, with *ten* as base, 342 represents 3 sets of *ten* times *ten*, 4 sets of *ten*, and 2 *single*, objects, so, with *six* as base, 342 would represent 3 sets of *six* times *six*, 4 sets of *six*, and 2 *single*, objects.

The number of digits employed in any such system would be the same as the number chosen as base.

Thus, if *six* were base, the only digits used would be 0, 1, 2, 3, 4, 5; for *six* would then be represented by the figures 10, *seven* by 11, and so on.

The figure 0, (called *nought*, *cipher*, or *zero*,*) has no *intrinsic* value, but is used to fill up a vacant place, and thus give *local* value to another digit.

The figure 1 is often called **unity.**

It is the principle of *local value* that constitutes the great superiority of the Arabic over other systems of notation.

In the Roman system, which was in general use in Europe before the introduction of the Arabic, the symbols are the letters I, V, X, L, C, D, M, representing 1, 5, 10, 50, 100, 500, 1000 respectively, a line placed over a letter increasing its value a thousandfold. Thus, $\overline{\mathrm{V}}$ represents 5000, $\overline{\mathrm{X}}$, 10000. The symbols IƆ, IƆƆ, CIƆ, CCIƆƆ, were also used for 500, 5000, 1000, 10000 respectively.

The Roman symbols have *no local* value. Thus, X always represents ten *ones* whatever its position, and XX means *ten and ten*, or twenty.

The symbols are arranged as a rule in order of value, the greater to the left, and their separate values are added together, thus CCLXXXVI represents 286. But in order to avoid to some extent repeated use of the same symbol, a symbol of *less* value sometimes stands to the left of one of greater value, in which case its value is *subtracted* from that of the greater; thus IV means *one less than five*, or 4; XL means *ten less than fifty*, or 40, and so on.

Even as a means of merely representing a number the Roman system, as compared with the Arabic, is very clumsy. Thus, for instance, the number MDCCCXCIII, or 1893, requires *ten* symbols instead of *four*. Moreover, owing to the absence of *local value* it is useless for the *operations* of arithmetic, these were performed mechanically, either by means of pebbles† or counters, or by help of an instrument called the abacus.

---

* "Cipher" and "zero" are both derived from the same Arabic word, signifying *vacant*.

† Whence the word "calculation," from the Latin *calculus*, a pebble.

To read, or write in words, a number expressed in *figures*

If the number consists of not more than four figures, we take each figure separately and mention both its intrinsic and its local value

For instance, 1372 represents

two *single*,
seven *sets of ten*,
three *sets of ten tens*, or *a hundred*,
and one *set of ten hundreds*, or *a thousand*, objects,

which we express shortly, thus

One *thousand*, **three** *hundred* and **seven-*ty*-two**

Again, 3502786 represents

six *ones*,	or units	of the	1st	order
eight *tens*,	„	„	2nd	„
seven *hundreds*,	„	„	3rd	„
two *thousands*,	„	„	4th	„
no *ten-thousands*,	„	„	5th	„
five *hundred-thousands*,	„	„	6th	„
and three *millions*,	„	„	7th	„

Here, instead of repeating the word *thousand* three times over, we group together the figures in the local value of which that word occurs, omitting to mention the cipher, and then, beginning with the units of the highest order, express the number thus

Three *million*, **five-hundred-and-two** *thousand*, **seven** *hundred* and **eigh-*ty*-six**

Similarly in a larger number we group together all the digits representing *millions*, &c

In order to facilitate the reading of large numbers commas may be inserted after the 3rd, 6th, and 12th figures, counting from right to left, then *the figures between the first and second commas are* **thousands**, and those *between the second and third commas* **millions**, and we read each group separately as a distinct number, adding the local value of the *group*

For instance, the number 57360942

Inserting commas between the 9 and 0, and between the 3 and 7, we see that there are 57 *millions*, and 360 *thousands* 57,360,942

Hence this number is expressed in words thus

Fifty-seven *million*, **three-hundred-and-sixty** *thousand*, **nine-hundred-and-forty-two.**

Again, to read the number 38764003201435

Inserting commas between the 4 and 1, the 2 and 3, and the 7 and 8, and remembering that a billion is a million millions, we see that there are 38 *billions*, 764003 *millions*, and 201 *thousands*

$$\overbrace{38}^{\text{Billions}},\overbrace{764003}^{\text{Millions}},\overbrace{201}^{\text{Thousands}},435$$

Hence we read this number thus

Thirty-eight *billion*, seven-hundred-and-sixty-four-thousand-and-three *million*, two-hundred-and-one *thousand*, four-hundred-and-thirty-five

To write in **figures** a number expressed in *words*.

The places of the *units*,* *tens*, *hundreds*, &c, may at first be indicated by little dashes and the commas inserted as before.

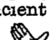
For instance, to write in figures the number *Forty-one millions, thirty-five thousand and three*

First arrange the commas, thus.—	,---,---
Next write the *millions*, 41, to the left of the left-hand comma; then the *thousands*, 35, to the left of the right-hand comma, then the 3 in the *units* place, thus —	41,-35,--3
Finally, fill up the *vacant* places with *ciphers*, thus —	41,035,003.

Note —It should be remembered that *seven* digits are needed to represent a million, *thirteen* to represent a billion. Also, that the *greatest* number consisting of *two* digits is 99, and the *least* is 10, the *greatest* number of *three* digits is 999, and the *least* is 100, and so on

Numbers taken *in their natural order* are called **consecutive.**

For instance, 17, 18, 19, are consecutive numbers

Note on the origin of the Roman symbols —Those denoting the smaller numbers are of very great antiquity, being derived from the hieroglyphics, or picture-writing, of the ancient Egyptians Thus the symbols I, II, III, IIII had their origin in  rough sketches of one, two, three, four *fingers*, V, of an entire *hand* , X, of two V's or hands The symbols for the higher numbers, *e g* C, M, are of much later origin, being the initials of the Latin names for the numbers they represent, and L was originally *half* of the letter C

* When used in connection with abstract numbers the word "units" implies *units of the* 1st *order*, i e *ones*

II. THE SIMPLE RULES.

The four fundamental operations of arithmetic—Addition, Subtraction, Multiplication, and Division—are called *simple* when the numbers dealt with are *abstract* numbers *

ADDITION

Addition is the operation of finding a number which shall contain as many units as there are in two, or more, given numbers together

The *result* is called the **sum**, the *total*, or the *amount*

The *sign* of Addition is +, and is read "*plus*"

For instance, 3 + 5 is read "Three *plus* five"

In primitive times the *sum* of numbers would be found by actual *counting*, each unit in each number would be represented perhaps by a pebble, and the whole heap so formed would then be counted

The use of the Arabic system of notation enables us to obtain a *sum* much more rapidly We commit to memory the Addition Table (*i e* the sum of every pair of the digits) and use its results in the addition of larger numbers In so doing we employ the following fundamental truth, or *axiom* —

The sum of numbers is the same as that of all their parts taken in any order

Hence, regarding each of the numbers to be added as consisting of the separate parts, *units*, *tens*, *hundreds*, &c, we first collect together all the *units*, then all the *tens*, and so on

If the resulting number of units exceeds *nine*, we remove *any complete sets of ten* at once and "**carry**" such sets forward to be counted in with the other *tens*, and so on

Note —For convenience it is usual to arrange the numbers to be added in a vertical column, so that all the *units* are in one straight line, all the *tens* in another, and so on It is well, however, to practise adding small numbers *across*

To test the result of Addition

Add the numbers in the reverse order, *i.e* from top to bottom, instead of from bottom to top, or, in the case of "cross" addition, from right to left, instead of from left to right

* Or when they refer to "simple" quantities See p 24

SUBTRACTION.

Subtraction is the operation of finding how many units remain when from a given number a smaller number is taken away.

The *result* is called the **difference**, or the **remainder**

The *sign* of Subtraction is —, and is read "*minus*"

For instance, 7 − 3 is read "Seven *minus* three"

It follows from the definition that the remainder, if added to the smaller number, will make up the larger number, hence Subtraction is the undoing of the Addition of two numbers, *i e* in Subtraction *we have the* **sum** *of two numbers given, we have also* **one** *of those numbers given; and we are required to find the* **other.**

Hence the Addition Table is available for Subtraction

For instance, as 8 + 7 is 15, conversely, 15 − 7 is 8, and 15 − 8 is 7.

The method used in the subtraction of large numbers corresponds to that used in addition; we take the *units*, *tens*, &c, of the smaller number from the *units*, *tens*, &c, respectively, of the greater But, as some digit of the smaller number may be greater than the corresponding digit of the larger number, it is often necessary to *rearrange* the larger number.

(i) For instance, in subtracting 35 from 81, 5 *units* cannot be taken away from 1 *unit*, but we can rearrange the greater number by changing one *ten* into *units*, thus, instead of regarding the number as 8 *tens* and 1 *unit*, we may regard it as 7 *tens* and 11 *units*

Now, taking 5 *units* from 11 *units*, there remain 6 *units*, and taking 3 *tens* from 7 *tens* there remain 4 *tens*, thus the required difference is 46

The written work then *might* be arranged thus, the figures 8, 1, being cancelled and replaced by 7 and 11

$$\begin{array}{cc} 7 & 11 \\ \cancel{8} & \cancel{1} \\ 3 & 5 \\ \hline 4 & 6 \\ \hline \end{array}$$

(ii) In practice, however, we change a ten into units *mentally*, and remember when we reach the *tens* column, that we must decrease the 8 *tens* by this 1 *ten* as well as by the 3 *tens*

But to take 1 and 3 *in succession* from 8 is equivalent to taking 4 from 8 *in one operation*

We therefore "carry 1" which we add to the 3 and then take 4 from 8

$$\begin{array}{r} 81 \\ 35 \\ \hline 46 \\ \hline \end{array}$$

Note —If a *cipher* occur in the greater number, the mental "rearrangement" required may be more complex For instance, in taking 267 from 305, as there are *no tens* in 305, we first change 1 *hundred* into 10 *tens*, and then, as before, change *one* of these tens into *units*

The method (ii) is based on the following axiom —

To take several numbers in succession from a given number is equivalent to taking their sum from that given number.

It follows from the foregoing axioms that the *order* in which a succession of additions and subtractions is performed does not affect the result

For instance, $4 - 3 + 2$ is correctly found, either
by first taking 3 from 4 and afterwards adding 2 to the result,
or, by first adding 2 to 4 and afterwards subtracting 3 from the result

Again, $7 - 3 + 8 - 5 + 12 - 9$ would involve *five* operations if each addition and subtraction were performed *separately in the order given*

But as $7 - 3 + 8 - 5 + 12 - 9$ is equivalent to $7 + 8 + 12 - 3 - 5 - 9$, and as to subtract 3, 5, and 9 in succession is equivalent to subtracting their *sum* in *one* operation, the result can be obtained in *three* steps thus —

the sum of 7, 8 and 12 is 27,
the sum of 3, 5 and 9 is 17,
and $27 - 17 = 10$

Also, in the question—*Find the value of* $7 - 13 + 23 - 41 - 19 + 64$, the operations *cannot be performed in the order given*

We proceed thus

$$\begin{aligned} 7 - 13 + 23 - 41 - 19 + 64 &= 7 + 23 + 64 - 13 - 41 - 19 \\ &= 94 - 73 \\ &= \underline{21\ \textit{Ans}} \end{aligned}$$

It is useful to be able to add several numbers together and subtract their sum from another number in *one* operation

EXAMPLE —*Take the sum of* 479, 1684, 592, *and* 2348 *from* 8126

Mental Work

(Only the figures in thick type are written down)

1st step

Beginning at the bottom of the units column, we say (or, *think*)
10, 14, 23 from 26 (changing *two* tens to units) leaves **3**

2nd step

Carry 2,
6, 15, 23, 30 from 32 (changing *three* hundreds to tens) leaves **2**

3rd step

Carry 3,
6, 11, 17, 21 from 21 (changing *two* thousands to hundreds) leaves **0**

4th step

Carry 2,
4, 5 from 8, leaves **3**

Written Work

$$\begin{array}{r} 8126 \\ \left.\begin{array}{r} 479 \\ 1684 \\ 592 \\ \underline{2348} \end{array}\right\} \\ \underline{3023}\ \textit{Ans} \end{array}$$

To test the result of Subtraction.

Add the result to the smaller number, this should give the larger.

MULTIPLICATION.

Multiplication is the operation of finding the *sum* which results when a given number is repeated as many times as there are units in another number

For instance, to multiply 23 by 4 is to find what 23 repeated *four* times amounts to, *i e* 23 multiplied by 4 = 23 + 23 + 23 + 23 = 92

The number to be multiplied is called the **multiplicand**; the number by which it is to be multiplied is called the **multiplier**; the *result* is called the **product**

For instance, in the example above,
23 is the *multiplicand*, 4 is the *multiplier*, and 92 is the *product*.

The *sign* of Multiplication is ×, and is read "multiplied by."

For instance, 5 × 3 is read "Five *multiplied by* three"

It follows from the definition of Multiplication that any product *might* be obtained by Addition.

For instance, 467 × 93 *might* (at great expense of time and labour) be found by setting down 467 *ninety-three* times and then adding these ninety-three numbers together

This we avoid by committing to memory the Multiplication Table* (*i e* the product of every pair of the digits), and using its results in the multiplication of larger numbers In so doing we employ the following principles —

(I)† *The product is the same, whether we multiply the first of two numbers by the second, or the second by the first*

For instance, to show that 4 multiplied by 3 is equivalent to 3 multiplied by 4

We know, from the definition of Multiplication, that

$$\begin{aligned} 4\times 3 &= 4 + 4 + 4 \\ &= 1+1+1+1+1+1+1+1+1+1+1+1 \\ &= 1+1+1 \quad +1+1+1 \quad +1+1+1 \quad +1+1+1 \\ &= 3 + 3 + 3 + 3 \\ &= 3\times 4 \end{aligned}$$

Or thus —

Take 12 counters and arrange them as in the accompanying diagram, then the total number of counters is the same whether we regard them as three *rows of four*,
or as four *columns of three* *i e* 4 × 3 = 3 × 4.

* * * *
* * * *
* * * *

---

* It is usual to learn the Multiplication Table beyond 9 *times* 9, as far as 12 *times* 12, and it is well to gradually learn it a little further still.

† Sometimes referred to as the *Commutative Law*

(II.) *The product of two numbers is the same as the sum of the products obtained by multiplying one of them by each of the parts of the other number*

For instance, considering 5 as made up of the parts 2 and 3, we know by the definition of multiplication, that

$$\begin{aligned} 4 \text{ multiplied by } 5 &= 4+4+4+4+4 \\ &= 4+4 \qquad\quad +4+4+4 \\ &= 4 \text{ multiplied by } 2 + 4 \text{ multiplied by } 3 \end{aligned}$$

Or thus —

Take 20 counters and arrange as in the accompanying diagram, then the total number of counters is the same whether we regard them as five *columns of four*, or as two *columns of four*, together with three more *columns of four*

```
*  *      *  *  *
*  *      *  *  *
*  *      *  *  *
*  *      *  *  *
```

i e, $4 \times 5 = 4 \times 2$ *plus* 4×3

Note —It is evident, from the same diagram, that, conversely,
4×5 *minus* $4 \times 3 = 4 \times 2$

If the product of two numbers be multiplied by a third number the result is called the **continued product** of the *three* numbers, and so on

Numbers multiplied together are called **factors** of their product.

For instance, the *continued product* of 2, 3, and 5 is 30, and 2, 3, and 5 are *factors* of 30

(III) *The order in which the factors are multiplied together does not affect the result*

For instance, $2 \times 3 \times 5 = 5 \times 3 \times 2 = 3 \times 5 \times 2$

(IV) *To multiply in succession by the factors is equivalent to multiplying by their product*

For instance, $7 \times 2 \times 3 = 7 \times 6$

To multiply a number by 10.

Affix one cipher to the right of the figure in the units place

Thus each figure is moved *one place towards the left*, and is consequently increased *tenfold* Hence, by (II), the number itself is multiplied by 10.

Similarly, **to multiply by 100, 1000, &c.**

Affix two, three, &c., ciphers

We will now illustrate the application of the foregoing principles —

EXAMPLE —*Multiply 243 by 75, and explain the process*

Regarding 75 as made up of the parts 5 and 70, we know, by (II), that we multiply 243 by 75 if we

(i.) multiply 243 by 5,
(ii.) then multiply 243 by 70,
(iii.) and add the results.

(i.) Now 243 is multiplied by 5 if each of its parts be multiplied by 5 Hence, regarding 243 as made up of 2 *hundreds*, 4 *tens*, and 3 *units*, we first multiply the 3 *units* by 5, carrying forward any *complete sets of ten* that result, and set down the remaining *units* in the units place; we next multiply the 4 *tens* by 5, adding in the "carried" *tens*, &c.

Here each small product is known from the Multiplication Table.

We thus obtain 1215 as the product of 243 and 5.

(ii) Again, we know by (IV), that we multiply 243 by 70, if we multiply in succession by 7 and 10, the factors of 70

Hence multiplying 243 by 7 (in the same way as we before multiplied by 5), we obtain the product 1701;

And we multiply 1701 by 10 if we affix a cipher.

Thus we obtain 17010 as the product of 243 and 70

(iii) Finally, adding 1215 to 17010 we obtain 18225 as the complete product

In practice *the cipher due to multiplication by 10 is omitted, and its place in the second line is left vacant*, the written work being arranged thus.

```
 243
  75
----
1215
1701
----
18225
```

Note —If we multiply 243 by 705, *two* places in the second line, due to multiplication by 100, must be left vacant; and so on

Hence, *each line of multiplication begins under the figure by which we multiply.*

Again, to multiply 243 by 7500, we first multiply 243 by 75 as before, and then affix two ciphers to the final result, *i e* multiply the final result by 100; thus

```
    243
   7500
  -----
  1215
 1701
-------
1822500
```

In the above example we multiplied, as is usual, by the *units* figure *first* We might equally well have multiplied by the *units* figure *last*

For instance, the work of 4731 × 524 might stand thus.

```
   4731
    524
-------
23655
  9462
  18924
-------
2479044
```

Multiplication by one figure may be combined with Addition, or Subtraction, in *one* operation

For instance, *Multiply* 473 *by* 8, *and subtract the result from* 3951

Mental Work

8 times 3 is 24, 24 from 31 leaves 7
Carry 3
8 times 7 is 56, 56 and 3 is 59, 59 from 65 leaves 6
Carry 6
8 times 4 is 32, 32 and 6 is 38, 38 from 39 leaves 1.

Written Work

$$\begin{array}{r} 3951 \\ \underline{473 \times 8} \\ \underline{167\ \textit{Ans}} \end{array}$$

The product of *two equal* factors is called the **square** (or *second* power) of one of those factors

For instance, 7×7, or 49, is the *square* of 7.

The continued product of *three equal* factors is called the **cube** (or *third* power) of one of those factors

For instance, $5 \times 5 \times 5$, or 125, is the *cube* of 5

The continued products of *four*, *five*, &c, *equal* factors are called the *fourth*, *fifth*, &c, *powers* of one of those factors respectively

For instance, $3 \times 3 \times 3 \times 3$, or 81, is the *fourth power* of 3.

A *power* of a number may be indicated by a small figure placed above the number, and called the **index** of the power

For instance, 7^2 means 7×7, or the *square* of 7,
and 10^5 means $10 \times 10 \times 10 \times 10 \times 10$, or the *fifth power* of 10

To test the result of Multiplication

Divide the product by one of the factors, then there should be no remainder, and the quotient obtained should be the other factor

Or, apply the test known as *casting out the nines*

In this test we first find the remainders when the *sum* of the figures in each of the factors is divided by 9, we then multiply these remainders together, and divide their product by 9 in order to obtain the remainder (R) We next find the remainder when the *sum* of the figures in the product is divided by 9, this should be the same as the previous remainder (R)

For instance, to show that $5642 \times 347 = 1957774$

First,

$5+6+4+2$ divided by 9 gives rem^r 8
and $3+4+7$ „ 9 „ 5 } and $40-9$ gives rem^r 4 (R).

Also,

$1+9+5+7+7+7+4$ divided by 9 gives rem^r 4, which agrees with (R).

Division

Division is the operation of finding how many times one number is contained in another.

The *sign* of division is $\div$, and is read "*divided by*"

For instance, $15 \div 3$ is read "Fifteen *divided by* three."

The number to be divided is called the **dividend**;
the number by which it is to be divided is called the **divisor**;
the result is called the **quotient**.

For instance, in the example above,
15 is the *dividend*, and 3 is the *divisor*

Also, as to divide 15 by 3 is to find how many *threes* there are in 15, and as we know from the Multiplication Table that 5 *threes* make 15, the *quotient* is 5

Thus Division is the *undoing* of Multiplication; *i e* when *the product of two factors is given, and one of those factors is also given, Division is the operation of finding the other factor* *

Hence the Multiplication Table supplies results in Division.

For instance, as 7×8 is 56, conversely $56 \div 8$ gives quotient 7 and $56 \div 7$ gives quotient 8

The *quotient*, (*i e* the *number of times* the divisor is contained in the dividend,) *might* always be obtained by *subtracting* the divisor from the dividend as often as possible, and then counting the number of such subtractions.

For instance, if we take 6 from 19, then 6 from the remainder, and so on, we find that after *three* subtractions have been performed the remainder is 1, from which we cannot again take 6

Thus $19 \div 6$ gives quotient 3 and remainder 1.

$$\begin{array}{rl} 19 & \\ \underline{6} & \text{1st} \\ 13 & \\ \underline{6} & \text{2nd} \\ 7 & \\ \underline{6} & \text{3rd} \\ 1 & \end{array}$$

But we know, that to subtract 6 three times in succession is equivalent to subtracting $6 + 6 + 6$, or 18, in one operation, and as we know from the Multiplication Table that *three* times 6 is less, and *four* times 6 is greater than 19, we can judge beforehand how many sixes to subtract, and so by help of the Multiplication Table obtain the result much more concisely thus.

$$\begin{array}{rl} 19 & \\ \underline{18} & 3 \textit{ sixes.} \\ 1 & \end{array}$$

* We shall see in Chapter XVIII that this view also applies to a division in which a final remainder occurs.

A division is called **exact** when *no final remainder* occurs.

In this case, as we have already seen, the divisor and quotient are the factors whose product is the dividend, *i e* in *exact* division,

$$\text{Divisor} \times \text{Quotient} = \text{Dividend} \qquad (i)$$

And, as we know, from (I), that in Multiplication the factors are interchangeable, it follows that in Division the divisor and quotient are interchangeable

$$\text{Hence, as Dividend} \div \text{Divisor} = \text{Quotient},$$
$$\text{so also Dividend} \div \text{Quotient} = \text{Divisor}$$

Thus Division may be regarded as the operation of finding either
the **number of times** one *given number* is contained in another,
or the **number** which is contained a *given number of times* in another

A division is called **inexact** when there *is* a final remainder

Hence, in *inexact* division,

$$\text{Divisor} \times \text{Quotient} + \text{Remainder} = \text{Dividend} \qquad (ii)$$

For instance, in the question *The divisor is* 6, *the quotient* 3, *and the remainder* 1, *find the dividend*, the required dividend $= 6 \times 3 + 1 = 19$

We will now illustrate the method known as "Long" Division.

EXAMPLE —*Divide* 18096 *by* 23, *and explain the process*

It is evident that 23 is contained less than 1000 times and more than 100 times in 18,096, the highest figure in the quotient will therefore represent *hundreds*, we find this figure first

1st step

We find on trial that 7 times 23 is less, but 8 times 23 is greater than 180, *i e* 23 is contained more than 700 times, and less than 800 times, in 18096

Thus the *hundreds* figure in the quotient is 7

Subtracting 700 times 23 from 18096, we obtain the remainder 1996

```
23)18096 (700
   16100    80
   -----     6
    1996
    1840
    ----
     156
     138
     ---
      18
```

We now proceed to find the next figure in the quotient, namely, that which represents *tens*

2nd step

We find on trial that 8 times 23 is less, but 9 times 23 is greater than 199. *i e* 23 is contained more than 80 times, and less than 90 times, in 1996

Thus the *tens* figure in the quotient is 8

Subtracting 80 times 23 from 1996, we obtain the remainder 156

3rd step

As 23 is contained less than 7 times, and more than 6 times in 156, the *units* figure in the quotient is 6

Subtracting 6 times 23 from 156, we obtain the final remainder 18.

We have now subtracted
first 700 times 23,
then 80 times 23,
and then 6 times 23,

which we know is equivalent to subtracting 23 successively 786 times from 18096 Also 18 remains, from which we cannot again take 23.

Hence the quotient is 786 and the final remainder is 18

In practice the written work stands thus—*the ciphers at the end of the successive products being omitted and their places being left vacant Also the figures of the dividend are brought down one by one, as required.*

```
23)18096(786
   161
    199
    184
     156
     138
      18
```

Abridged method of working Long Division *

This consists in subtracting each figure of the various products *as soon as it is obtained* and writing down the *differences only*

For instance, the above example by this method is worked thus.

Mental Work

(Only the figures in thick type are written down)

1st step.
7 times 3 is 21, 21 from 30 leaves **9.**
Carry 3
7 times 2 is 14, 14 and 3 is 17, 17 from 18 leaves **1.**

2nd step Bring down **9**
8 times 3 is 24, 24 from 29 leaves **5**
Carry 2
8 times 2 is 16, 16 and 2 is 18, 18 from 19 leaves **1.**

3rd step Bring down **6**
6 times 3 is 18, 18 from 26 leaves **8**
Carry 2
6 times 2 is 12, 12 and 2 is 14, 14 from 15 leaves **1.**

Written Work.

```
23)18096(786
    199
     156
      18
```

Note—The advantages claimed for this method are that it saves time and is very compact in appearance On the other hand, the greater complexity of the mental process increases the liability to err, and errors if made are also less easy to detect Moreover when the same figure occurs more than once in the quotient, the labour of multiplying the divisor by that figure has to be repeated

---

When the divisor is not greater than 12, the whole of the work is always performed mentally, and the figures of the quotient are written in succession below the corresponding figures of the dividend, this is called "Short" Division

* Sometimes called the Italian, or Continental, method

Division by Factors.

It follows conversely from (IV) that to divide in succession by the factors is equivalent to dividing by their product

Hence, whenever the divisor is known to be the product of factors not greater than 12, *short* division can be used.

For instance, *to divide* 3621 *by* 24, *i e* 4 × 6.

Dividing first by 4 and then by 6, we obtain the final quotient 150, and the remainders 1 and 5

$$24\left\{\begin{array}{r|l} 4 & \underline{3621} \\ 6 & \underline{905} \\ & \underline{150} \end{array}\right.\quad \left.\begin{array}{l} \\ 1 \\ 5 \end{array}\right\}\ \underline{21\ \textit{rem}^{r}.}$$

We have now to find the complete remainder

When we divide 3621 by 4 we find how many *sets of four* there are in 3621 *units,* namely, 905 *fours* and 1 *unit* over.

When we divide 905 *fours* by 6, we find how many *sets of six-fours,* (*i e* how many *twenty-fours,*) there are in 905 *fours,* namely, 150 *twenty-fours* and 5 *fours* over

Thus the complete remainder is 5 *fours* + 1 *unit,* or 21 *units*

Hence, in division by *two* factors, to obtain the **complete remainder** *Multiply the second remainder by the first divisor and add the result to the first remainder* *

Again, *to divide* 6787 *by* 3 × 5 × 7, *i e by* 105

1st step

6787 *units* contain 2262 *threes* and 1 *unit* over.

2nd step

2262 *threes* contain 452 *fifteens,* and 2 *threes* over

$$105\left\{\begin{array}{r|l} 3 & \underline{6787} \\ 5 & \underline{2262} \\ 7 & \underline{452} \\ & \underline{64} \end{array}\right.\quad \left.\begin{array}{l} \\ 1\ \textit{unit} = 1 \\ 2\ \textit{threes} = 6 \\ 4\ \textit{fifteens} = 60 \end{array}\right\}\underline{67\ \textit{rem}^{r}.}$$

3rd step

452 *fifteens* contain 64 *hundred-and-fives,* and 4 *fifteens* over

Thus the complete remainder is 4 *fifteens* + 2 *threes* + 1 *unit, i e.* 67 *units*

Hence, in the case of *three* factors, to obtain the **final remainder** *Multiply the third remainder by the product of the first two divisors, the second remainder by the first divisor, and add these results to the first remainder.*

Similarly we might proceed in the case of four, or five, factors

---

* This rule should be illustrated practically with counters to young beginners, thus — Take, say, 74 counters and arrange them in *stacks of five,* we thus find that we can make 14 stacks of five and have 4 *single* counters over Now arrange the stacks in *groups of three stacks,* we thus obtain 4 *groups* and have 2 *stacks* over Thus 74 divided by the factors of 15 (*i e* 5 and 3) yields quotient 4 and remainder 4 *single* counters + 2 *stacks of five* counters, *i e* 14 *single* counters

To divide a number by 10.

Cut off the figure which stands in the units place

For, by so doing, each of the remaining figures is moved *one place toward the right*, and is, consequently, *decreased tenfold*, hence these remaining figures give the quotient, and the figure cut off, which was originally in the units place, is the remainder.

For instance, 23,4 divided by ten gives quotient 23, remainder 4.

Similarly, **to divide by 100, 1000, &c**,

Cut off two, three, &c, figures to the right of the number.

For instance, 364,27 divided by 100 gives quotient 364, remainder 27.

Also, **to divide by 20, 200, &c., or by 30, 300, &c.,**

Cut off one, two, &c., figures, and divide by 2, 3, &c

For instance, *to divide* 3658 *by* 70

The remainder 18 results from 1 *ten* remaining from the division of 365 *tens* by 7, together with the 8 *units* remaining from the previous division by 10

```
7,0 | 365,8
       52.  18 remr.
```

Again, *to divide* 38821 *by* 1200.

The remainder 421 results from 4 *hundreds* remaining from the division of 388 *hundreds* by 12, together with 21 *units* remaining from the previous division by 100

```
12,00 | 388,21
         32   421 remr.
```

Also, *to divide* 305715 *by* 19000

The remainder 1715, results from 1 *thousand* remaining from the division of 304 *thousands* by 19, together with 715 *units* remaining from the previous division by 1000

```
19,000)305,715(16
       19
       115
       114
         1715 remr.
```

The following principle is important

(V.) *The quotient is not altered by multiplying, or dividing, both dividend and divisor by the same number.*

For instance, the quotient of 6÷2 is the same as that of 7 *times* 6÷7 *times* 2.

To test the result of Division.

Multiply the divisor and quotient together, and add the remainder (if any) to the result, this should give the dividend

Or, subtract the remainder (if any) from the dividend, and then cast out the nines as in proving multiplication.

Use of Signs.

The sign of equality is =, and is read "equals"
It should, of course, never connect *un*equal amounts

For instance, in the question *Multiply* 8 *by* 4 *and add* 3 *to the result*, many a beginner will write $8 \times 4 = 32 + 3 = 35$, which is absurd He *means*, and should be careful to *write*, $8 \times 4 = 32$, $32 + 3 = 35$

The sign $\therefore$ *is used for the word "therefore"*

A line* *placed between two numbers indicates division*

Thus, $\frac{14}{2}$, and $14/2$, are each equivalent to $14 \div 2$

The sign which stands to the **left** *of a number indicates the operation to be performed with that number*

Thus, $7 - 3$ means that 3 is to be taken from 7. It will *not* do to write $3 - 7$

Also, in $8 - 3 + 1$, it will *not* do to add the 1 to the 3 and subtract the result from 8, for this is equivalent to *subtracting* the 1 (See p 8)

When multiplication, or division, is indicated as well as addition, or subtraction, the multiplication, or division, must be performed **before** *the addition, or subtraction*

Thus, $4 \times 3 + 2$ means that 4 is to be multiplied by 3 and 2 added to the result, *not* that 2 is to be added to 3 and 4 multiplied by the result

Similarly, $8 + 12 \div 4$ means that 12 is to be divided by 4 and the result added to 8, *not* that 12 is to be added to 8 and the result divided by 4

Brackets indicate that their contents are to be regarded as a whole

Thus, $(14 - 3) \times 4$ means 11×4, *i e* 44
Similarly, $18 \div (6 - 4)$ means $18 \div 2$, *i e* 9

When multiplication and division are both indicated and no brackets are used, the operations must be performed in order **from left to right.**

Thus, $30 \div 10 \times 2 = 3 \times 2 = 6$ And $5 \times 8 \div 4 = 40 \div 4 = 10$

When **no** *sign is placed between a number and a bracket, or when a* **dot** *is placed between two numbers, multiplication is implied*

Thus, $3(5-1)$ means $3 \times (5-1)$, *i e* 3×4, or 12
And $(7-3)(9-2)$ means $(7-3) \times (9-2)$, *i e* 4×7, or 28.
Also, $2 \cdot 3 \cdot 4$ means $2 \times 3 \times 4$

If one pair of brackets stands within another pair the value of the contents of the inner pair should first be found

Thus, $4 \div \{7 - (8 - 3)\} = 4 \div \{7 - 5\} = 4 \div 2 = 2$

A line placed above (or below) numbers is equivalent to brackets.

Thus, $7 \times \overline{8 - 5} = 7 \times (8 - 5) = 7 \times 3 = 21$

---

* The dots in the sign ÷ were probably intended to represent the dividend and divisor

III. MISCELLANEOUS EXAMPLES.

The examples marked (A) which follow show how, in certain cases of Multiplication and Division, labour may be saved by taking advantage of the fact that any number can be multiplied or divided by 10, 100, 1000, &c., *at sight*

A (i) *Multiply* 3675 *by* 25.

As $100 = 25 \times 4$, if we multiply 3675 by 100 the result will be *four* times as great as that required, which is rectified by dividing this result by 4, thus —

$$4 \,\underline{|\,367500}$$
$$91875 \text{ Ans.}$$

Hence, to multiply by 25, affix *two* ciphers and then divide by 4.

A (ii). *Multiply* 7398 *by* 125

As $1000 = 125 \times 8$, if we multiply 7398 by 1000, the result will be *eight* times as great as that required, which is rectified by dividing this result by 8, thus —

$$8 \,\underline{|\,7398000}$$
$$924750 \text{ Ans.}$$

Hence, to multiply by 125, affix *three* ciphers, and then divide by 8

Similarly to multiply by 625 affix *four* ciphers and divide the result by 16

A (iii) *Divide* 47986 *by* 25

We know (by V., page 17) that 47986 divided by 25 gives the same quotient as 47986×4 divided by 25×4 (*i e* 100)

Hence, we multiply by 4 and cut off two figures, obtaining quotient 1919 The two figures cut off, *when divided by* 4, give the rem^r 11

$$\begin{array}{r} 47986 \\ 4 \\ \hline 1919,44 \end{array}$$

Ans 1919 + 11 *rem*^{r}.

Similarly we may deal with divisors 125 and 625

A (iv) *Multiply* 5876 *by* 11

As $11 = 10 + 1$, we multiply by 11 if we first *affix* one *cipher* (*i e* multiply by 10), *and then add the result to the given number,* thus —

$$\begin{array}{r} 5876 \\ 58760 \\ \hline 64636 \text{ Ans.} \end{array}$$

A (v) *Multiply* 8367 *by* 99

As $99 = 100 - 1$, we multiply by 99 if we *affix* two *ciphers, and then subtract the given number,* thus

$$\begin{array}{r} 836700 \\ 8367 \\ \hline 828333 \text{ Ans.} \end{array}$$

A (vi) *Multiply* 3768 *by* 996.

As $996 = 1000 - 4$, we multiply by 996 if we affix *three* ciphers and then subtract *four* times the given number from the result, thus

$$\begin{array}{r} 3768000 \\ 15072 \\ \hline 3752928 \; \textit{Ans} \end{array}$$

Similarly we may deal with such multipliers as 9, 98, 999, &c , and with many multipliers in which the figure 9 occurs, for instance —

A (vii) *Multiply* 8347 *by* 594

As $594 = 600 - 6$, we multiply 8347 by 600 and then subtract 6 times 8347 from the result, thus

$$\begin{array}{r} 8347 \\ 600 \\ \hline 5008200 \\ 50082 \\ \hline 4958118 \; \textit{Ans} \end{array}$$

A (viii) *Divide* 387654 *by* 99

If we divide 387654 by 100 we obtain quotient 3876 *hundreds* and 54 *units* over But in 3876 *hundreds* there are 3876 *ninety-nines* and 3876 *units* besides Hence, we now divide 3876 by 100, obtaining 38 more *ninety-nines* and 38 more *units* over We now add the three rem^{rs} 54, 76, and 38, obtaining a total rem^{r} 168, which divided by 100, yields 1 more *ninety-nine* and 1 more *unit* Hence, adding the three quotients, we obtain the complete quotient 3915 and the complete rem^{r} 69

$$\begin{array}{r} 3876,54 \\ 38,76 \\ ,38 \\ \hline 1,68 \\ 1 \\ \hline 3915,69 \end{array}$$

Ans $3915 + 69$ *rem*^{r}

Similarly we may deal with divisors 999, 9999, &c

The next two examples illustrate another case in which the work of multiplication may be shortened —

B (i) *Multiply* 2341 *by* 567

Observing that 7 is a factor of 56, we regard 567 as $7+560$, *i e* as $7+7\times 80$

Hence, if we first multiply 2341 by 7 (i), and then *this* result by **80** (ii), the sum of the two results is the req^{d} product

$$\begin{array}{rl} 2341 & \\ 567 & \\ \hline 16387 & \text{(i)} \\ 1310960 & \text{(ii)} \\ \hline 1327347 & \textit{Ans} \end{array}$$

B (ii) *Multiply* 325146 *by* 189273 *in three lines*

Observing that 27 is 3×9, and that 189 is 27×7, we regard 189273 as $3+270+189000$, *i e* as $3+3\times 90+270\times 700$

Hence, if we multiply 325146 by 3 (i), then this result by 90 (ii), and then the new result by 700 (iii), the sum of all three results is the req^{d} product.

$$\begin{array}{rl} 325146 & \\ 189273 & \\ \hline 975438 & \text{(i)} \\ 87789420 & \text{(ii)} \\ 61452594000 & \text{(iii)} \\ \hline 61541358858 & \textit{Ans} \end{array}$$

The three following are examples of "*unequal*" Division —

C (i) *Divide* 34722 *into two parts, one of which shall be* 8 *times as great as the other.*

Here we divide by **9**, obtaining 3858 for the smaller part

$$9\,\underline{|34722}\quad \underline{3858}$$

Hence, $34722 - 3858 = \underline{30864}$ is the greater

C (ii) *Divide* 1424 *into* 7 *parts, one of which shall exceed each of the others by* 17

Here we first *subtract the given excess*, 17, from 1424, and then divide the remainder into 7 *equal* parts, obtaining 201 as one of the six equal parts

Hence, $201 + 17 = \underline{218}$ is the other part

$$\begin{array}{r} 1424 \\ \underline{17} \\ 7\,\underline{|1407} \\ \underline{201} \end{array}$$

C (iii) *Find three numbers whose sum is* 757, *such that the first exceeds the second by* 12, *and the second exceeds the third by* 17.

It is evident that the 1st exceeds the *3rd* by 12+17, or **29**.

And the 2nd exceeds the 3rd by **17**

Hence, if we take 29+17, or **46**, from 757, the remainder, 711, is *three* times the 3rd number

Thus, the required numbers are <u>237, 254, and 266</u>

$$\begin{array}{r} 757 \\ \underline{46} \\ 3\,\underline{|711} \\ \underline{237} \end{array}$$

D. *Find the average of the numbers* 8, 12, 6, 7, 5, 13, 14, 9, *and* 16

<u>The *average* (or mean) of two or more given numbers is that number which, when repeated as many times as there are numbers given, amounts to their sum</u>

Hence, to find the average, we *add together all the given numbers and divide their sum by the number of them*

Here. $8 + 12 + 6 + 7 + 5 + 13 + 14 + 9 + 16 = 90$,
and there are nine numbers
∴ the req^d average is $90 \div 9$, i e <u>10 *Ans*</u>

E. *If* 23 *be added to a certain number, and the sum be divided by* 13, *the double of the quotient, decreased by* 22, *is* 1000· *find the number*

In questions of this kind where the *result* of a series of operations is given, and it is required to find the original number operated upon, we *must perform upon the given result the* converse *of each operation mentioned, and in the* reverse order

Here, the given result is 1000
Hence, we first *increase* 1000 by 22, obtaining 1022;
then *divide* 1022 by 2, obtaining 511,
next *multiply* 511 by 13, obtaining 6643,
and finally *subtract* 23 from 6643, obtaining <u>6620, the req^d number</u>

F. *If* 4793 *be multiplied by a certain number of two digits the units' figure in the product is* 2, *and the tens' figure is* 5, *find the multiplier.*

The units' figure of the req^{d} multiplier must be **4**, for this is the only figure by which 3 can be multiplied, so as to give a result ending in 2, hence the first line of multiplication ends thus 72.

We have now to find a figure which, added to 7, gives a sum which ends in 5, and this figure is **8**.

Hence the tens' figure in the multiplier must be **6**, for this is the only figure by which 3 can be multiplied so as to give a result ending in 8

$$\begin{array}{r} 4793 \\ \underline{64} \\ 72 \\ \underline{8} \\ \underline{52} \end{array}$$

∴ the req^{d} multiplier is 64

G. *How many four-figure numbers can be represented by the four digits* 1, 2, 3, 0?

There are *six* numbers in which 1 stands first, namely,

1230, 1203, 1320, 1302, 1023, 1032,

and there are *six* numbers in which 2, and *six* in which 3 stands first, namely, 2301, 2310, 2013, 2031, 2130, 2103,

and 3012, 3021, 3120, 3102, 3201, 3210

But there are *no* numbers in which 0 stands first

Hence, there are 18 different numbers.

H. *The sum of two numbers is* 473, *their difference is* 87, *find them*

If we take two numbers (say 8 and 5), and add their difference (3) to the smaller number (5) we obtain the greater (8)

Hence, if we *add the* difference *to the* sum *of two numbers we obtain the* double *of the greater number*

Here, $473 + 87 = 560$, and $560 \div 2 = 280$, the greater of the required numbers Hence, $280 - 87 = 193$, the less.

K. *Find the sum of* 14 *consecutive numbers, the least of which is* 173

The numbers are 173, 174, 175, &c

i.e. 173, 173 + 1, 173 + 2, &c

Hence, the req^{d} sum is

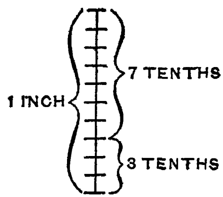
$$173 \times 14 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13$$
$$= 2422 + 91$$
$$= 2513 \ \textit{Ans}$$

III^A. ELEMENTARY DECIMALS.

So far, we have regarded a number as made up of *complete* units But the operation of **dividing** a *group of units* into parts naturally suggests the division of a *single unit* into parts, thus extending our idea of the range of numbers so as to include, not only *whole* numbers, or groups of complete units, but also groups of *parts of a unit*

Now, if the unit be divided into **two** *equal parts*, each part is called a **half**, if into **four** *equal parts*, each part is called a **fourth**, if into **ten** *equal parts*, each part is called a **tenth**, of the unit, and so on.

For instance, if 1 *inch* be the unit, and this length be divided into *ten equal* parts, then the length of *one* part is *one*-tenth,
of *three* parts is *three*-tenths,
of *seven* parts is *seven*-tenths, of an inch



Similarly, if each *part* in the above diagram were, in its turn, divided into *ten* tiny equal portions, the whole inch would be divided into a *hundred* such portions, and one of these tiny portions, that is, *one-tenth* of *one of the former parts*, or *one-tenth* of *a tenth*, would be *one*-hundredth of the *whole*, inch; and so on

The idea of dividing the unit into *ten*, a *hundred*, &c , equal parts leads to the following important extension of our system of Notation.

We know (see page 2) that, if a figure be moved one place towards the *left*, its local value is thus *multiplied by ten*

Consequently, if it be moved one place towards the *right*, its local value is *divided by ten*; that is, its new local value becomes **one-tenth** of what it was before.

For instance, in the number 333,
the right-hand figure represents three units, *i e three*;
the middle figure, three tens, or *thirty*,
the left-hand figure, *three hundreds*,

and three is *one-tenth* part of thirty, also thirty is *one-tenth* part of three hundred,

thus each figure has **one-tenth** *of the value of its* left-*hand neighbour*

If, then we *mark the position of the* **units'** *place, and keeping that fixed*, set another figure on its *right*-hand side, this new figure will. by the above law of place-values, represent some **tenths of**

the *unit*; and a second new figure on the right will represent some *tenths of these tenths*, or **hundredths** of the *unit*, and so on

Parts of the unit, thus represented, are called **Decimals**,* and the *position of the units' place* is marked by a *dot*, called the **decimal point**, *placed on the right-hand side of the units' figure*

Thus, 3·3 means three *units* and three *tenths* of the unit,
also 3·33 means 3 *units*, 3 *tenths* and 3 *hundredths*, and so on.

Hence we have the following extended table of place values·—

Whole Numbers						*Decimals*			
&c.	Thousands.	Hundreds	Tens	Units.	Decimal Point	*Tenths*	*Hundredths*	*Thousandths.*	&c
Thus,			1	2	·	0	7	5	

represents 1 *ten*, 2 *units*, 0 *tenths*, 7 *hundredths*, 5 *thousandths*

Decimals are often read figure by figure, the word "decimal" being said when the *point* is reached

Thus the above would be read "one, two, decimal, nought, seven, five".

N B —(i) But, being *written* on the same plan, decimals may also be read as we read whole numbers.

Thus, just as 1800 may be read either as "One *thousand*, eight *hundred*", or as "Eighteen *hundred*",
so ·34 may be read either as "Three *tenths*, four *hundredths*", or as "Thirty-four *hundredths*",
and 12·075 may be read "Twelve, and seventy-five *thousandths*", for 7 *hundredths* is the same as 70 *thousandths*

Hence, to write down the decimal expressed in words by, say, "Seventeen thousandths", we put 7 in the *thousandths'* place, 1 in the *hundredths'* place, 0 in the *vacant tenths'* place, and then mark the decimal point, thus ·017

N B —(ii) Noughts on the extreme *right*-hand side of a decimal have no effect upon its value

Thus, ·3 and ·30 have the same meaning,
for ·3 means *three tenths*,
and ·30 means *three tenths* and no *hundredths*

* From the Latin for *tenth*

Therefore we may, when we find it convenient to do so, either remove, or append, such noughts without altering the value of the decimal.

In order to *multiply* a decimal by **10**, we have but to move the decimal point *one* place to the **right**, for then the units become *tens*, the *tenths* become units, and so on, *i.e.* every figure in the number is given *ten* times its former value.

Thus, $4 \cdot 253 \times \mathbf{10} = 42 \cdot 53$.

Similarly, to multiply by 100, 1000 ... we move the decimal point *two*, *three* . . places to the *right*

Thus, $4 \cdot 253 \times \mathbf{100} = 425 \cdot 3$, also $17 \cdot 5 \times \mathbf{100} = 1750$

Consequently, to *divide* a decimal by 10, 100, 1000. . we move the decimal point *one*, *two*, *three*... places to the **left**

Thus, $46 \cdot 8 \div \mathbf{10} = 4 \cdot 68$, $317 \cdot 2 \div \mathbf{100} = 3 \cdot 172$, $9 \cdot 5 \div \mathbf{1000} = \cdot 0095$.

ADDITION AND SUBTRACTION

Decimals, being written on the same plan as whole numbers, are added in the same way. If then we arrange them so that the *decimal points are in column*, the columns of figures are added as in Simple Addition, the *decimal point in the result being placed in column with the other decimal points.*

EXAMPLE i.—*Add* 4·805, 31·6, ·085 *and* 7.

Observe here that the 7 is not a decimal; also that the final 0 in the result is discarded (See N B (ii), page 24*.)

$$\begin{array}{r} 4 \cdot 805 \\ 31 \cdot 6 \\ \cdot 085 \\ 7 \cdot \\ \hline 43 \cdot 490 \\ \textit{Ans}\ \ 43 \cdot 49 \end{array}$$

EXAMPLE ii.—*Subtract* 9·086 *from* 14·3

Observe here that we append two noughts to 14·3 before beginning the work

$$\begin{array}{r} 14 \cdot 300 \\ 9 \cdot 086 \\ \hline \textit{Ans.}\ \ 5 \cdot 214 \end{array}$$

MULTIPLICATION AND DIVISION BY A WHOLE NUMBER

The operation of multiplying a *decimal by a whole number* exactly corresponds with Simple Multiplication.

Thus, just as **3** times 2 *tens* is 6 *tens*,
so also **3** times 2 **tenths** is 6 **tenths**

EXAMPLE III —*Multiply* 4·23 *by* 8, *also* ·034 *by* 65.

Observe here that *the decimal point in each result stands vertically below that in the decimal multiplied.*

$$\begin{array}{r} 4{\cdot}23 \\ 8 \\ \hline \textit{Ans}\ \ 33{\cdot}84 \end{array} \qquad\qquad \begin{array}{r} {\cdot}034 \\ 65 \\ \hline 170 \\ 2\ 04 \\ \hline \textit{Ans}\ \ 2{\cdot}21 \quad 2\ 210 \end{array}$$

Note —If the *multiplier* (or divisor) end in 0, we begin by multiplying (or dividing) by 10, or 100, &c, *at sight,* and then proceed without any final 0

Thus, 3·26 × 470 = 32·6 × 47, &c , also 64·15 — 400 = ·6415 — 4, &c

The operation of dividing a decimal by a whole number is similar to that of Simple Division

Thus, just as 8 *hundred* — 2 = 4 *hundred,*
so also 8 **hundredths** — 2 = 4 **hundredths**

EXAMPLE IV —*Divide* 8·538 *by* 6, *also* ·544 *by* 16.

Observe here that *the decimal point in each result stands vertically below that in the dividend*

$$\begin{array}{r} 6\,\underline{|8{\cdot}538} \\ \textit{Ans}\ \ 1{\cdot}423 \end{array} \qquad\qquad 16\begin{cases} 4\,\underline{|\ {\cdot}544} \\ 4\,\underline{|{\cdot}136} \end{cases} \quad \textit{Ans}\ \ 034$$

EXAMPLE V —*Divide* 1·7 *by* 6 *as far as three places of decimals*

Observe here, that as noughts may be *appended to the dividend without altering its value* (see page 24*), we can continue the division beyond *tenths,* thus we obtain the quotient ·283, with a *remainder* of 2 *thousandths,* which remʳ not being asked for, is omitted in the Ans

$$\begin{array}{r} 6\,\underline{|1{\cdot}700} \\ \textit{Ans}\ \ {\cdot}283 \end{array}$$

EXAMPLE VI —*Divide* 4·5 *by* 17, *to three places of decimals*

As 17 is not contained in the *whole* number 4, there are no *complete units* in the *quotient,* so we *at once mark* the *decimal point* there

Now 4 *units* and 5 *tenths* make 45 *tenths* (see page 24*), and 45 *tenths* — 17 gives *quotient* 2 *tenths,* so we set 2 next the decimal point in the quotient, and then proceed, as in Simple Division, until the second of the appended noughts has been used

Thus we obtain the quotient ·264 with the *remʳ* 12 *thousandths*

$$\begin{array}{r} 17)4{\cdot}500(\ 264 \\ 3\ 4 \\ \hline 1\ 10 \\ 1\ 02 \\ \hline 80 \\ 68 \\ \hline 12 \\ \textit{Ans}\ \ 264 \end{array}$$

Note —For methods of multiplying and dividing a decimal *by a decimal,* the student is referred to pages 108–113

III^{B} THE METRIC SYSTEM

[Before beginning this chapter the student should read the first page and a half of Chapter IV]

The Metric System is the *decimal* system of weights and measures in general use in most of the countries on the continent of Europe.

In this system the chief unit of length is called a *metre* *

Being a *decimal* system, the only numbers used in the Tables are *powers of ten*, namely, 10, 100, 1000; and, when once the various *names* employed are understood, the Tables of this system impose no tax on the memory.

The Greek prefixes **deca-**, **hecto-**, **kilo-**, signify the *multiplication* by 10, 100, 1000, respectively, of the unit to which they are prefixed;

and the Latin prefixes **deci-**, **centi-**, **milli-**, signify the *division* by 10, 100, 1000, respectively, of the unit to which they are prefixed.

Thus, a *deca*-metre is *ten* metres, but a *deci*-metre is one *tenth part* of a metre

Also, a *hecto*-metre is a *hundred* metres, but a *centi*-metre is one *hundredth part* of a metre

METRIC TABLE OF LENGTH

10 *milli*-metres (*mm*)	=	1 *centi*-metre
10 *centi*-metres (*cm*)	=	1 *deci*-metre.
10 *deci*-metres (*dm*)	=	1 **metre** (*m*)
10 **metres**	=	1 *deca*-metre (*Dm.*).
10 *deca*-metres	=	1 *hecto*-metre (*Hm.*).
10 *hecto*-metres	=	1 *kilo*-metre (*Km*).

Now, as the *name* implies, a *kilometre* is 1000 *metres*,
∴ 1 *metre* is **one-thousandth** of a *kilometre*,
that is, 1 *metre* = $\cdot 001$ of a *kilometre*.

Again, as the *name* implies,
a *centimetre* is **one-hundredth** of a *metre*,
that is, 1 *centimetre* = $\cdot 01$ of a *metre*

And so on.

Hence, with the above simple Table, "Reduction" from one denomination to another involves no "work," but can always

* The *metre* is about 3 inches longer than our *yard*

be performed *at sight*, for it consists merely in either multiplying, or dividing, by 10, or 100, or 1000

For instance, it is *evident* that the length

2 *Km.*, 4 *Hm*, 3 *Dm*, 5 *m* = 2435 *metres*,

(for we know that the abbreviation "Km" stands for the word "kilometre," which means *thousand* metres, that "Hm." stands for "hectometre," which means *hundred* metres, &c)

Similarly, that the length 6 *m*, 1 *dm*, 7 *cm* = 617 *centimetres*

Again, it is *evident* that

1875 *m* = **1·875** *Km*

(for in order to "reduce" *metres* to *kilometres* we *divide* by 1000),

also that 903 *cm* = **9·03** *m*

Note—The kilometre, metre, centimetre, are the units commonly employed* in stating lengths, the others are seldom used Thus the length 2685 metres is more conveniently expressed by "2 *Km* 685 *m*" than by "2 *Km* 6 *Hm* 8 *Dm* 5 *m*"

EXAMPLE i—*Express in metres* (i) ·38 *Km*, (ii) 205 *cm*

(i) [As a kilometre is 1000 metres, *we reduce kilometres to metres if we multiply by* 1000, hence—]

·38 *Km* = 380 *metres*

(ii) [As a metre is 100 centimetres, *we reduce centimetres to metres if we divide by* 100, hence—]

205 *cm* = 2·05 *metres*

EXAMPLE ii—*Express* (i) *the length* 7 *Km* 24 *m* *in decametres;* (ii) *the length* 5 *Dm* 7 *dm* *in centimetres*

(i) [7 *Km* 24 *m* = 7024 *metres*, and as *ten* metres make a *decametre*, we now *divide* by 10, hence—]

7 *Km* 24 *m* = 702·4 *decametres*.

(ii) [5 *Dm* is 50 *metres*, that is 5000 *centimetres*, and 7 *dm* is 70 *centimetres*, hence—]

5 *Dm* 7 *dm* = 5070 *centimetres*

We have seen that a length expressed in terms of several metric units can at once be expressed in terms of any *one single* unit, hence it follows that, where the metric system is adopted,

---

* Much as we use mile, yard, inch A kilometre is considerably less than a mile, and a centimetre than half an inch 8 *Km.* = 5 miles, nearly, and 5 *cm* = 2 inches, nearly

operations involving these metric measures are performed by means of "simple" addition, multiplication, &c, either of integers or decimals.

EXAMPLE iii.—*Add the lengths* 5 *Km* 185 *m*, 18 *Km* 230 *m*, *and* 2 *Km.* 45 *m.*

1st method

If we write each of the three given lengths in *metres*, we obtain, by simple add^{n}, the result 25460 *m*, that is, 25 *Km* 460 *m*

metres
5185
18230
2045
25460

2nd method.

Or, if we write each length in *kilometres*, we obtain, by add^{n} of decimals, the result 25·460 *Km*, that is, 25 *Km* 460 *m*

kilometres
5·185
18·230
2·045
25·460

EXAMPLE iv —(i) *Multiply* 5 *m.* 8 *cm* *by* 12;
(ii) *divide* 3 *Dm.* 1 *m* 5 *dm.* *by* 18.

(i) 5 *m* 8 *cm.* = 508 *cm.*
Multiplying this by 12, we obtain the result 6096 *cm*, that is, 60 *m* 96 *cm*

centimetres
508
12
6096

(ii) 3 *Dm* 1 *m* 5 *dm* = 31·5 *metres*
Dividing this by 18, we obtain the result 1·75 *metres*, that is, 1 *m* 75 *cm*

metres
3 | 31·5
6 | 10·50
1·75

EXAMPLE v.—*Divide* (i) 6 *m.* 5 *cm* *by* 12; *also* (ii) 2 *m.* 3 *dm.* *by* 7, *obtaining the results to the nearest millimetre.*

(i) 6 *m* 5 *cm* = 6050 *mm*
Dividing this by 12, we obtain 504 *whole* millimetres together with some decimal part, *less than half*, of a millimetre

Hence the req^{d} result is 504 *mm* = 50 *cm* 4 *mm*

millimetres.
12 | 6050·0
504·1 . .

(ii) 2 *m* 3 *dm* = 2300 *mm*
Dividing by 7 we obtain 328 *whole* millimetres, together with some decimal part, *greater than half*, of a millimetre

Hence the req^{d} result is 329 *mm.* = 32 *cm* 9 *mm.*

millimetres
7 | 2300·00
328·57 . . .

WEIGHT

The chief unit of weight in the metric system is the *gramme*, and the Table of Weight exactly corresponds to the above Table of Length, the word "gramme" taking the place of "metre".

Thus, a *kilogramme* (*Kg*) = 1000 *grammes* (*g*), &c.

Note —The gramme is a very small unit, hence the weights of common substances are expressed in kilogrammes.*

CAPACITY

The chief unit of capacity is the *litre* † The Table corresponds to that of length, the word "litre" taking the place of "metre"

Thus, a *hecto litre* (*Hl*) = 100 *litres* (*l*), &c

Note —For the Metric Tables of Area and Volume see Part II, page 231

DECIMAL COINAGES

All countries which have adopted the metric system of weights and measures have also adopted a *decimal coinage*, in which there are *two principal* ‡ coins, one being a *decimal* part of the other.

Thus, in France these coins are the *franc* and *centime*, and 1 *franc* (*fr*) = 100 *centimes* (*c*), in Germany these coins are the *mark* and *pfennig*, and 1 *mark* (*M*) = 100 *pfennige* (*pf*)

For purposes of calculation a decimal coinage has the same advantages as a decimal system of weights and measures

EXAMPLE vi —*Find the cost of* 24 *metres of silk at* 4 *francs* 35 *centimes per metre*

	francs	
4 *fr* 35 *c* = 4·35 *fr*,	4·35	
and this, the cost of *one* metre,	24	
we multiply by 24	104 40	*Ans* 104 *fr* 40 *c*.

EXAMPLE vii —*Find, to the nearest centime, the cost of* 5 *kilogrammes* 120 *grammes of butter at* 2 *francs* 35 *centimes per kilogramme.*

	centimes
5 *Kg* 120 *g*. = 5·12 *Kg*,	5 12
and 2 *fr* 35 *c* = 235 *c*	2 35
Now, 235 centimes is the cost of one kilogramme, hence the req^d cost is given in centimes by the *product* of 235 and 5·12	25 60
	153 6
Thus, we obtain 1203·2 centimes	1024
∴ the req^d result is 1203 centimes = 12 *fr* 3 *c*	1203 20

* A kilogramme is about 2 2 lb † A measure rather less than our *quart*
‡ Those in which money accounts are written, thus in England we have *three principal* coins, namely, £, *s* *d*

IV. REDUCTION.

CONCRETE QUANTITIES.

We estimate the size of a group of concrete units which are all alike and *separate* (such, for instance, as *sheep* or *shillings*) by counting them

This process we extend to magnitudes (*i e.* things possessing *size*) which are *not* composed of separate units, but which are *all in one piece*, or continuous. We choose another thing of the same kind (as large or as small as we find convenient) which we regard as the *unit*, and we then compare the thing we wish to estimate with this unit in order to discover *how many such units it would, if cut up, yield* This is called **measuring** it, and the number of such units which the magnitude measured is found to contain is called its **quantity.**

It almost always happens that, whatever the size of the unit of measurement we choose, there remains, after we have counted off as many such units as possible, a portion of the thing we measure too small to yield another complete unit. This portion may, in its turn, be measured in terms of a *smaller* unit, and so on, to any degree of accuracy which (if the means at our disposal permit) the case requires.

For instance, suppose we wish to discover which of two maps is the longer, and suppose that they are fixed on opposite sides of a room so that we cannot bring them together in order to compare them We *measure* them in terms of any convenient unit—say the length of this book Suppose, now, that after marking off the length of this book six times along the side of each map there remains a portion of the length of each too small to allow of this being done a seventh time Then the question is, as yet, unsolved, and we proceed to measure these two remaining portions in terms of some smaller unit—say the length of a postage-stamp If we find, on trial, that the first of these portions contain the length of a stamp 5 times, but not 6 times, and that the second portion contain it 4 times, but not 5 times, we need proceed no further, as we now know that the first of the two maps is the longer, the measure of its length, expressed in terms of the units selected,

being 6 *book-lengths* + 5 *stamp-lengths,*
and that of the second 6 *book-lengths* + 4 *stamp-lengths.*

It should be noticed that in neither case does the measure express the *exact* length of the map, for we neglected, as inconsiderable, the very small portions remaining over after the second pair of measurements.

The unit of measurement must always be of like *kind* with the thing measured

For instance, any solid substance can be measured by *weight* (*i e.* by discovering how many times as heavy it is as a fixed amount of some particular substance), but we cannot measure *time* by the *pound,* nor *distance* by the *gallon.*

A quantity expressed in terms of *one unit only* is called a **simple quantity.**

For instance, the length 23 *feet*

A quantity expressed in terms of *more than one unit* is called a **compound quantity.**

For instance, the length 5 *yards*, 2 *feet*, 7 *inches*

Reduction is the operation of *changing the unit*, or *units*, in which the measure of a quantity is expressed

The *name* of the unit in terms of which a quantity is measured is called its **denomination.**

A quantity is said to be reduced to a *higher* or *lower denomination* according as the change is made from a smaller to a larger unit, or *vice versa*

For instance, when we change 2 *feet* into 24 *inches*, we reduce this length to a *lower* denomination

The connections which exist between various units are called **Tables.**

The following are the chief Tables in use in the British Isles *

MONEY

4 farthings (*f*) = 1 penny (*d*)
12 pence = 1 shilling (*s*).
20 shillings, or 240 pence = 1 pound (£)

Beginners should be practised in reducing small numbers of *pence* to *shillings*, and of *shillings* to *pounds*, mentally until great speed and accuracy is acquired Hence it is well to *commit to memory* the following Pence Table —

20 *pence* = 1*s* 8*d*	50 *pence* = 4*s*. 2*d*.	80 *pence* = 6*s* 8*d*
30 ,, = 2*s* 6*d*	60 ,, = 5*s*	90 ,, = 7*s* 6*d*
40 ,, = 3*s* 4*d*	70 ,, = 5*s* 10*d*	100 ,, = 8*s* 4*d*

The following equivalents are also noteworthy —

1 shilling = 3 *fourpences*, 4 *threepences*, 6 *twopences*, 8 *three-halfpences*, 24 *halfpence*, or 48 *farthings*

1 half-crown = 5 *sixpences*, 10 *threepences*, or 30 *pence*

1 pound = 4 *crowns*, 8 half-crowns, 10 *florins*, 40 *sixpences*, 60 *fourpences*, 80 *threepences*, 120 *twopences*, 480 *halfpence*, or 960 *farthings*

* For Tables of Foreign Money and the Metric System, see Part II

The English coins now *current* (*i e* in common use) are —

The *sovereign* (20*s*) and *half-sovereign*, in gold.

The *crown* (5*s*), *four-shilling piece*, *half-crown*, *florin* (2*s*), *shilling*, *sixpence*, and *threepence* } in silver.

The *penny*, *halfpenny*, and *farthing*,* in copper

The smallest piece of "paper money" used in England is the *five-pound note* In Scotland and Ireland one-pound notes are current.

A sum of money is still often expressed by the name of coins now no longer used, namely, the *guinea* (21*s*) and the *half-guinea*

The letters *£*, *s*, *d* are the initials of the Latin names *libra*, *solidus*, *denarius* respectively The use of the word *pound* is due to the primitive custom of *weighing*, not coining, money A "pound" originally meant 1 lb weight of silver

The notation of Fractions is used in writing farthings thus, 1 farthing and 3 farthings are written $\frac{1}{4}d$ and $\frac{3}{4}d$, 2 farthings, or a halfpenny, thus, $\frac{1}{2}d$. The word *farthing* (*i e* "fourthing") means a fourth part.

We estimate the value of all articles of commerce in comparison with gold Consequently while their market value rises or falls according as the supply is less or more plentiful, that of gold remains constant.

The gold of which a sovereign is composed is not absolutely pure, but consists of 22 parts by weight of pure gold mixed with 2 parts of *alloy* This is the *standard of value* The alloy is a mixture of silver and copper used to make the coins harder †

One ounce of *standard gold* is always worth £3, 17*s*. $10\frac{1}{2}d$, and gold coins are worth their weight ‡

Silver coins contain 37 parts of pure silver mixed with 3 parts of alloy Copper coins contain 95 parts of copper, 4 parts of tin, and 1 of zinc Silver and copper coins are *tokens*, *i e* they represent certain values, but do not contain that worth of metal, for instance, the silver obtained from 20 shillings melted down would not be worth £1 On this account silver is not "legal tender" for payment of a debt of more than £2, nor copper for more than 1*s*

Precious metal, (*i e.* gold or silver,) when uncoined is called *bullion*, when coined *specie*

The government establishment for coining is called the *Mint*

The word *sterling* ‖ applied to a sum of money signifies that it is of *standard* value

---

* Only a limited use is made of farthings, namely, for small ready-money payments

† Australian sovereigns are lighter in colour than English owing to the alloy used containing less copper

‡ Jeweller's gold contains much more alloy, thus 18 *carat* gold (the finest used by jewellers) contains but 18 parts out of 24 of pure gold

‖ Sterling is a corruption of *Easterlings* (*i e* men from the East), a name given to the German merchants in England in the time of Edward III, and to money coined by them.

TIME

60 seconds (*sec*) = 1 minute (*min*).
60 minutes = 1 hour (*hr*)
24 hours = 1 day.
7 days = 1 week (*wk*)
365 days = 1 (common) year (*yr*).
366 days = 1 leap year
100 years = 1 century.

Hence, 52 weeks + 1 day = 1 (common) year.

The year is divided into 12 **Calendar* Months**, of which **February** contains { in *common* years, 28 days, in *leap* years, 29 days.

Of the others "Thirty days hath September,†
April, June, and November,"
and the remaining months each contain 31 days

For legal purposes the year is divided into 4 quarters The English Quarter Days, on which rents, &c, for the preceding period, are due, being —

March 25th (Lady Day),
June 24th (Midsummer),
September 29th (Michaelmas),
December 25th (Christmas)

Leap year occurs (as a rule) once every *four* years, those years being counted as leap years the dates of which can be divided by 4 without remainder.

For instance, 1892 was a leap year, for 1892 ÷ 4 yields *no remainder*.

If, however, the year completes a century it is *not* counted as a leap year, *unless the* number of centuries *can be divided by* 4 *without remainder*

For instance, 1900 will *not* be a leap year, for 19 ÷ 4 yields a remainder, but the year 2000 *will be* a leap year, for 20 ÷ 4 yields *no* remainder

* So called to distinguish them from Lunar Months A *Lunar* month (Latin *luna*, the moon), the time the moon takes to make its journey round the earth, consists of about 4 weeks

† The Roman year began in March, hence the names September, October, November, December, derived from the Latin numerals, *septem, octo, novem, decem*, mean 7th, 8th, 9th, 10th month, respectively

This complex arrangement of the calendar is owing to the fact that the earth does not complete its annual journey round the sun in an *exact* number of days

The average length of the solar* year is 365 days 5 hours 48 minutes 49 seconds Thus the common (or civil) year of 365 days is *very nearly* $\frac{1}{4}$ *of a day too short*, and so, if every year were taken as 365 days, the error would in time mount up to such an extent that the months would cease to correspond with their seasons, and *January*, for instance, would gradually travel round into the *summer*

In order to correct this, Julius Cæsar, in the year 46 B C, enacted that *every fourth* year should contain 366 days, an arrangement called the Julian Calendar, or **Old Style**

In the Julian Calendar, then, the average length of the *civil* year is 365$\frac{1}{4}$ days, or 365 days 6 hours, while that of the *solar* year is

365 days 5 hours 48 min 49 secs

The civil year was thus made a little *too long* (*i e* the calendar, which before had *gained*, was made *to lose* time slowly)

And this loss of 11 min. 11 secs a year amounts in 400 years to a very little more than 3 days

As a further correction Pope Gregory, in 1582, ordered that *in every 400 years three of the leap years* in the Julian Calendar should be replaced by *common* years, namely, those which contained an exact number of centuries, unless that number of centuries could be divided by 4 without remainder, an arrangement called the Gregorian Calendar, or **New Style.**

Thus in the New Style there are 97 leap years in every 400 years

The New Style was immediately adopted in all Roman Catholic countries, but not till much later in others In England the change was not made till September 2nd, 1752, when (as the Julian Calendar had by that date become 11 days behind time) *eleven days were omitted* from that month, and the day following Sept 2nd was reckoned as Sept 14th

Traces of the Old Style may still be seen in the names Old Lady Day, Old Michaelmas Day, &c , and in the date April 5th (*i e* Old Lady Day) on Government tax-papers, &c , as the last day of the Financial year

In Russia the old style is still in use. Hence, as the Old Style Calendar is now 12 days behind the New, Christmas Day is kept nearly a fortnight later in Russia than in England.

The Gregorian Calendar is not absolutely *perfect*, but the error is very small, amounting only to 1 day in about 5000 years

The *standard unit of measurement* of time is the **day**, the average interval between two successive transits of the sun across the meridian of any place, *i e* between two *noons*

A day is considered as beginning at midnight

---

* Latin *sol*, the sun

Avoirdupois Weight.

16 drams (*dr*)	=	1 ounce (*oz*)
7000 grains, or 16 ounces	=	1 pound (*lb*)
14 pounds	=	1 stone (*st*)
28 pounds, or 2 stones	=	1 quarter (*qr*).
112 pounds, 8 stones, or 4 quarters	=	1 hundredweight (*cwt*).
20 hundredweights	=	1 ton.

Avoirdupois* weight is used for *all common substances* subject to waste, such as coal, meat, butter, tea, &c

In addition may be mentioned—

a *firkin* of butter = 56 *lbs*	a *quartern* loaf = 4 *lbs*
a *pack* of wool = 240 *lbs*	a *sack* of coal = 2 *cwt*

The abbreviation "*cwt*" for hundredweight consists of the Roman numeral C (100), with the first and last letters of the word "weight"

Troy Weight

24 grains (*gr*)	=	1 pennyweight (*dwt*)
480 grains, or 20 dwts	=	1 ounce Troy (*oz Tr*).
12 ounces Troy	=	1 pound Troy (*lb Tr*)

Troy† weight is only used for gold, silver, and precious stones ‡

The pound Troy is no longer a "legal" measure of weight

The ounce and pound Avoirdupois differ from the ounce and pound Troy, *the only connection is through the grain*

Thus 1 lb. Avoirdupois contains 7000 grains,
but 1 lb. Troy = $24 \times 20 \times 12$, or 5760 grains.

Apothecaries' Weight
(used for drugs sold by retail)

20 *grains* = 1 *scruple* (℈), 3 *scruples* = 1 *dram* (ʒ), 8 drams = 1 oz (℥) ‖

The *standard unit of weight* is the **Imperial pound** Avoirdupois, a piece of platinum very carefully preserved in the wall of the Houses of Parliament

---

* French *avoir du pois*, to have some weight † From the town Troyes in France
‡ The *carat* used in weighing diamonds = $3\frac{1}{6}$ grs ‖ Troy

LENGTH.

12 inches (*in.*) = 1 foot (*ft.*).
36 inches, or 3 feet = 1 yard (*yd.*).
1760 yards = 1 mile (*mi.*).

$5\frac{1}{2}$ yards, or 11 half-yards = 1 pole (*po.*), rod, or perch (*per.*).
220 yards, or 40 poles = 1 furlong (*fur.*).
8 furlongs = 1 mile.†

The chain used in land-surveying is 22 yards long and contains 100 links.
Hence, 100 links, or 22 yards = 1 chain (*ch.*),
25 *links* = 1 *pole*,
10 *chains* = 1 *furlong*,
80 *chains* = 1 *mile*

In addition to the above may be mentioned—
the *fathom* = 6 *feet*, used for soundings at sea.
the *nautical mile*, or *knot* = 6080 *feet* „ the rate of ships.
the *cable's length* = 120 *fathoms*.
the *hand* = 4 *inches* used for measuring horses
the Irish *pole* = 7 *yards*

Also, 5 *feet* = 1 *pace*,	9 *inches* = 1 *span*,
3 *miles* = 1 *league*	18 *inches* = 1 *cubit*.

CLOTH MEASURE

$2\frac{1}{4}$ *inches* = 1 *nail*,
9 *inches*, or 4 *nails* = 1 *quarter*,
4 *quarters* = 1 *yard*,
5 *quarters* = 1 *ell*.

Our units of measurement originated in those provided by nature, as we see in the names "hand," "foot," "span," &c

Also, "yard" means the length of the arm, "furlong," furrow-length, "cubit" is from the Latin *cubitus*, the fore-arm, and "mile" from the Latin *mille*, a thousand (paces)

The *standard unit of length* is the **Imperial yard**, being the distance between two gold pegs in a bronze bar which is kept in the Houses of Parliament with other "standards"

---

* The second part of this Table and exercises upon it are often deferred until Fractions have been learned

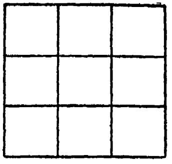
† Often called a "statute" mile, *i e* of the length fixed by *act* of Parliament

Square Measure

144 square inches (*sq in*) = 1 square foot (*sq. ft*)
9 square feet = 1 square yard (*sq yd*).
$30\frac{1}{4}$ square yards, or 121 square quarter-yards = 1 square pole (*sq po*, or P).
40 square poles = 1 rood (*ro*, or R)
4840 square yards, or 4 roods = 1 acre (*ac*, or A)
640 acres = 1 square mile (*sq mi*)

Square measure is used in measuring surfaces or areas, *e g* fields, floors, &c

The first part of the table may be found by *squaring* Long Measure, as may be seen from the accompanying diagram



If we take nine squares, having their sides each a foot long, we can arrange them so as to form one larger square, having each of its sides 3 feet, or 1 yard long, thus —

Hence, 3 × 3, or 9, square feet make 1 square yard

Similarly, 12 × 12, or 144, square inches make 1 square foot

In this way, then, we can supplement the table of Square Measure from the table of Long Measure

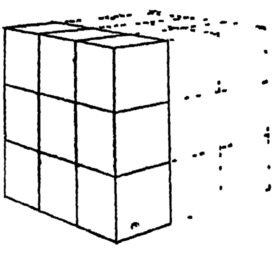
For instance, as 22 yards = 1 chain, 22 × 22, or 484, sq yds = 1 sq chain
And, as 100 links = 1 chain, 100 × 100, or 10000, sq links = 1 sq chain
Also, as 1760 yards = 1 mile, . 1760 × 1760 sq yds = 1 sq mile

Cubic Measure

1728 cubic inches (*cub in*) = 1 cubic foot (*cub ft*)
27 cubic feet = 1 cubic yard (*cub yd*)

Cubic Measure is used for measuring space, also the volume of certain solids, *e g* stone, timber, &c

This table is formed by *cubing* the corresponding part of Long Measure.



For instance, if *nine* cubes (blocks measuring a foot each way) were arranged, as in the diagram, in a stack 3 feet long, 3 feet high, and 1 foot wide, and if two more such stacks were placed alongside, as indicated by the dotted lines, the whole pile so formed would contain 3 × 9, or 27, blocks, and would measure 3 feet, or 1 yard, each way

Hence, 3 × 3 × 3, or 27, cubic feet make 1 cubic yard

Measures of Capacity

2 pints (*pt.*) = 1 quart* (*qt.*).	}	Liquids
8 pints, or 4 quarts = 1 gallon (*gal.*).	}	(*e g* milk).
2 gallons = 1 peck (*pk.*)	}	Dry goods
8 gallons, or 4 pecks = 1 bushel (*bus.*).	}	(*e g.* corn)
8 bushels = 1 quarter (*qr.*).	}	

In addition may be mentioned—

the *firkin* = 9 *gallons*,	Beer	the *hogshead* = 63 *gallons*	Wine.
kilderkin = 18 *gallons*,	Beer	*pipe* = 2 *hogsheads*,	Wine.
barrel = 36 *gallons*,	Beer	Also, 4 *gills* = 1 *pint*	
hogshead = 54 *gallons*,	Beer	2 *bushels* = 1 *strike*.	
butt = 2 *hogsheads*,	Beer	4 *bushels* = 1 *sack*	
tun = 2 *butts*,	Beer	5 *quarters* = 1 *load*	

A bushel of English wheat = 63 lbs weight The hogshead, pipe and butt vary for some particular kinds of wine

The *standard* unit of capacity is the **Imperial gallon**, the volume of 10 lbs. of pure water

Hence the rhyme, "A pint of pure water, weighs a pound and a quarter"

Also, as a gallon occupies about $277\frac{1}{4}$ cubic inches of space, **1 cubic foot of water weighs 1000 ounces**, nearly

---

Miscellaneous

Number.

12 *units* = 1 *dozen*.
12 *dozens* = 1 *gross*
20 *units* = 1 *score*

Paper.

24 *sheets* = 1 *quire*.
480 *sheets*, or 20 *quires* = 1 *ream*

A sheet of "Foolscap" is $16\frac{1}{2}$ inches long and $13\frac{1}{2}$ inches wide, which is a size often used in the printing of books Thus in a volume of the size called Quarto (4*to*) a sheet is folded into *four* leaves, in Octavo (8*vo*), into 8 leaves, in Duodecimo (12*mo*), into 12 leaves

Apothecaries Fluid Measure

60 *minims* (M) = 1 *fluid drachm* (f ʒ)
8 *drachms* = 1 *fluid ounce* (f ℥).
20 *ounces* = 1 *pint* (O)

Angular Measure

60 *seconds* (60″) = 1 *minute* (1′).
60 *minutes* = 1 *degree* (1°)
90 *degrees* = 1 *right angle*.

The distance on the surface of the globe corresponding to one *degree of latitude* = 60 *nautical* miles, or nearly 70 *statute miles*

A *ton of shipping* is not a weight but a measure of volume, namely, 40 *cubic feet* of space

---

* Often called *Imperial* pint and quart to distinguish them from the *Reputed* pint and quart measures commonly used for bottled wines 6 *reputed quarts* = 1 *gallon*

The following examples illustrate the process of Reduction:—

EXAMPLE I.—*Reduce £17, 5s. 3d. to pence.*

Explanation.

As in £1 there are 20*s*,
∴ in £17 there are 17 *times* 20*s*. = 340*s*.
∴ in £17, 5*s* there are 340 + 5 = 345*s*.

As in 1*s* there are 12*d*,
∴ in 345*s* there are 345 × 12 = 4140*d*
∴ in 345*s*. 3*d*. there are 4140 + 3 = 4143*d*

Written Work.

$$\begin{array}{rrr} \pounds & s & d \\ 17\,. & 5 & 3 \\ 20 & & \\ \hline 345 & & \\ 12 & & \\ \hline 4143 & & \end{array}$$

Ans. 4143*d*

EXAMPLE II —*Reduce £312, 12s. 6d. to half-crowns.*

As 8 half-crowns = £1, we multiply 312 by 8, adding in 5, *the number of half-crowns in* 12*s* 6*d*

$$\begin{array}{rcc} \pounds & s & d \\ 312 & \underbrace{12 \quad 6}_{5\ hf\text{-}cr} & \\ 8 & & \\ \hline 2501 & & \end{array}$$

Ans 2501 *hf-cr*

EXAMPLE III.—*Reduce* 47135 *farthings to £ s d*

As 4 farthings = 1 penny, the number of *farthings* in any sum of money must be *four* times the number of *pence* in that same sum

We therefore divide the number of farthings by 4, obtaining 11783 *sets of four farthings, i e pence,* with 3 *farthings* over And so on

$$\begin{array}{r|l l} & f & \\ 4 & 47135 & \\ \hline 12 & 11783 & 3f. \\ \hline 2,0 & 98,1 & .\ 11d \\ \hline & 49 & .\ 1s \end{array}$$

Ans £49, 1*s* 11¾*d*.

EXAMPLE IV —*Reduce* 392 *half-guineas to half-crowns.*

When we cannot proceed directly from one of the given denominations to the other, we must first find some *common* denomination to which both can easily be reduced, taking care that this is not *lower* than it need be

In this case we choose *sixpences* Hence, multiplying 392 by 21 (the number of sixpences in half-a guinea) we obtain 8232 as the number of sixpences in 392 half-guineas This we divide by 5 (the number of sixpences in half-a-crown), thus obtaining 1646 as the required number of half crowns with 2 *sixpences, i e* 1 *shilling*, over

$$\begin{array}{r|l l} & hf\text{-}gs & \\ & 392 & \\ & 21 & \\ \hline & 392 & \\ & 784 & \\ \hline 5 & 8232 & \\ \hline & 1646 & \ldots 2\ sixpences \end{array}$$

Ans 1646 *hf-cr* + 1*s*

EXAMPLE v.—*Reduce* 475302 *seconds to days, &c.*

6,0 | 47530,2
6,0 | 792,1 . .. 42 *secs*
24 { 4 | 132 ...... 1 *min.*
{ 6 | 33...0 } 12 *hrs.*
5.. 3 }

Ans 5 *days* 12 *hrs* 1 *min.* 42 *secs.*

EXAMPLE vi.—*Reduce* 7 *tons* 13 *cwts.* 23 *lbs* 9 *ozs to ounces.*

1st Method

tons	cwts	qrs	lbs	ozs
7	13 .	0 .	23 .	9

20
153
4
612
28
4919
1224
17159
16
102963
17159
274553

Ans 274553 *ozs*

2nd Method

tons	cwts	lbs	ozs.
7	13 .	23 .	9

20
153
112
329
153
153
17159
16
274553

Here, as no *quarters* are mentioned, we step directly from *cwts* to *lbs.*, multiplying by 112, instead of by 4 and 28 in succession Also, we multiply by 16 in *one* line *

EXAMPLE vii.—*Reduce* 3754219 *ozs. to tons, cwts., &c.*

16 { 4 | 3754219
{ 4 | 938554 . .. 3 } 11 *ozs.*
28 { 4 | 234638 2 }
{ 7 | 58659 . 2 } 26 *lbs*
4 | 8379 . 6 }
2,0 | 209,4. . 3 *qrs*
104 . 14 *cwt.*

Ans 104 *tons* 14 *cwt* 3 *qrs* 26 *lbs* 11 *ozs.*

N B.—Here short *divisions should always be used.*

* This may be done either by committing to memory the Multiplication Table as far as 16 times 9, or by multiplying by 6 and adding the result, figure by figure, to 171590 (*i e* 10 times 17159) See page 12.

EXAMPLE VIII —*Reduce 5 mi. 3 fur. 24 po 4 yds to yds.*

1st Method

mi	fur	po	yds
5 .	3	24 .	4

8
43
40
2 | 1744
5
8724
872
9596

Ans 9596 *yds*

As $5\frac{1}{2}$ yds make 1 pole, and, as $1744 \times 5\frac{1}{2} = 5$ times $1744 +$ half of 1744, we may multiply 1744 by 5, then divide 1744 by 2, and add these results

2nd Method

mi	fur	po	yds
5	3	24	4

8
43
40
1744
17448
2 | 19192
9596

Ans 9596 *yds*

As 11 *half-yards* make 1 pole, we multiply 1744 by 11, adding in 8 *half-yards*, and then divide the result by 2 In this case the multiplier 11 is not written See A (iv) p. 19.

EXAMPLE IX —*Reduce 3 mi 5 fur 17 po 3 yds 2 ft 7 in to inches.*

1st Method

mi	fur	po	yds	ft	in
3	5	17 .	3	2	7
8				1	6

29
40
2 | 1177
5
5888
$588\frac{1}{2}$
6476
3
19431
12
233185

Ans 233185 *inches*

Here, in dividing 1177 by 2 we obtain quotient 588 and 1 *half-yard* over, this we replace by 1 ft 6 in, which we set beside the 2 ft. 7 in and add in with them when we come to multiply by 3 and 12.

2nd Method.

mi	fur	po	yds	ft	in
3 .	5	17 .	3	2 .	7

$\underbrace{2 . 7}_{31\ ins}$

8
29
40
1177
11776
12953
18
103655
12953
233185

Ans 233185 *inches*

Here, multiplying 1177, the number of poles, by **11**, and adding in **6** half-yards, we obtain 12953 as the number of *half-yards* We then multiply by **18**, *the number of inches in half-a-yard*, and add in 2 ft. 7 in as **31** inches

EXAMPLE x —*Reduce* 12 *mi.* 2 *fur.* 135 *yds. to yards.*

Here, as no *poles* are given, we can avoid multiplying by the troublesome $5\frac{1}{2}$.

1st Method.

```
mi    fur    yds
12     2     135
 8
 98
 220
 1960
196
 135
21695      Ans. 21695 yds
```

2nd Method

```
mi.    fur    yds
12      2     135
1760          440
21695
```

Here we convert 2 *furs* to *yards*, and then multiply 1760 by 12 (the multiplier standing *above* the multiplicand), adding in 135 + 440 yards.

EXAMPLE xi —*Reduce* 1345732 *inches to miles, furlongs, poles, &c.*

As 11 *half-yards* make 1 pole, we first reduce the given inches to *half-yards*, *i.e.* we divide by 18 (3 × 6), obtaining 74762 as the number of *half-yards* and 16 *inches* over We next divide 74762 by 11, obtaining 6796 as the number of *poles* and 6 *half-yards* over

```
                in
18 { 3 | 1345732
   { 6 | 448577.... .1 }
    11 | 74762 .... 5  } 16 in. = 1 ft 4 in
    4,0 | 679,6 . .. 6 half-yds. = 3 yds.
      8 | 169  .. . 36 poles.
          21   .    1 fur
```

Ans 21 *mi* 1 *fur.* 36 *po* 3 *yd* 1 *ft.* 4 *in*

N B.—In writing down the answer the beginner must be careful *not to forget to* change *half-yards* to *yards.*

EXAMPLE xii —*Reduce* 457115 *feet to miles, furlongs, poles, &c*

Dividing by 3 we obtain 152371 as the number of *yards* We now multiply by 2, obtaining 304742 as the number of *half-yards* We next divide by 11 (the number of *half-yards* in 1 *pole*), when the 9 *half-yards* over yield $4\frac{1}{2}$ *yards*

```
       ft
   3 | 457115
       152371 ..  ...............2 ft.
            2
  11 | 304742
  4,0 | 2770,3  .9 half-yds = 4½ yds
    8 | 692 .    . . .   23 po
        86               4 fur
```

Thus 457115 *feet* = 86 *mi* 4 *fur* 23 *po.* $4\frac{1}{2}$ *yds* 2 *ft*;

or, taking *half a yard* from the 2 *ft* and adding it to the $4\frac{1}{2}$ *yards,* we have the neater form of *Ans* 86 *mi.* 4 *fur* 23 *po* 5 *yds* 0 *ft* 6 *in*

EXAMPLE XIII.—*Reduce 5 ac 3 ro 31 po 14 sq yds 7 sq ft to square feet*

To find the number of sq yds equivalent to 951 poles we multiply 951 by 30, divide 951 by 4, and add the results

The $\frac{3}{4}$ sq yd remaining over from this division yields $\frac{27}{4}$ sq ft or $6\frac{3}{4}$ sq ft, which we place against the 7 sq ft and add in with it when we multiply by 9

[This process corresponds to the 1st method of Ex IX We should gain nothing here by adopting the 2nd method, as that would involve multiplying 951 by 121 and then dividing the result by 4]

```
   ac    ro    po.   sq yd   sq ft
    5     3    31     14      7
    4                         6¾
   23
    40
4 | 951
      30
  28544
    237¾
  28781
      9
 259042¾
```

Ans 259042¾ *sq ft*

EXAMPLE XIV.—*Reduce 17 ac 1 ro to square yards.*

Here, as no *poles* are given, we avoid multiplying by the troublesome $30\frac{1}{4}$, and step directly from acres to yards, remembering that 4840 sq yds make 1 ac, and that, consequently, 1210 sq yds make 1 rood

(Here, also, the multiplier, 17, stands *above* the multiplicand)

```
  ac    ro
  17 .  1
 4840
33880
4840
 1210
83490
```

Ans 83490 *sq yds*

EXAMPLE XV.—*Reduce 14170 sq yds. to acres, roods, poles, &c.*

We first change 14170 sq yds to *sq quarter-yds*; then, as 121 *sq quarter-yds* make 1 *sq pole*, we divide by 11 × 11, obtaining 468 poles and 52 *sq quarter-yds*, *i e* 13 *sq yds* over, &c

```
          sq yd
          14170
              4
121 { 11 | 56680
    { 11 | 5152     8 } 52 sq qr-yds = 13 sq. yds
       4,0 | 46,8   4 }
         4 | 11             . .     28 po
              2            . . .    3 ro
```

Ans 2 *ac* 3 *ro* 28 *po* 13 *sq yds*

EXAMPLE XVI.—*Reduce* 1470358 *sq in to poles, &c.*

$$
\begin{array}{r}
\text{sq in} \\
144 \left\{ \begin{array}{r} 12 \,|\, \underline{1470358} \\ 12 \,|\, \underline{122529} \ldots 10 \end{array} \right. \\
9 \,|\, \underline{10210} \ldots \; 9 \\
1134 \ldots\ldots\ldots\ldots 4 \; sq.\,ft. \\
\underline{4} \\
121 \left\{ \begin{array}{r} 11 \,|\, \underline{4536} \\ 11 \,|\, \underline{412} \quad 4 \end{array} \right. \\
37 \quad 5
\end{array}
\quad
\begin{array}{l}
\left.\begin{array}{r} 10 \\ 9 \end{array}\right\} 118 \; sq \; in. \\
\\
\left.\begin{array}{r} 4 \\ 5 \end{array}\right\} 59 \; sq \; quarter\text{-}yds = 14\tfrac{3}{4} \; sq \; yds.
\end{array}
$$

Hence 1470358 *sq in* = 37 *po* $14\frac{3}{4}$ *sq yds* 4 *sq ft* 118 *sq in*;
or, as $\frac{1}{4}$ of a sq yd = $\frac{1}{4}$ of 9 sq ft = $2\frac{1}{4}$ sq ft = 2 sq ft. 36 sq. in., if we take 2 sq ft 36 sq in from the 4 sq ft 118 sq in., and add it to the $14\frac{3}{4}$ sq yds, we obtain in improved form the *Ans* 37 *po* 15 *sq yds* 2 *sq ft* 82 *sq in.*

EXAMPLE XVII.—*Reduce* 3754813 *cub. in to cubic yards*

We first reduce the cub in to cub ft, dividing by 12 × 12 × 12 (*i e* 1728), and obtaining the remainder, 1597 cub in. (See p 16.) We then divide the 2172 cub ft by 3 × 9 (*i e* 27), &c.

$$
\begin{array}{r}
\text{cub in} \\
1728 \left\{ \begin{array}{r} 12 \,|\, \underline{3754813} \\ 12 \,|\, \underline{312901} \\ 12 \,|\, \underline{26075} \end{array} \right. \\
27 \left\{ \begin{array}{r} 3 \,|\, \underline{2172} \\ 9 \,|\, \underline{724} \end{array} \right. \\
80 \ldots
\end{array}
\quad
\begin{array}{l}
\left.\begin{array}{rcr} 1 & = & 1 \\ 1 \times 12 & = & 12 \\ 11 \times 144 & = & 1584 \end{array}\right\} 1597 \; cub. \; in. \\
\left.\begin{array}{r} 0 \\ 4 \end{array}\right\} 12 \; cub. \; ft.
\end{array}
$$

Ans 80 *cub yds* 12 *cub ft.* 1597 *cub in*

EXAMPLE XVIII.—*Express* 13 *lbs* 4 *ozs Avoirdupois in grains.*

As 7000 *grains* make 1 *lb* Avoirdupois, and as 4 *ozs* is $\frac{1}{4}$ of a *lb* Av, there are 1750 *grains* in 4 *ozs* Hence we multiply the 13 by 7000 and add in 1750, obtaining 92750 as the number of *grains*

$$
\begin{array}{rr}
\text{lbs} & \text{ozs} \\
13 & 4 \\
\underline{7000} & \\
91000 & \\
\underline{1750} & \\
\underline{92750} &
\end{array}
$$

Ans 92750 *grs.*

To test the work of Reduction.

Reduce the answer back to the form given in the question.

V. THE COMPOUND RULES.

The principles involved in the addition, subtraction, multiplication, and division of *compound quantities* are those explained in Chapter II. The examples which follow illustrate the methods used.

Compound Addition

It is evident that the quantities to be added must be of the *same kind*

For instance, 3 *acres* + 4 *acres* make 7 *acres*,
but 3 *acres* + 4 *tons* make neither 7 *acres* nor 7 *tons*

Also, that when the quantities *are* of the same kind, each *denomination* must be dealt with separately

For instance, 3 *pence* + 4 *pence* make 7 *pence*,
but 3 *shillings* + 4 *pence* make neither 7 *shillings* nor 7 *pence*

Hence, we arrange the quantities in columns so that all the quantities in the same column are of the same denomination, we then add together the numbers in the column of lowest denomination, and carry forward to the next column as many complete units of that next higher denomination as the sum yields, setting the remainder below the column added; and so on

The following example will serve to illustrate the process with which, at least in the case of *money*, we assume that the student is already familiar *

Example 1 —*Add together* 3 *mi* 1 *fur* 17 *po* 3 *yds*, 1 *mi* 7 *fur.* 23 *po* 4 *yds*, 11 *mi* 6 *fur* 10 *po* 1 *yd*, 8 *mi* 2 *fur* 35 *po* 5 *yds*, 19 *mi* 0 *fur* 7 *po* 2 *yds*, *and* 2 *mi* 4 *fur* 13 *po* 4 *yds.*

mi	fur	po	yds
3	1	17	3
1	7	23	4
11	6	10	1
8	2	35	5
19	0	7	2
2	4	13	4
46	6	28	$2\frac{1}{2}$ *Ans*

Adding the numbers in the column headed "yards" we obtain the sum 19 yds We now remove from the 19 *yards* as many complete *poles* as we can (in this case 3, for 3 times $5\frac{1}{2}$ is $16\frac{1}{2}$), and set down the remaining $2\frac{1}{2}$ yds, carrying forward 3 *poles*

We next add the numbers in the column headed "poles," adding in the 3 carried, and divide the sum, 108 *poles*, by 40, obtaining 2 *furlongs* to carry forward, and 28 *poles* remaining, which we set down

And so on.

---

* Addition and Subtraction of money may be illustrated *practically* to young beginners, by using counters of different colours (say yellow, white, and brown) to represent pounds, shillings, and pence

COMPOUND SUBTRACTION.

EXAMPLE ii.—*From 14s. 3d. take 7s 10d, and explain the process.*

The Pence

We cannot take 10 *pence* from 3 *pence*, so we change one of the 14 *shillings* into 12 *pence*, which we mentally add to the 3 *pence*, thus making up 15 *pence*

Then 10 *pence* from 15 *pence* leaves **5** *pence.*

s	*d*	
14 .	3	
7	10	
6 .	5	*Ans*

The Shillings

Remembering that one of the 14 shillings has been changed into pence, as we have now to take away 7 other shillings, it is evident that

$14-1-7$ will be the number of shillings left

But *to take* 1 *and* 7 *in succession from* 14 *is equivalent to taking* **8** *from* 14 *in* one *operation*

So when we pass from the pence to the shillings column, we "carry" **1**, which we *add to the 7 shillings.*

Then 8 *shillings* from 14 *shillings* leaves **6** *shillings.*

Addition and Subtraction (as was shown with abstract numbers, on page 8), can often be conveniently combined in *one* operation.

EXAMPLE iii.—*Take the sum of* £2, 15*s.* 7½*d*, £16, 9*s* 0¼*d.*, £13, 18*s* 9¼*d*, £5, 13*s* 2¾*d*; *and* £1, 3*s* 7¾*d. from* £45, 13*s* 5¼*d*

£	*s*	*d*	
45	13 .	5¼	
2	15 .	7½	
16	9 .	0¼	
13 .	18	9¼	
5 .	13 .	2¾	
1 .	3	7¾	
5 .	13	1¾	*Ans.*

1st step

Adding the farthings in the five lowest lines, we obtain the sum 2½*d*,
and 2½*d* from 3¼*d* leaves ¾d.
Carry 3

2nd step

Adding the pence in the same five lines and including the 3 carried, we obtain the sum 2*s* 4*d*,
and 2*s* 4*d.* from 2*s* 5*d* leaves **1d.**
Carry 2

3rd step.

Adding the *units* column of the shillings, and including the carried 2, we obtain the sum 30, and 30 from 33 leaves **3.** Carry 3.

Then adding the *tens* column, and including the carried 3, we obtain the sum 6, and 6 from 7 (changing £3) leaves **1.**

Carry 3

4th step

Adding the *units* column of the pounds, and including the carried 3, we obtain the sum 20, and 20 from 25 leaves **5.** Carry 2

Then adding the *tens* column of the pounds, and including the carried 2, we obtain the sum 4; and 4 from 4 leaves 0

Compound Multiplication.

We know from the definition of Multiplication (see page 9) that the multiplier is the *number of times* the multiplicand is repeated

Thus **the multiplier can never be concrete, but is always an abstract number.**

Hence, in Compound Multiplication the *multiplicand* is a *compound quantity,* the *multiplier* is an *abstract number,* and the *product* a quantity of the same kind as the multiplicand

For instance,

7 tons 2 *cwt* × 3 = *7 tons* 2 *cwt* + 7 *tons* 2 *cwt* + 7 *tons* 2 *cwt* = 21 *tons* 6 *cwt*

Again, in the question, *If* 1 lb *of tea cost 2s, what will* 7 lbs *cost?* The answer, 14*s* is the result of the following *mental* process —

"As 7 lbs will cost 7 *times as much* as 1 lb, the cost of 1 lb. must be repeated 7 times" We do *not* multiply 2*s* by 7 *lbs*, but by 7

The following examples illustrate various cases that occur:—

(1) When the multiplier is not greater than one of the factors of the Multiplication Table

Example iv —*Multiply* 3 *fur* 29 *po* 4 *yds.* 1 *ft by* 7.

1st step.

7 times 1 *ft* = 7 *ft* = 2 *yds* 1 *ft*
Set down 1, and carry 2

2nd step

7 times 4 *yds* = 28 *yds*, 28+2 (carried) = 30 *yds* = 5 *po* 2½ *yds*
Set down 2½, and carry 5
And so on

mi	fur	po	yds	ft
	3	29	4	1
				7
3	2	8	2½	1

Finally, converting the *half-yard* in the result into 1 *ft* 6 *in* we write the answer thus —

Ans 3 *mi* 2 *fur* 8 *po* 2 *yds* 2 *ft* 6 *in*

(2) When the factors of the multiplier are known from the Multiplication Table

Example v —*Multiply* 2 *tons,* 13 *cwts* 1 *qr* 17 *lbs by* 63

As 63 = 7 × 9, we multiply first by 7, and then multiply the result by 9 (or *vice versa,* first by 9 and then the result by 7), thus obtaining the required result See (IV), page 10

tons	cwts	qrs	lbs	
2	13 .	1	17	
			7	
18	13 .	3	7	
			9	
168 .	4 .	1	7	*Ans*

(3) When the multiplier is rather less, or rather greater, than the product of some two factors of the Multiplication Table.

EXAMPLE VI.—*Multiply* £1, 5s 7d. *by* 93

Here, as the Multiplication Table provides no factors which yield a product nearer to 93 than 9 × 10, we multiply in succession by 9 and 10, thus obtaining 90 times £1, 5s 7d (i)

We next multiply £1, 5s 7d. by 3, setting the result (ii) below (i)

Finally, we add the results (i) and (ii), thus obtaining 93 times £1, 5s 7d

£	s	d	
1 .	5 .	7	×3
		9	
11 .	10	3	
		10	
115 .	2 .	6 .	(i)
3 .	16 .	9 .	(ii)
£118	19 .	3	*Ans.*

EXAMPLE VII —*Multiply* £3, 17s. 5d. *by* 39

The factors in the Multiplication Table which *most nearly* produce 39 are 4 × 10

We therefore multiply in succession by 4 and 10, thus obtaining 40 times £3, 17s 5d (i)

We then subtract £3, 17s 5d from (i), thus obtaining 39 times £3, 17s 5d

£	s	d	
3 .	17 .	5	
		4	
15 .	9 .	8	
		10	
154 .	16 .	8..	(i)
3 .	17 .	5	
£150	19 .	3	*Ans.*

(4) When the multiplier exceeds the limit of the Multiplication Table

In such cases we may regard the multiplier as composed of the parts *units*, *tens*, *hundreds*, &c., multiply by each of these parts separately, and then add the results See (II) on page 10

EXAMPLE VIII —*Multiply* £1, 5s 7d *by* 283

Here, 283 = 200 + 80 + 3, so we first multiply in succession by 10, 10, and 2, obtaining 200 times £1, 5s 7d. (i)

We next multiply the second line by 8, obtaining 80 times £1, 5s 7d, and set the result (ii) below (i)

We then multiply the top line by 3, obtaining 3 times £1, 5s 7d (iii), and set the result below (ii)

Finally, we add these three results together, thus obtaining 283 times £1, 5s. 7d.

£	s	d	
1 .	5 .	7	×3
		10	
12 .	15 .	10	×8
		10	
127	18	4	
		2	
255	16	8 .	(i)
102	6 .	8	(ii).
3	16	9	(iii)
£362 .	0	1	*Ans.*

The following method may be used instead of that of Example viii, and, when the multiplier is *very large*, is to be preferred.—

EXAMPLE IX.—*Multiply* £11, 13*s* $3\frac{1}{4}d$ *by* 4957.

1st step
Multiply 1 *farthing* by 4957, reducing the result to *pence* (i)

2nd step
Multiply 3 *pence* by 4957, set the result under (i) and add it to (i), reducing the sum to *shillings* (ii)

3rd step
Multiply 13 *shillings* by 4957, set the result under (ii), and add it to (ii), reducing the sum to *pounds* (iii)

4th step
Multiply £11 by 4957, set the result under (iii), and then add it to (iii)

Written Work

4	4957			
	$1239\frac{1}{4}$			.... (i)
	14871			
12	$16110\frac{1}{4}$			
	1342	..	$6\frac{1}{4}$	(ii)
	64441			
20	6578,3		$6\frac{1}{4}$	
	3289	3 .	$6\frac{1}{4}$	..(iii)
	54527			
	£57816	3	$6\frac{1}{4}$	*Ans.*

We have supposed that all the work, except what is given on the right, has been performed mentally

Note—The multiplication of compound quantities by very large numbers is of no great importance, as such products can be more concisely obtained, as we shall see later, by the method of *Practice*

The method of the following example is important, and may often be employed with advantage —

EXAMPLE X—*Multiply* £2, 19*s* $7\frac{1}{2}d$ *by* 1203

	1203	
	9	
2	10827	
12	5413	$\frac{1}{2}d.$
20	45,1 .	1*d*
	22	11*s*

£	*s*	*d*	
1203 .	0 .	0	
		3	
3609	0 .	0	
22	11	$1\frac{1}{2}$	
3586	8	$10\frac{1}{2}$	*Ans*

Here, observing that £2, 19*s* $7\frac{1}{2}d$ only differs from £3 by $4\frac{1}{2}d$,
we first multiply £3 by 1203,
we then multiply $4\frac{1}{2}$d by 1203 (*i e* multiply 1203 by 9, and reduce this number of *halfpence* to £ *s d*)

Finally we subtract these results

Note—If we *add* the two results in this example we obtain 1203 times £3, 0*s.* $4\frac{1}{2}d.$

COMPOUND DIVISION.

We know (see page 13) that Division, being the inverse of Multiplication, may be regarded either as the operation of finding

(i) the *multiplicand* when product and multiplier are given,
or (ii) the *multiplier* when product and multiplicand are given

Also (see page 40), if the multiplicand be a *concrete* quantity, the product is also a concrete quantity of the same kind, but the *multiplier* must always be an *abstract* number.

For instance, (i) *Divide 77 feet into 11 equal parts.*
Here we have given the *product 77 feet*, and the *multiplier* 11, and are asked to find the multiplicand 7 *feet*

(ii) *How many times is 3 shillings contained in 93 shillings?*
Here we have given the *product* 93 *shillings*, and the *multiplicand* 3 *shillings*, and are asked to find the abstract multiplier **31.**

Hence, in Compound Division, the dividend is a concrete quantity, and either

(i) the *divisor* an *abstract number*, and the quotient a concrete quantity of the same kind as the dividend,

or (ii) the *divisor a concrete quantity* of the same kind as the dividend, and the quotient an abstract number.

It is very important to notice that the *divisor* and the *quotient cannot* both *be concrete* together

Note —Division of the first of these two classes is sometimes called *Partition*, or *Sharing*, division of the second kind, *Quotition*, or *Measuring*.

CLASS I (*Abstract Divisor*).

Short Division.

EXAMPLE XI —*Divide* 9 *po* 16 *sq yds* 7 *sq ft* 6 *sq in. by* 6.

	po	sq yds	sq ft	sq in	
6)	9	16 .	7 .	6	
	1 .	17	8 .	43	*Ans*

1st step
6 in 9, 1, and 3 *po*, *i e* $90\frac{3}{4}$ *sq yds* over,
$90\frac{3}{4} + 16 = 106\frac{3}{4}$ *sq yds*

2nd step
6 in $106\frac{3}{4}$, **17**, and $4\frac{3}{4}$ *sq yds. i e*
$42\frac{3}{4}$ *sq ft* over, $42\frac{3}{4} + 7 = 49\frac{3}{4}$ *sq ft*

3rd step
6 in $49\frac{3}{4}$, **8**, and $1\frac{3}{4}$ *sq ft* over
Now, $1\frac{3}{4}$ *sq ft* $= 1\frac{3}{4} \times 144$ *sq in* $= 144 + 108 = 252$ *sq. in.*,
$252 + 6 = 258$ *sq in.*

4th step.
6 in 258, **43.**

When the factors of the divisor are known from the Multiplication Table "Short" Division may be used

EXAMPLE xii.—*Divide £213, 16s 8½d by 42.*

Here, as $42 = 6 \times 7$, we may divide in succession by 6 and 7, using "short" division.

The first rem^r is $4f$, and the second rem^r is also $4f$

Hence, the complete rem^r (see p 16) is $4 \times 6 + 4 = 28f = 7d$.

$$
42\left\{\begin{array}{r|rrrl}
 & \pounds & s & d & \\
6 & 213 & 16 & 8\tfrac{1}{2} & \\ \hline
7 & 35 & 12 & 9\tfrac{1}{4} & 4f \\ \hline
 & 5 & 1\;. & 9\tfrac{3}{4} & 4f
\end{array}\right. \left.\right\}\ \underline{28f}
$$

Ans £5, 1s $9\tfrac{3}{4}d + 7d$ *rem^r*

Or we may proceed by the method of "Long" Division, thus:

Long Division.

EXAMPLE xiii.—*Divide £213, 16s 8½d. by 42.*

1st step

Dividing £213 by 42 we obtain quotient £5 and rem^r £3

Reducing the rem^r, £3, to *shillings*, and adding in 16s, we obtain 76s

2nd step.

Dividing 76s by 42 we obtain quotient 1s and rem^r 34s

Reducing the rem^r 34s to *pence*, and adding in 8d, we obtain 416d

3rd step

Dividing 416d by 42 we obtain quotient 9d and rem^r 38d

Reducing the rem^r 38d to farthings, and adding in 2f, we obtain 154f

4th step

Dividing 154f by 42 we obtain quotient 3f and rem^r 28f, i e 7d

$$
\begin{array}{l}
\qquad\ \ \pounds \qquad s \qquad d \quad \pounds \\
42)\,213 \quad 16 \quad 8\tfrac{1}{2}\,(5 \\
\quad\ \ \underline{210} \\
\qquad\ 3 \\
\quad\ \ \underline{\ 20\ }\ s \\
42)\,76\,(1 \\
\quad\ \ \underline{42} \\
\quad\ \ 34 \\
\quad\ \ \underline{\ 12\ }\ d \\
42)\,416\,(9 \\
\quad\ \ \underline{378} \\
\quad\ \ \ 38 \\
\quad\ \ \underline{\ \ 4\ }\ f \\
42)\,154\,(3 \\
\quad\ \ \underline{126} \\
\quad\ \ \ \underline{28}f. = 7d
\end{array}
$$

Ans £5, 1s $9\tfrac{3}{4}d + 7d$ *rem^r.*

Note—In Compound Long Division the *written* work may be considerably curtailed by using the abridged method described on page 15

EXAMPLE xiv.—*Divide* 81 *cwts* 1 *qr* 7 *lbs* 5 *ozs* *by* 53.

1st Method.

```
    cwt    qr   lb   ozs   dr cwt
53)81  .  1  .  7  .  5  .  0(1
   53
   28
    4      qr
53)113( 2
   106
     7
    28     lb
53)203( 3
   159
    44
    16
   269
    44     oz
53)709( 13
   53
   179
   159
    20
    16     dr
53)320( 6
   318
     2
```

2nd Method

```
    cwt   qr   lb   oz   dr cwt
53)81  .  1  .  7  .  5  .  0(1
   28      qr
   113( 2
     7     lb
   203( 3
    44     oz
   709( 13
   179
    20     dr.
   320( 6
     2
```

In the *1st method* the mental work corresponds to that described in Ex xiii, except that the Table of Avoirdupois Weight is used in place of the Money Table

In the *2nd method* abridged division is used Also in the successive reductions the multipliers are omitted.

Ans. 1 *cwt.* 2 *qr* 3 *lb.* 13 *oz* 6 *dr.*+2 *dr. remr.*

Division by 100.

EXAMPLE xv.—*Divide* £322, 12*s* 1*d* *by* 100.

In this case it is *not necessary to write down the divisor*, for we know (see p 17), that division by 100 is performed at once, by striking off *two* figures on the right of the number divided.

1st step

Cutting off *two* figures, on the right, from £322, we obtain quotient £3 and remr £22 Reducing £22 to shillings, and adding in 12*s*, we obtain 452*s*

2nd step

Cutting off *two* figures, on the right, from 452*s*, we obtain quotient 4*s*. and remr 52*s* Reducing 52*s* to pence, and adding in 1*d*, we obtain 625*d*

And so on

```
  £      s     d.
3,22  .  12  .  1
   20
 4,52
   12
 6,25
    4
 1,00
```

Ans £3, 4*s*. 6¼*d*.

Similarly we proceed in the case of divisors 1000, 10000, &c., cutting off, at each stage, *three*, *four*, &c , figures respectively

CLASS II. (*Abstract Quotient*).

When the divisor and the dividend are both *compound* quantities, we must, before the division can be performed, reduce them both to the *same denomination*

In choosing this common denomination, we take care, if we would avoid unnecessary labour, that it is as *high* as possible

EXAMPLE XVI —*How many times is* 7 *lbs* 4 *ozs contained in* 4 *cwts* 2 *qrs* 10 *lbs* 12 *ozs ?*

In this case we reduce both quantities to *quarter-pounds*

Multiplying 7 by 4 and adding in the 4 ozs as 1 *quarter-lb* we obtain 29 *quarter lbs*

Reducing 4 cwt 2 qrs 10 lbs 12 ozs also to *quarter-lbs* and adding in 12 *ozs* as 3 *quarter-lbs* we obtain 2059 *quarter-lbs.*

Dividing 2059 by 29 we find that 29 *qr -lbs* is contained 71 times in 2059 *qr.-lbs.*

```
lbs  ozs        cwts  qrs  lbs   ozs
 7    4           4  .  2   10  .  12
 4                4
29               18
                 28
                154
                36
                514
                  4
           29)2059(71
              203
                29
                29
```

Ans 71 *times.*

EXAMPLE XVII —*How many men could each receive* 14*s* 7*d. from a fund of* £6, 18*s ?*

Reducing both quantities to the common denomination *pence*, we obtain 175*d* and 1656*d*

Dividing 1656 by 175 we find that 175*d* is contained 9 *times* in 1656*d*, and that 81*d*, *i e* 6*s* 9*d* is left over

Note.—Here, again, the quotient 9 is an *abstract number* The answer 9 *men* results from the following *mental* process —

"As 175 *pence* is contained 9 *times* in 1656 *pence*, therefore 9 *men* could each receive that sum"

```
 s    d        £    s    d
14 .  7        6 . 18 .  0
12            20
175          138
              12
         175)1656(9
             1575
               81d = 6s. 9d.
```

Ans 9 men, and 6*s* 9*d* *over*

VI. MISCELLANEOUS EXAMPLES.

A. *If 123 articles, bought at £2, 17s. 9d each, are sold at £3, 2s. 3d. each, what profit is made?*

We first subtract £2, 17s 9d. from £3, 2s. 3d. to find the profit on *one* article, namely, 4s 6d

Hence, the total profit is 123 times 4s. 6d = £27, 13s. 6d *Ans.*

Note—Much labour would be wasted if we multiplied £2, 17s. 9d and £3, 2s 3d. *separately* by 123 and *afterwards* subtracted the results.

B. *A merchant bought 28 gallons of spirits at 36s a gallon, diluted it with water, and then sold the mixture at 32s a gallon, making a profit of £4 How much water did he add?*

The total cost was 36 × 28 = 1008s
The total profit was £4 = 80s
∴ the whole mixture was sold for 1088s
But one gallon was sold for 32s,
And 32s is contained 34 times in 1088s.

i e. 34 gallons of *mixture* were sold, of which 28 was *spirit.*
∴ 34 − 28 = 6 gallons of water must have been added.

C. (i) *Divide £16, 8s. 6d between A and B so that for every half-crown A has B may have a florin*

Each time A receives 5 *sixpences* B is to receive 4 *sixpences*,
∴ with 9 sixpences this could be done once
But in £16, 8s 6d there are 657 sixpences
And 9 *sixpences* is contained 73 times in 657 *sixpences*.

Hence, A's share is 73 *half-crowns* = £9, 2s 6d
and B's „ 73 *florins* = £7, 6s 0d. } *Ans*

C. (ii) *Divide £1, 16s 2d among 7 men, 12 women, and 17 boys, so that each woman may have twice as much, and each man three times as much, as each boy*

Here the 7 men receive	as much	as	21 boys,
„ 12 women	„	„	24 boys,
„ 17 boys	„	„	17 boys;
∴ all the persons	„	„	62 boys.

Hence, as all the persons receive £1, 16s 2d among them, a boy's share is found by dividing this sum by 62, whence we obtain quotient 7d

Thus a boy's share is 7d, a woman's 14d, and a man's 21d. *Ans*

C. (111). *Divide £10, 15s 6d among 8 men, 9 women, and 10 boys, giving each woman 2s. less than each man, and 3s more than each boy.*

Here each *man* has 5*s* *more* than each *boy*
Also „ *woman* „ 3*s* „ „ *boy*.

∵ 40*s* would supply the extra payment for 8 men, and 27*s* would supply the extra payment for 9 women

Now 40*s* + 27*s* = 67*s* = £3, 7*s*

Hence we subtract £3, 7*s* from £10, 15*s* 6*d*, and divide the remainder *equally* among the total number of persons

Thus a boy's share is 5*s* 6*d*,
Hence a woman's „ 8*s* 6*d*,
and a man's „ 10*s* 6*d* } *Ans*

£	s	d
10	15	6
3	7	0
7	8	6

$$\begin{array}{l} 7 \\ 20 \\ \hline 27)148(5\ s \\ 135 \\ \hline 13 \\ 12 \\ \hline 27)162(6\ d \\ 162 \end{array}$$

D. *A sum of £26, 9s was made up of a certain number of half-sovereigns, twice as many shillings, three times as many half crowns, and seven times as many sixpences, how many coins were there altogether?*

Here 1 *half-sovereign* + 2 *shillings* + 3 *half-crowns* + 7 *sixpences*
= (20 + 4 + 15 + 7) *sixpences*
= 46 *sixpences*

Also, £26, 9*s* = 1058 *sixpences*
And 46 *sixpences* is contained 23 times in 1058 *sixpences*

∴ the total number of half-sovereigns was 25
„ „ shillings „ 46
„ „ half-crowns „ 69
„ „ sixpences „ 161
∴ the total number of coins „ 299 *Ans*

$$\begin{array}{ll} £ & s \\ 26 & 9 \\ 40 & \\ \hline \end{array}$$

$$\begin{array}{l} 46)1058(23 \\ 92 \\ \hline 138 \\ 138 \end{array}$$

E. *A sum of £4, 5s was distributed among 101 school-children so that each boy had 9d and each girl 1s. How many boys were there?*

We first consider how much money would be used in giving 9*d* to *every child*

This would take 101 × 9 = 909*d* = 75*s* 9*d*

There would now remain the difference between £4, 5*s* and 75*s* 9*d*, that is, 9*s* 3*d*

And this, by the question, was sufficient to provide an extra 3*d*. for each *girl*

Hence, as 9*s* 3*d* = 37 *threepences*, there must have been 37 *girls*.

∴ the number of *boys* was 101 − 37 = 64 *Ans*

F. *A and B have* 8s *between them, B and C have* 11s; *A and C have* 15s *How much has each?*

Twice A's money + *twice* B's + *twice* C's = 8*s* + 11*s* + 15*s*. = 34*s*

∴ A's money + B's + C's = 17*s*.

But, by the question, B's + C's = 11*s*;

∴ A's money is 17*s* − 11*s*. = 6*s*

Hence, B's money is 8*s* − 6*s* = 2*s*,

And C's money is 11*s*. − 2*s* = 9*s*.

G (i). *Find the number of days from September 7th,* 1891, *to June* 16*th*, 1892

		days
In 1891,	from September 7th to the end of the month	23
"	in October	31
"	" November	30
"	" December	31
In 1892,	" January	31
"	" February (leap-year)	29
"	" March	31
"	" April	30
"	" May	31
"	from June 1st to June 16th	16
	the total number of days was	283 *Ans.*

N B—In calculating the number of days between two given dates the *first of the days mentioned is not counted*

For instance, from May 5th to May 6th is 1 day,
and from May 23d to May 27th is 4 days.

G (ii) *The "Saturday Review" of October* 12*th*, 1889, *was numbered* 1772 *What was the number on a copy bearing the date December* 31*st*, 1892?

Here we first find the number of *weeks* in the period.

Now, from Oct 12th, 1889, to Oct 12th, 1890, there were 365 days.
" " 1890, " 1891, " 365 "
" " 1891, " 1892 (leap-year), 366 "
and from Oct 12th, 1892, to Dec 31st, 1892,
there were 19 + 30 + 31 = 80 "
Hence, the total length of the period was 1176 "

But 1176 *days* = 168 *weeks*

∴ the required number is 1772 + 168 = 1940 *Ans*

G (iii) *August 8th, 1892, fell on a Monday, what day of the week was April 14th, 1891?*

As $365 \div 7$ gives quotient 52 and rem^r 1, it follows that if any particular date in one year fall on a *Monday* it will fall on a *Tuesday* in the next year, unless that be leap-year, in which case it will fall on a *Wednesday*, and so on

Now, 1892 was leap-year

Hence, counting *back* two days, as Aug 8th, 1892, was a Monday, Aug 8th, 1891, was a *Saturday*

Again, the number of days between April 14th and Aug 8th in 1891, was $16 + 31 + 30 + 31 + 8 = 116$ days,

And $116 \div 7$ gives quotient 16 and rem^r 4

i e, there were 16 complete weeks and 4 days in this period

Hence, counting *back* four days from Saturday, we find that April 14th, 1891. fell on a Tuesday

---

G (iv). *On September 13th, 1891, a man had lived 14000 days; find the date of his birth*

$$\begin{array}{r} 365\,)\,14000\,(\,38 \\ 1095 \\ \hline 3050 \\ 2920 \\ \hline 130 \\ 9 \\ \hline 121 \end{array}$$

Dividing 14000 by 365 we obtain quotient 38 and rem^r 130

Thus the period contains 38 common years + 130 days

But in 38 years there were 9 leap-years, which would account for 9 extra days

Subtracting 9 from 130 we obtain rem^r 121

And $1891 - 38 = 1853$

Hence, the man was born 121 days *before* Sept 13th, 1853

Now, counting back from Sept 13th,

there were in Sept, 1853 (*up to the 13th*),		13	days.
" " Aug,		31	"
" " July,		31	"
" " June,		30	"
	Total	105	"

And $121 - 105 = 16$ days

Thus the man was born 16 days *before* June 1st, 1853

i e., he lived 16 days in May, 1853,

∴ the date of his birth was May 15th, 1853

VII. FACTORS, MULTIPLES, PRIMES.

A **Factor** of a number *divides* that number *exactly* (*i.e* without remainder)

For instance, 7 is a *factor* of 35; 13 is a *factor* of 91; and 2, 3, 8, 12, are all *factors* of 24

A **Multiple** of a number *contains* that number *exactly*.

For instance, 35 is a *multiple* of 7, 91 is a *multiple* of 13; and 24 is a *multiple* of each of the numbers 2, 3, 8, 12

Note—As every number is exactly divisible by itself and by 1, *every* number has, strictly speaking, at least two factors, hence when a number is spoken of, shortly, as having *no* factors, the words "except itself and unity" are implied

Also the words *divisible* and *divisor* are commonly used with the sense "exactly divisible" and "exact divisor"

A **Prime number** (or *prime*) has *no factors*, except itself and unity.

For instance, 5, 13, 29. 53, are *primes*

Hence, the *prime factors* of a number are those *prime* numbers which exactly divide that number

For instance, 5 and 7 are *prime* factors of 35; but 4 and 6, though *factors*, are not prime *factors* of 12.

A number which *has* factors, other than itself and unity, is called *composite*.

A **common factor** of two, or more, numbers *divides each* of them exactly.

For instance, 7 is a *common* factor of 63 and 35, and 4 is a *common* factor of 36, 84, and 100.

A **common multiple** of two, or more, numbers *contains each* of them exactly.

For instance, 48 is a *common* multiple of 6, 8, and 12.

Two numbers are *prime to each other*, when they have *no common* factor, except unity.

For instance, 7 is prime to 11, 8 is prime to 13; 12 is prime to 25.

Hence two *composite* numbers *may* be, but two *prime* numbers *must* be, prime to each other.

A number is called **even** or **odd** according as it is, or is not, exactly divisible by 2.

Thus, 0, 2, 4, 6, &c are *even*, and 1, 3, 5, 7, &c are *odd*.

Tests of Divisibility *

The following tests enable us to discover at once, without the labour of a trial division, whether a given number is, or is not, exactly divisible by any number less than 12 (except 7)

Also, to obtain the remainder (if any) without actually dividing

(I) *A number is divisible by* 2, *if the right-hand digit is* **even**, *and not unless*

Thus, we know at a glance, that 754037298 is, and that 38402283 is *not*, divisible by 2

(II) *A number is divisible by* 5, *if the right-hand digit is either* 0 *or* 5, *and not unless.*

Thus, we know at a glance, that 1456310 and 7846365 *are* divisible by 5, and that 3705254 is *not* divisible by 5

(III) *A number is divisible by* 4, *or by* 25, *if the number formed by the* **two** *last digits is divisible by* 4, *or by* 25, *and not unless* †

Thus, by this test we quickly discover that 387645372 *is* divisible by 4, and that 760889326 is *not* divisible by 4

Also, that 2476375 *is*, and that 3867385 is *not*, divisible by 25

(IV) *A number is divisible by* 3, *or by* 9, *if the* **sum of its digits** *is divisible by* 3, *or by* 9, *and not unless*

For instance, 725634 *is* divisible by 9, for the *sum of its digits* is 27, and 137625 is *not* divisible by 9, but *is* divisible by 3, for the *sum of its digits* is 24.

N B —In practice it is not necessary actually to add *all* the digits of the number together, we may "cast out" the nines one by one as they are obtained, since all we require to know is the remainder, if any

Thus we can see at once that 725634 *is* divisible by 9, since its digits can be arranged in pairs, each of which pairs makes up nine

Also, we can see that 3472856 *is not* divisible by 9 (nor by 3) for, after discarding three pairs each amounting to 9, the digit 8 is left alone

Hence also we know, without actually dividing, that 3472856—9 yields *remainder* 8

Again, if we "cast out" two *threes* from this rem^r^ 8, we also know that 3472856—3 yields rem^r^ 2

---

* For *proofs* of these tests see Appendix

† Similar tests applied to the *three* last figures hold for 8 and 125

The test for 6 is evidently that the number be divisible by both 2 and 3, *i e* that it is *even*, and that the *sum of its digits* is divisible by 3

(V.) *A number is divisible by* 11 *if the difference between the sums of the digits in the units', hundreds', &c places, and those in the alternate tens', thousands', &c. places, is divisible by* 11, *and not unless*

For instance, in the number 745432809

as $9+8+3+5+7=32$, and as $0+2+4+4=10$;
and as $32-10=22$, which is divisible by 11,

$\therefore$ 745432809 *is* divisible by 11

EXAMPLE I —*Resolve* 42840 *into its prime factors*

1st Method

Here we divide in succession by 2 as long as the quotients are *even*, then by 5 because the quotient ends in 5, then by 3 as long as the *sums of the digits* of the quotients are divisible by 3 We then find *by trial* that 7 is a factor and yields quotient 17, a prime number

$$\begin{array}{r|l} 2 & 42840 \\ 2 & 21420 \\ 2 & 10710 \\ 5 & 5355 \\ 3 & 1071 \\ 3 & 357 \\ 7 & 119 \\ & 17 \end{array}$$

$\therefore$ $42840 = \underline{2\times2\times2\times5\times3\times3\times7\times17}$ *Ans*

2nd Method

1st step Cut off the 0 (*i e* divide by 10)
2nd step Divide by 4, since 84 is divisible by 4
3rd step Divide by 9, since the *sum of the digits* of 1071 is divisible by 9
4th step Obtain the factor 7 by trial

$$\begin{array}{r|l} 4 & 4284,0 \\ 9 & 1071 \\ 7 & 119 \\ & 17 \end{array}$$

$\therefore$ $42840 = 10\times4\times9\times7\times17 = 2\times5\times2\times2\times3\times3\times7\times17.$

Or, with the index notation (see p 12), $42840 = \underline{2^3\times3^2\times5\times7\times17}$ *Ans*

EXAMPLE II —*Is* 823 *a prime number?*

We know at once that 823 is not divisible by either 2, 3, 5 or 11.

We now try the other primes, namely, 7, 13, 17, &c, in succession, up to 29, and obtain a remainder in each case

Also, when we divide by 29, we again obtain a remainder And the quotient in this case is 28, a number *less than the trial divisor* 29 We need therefore proceed no further, for if any number greater than 29 were a factor of 823 the quotient would be one of the numbers already tried and discarded.

$$\begin{array}{l} 29)823(28 \\ \quad\;\; 58 \\ \quad\;\; \overline{243} \\ \quad\;\; 232 \\ \quad\;\; \overline{\;\;11} \end{array}$$

$\therefore$ 823 *is* a prime.

VIII. GREATEST COMMON FACTOR.*

The Greatest Common Factor (G C F) of two or more numbers is the *greatest* number which *divides* each of them exactly.

When the prime factors of the numbers are known their G C.F. is evident.

For instance, as $35 = 5 \times 7$,
and $77 = 11 \times 7$,
7 is the *only* common factor of 35 and **77**.
$\therefore$ 7 is their *greatest* common factor

Again, as $36 = 2 \times 2 \times 3 \times 3$,
$40 = 2 \times 2 \times 2 \times 5$,
and $56 = 2 \times 2 \times 2 \times 7$,

and as the number 2 occurs as a common factor twice over, but no other prime factor is common to all three numbers,

$\therefore$ the G C F of 36, 40, and 56 is 2×2, or **4**.

Note—After a little practice the G C F of small numbers such as the above can easily be found *mentally*.

EXAMPLE i—*Write down the G C F. of* 21×11 *and* 35×11.

Ans 7×11, or 77

EXAMPLE ii—*Find, by resolving the numbers to their prime factors, the G C F. of* 126, 210, *and* 231.

Here we observe that there are two, and only two, factors *common to all three* of the given numbers, namely, 3 and 7

$\therefore$ the G C F. is 3×7, or 21.

$$\begin{array}{r|l} 2 & 126 \\ 3 & 63 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|l} 2 & 210 \\ 5 & 105 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|l} 3 & 231 \\ 7 & 77 \\ & 11 \end{array}$$

$3 \times 7 = 21$ *Ans*

The labour of resolving large numbers to their prime factors in order to discover their G C F. is avoided in practice by adopting the following method, or some modification of it

---

* The word *measure* is often used instead of *factor*, and Greatest Common Measure (G C M), or Highest Common Divisor (H C D), instead of Greatest Common Factor (G C F)

EXAMPLE III.—*Find the G C F. of* 1517 *and* 5986.

1st step
Dividing the greater number by the less we obtain the rem^r 1435

2nd step
Dividing the previous divisor 1517 by 1435 we obtain the rem^r 82.

3rd step
Dividing the previous divisor 1435 by 82 we obtain the rem^r 41.

4th step
Dividing the previous divisor 82 by 41, we obtain *no* rem^r. The *last divisor*, 41, is the G C F.

```
1517)5986(3
     4551
     ----
     1435)1517(1
          1435
          ----
            82)1435(17
                82
               ---
               615
               574
               ---
                41)82(2
                   82
                   --
```

Ans. 41

The above method depends upon the following principles:—

(I.) *Every common factor of two numbers is a factor of the* **sum** *of* **any multiples** *of those numbers*

For instance, as 7 is a common factor of 14 and 21,
∴ 7 is a factor of $14 \times 4 + 21 \times 3$, *i e* of 119,
also 7 is a factor of $14 \times 5 + 21 \times 1$, *i e* of 91, and so on

(II.) *Every common factor of two numbers is a factor of the* **difference** *of* **any multiples** *of those numbers.*

For instance, as 7 is a common factor of 14 and 49,
∴ 7 is a factor of $49 \times 3 - 14 \times 5$, *i e* of 77,
also 7 is a factor of $49 \times 1 - 14 \times 2$, *i e* of 21, and so on.

Hence, in Example III., it follows from (II.) that every common factor of 5986 and 1517 is a factor of $5986 \times 1 - 1517 \times 3$, *i.e* of 1435.

And from (I.) that every common factor of 1517 and 1435 is a factor of $1517 \times 3 + 1435 \times 1$, *i e.* of 5986

∴ the G C F of 1517 and 5986 is also the G C F of 1435 and 1517

In other words, the *second* pair (divisor and dividend) have the same G C F. as the *first* pair. Similarly for the *third* pair, &c.

Hence, finally, we conclude that the G C F of 1517 and 5986 is *the same* as that of 41 and 82, which is evidently 41.

In the same way the G C F of *three*, or more, numbers may be obtained by first finding the G C F of two of them, then the G C F. of *the first result* and the *third number*, and so on

EXAMPLE IV —*Find the G C F. of* 3157, 3731, *and* 4715

```
3157)3731(1                          287)4715(16
     3157                                287
      574)3157(5                         1845
          2870                           1722
1st result, 287)574(2                     123)287(2
                574                           246
                                               41)123(3
                                 Ans 41           123
```

Note —If the abridged method of division (see page 15) be employed, Example III would be worked thus —

Mental Work / *Written Work.*

1st step
3 times 7 is 21, 21 from 26, **5**
Carry 2.
3 times 1 is 3, 3 and 2 is 5, 5 from 8, **3**
3 times 5 is 15, 15 from 19, **4**
Carry 1
3 times 1 is 3, 3 and 1 is 4, 4 from 5, **1**.
&c

```
1517)5986(3
     1435)1517(1
            82)1435(17
                615
         Ans 41)82(2
```

The work of finding a G C F may often be much condensed by a judicious use of the principles (I.) and (II) on page 55, combined with a knowledge of the Tests of Divisibility on page 52

Thus (I) Any factors which *evidently* divide *all* the given numbers may be first removed and *reserved* as factors of the G C F

(II) Any factor which evidently divides *one* or more of the given numbers, but does not divide *all* of them, may be *rejected* at any stage of the work, as it cannot be a factor of the G C F

(III) As the remainder resulting from any of the subtractions contains the G C F, our object is to make this remainder as *small* as we can. Hence it is often convenient to *reverse* the order in such subtractions, when by so doing the difference which results is smaller than would be obtained by following the ordinary process of division

The examples which follow illustrate such modifications of the process *

---

* It is best for young beginners to confine their attention to the methods of Examples I II. III and IV

EXAMPLE V.—*Find the G.C.F. of* 9063 *and* 14787.

Here we observe that 9 is a factor of both the numbers (see page 52). We therefore first divide by 9 and reserve 9 as a factor of the G.C.F.

We next operate upon 1007 and 1643.

1st step.

Double 1007 and subtract (reverse way) 1643 from it, thus obtaining a difference, 371, less than what would have been obtained by taking 1007 from 1643.

2nd step.

Treble 371 and subtract (reverse way) 1007 from it, obtaining the difference 106.

3rd step.

Treble 106 and subtract from 371, obtaining the difference 53.

4th step.

Double 53 and subtract from 106, when no remainder occurs.

9	9063	14787
	1007	1643
		2014
	1113	371
	106	318
	106	53

$9 \times 53 = 477$ *Ans.*

Hence the G.C.F. of 1007 and 1643 is 53,

∴ the G.C.F. of 9063 and 14787 is 9×53, *i.e.* 477.

EXAMPLE VI.—*Find the G.C.F. of* 40457 *and* 420325.

Here we first reject 25 from 420325, for we see that 5 cannot be a factor of the G.C.F.

Dividing 420325 by 25, *i.e.* multiplying 420325 by 4 and cutting off the last two figures (see p. 19), we obtain quotient 16813.

We next operate upon 16813 and 40457.

1st step.

Double 16813, and subtract from 40457, obtaining rem[r] 6831.

2nd step.

Treble 6831 and subtract (reverse way) 16813 from the result, obtaining rem[r] 3680.

3rd step.

Reject in succession the factors 10, 4, 4, none of which contain a factor of the G.C.F. (for the two given numbers are *odd*, and do not both end in 5), we now know that the req[d] G.C.F. is either 23 or 1, for 23 is a *prime*.

4th step.

Divide 6831 by 23, when no remainder occurs. ∴ 23 is the req[d] G.C.F.

	420325	
	4	
	16813,00	40457
	20493	33626
4	368,0	6831(297
4	92	46
	23	223
		207
		161
		161

Ans. 23.

Note.—For the G.C.F. of compound quantities see Chapter X.

IX. LEAST COMMON MULTIPLE.

The Least Common Multiple (L C M) of two or more numbers is the *least* number which *contains* each of them exactly

When the prime factors of the numbers are known those of their L C M can be written down at once

For instance, as $35 = 5 \times 7$, and $77 = 7 \times 11$, it is evident that *every* common multiple of 35 and 77 must contain each of the primes 5, 7, and 11

$\therefore$ the *least* common multiple of 35 and 77 is $5 \times 7 \times 11$, or 385.

$$\begin{aligned} \text{Again, as } 36 &= 2 \times 2 \times 3 \times 3, \\ 40 &= 2 \times 2 \times 2 \times 5, \\ \text{and } 56 &= 2 \times 2 \times 2 \times 7, \end{aligned}$$

it is evident that in *every* common multiple of 36, 40, and 56
the prime 2 must occur as a factor *not less than* **three times** over,
,, 3 ,, ,, ,, **twice** over,
and 5 and 7 ,, ,, ,, **once**.

$\therefore$ the *least* common multiple is $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$, or 2520.

Hence all the different prime factors to be found in the numbers must appear in their L C M

Also, if any particular prime factor occurs *twice over* in one or more of the numbers but does not occur *more than twice* over in any one of them, it *must occur twice*, but *must* **not** *occur more than twice* over in their L C M And so on.

EXAMPLE 1 —*Find, by resolving each of the numbers to its prime factors, the L C M of* 126, 210, 231

Here we observe that 3 is the only factor which occurs twice over in any one of the given numbers, and that no prime occurs more than twice over in any of the numbers

$$\begin{array}{r|l} 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 210 \\ \hline 5 & 105 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 231 \\ \hline 7 & 77 \\ \hline & 11 \end{array}$$

Hence the L.C M. is $2 \times 3 \times 3 \times 7 \times 5 \times 11 = 6930$ *Ans*

It is not always necessary to resolve each of the numbers *separately* into its prime factors, as it often happens that one or more of the numbers whose L C.M is required is contained in another of them

For instance, in order to find the L C M. of 21, 35, 63, and 105, it is not necessary to consider 21 and 35, for these are both contained in 105, and,

consequently, *any number which contains* 105 *must also contain* 21 and 35. Here then we need only consider 63 and 105

Now $63 = 7 \times 3 \times 3$, and $105 = 5 \times 3 \times 7$.

Hence the L C M of 21, 35, 63, and 105 is $7 \times 3 \times 3 \times 5$, or 315

In the case of numbers which can easily be resolved into their prime factors, the process is commonly arranged as follows —

EXAMPLE ii.—*Find the L.C.M. of* 7, 8, 12, 14, 16, 24, 30, *and* 35.

$$\begin{array}{r|l}
2 & \cancel{7},\ \cancel{8},\ \cancel{12},\ 14,\ 16,\ 24,\ 30,\ 35 \\ \hline
2 & \cancel{7},\ 8,\ 12,\ 15,\ 35 \\ \hline
2 & 4,\ 6,\ 15,\ 35 \\ \hline
5 & 2,\ \cancel{3},\ 15,\ 35 \\ \hline
 & 2,\ 3,\ 7
\end{array}$$

L.C M. is $2 \times 2 \times 2 \times 5 \times 2 \times 3 \times 7$
$= 80 \times 21 = $ 1680 *Ans*

1st line

Cut out 7, 8, 12 as they are contained in 14, 16, 24, respectively.

Divide by 2, *i e* by any *prime* which is contained in *two at least* of the remaining numbers

Bring down 35, which is not exactly divisible by 2, to the next line

2nd line.

Cut out 7, as it is contained in 35.

Divide again by 2

Bring down 15 and 35 to the next line.

Continue this process until a line is reached in which *no two* numbers have *any common factor.*

Finally, multiply together all the divisors and the numbers in this last line, and the result is the L C M. required

---

In many cases the above process may be dispensed with, for the factors of the L C M can easily be found *by inspection,* and be written down one by one as follows.—

EXAMPLE iii.—*Find the L C M of* 15, 21, 27, 30, 35, *and* 42.

Written Work

$3 \times 5 \times 7 \times 3 \times 3 \times 2 = 21 \times 90 =$ 1890 *Ans*

Mental Work.

1st step. As $15 = 3 \times 5$, we write down **3×5**, as two factors of the L.C M.

2nd step As $21 = 3 \times 7$, and we already have a 3 set down, we append 7 only

3rd step As $27 = 3 \times 3 \times 3$, and we already have one 3 set down, we now need but *two* more, and so append **3×3**

4th step As $35 = 5 \times 7$, and as we already have both these factors written down, we append no fresh factor at this stage

5th step As $42 = 7 \times 3 \times 2$, we need only to append the fresh factor **2.**

Finally, $3 \times 5 \times 7 \times 3 \times 3 \times 2 = 21 \times 9 \times 10 = 1890$.

When the numbers whose L C M is required are not easily resolved into factors *at sight*, we first find their G C.F, by help of which the factors of their L C M. are found

For instance, in order to find the L C M of 391 and 437,
we first find their G.C F, namely, 23,
we then obtain by division, $391 = 23 \times 17$
and $437 = 23 \times 19$
Thus the required L C M is $23 \times 19 \times 17 = 437 \times 17 = 7429$.

It follows from the above that *the L C M of* **two** *numbers is equal to their product divided by their G C F.*

For the L C.M of 391 and 437 is 437×17
$= 437 \times 391 - 23$
$=$ product $-$ G C F

Hence, to find the L C.M of **two** large numbers we first find their G C F, then *divide one of the given numbers by their G.C F., and multiply the quotient obtained by the other given number*

EXAMPLE IV —*Find the L C M of* 1333 *and* 1767.

```
1333)1767(1
     1333
     ----
      434)1333(3
          1302
          ----
      G C F 31)434(14
                31
                ---
                124
                124
                ---
```

```
31)1333(43
   124
   ---
    93
    93
    --

   1767
     43
   ----
   5301
  7068
  -----
L C M = 75981 Ans
```

Here we first find the G C F, namely, 31, we then divide 1333 by 31, obtaining quotient 43, finally, we multiply 1767 by 43, obtaining the L.C M 75981

EXAMPLE V —*Find the L C.M of* 1353, 1517, *and* 2829.

Here the G C F is found to be 41
Dividing each of the given numbers by 41, we obtain

$$1353 = 41 \times 33 = 41 \times 11 \times 3$$
$$1517 = 41 \times 37$$
$$2829 = 41 \times 69 = 41 \times 23 \times 3$$

Hence the L.C M is $41 \times 37 \times 23 \times 11 \times 3$
$= 1517 \times 23 \times 33 = \underline{1151403}$ *Ans.*

Note —In this example, in order to save space, the work done in finding the G C F, and in the succeeding divisions and multiplications, is not given. The student, however, should *in all cases show all the written work*

X. MISCELLANEOUS EXAMPLES.

In order to find the G C F., or L C M., of *compound quantities*, we must express them all in *one single denomination*, and then find the G C F, or L C M., in that denomination

A (i) *Find the largest sum of money which is exactly contained in both £23, 3s 6d and £14, 3s 3d.*

In other words, find the G C F of £23, 3s 6d and £14, 3s 3d

Here we choose the common denomination *threepences* in which to express both quantities.

We find that £23, 3s 6d = 1854 *threepences*,
and £14, 3s 3d = 1133 *threepences*

We next find the G C F of 1854 and 1133, namely, 103

Hence the G C F of 1854 *threepences* and 1133 *threepences* is 103 *threepences* = £1, 5s. 9d *Ans.*

A (ii). *Find the smallest debt which could be exactly discharged either in dollars (4s 2d), half-crowns, florins, shillings, francs (10d.), or threepences.*

The req^d^ sum of money is the L C M of a dollar, a half-crown, a florin, a shilling, a franc, and a threepence;

i e the L C M of 50d, 30d, 24d, 12d, 10d, and 3d
$= 5 \times 5 \times 2 \times 3 \times 2 \times 2$ *pence* = 50s = £2, 10s *Ans*

B (i) *Find the greatest number which will divide both 4659 and 1068 and leave the remainder 4 in each case*

We first subtract 4 from each number, obtaining 4655 and 1064

We then find the G C F of 4655 and 1064, namely, 133

i e 133 is the greatest number which divides 4655 and 1064 *without rem^r^.*

∴ 133 is the greatest number which divides both 4659 and 1068, *with rem^r^* 4

B (ii). *Find the least number which, when divided by 35 yields remainder 34, when divided by 42 yields remainder 41, and when divided by 56 yields remainder 55.*

The L C M of 35, 42, and 56 is $5 \times 7 \times 3 \times 2 \times 4 = 70 \times 12 = 840$;

i e 840 is *exactly* divisible by each of the numbers 35, 42, 56.

Hence the req^d^ number is $840 - 1 = 839$ *Ans*

C. *Prove that 421 and 536 are prime to each other.*

In other words, show that their G C F is *unity*.

D. *Find* (i) *the multiple of* 67 *which is nearest to* 10000. *Also* (ii) *the multiple of* 67 *which is nearest to* 1000

(i) Dividing 10000 by 67 we obtain quotient 149 and rem^{r} 17
Now 17 is *less than half* of 67
Hence, the req^{d} multiple
is $10000 - 17$, or 9983 *Ans* (i)

```
67)10000(149
   67
   330
   268
    620
    603
     17
```

(ii) Again, from the above division, we see that $1000 \div 67$ gives quotient 14, and a rem^{r} 62, which is *more than half* of 67 (*i e* 1000 falls short of containing 67, just fifteen times by 5)

Hence, in this case, the req^{d} multiple is $1000 + 5 =$ 1005 *Ans.* (ii)

When the sum, or difference, of products or quotients is required, we may often save labour if we detect the presence of a *common factor*. For instance —

E (i) *Find the sum of* 87×23 *and* 29×37

Here, instead of multiplying 87 by 23 and 29 by 37, and then adding the results, we observe that 29 is a common factor of 87 and 29 We, therefore, *first add* 3×23 to 37, and *afterwards multiply* the result by 29 (See Theorem II on p 10)

Thus, $87 \times 23 + 29 \times 37 = 29 \times (69 + 37) = 29 \times 106 =$ 3074 *Ans.*

E (ii) *Find the difference between* $24552 \div 62$ *and* $11563 \div 31$.

Here, as 31 is a common factor of both divisors, we first find the difference between $24552 \div 2$ and 11563, and *afterwards divide* the result by 31

Thus, $24552 \div 62 - 11563 \div 31 = (12276 - 11563) \div 31$
$= 713 \div 31 =$ 23 *Ans*

F (i). *Find all the divisors of* 1155.

Resolve 1155 into prime factors then, if we include 1 and 1155, we see that there are, in all, *sixteen* divisors, namely:—

```
5|1155
 3|231
  7|77
    11
```

1, 5, 3, 7, 11,
5×3, 5×7, 5×11, 3×7, 3×11, 7×11,
$5 \times 3 \times 7$, $5 \times 3 \times 11$, $5 \times 7 \times 11$, $3 \times 7 \times 11$,
and $5 \times 3 \times 7 \times 11$

i e 1, 3, 5, 7, 11, 15, 35, 55, 21, 33, 77, 105, 165, 231, 385, 1155

F (ii). *Find all the common factors of* 3003 *and* 1287.

Resolve into prime factors

$$\begin{array}{r|l} 3 & \underline{3003} \\ 7 & \underline{1001} \\ 11 & \underline{143} \\ & 13 \end{array} \qquad \begin{array}{r|l} 3 & \underline{1287} \\ 3 & \underline{429} \\ 11 & \underline{143} \\ & 13 \end{array}$$

Hence *all* the factors common to both numbers are

3, 11, 13,

3×11, 3×13, 11×13;

and $3 \times 11 \times 13$

i.e 3, 11, 13, 33, 39, 143, and 429.

G. *The G.C.F of two numbers is* 440; *their L.C.M. is* 19360, *one of the numbers is* 1760, *find the other.*

The L.C.M of two numbers = their product — their G.C.F (See p. 60.)

$\therefore$ the product of the numbers = their L.C.M × their G.C.F

Hence the product of the req^d numbers $= 19360 \times 440$.

But one of the numbers is 1760;

$$\begin{aligned} \therefore \text{ the other is } & (19360 \times 440) \div 1760 \\ &= (19360 \times 440) \div (4 \times 440) \\ &= 19360 \div 4^* \\ &= \underline{4840 \textit{ Ans}} \end{aligned}$$

H. *The continued product of three consecutive numbers is* 35904; *find them.*

Resolving into factors we find that

$$\begin{aligned} 35904 &= 4 \times 4 \times 4 \times 3 \times 11 \times 17 \\ &= 4 \times 4 \times 2 \times 3 \times 11 \times 17 \times 2 \\ &= (4 \times 4 \times 2) \times (3 \times 11) \times (2 \times 17) \\ &= \underline{32 \times 33 \times 34 \textit{ Ans}} \end{aligned}$$

$$\begin{array}{r|l} 4 & \underline{35904} \\ 4 & \underline{8976} \\ 4 & \underline{2244} \\ 3 & \underline{561} \\ 11 & \underline{187} \\ & 17 \end{array}$$

K. *By what number must* 23958 *be multiplied that the product may be a perfect cube?*

[A *perfect* cube is a number which can be expressed as the continued product of *three equal* factors See page 12]

Resolving into prime factors we find that

$$\begin{aligned} 23958 &= 2 \times 3^2 \times 11^3. \\ \text{Hence, } 23958 \times 2^2 \times 3 &= 2^3 \times 3^3 \times 11^3, \\ &= (66)^3. \end{aligned}$$

$\therefore$ the req^d multiplier is $2^2 \times 3$, or $\underline{12 \textit{ Ans}}$

$$\begin{array}{r|l} 2 & \underline{23958} \\ 9 & \underline{11979} \\ 11 & \underline{1331} \\ & 121 \end{array}$$

---

* It would be a waste of labour to multiply 19360 by 440 and then divide the result by 1760, for 1760 is 440×4, and to multiply by 440 and afterwards to divide by 440×4 is equivalent to simply dividing by 4

XI. FRACTIONS.

Notation

If any *unit* (or *quantity*) be divided into a number of *equal parts*, one or more of those parts is called a **Fraction** of that unit (or quantity)

If the whole be divided into

two	equal parts,	the parts are	called	*halves,*
three	,,	,,	,,	*thirds;*
four	,,	,,	,,	*fourths*, or *quarters;*
five	,,	,,	,,	*fifths,*
six	,,	,,	,,	*sixths,*

and so on

For instance, the abstract fraction **Three-eighths** indicates that, *unity* being divided into *eight* equal parts, *three* of those parts are taken, and **Eleven-sixteenths** indicates that, *unity* being divided into *sixteen* equal parts, *eleven* of those parts are taken

Again, **Seven-twentieths** of £1 indicates that, the concrete unit £1 being divided into *twenty* equal parts, *seven* of those parts are taken,

hence, as £1 = 20 shillings,
one-twentieth of £1 is 1 shilling,
and *seven*-twentieths of £1 is 7 shillings

Also, **Nine-tenths** of 4 tons indicates that, the quantity 4 tons being divided into *ten* equal parts, *nine* of those parts are taken,

hence, as 4 tons = 80 cwts,
. *one*-tenth of 4 tons is 8 cwts,
and *nine*-tenths of 4 tons is 72 cwts, or 3 tons 12 cwts.

Any fraction may be represented in figures by two numbers, one of which is written above the other, with a line between them

The *lower* number is called the **Denominator** (*i e* the *Namer*); for it shows into *how many equal parts the unit is divided, i.e.* it *names* the kind of part, whether *fifths, twelfths,* &c.

The *upper* number is called the **Numerator** (*i e* the *Counter*), for it shows how many of these parts there are in the fraction, *i e.* it *counts* the number of *fifths*, *twelfths*, &c , as the case may be, which form the fraction.

For instance, *Three-eighths* is written thus $\frac{3}{8}$, *Eleven-sixteenths*, thus $\frac{11}{16}$.

Here 8 and 16 are *denominators*, showing that unity is divided into *eight* and *sixteen* equal parts respectively,

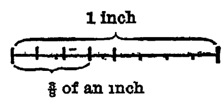
and 3 and 11 are *numerators*, showing that the fractions consist of *three* and of *eleven* parts of such size as their respective denominators indicate

Fractions represented in the manner described above are called **Vulgar** (Latin *vulgus*, common) Fractions *

A number regarded as made up of units *each of which units is unbroken* and entire is called a *whole* number, or **Integer**; (Latin *integer*, whole), as distinguished from a Fraction (Latin *fractus*, broken)

Note —A convenient way of illustrating the definition of a fraction is to take a unit of *length*.

For instance, if *one inch* be regarded as the unit, then $\frac{3}{8}$ of an inch indicates the length shown in the accompanying diagram



Another practical illustration, which has also the advantage of allowing each pupil to experiment for himself, is to provide a number of sheets of paper of the same size and shape These are the units, and each pupil may easily, by folding, divide a sheet into *fourths*, and be then called upon to exhibit, first one fourth, then three fourths Then from another set of sheets *eighths*, or *sixteenths*, may be obtained

The way in which the sheets are divided may also be varied; on one occasion each sheet being divided into the same number of equal squares, on another occasion, into the same number of equal strips Also the sheets need not be actually separated into parts, the division can be indicated by the creases made in folding them, so that *five-eighths* (for instance) may be exhibited as a single piece of paper

In the same way, by dividing several sheets each into the same number of equal parts, simple experimental proofs of the processes of Chapter XII may easily be devised

* In contradistinction to *Decimal* fractions, which, as we shall see later, are represented in another manner

XII. FRACTIONS.

Proper and Improper Fractions, Mixed Numbers.

It is evident that, if we divide a unit into any number of equal parts, all these parts together make up the complete unit

For instance, Seven *sevenths* is unity, *i e* $\frac{7}{7} = 1$

Similarly, $\frac{10}{10} = 1$, $\frac{19}{19} = 1$, $\frac{31}{31} = 1$, and so on

Also, if we divide *two*, *three*, &c , units each into the same number of equal parts, all these parts together make up *two*, *three*, &c. complete units respectively

For instance, $\frac{14}{7} = 2$, $\frac{20}{10} = 2$, $\frac{38}{19} = 2$, and so on

Also, $\frac{21}{7} = 3$; $\frac{30}{10} = 3$, $\frac{57}{19} = 3$, and so on

Hence, a fraction whose numerator exactly contains its denominator represents an *integer*, and this integer is the quotient obtained by dividing the numerator by the denominator

Again, if the *numerator* of a fraction be *less* than the *denominator*, the fraction must be *less than unity*

For instance, just as 5 *shillings* is less than 7 *shillings*, so 5 *sevenths* is less than 7 *sevenths*, or unity,

i e $\frac{5}{7}$ is less than $\frac{7}{7}$, or 1

But if the *numerator* be *greater* than the *denominator*, the fraction must be *greater than unity*

For instance, 9 *sevenths* is greater than 7 *sevenths*, or unity,

i e $\frac{9}{7}$ is greater than $\frac{7}{7}$, or 1

A fraction whose numerator is *less* than its denominator is called a **proper fraction.**

For instance, $\frac{1}{5}$, $\frac{2}{3}$, $\frac{7}{11}$ and $\frac{19}{20}$ are all *proper* fractions

A fraction whose numerator is *greater* than its denominator is called an **improper fraction.**

For instance, $\frac{7}{5}$, $\frac{4}{3}$, $\frac{16}{11}$ and $\frac{21}{20}$ are all *improper* fractions

Hence a *proper* fraction is *less*, and an *improper* fraction is *greater*, then unity

A number which is partly integral and partly fractional is called a mixed number

For instance, $2\frac{4}{7}$ (read "Two and four-sevenths") is a *mixed* number.

A mixed number can always be expressed as an improper fraction.

For instance, $2\frac{4}{7} = 2$ units $+ 4$ sevenths of a unit,
and as each unit contains 7 *sevenths*,
$\therefore$ 2 *units* + 4 *sevenths* = 14 *sevenths* + 4 *sevenths* = 18 *sevenths*,

$$\text{i e } 2\tfrac{4}{7} = \tfrac{18}{7}.$$

Hence, to express a mixed number as an improper fraction, *we multiply the integer by the denominator, and add the product to the numerator, of the fractional part of the mixed number,* the result is the numerator of the improper fraction, and its denominator is the same as that of the fractional part of the mixed number.

Conversely, an improper fraction can always be expressed as a mixed number

For instance, $\frac{11}{5}$ is 11 *fifths* = 10 *fifths* + 1 *fifth*,
and as 10 *fifths* is 2 complete units,
$\therefore$ 10 *fifths* + 1 *fifth* = 2 *units* + 1 *fifth*, i e. $\frac{11}{5} = 2\frac{1}{5}$.

Hence, to express an improper fraction as a mixed number, *we divide the numerator by the denominator,* the *quotient* obtained is the *integer*, the *remainder* is the *numerator* of the *fractional* part of the mixed number, and its denominator is the same as that of the improper fraction

---

EXAMPLE i.—*Express* $17\frac{5}{23}$ *as an improper fraction.*

The product of 17 and 23 is 391.

$$\text{Hence } 17\tfrac{5}{23} = \frac{391+5}{23} = \frac{396}{23} \quad \underline{Ans}$$

$$\begin{array}{r} 23 \\ \underline{17} \\ 161 \\ \underline{23} \\ \underline{391} \end{array}$$

EXAMPLE ii —*Express* $\frac{684}{53}$ *as a mixed number.*

Dividing 684 by 53 we obtain quotient 12 and rem^r^ 48

$$\text{Hence } \frac{684}{53} = 12\tfrac{48}{53} \quad \underline{Ans}$$

$$\begin{array}{r} 53)684(\underline{12} \\ \underline{53} \\ 154 \\ \underline{106} \\ \underline{48} \end{array}$$

XIII. FRACTIONS.

Reduction.

Whatever be the number of equal parts into which we divide the unit, *each part must be twice as large* as it would have been if the unit had been divided into *twice that number of equal parts*

For instance, 4 *fifths* = twice 4 *tenths* = 8 *tenths*,

$$\text{i e } \frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}.$$

Also, *each part must be three times as large* as it would have been if the unit had been divided into *three times that number of parts*

For instance, 4 *fifths* = three times 4 *fifteenths* = 12 *fifteenths*

$$\text{i. e. } \frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}.$$

And so on.

Hence the following important principles.—

(I.) *The value of a fraction is not altered by* multiplying *both its numerator and denominator by the same number.*

$$\text{For instance, } \frac{3}{7} = \frac{3 \times 5}{7 \times 5} = \frac{15}{35},$$

$$\text{Similarly, } \frac{3}{7} = \frac{3 \times 11}{7 \times 11} = \frac{33}{77},$$

$$\text{also } \frac{3}{7} = \frac{3 \times 29}{7 \times 29} = \frac{87}{203}, \text{ and so on.}$$

i e the fractions $\frac{3}{7}$, $\frac{15}{35}$, $\frac{33}{77}$, $\frac{87}{203}$ are all equal to one another

And conversely,

(II) *The value of a fraction is not altered by* dividing *both its numerator and denominator by the same number*

$$\text{For instance, since } \frac{3}{7} = \frac{3 \times 5}{7 \times 5} = \frac{15}{35},$$

$$\therefore \text{, conversely, } \frac{15}{35} = \frac{15 - 5}{35 - 5} = \frac{3}{7},$$

$$\text{Similarly, } \frac{33}{77} = \frac{33 - 11}{77 - 11} = \frac{3}{7},$$

$$\text{also } \frac{87}{203} = \frac{87 - 29}{203 - 29} = \frac{3}{7}, \text{ and so on}$$

Thus we see that the *same portion* of a unit may be represented in innumerable ways

The process of changing the denominator is called *Reduction* of Fractions.

EXAMPLE i—*Express 7 with denominator 4.*

As $1 = 4$ *fourths*
$\therefore 7 = 7$ times 4 *fourths* $= 28$ *fourths.*

i e $7 = \frac{28}{4}$ *Ans.*

EXAMPLE ii.—*Express* $\frac{7}{13}$ *with the denominator* 481.

$$\begin{array}{l} 13)481(37 \\ 39 \\ \overline{91} \\ 91 \end{array}$$

Dividing 481 by 13 we find that 13 is contained exactly 37 times in 481.

Hence $\frac{7}{13} = \frac{7 \times 37}{13 \times 37} = \frac{259}{481}$ *Ans*

EXAMPLE iii.—*Express* $\frac{7}{9}$ *with numerator* 602

Dividing 602 by 7 we find that 7 is contained exactly 86 times in 602.

Hence $\frac{7}{9} = \frac{7 \times 86}{9 \times 86} = \frac{602}{774}$ *Ans*

EXAMPLE iv.—*Find the fraction with denominator 7 which is equal to* $\frac{129}{301}$.

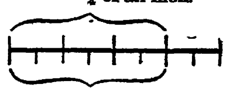
Dividing 301 by 7 we find that 7 is contained exactly 43 times in 301.

Hence $\frac{129}{301} = \frac{129 - 43}{301 - 43} = \frac{3}{7}$ *Ans*

Note.—The equality of two fractions may be illustrated *visibly* in various ways

For instance, if we take an *inch* as unit, it is evident from the accompanying diagram that $\frac{3}{4}$ of an inch, and $\frac{6}{8}$ of an inch, both represent the same length

$\frac{3}{4}$ of an inch.



$\frac{6}{8}$ of an inch

Or, if we take several sheets of paper of the same size and shape and regard these as units, they may easily, by folding, be divided into *fourths*, *eighths*, *sixteenths*, &c respectively, and the equality of two such fractions as $\frac{3}{4}$ and $\frac{12}{16}$, or of $\frac{5}{8}$ and $\frac{10}{16}$, &c, may then be visibly demonstrated by superposition.

XIV. FRACTIONS.

Reduction to Lowest Terms.

The numerator and denominator of a fraction are called its *Terms*

To reduce a fraction to its *lowest* terms is to represent the fraction by means of as small a numerator and denominator as possible

Now we know, from Chapter XIII, that if we divide both numerator and denominator of a fraction by the same number, the value of the fraction remains unaltered

Hence, so long as the numerator and denominator of the fraction have a *common factor* we can, by dividing both of them by that factor, reduce the fraction to *lower* terms, and the fraction will be in its *lowest* terms when *all* such common factors have been removed

Thus a fraction is in its **lowest terms** when its numerator and denominator have **no common factor.**

Hence, to reduce a fraction to its lowest terms, *we divide both its numerator and denominator*

either (i) *by their G C F*,
or (ii) *by the factors of their G C F in succession.*

We adopt the latter method when common factors of the numerator and denominator can be easily found by inspection

For instance, the fraction $\frac{35}{80}$ is evidently not in its lowest terms, for 35 and 80 are both divisible by 5

$$\text{Hence } \frac{35}{80} = \frac{35 \div 5}{80 \div 5} = \frac{7}{16}$$

And as 7 and 16 have no common factor, the fraction is now in its lowest terms.

Example i—*Reduce $\frac{72}{1368}$ to its lowest terms*

Mental Work

1st step

As 9 is evidently a common factor of 72 and 1368 (see p 52), we divide both by 9, obtaining quotients 8 and 152 respectively

2nd step

As 8 is evidently a common factor of 8 and 152, we divide both by 8, obtaining quotients 1 and 19 respectively.

Written Work

$$\frac{72}{1368} = \frac{8}{152}$$

$$= \frac{1}{19} \ \underline{\textit{Ans.}}$$

EXAMPLE ii.—*Express the fraction* $\frac{7254}{9126}$ *in its lowest terms.*

Written Work

$$\frac{7254}{9126} = \frac{3627}{4563} = \frac{403}{507} = \frac{31}{39} \ \underline{\textit{Ans}}$$

Mental Work.

1st step

As 7254 and 9126 are even, we divide both by 2, obtaining quotients 3627 and 4563

2nd step

As 3627 and 4563 are evidently both divisible by 9, we divide both by 9, obtaining quotients 403 and 507

3rd step

We know [by (ii), page 55] that if 403 and 507 have a common factor that common factor divides their difference 104 Now $104 = 8 \times 13$ Hence as 403 and 507 are both *odd*, we know that if they have any common factor it must be 13.

We find, on trial, that 13 *is* a factor of both numbers, and on dividing 403 and 507 by 13 we obtain quotients 31 and 39.

Also, as 31 is a prime, the fraction must now be in its lowest terms

Note—In such cases as this the beginner must be careful not to fall into the common error of *assuming that a pair of odd numbers*, such as 403 and 507, *are necessarily prime to each other*, or, though he may succeed in reducing the given fraction to *lower* terms, he will fail to reduce it to its *lowest* terms

---

EXAMPLE iii—*Reduce* $\frac{6157}{10349}$ *to its lowest terms*

Here no small common factors of numerator and denominator are apparent

We therefore perform the operation of finding the G C F of 6157 and 10349, namely, 131.

$$\text{Hence } \frac{6157}{10349} = \frac{6157 - 131}{10349 - 131} = \frac{47}{79} \ \underline{\textit{Ans}}$$

Note—In order to save space, the work done in finding the G C F. has been omitted here The student, however, is reminded that, in an examination, no part of the written work should be omitted

---

EXAMPLE iv—*Simplify* $\frac{7007}{3003}$

Here it is evident, by inspection, that 1001 is the G C F of 7007 and 3003

$$\text{Hence } \frac{7007}{3003} = \frac{7}{3} = 2\tfrac{1}{3} \ \underline{\textit{Ans.}}$$

XV. FRACTIONS.

Reduction to the Least Common Denominator.

We know that if we multiply both numerator and denominator of a fraction by any number the value of the fraction is not altered Hence, by choosing suitable multipliers, it is possible to reduce several fractions having different denominators to equivalent fractions having the same denominator

For instance, as $\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$, and $\frac{4}{7} = \frac{4 \times 5}{7 \times 5} = \frac{20}{35}$
$\therefore \frac{3}{5}$ and $\frac{4}{7}$, when reduced to the *common* denominator 35, become respectively $\frac{21}{35}$ and $\frac{20}{35}$.

It is evident that the *common denominator* must be a *multiple* of each of the denominators of the given fractions, also that if the common denominator is as small as possible it must be the **least** *common multiple* of the denominators of the given fractions.

For instance, the least common denominator for the fractions $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ is the L C M of 3, 4 and 6, *i e* 12

Now, as we have seen in Chapter XIII, to discover the number by which we must multiply both numerator and denominator in order to reduce the fraction $\frac{2}{3}$ to *twelfths*, we divide the new denominator 12 by the denominator of the given fraction, 3, and the quotient, 4, is the required multiplier

Thus, $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ Similarly, $\frac{3}{4} = \frac{9}{12}$, and $\frac{5}{6} = \frac{10}{12}$

The letters L C D are sometimes used for the words Least Common Denominator

---

Example 1 —*Reduce the fractions* $\frac{3}{8}$, $\frac{7}{16}$ *and* $\frac{11}{24}$ *to their least common denominator*

The L C M of 8, 16 and 24 is **48.**

Now $48 \div 8 = 6$, hence $\frac{3}{8} = \frac{3 \times 6}{8 \times 6} = \frac{18}{48}$;

Again, $48 \div 16 = 3$; hence $\frac{7}{16} = \frac{7 \times 3}{16 \times 3} = \frac{21}{48}$,

Also $48 \div 24 = 2$; hence $\frac{11}{24} = \frac{11 \times 2}{24 \times 2} = \frac{22}{48}$.

<u>*Ans*</u> $\frac{18}{48}$, $\frac{21}{48}$, $\frac{22}{48}$

EXAMPLE ii.—*Reduce* $\frac{9}{20}$, $\frac{13}{24}$ *and* $\frac{23}{56}$ *to their L C D.*

The L.C M of 20, 24 and 56 is found to be 840.

$$\left.\begin{array}{l} \text{Now } 840 \div 20 = 42;\ \text{hence } \frac{9}{20} = \frac{9 \times 42}{20 \times 42} = \frac{378}{840} \\ \text{Again, } 840 \div 24 = 35,\ \text{hence } \frac{13}{24} = \frac{13 \times 35}{24 \times 35} = \frac{455}{840} \\ \text{Also } 840 \div 56 = 15;\ \text{hence } \frac{23}{56} = \frac{23 \times 15}{56 \times 15} = \frac{345}{840} \end{array}\right\} \underline{\textit{Ans.}}$$

Note—Labour is saved by writing the common denominator in factors as in the following Example, for by so doing the quotients resulting from the division of the common denominator by the denominators of the fractions are obtained at *sight*

EXAMPLE iii—*Reduce* $\frac{11}{14}$, $\frac{13}{18}$ *and* $\frac{16}{21}$ *to their least common denominator.*

Mental Work.	*Written Work*
1st step By inspection, the L.C.M of 14, 18 and 21 is **7×2×3×3.**	$\frac{11}{14}, \frac{13}{18}, \frac{16}{21}$
2nd step $7 \times 2 \times 3 \times 3 \div 14 = 9$ (for $7 \times 2 \times 3 \times 3$ is evidently 14×9, and $9 \times 14 \div 14$ is 9), $11 \times 9 =$ **99.**	$\frac{99,\ 91,\ 96}{7 \times 2 \times 3 \times 3}$.
3rd step $7 \times 2 \times 3 \times 3 \div 18 = 7$, $13 \times 7 =$ **91**	$\frac{99,\ 91,\ 96}{126}$ $\underline{\textit{Ans.}}$
4th step $7 \times 2 \times 3 \times 3 \div 21 = 6$, $16 \times 6 =$ **96.**	

We can *compare the size* of fractions if we reduce them to a common denominator.

For instance, $\frac{3}{5} = \frac{21}{35}$, and $\frac{4}{7} = \frac{20}{35}$, $\therefore \frac{3}{5}$ is greater than $\frac{4}{7}$; for 21 *thirty-fifths* is evidently greater than 20 *thirty-fifths.*

EXAMPLE iv.—*Compare* $\frac{5}{7}$, $\frac{16}{21}$ *and* $\frac{23}{30}$.

Mental Work	*Written Work.*
The L C M of 7, 21, 30 is **7×3×10.**	$\frac{5}{7}, \frac{16}{21}, \frac{23}{30}$
$7 \times 3 \times 10 \div 7 = 30$, $5 \times 30 =$ **150**	$\frac{150,\ 160,\ 161}{7 \times 3 \times 10}$
$7 \times 3 \times 10 \div 21 = 10$, $16 \times 10 =$ **160**	
$7 \times 3 \times 10 \div 30 = 7$, $23 \times 7 =$ **161**	$\underline{\textit{Ans}}$ $\frac{5}{7}$ is least, $\frac{23}{30}$ is greatest

Note—Thus the fractions, as they stand in the question, are arranged in *ascending order* of magnitude, *i e.* the least first, and so on.

XVI. ADDITION OF FRACTIONS.

The principle involved in Addition of Fractions is the same as in all other additions, namely, the numbers added must all bear the same *name*.

For instance, just as, in Simple Addition,
3 *tens* + 4 *hundreds* make neither 7 *tens* nor 7 *hundreds*,
and, in Compound Addition,
3 *shillings* + 4 *pence* make neither 7 *shillings* nor 7 *pence*,
so, in Fractions,
3 *fifths* + 4 *tenths* make neither 7 *fifths* nor 7 *tenths*
But just as, in Simple Addition, 3 *tens* + 4 *tens* make 7 *tens*,
and, in Compound Addition, 3 *shillings* + 4 *shillings* make 7 *shillings*,
so, in Fractions 3 *fifths* + 4 *fifths* make 7 *fifths*

Hence if the fractions to be added have different denominators, *we first reduce them to their* **least common denominator**, and then *add the resulting numerators.*

For instance, 3 *fifths* + 2 *sevenths* = 21 *thirty-fifths* + 10 *thirty-fifths*
= 31 *thirty-fifths*

$$i\,e \quad \frac{3}{5}+\frac{2}{7}=\frac{21}{35}+\frac{10}{35}=\frac{31}{35}.$$

EXAMPLE i.—*Add* $\frac{7}{12}$, $\frac{11}{16}$ *and* $\frac{13}{20}$ *together.*

1st step
The L C M of 12, 16 and 20 is **240.**
$240 - 12 = 20$, $7 \times 20 = \mathbf{140}$.
$240 - 16 = 15$, $11 \times 15 = \mathbf{165}$.
$240 - 20 = 12$, $13 \times 12 = \mathbf{156}$.

2nd step
Adding across we obtain the sum **461.**
As 461 is evidently not divisible by 2, 5 or 3, the only factors contained in the den^r, we see that the result cannot be reduced to *lower* terms

3rd step.
Finally we reduce the improper fraction $\frac{461}{240}$ to a mixed number

Written Work

$$\frac{7}{12}+\frac{11}{16}+\frac{13}{20}$$
$$=\frac{140+165+156}{240}$$
$$=\frac{461}{240}$$
$$=1\tfrac{221}{240}\ \underline{Ans.}$$

N B—The answer should always be expressed in its **lowest terms,** and when an improper fraction results it should be reduced to a mixed **number.**

As was pointed out in the last chapter, we may often save labour by writing the common denominator in *factors.*

EXAMPLE ii.—*Find the sum of* $\frac{17}{28}$, $\frac{5}{12}$ *and* $\frac{11}{42}$.

1st step

The L.C.M. of 28, 12, 42 is $4\times7\times3$.

$4\times7\times3\div28=3,\quad 17\times3=51,$

$4\times7\times3\div12=7,\quad 5\times7=35,$

$4\times7\times3\div42=2,\quad 11\times2=22.$

2nd step

Adding across, we obtain the sum 108.

3rd step

Dividing both num^r and den^r of the fraction $\frac{108}{4\times7\times3}$ by 12, we reduce this fraction to its lowest terms $\frac{9}{7}$, &c

Written Work

$$\frac{17}{28}+\frac{5}{12}+\frac{11}{42}$$
$$=\frac{51+35+22}{4\times7\times3}$$
$$=\frac{108}{4\times7\times3}$$
$$=\frac{9}{7}$$
$$=1\frac{2}{7}\ \underline{Ans}$$

In adding **mixed numbers**, the *integers* and *fractions* should be dealt with *separately* The mixed numbers should *not* be reduced to improper fractions, for this involves much needless labour. We do not reduce all the *pence* to *farthings* before attempting an addition of money!

EXAMPLE iii.—*Add together* $27\frac{5}{8}$, $4\frac{5}{12}$ *and* $11\frac{13}{24}$

$$27\tfrac{5}{8}+4\tfrac{5}{12}+11\tfrac{13}{24}=42+\frac{15+10+13}{24}$$
$$=42+\frac{38}{24}$$
$$=42+\frac{19}{12}$$
$$=42+1\tfrac{7}{12}=43\tfrac{7}{12}\ \underline{Ans}$$

If any of the fractions to be added are not in lowest terms, or are improper fractions, it is generally best to reduce them *before beginning the addition.*

EXAMPLE iv.—*Add* $3\frac{13}{15}$, $\frac{238}{25}$, $1\frac{49}{175}$, $\frac{104}{80}$.

$$3\tfrac{13}{15}+\tfrac{238}{25}+1\tfrac{49}{175}+\tfrac{104}{80}=3\tfrac{13}{15}+9\tfrac{13}{25}+1\tfrac{7}{25}+1\tfrac{3}{10}$$
$$=14\frac{130+78+42+45}{5\times3\times5\times2}$$
$$=14\frac{295}{5\times3\times5\times2}$$
$$=14\tfrac{59}{30}=15\tfrac{29}{30}\ \underline{Ans}$$

XVII. SUBTRACTION OF FRACTIONS.

As in Addition, so in Subtraction, we first reduce the given fractions to their least common denominator

EXAMPLE i —*From $\frac{5}{7}$ take $\frac{3}{5}$.*

$$\frac{5}{7}-\frac{3}{5}=\frac{25-21}{35}$$
$$=\frac{4}{35}\ \underline{Ans}$$

EXAMPLE ii —*Take $\frac{3}{8}$ from $\frac{7}{12}$.*

$$\frac{7}{12}-\frac{3}{8}=\frac{14-9}{24}$$
$$=\frac{5}{24}\ \underline{Ans}$$

In the subtraction of **mixed numbers** the *integers* and *fractions* should be treated *separately*

EXAMPLE iii.—*Find the difference between $17\frac{13}{21}$ and $43\frac{17}{24}$.*

$$43\tfrac{17}{24}-17\tfrac{13}{21}=26\frac{119-104}{8\times3\times7}$$
$$=26\frac{15}{8\times3\times7}$$
$$=26\tfrac{5}{56}\ \underline{Ans}$$

In the subtraction of mixed numbers it often happens that the *fractional* part to be subtracted is greater than the fractional part of the mixed number from which it is to be taken in which case we *change a unit* of the latter into fractional parts—just as, in Compound Subtraction, we change, if need be, a *penny* into *farthings*

EXAMPLE iv.—*Take $19\frac{11}{12}$ from $25\frac{3}{8}$.*

Here, after subtracting the integers and reducing the fractions to their L C D we obtain the result $6\frac{9-22}{24}$

Now 22 *twenty-fourths* cannot be taken from 9 *twenty-fourths* We therefore *change one* of the 6 units into *twenty-fourths*

Note—After a little practice the step marked with an asterisk may be performed mentally, and would then be omitted from the written work.

Written Work

$$25\tfrac{3}{8}-19\tfrac{11}{12}$$
$$=6\frac{9-22}{24}$$
$$=5\frac{24+9-22}{24}*$$
$$=5\tfrac{11}{24}\ \underline{Ans}$$

EXAMPLE v.—*From 17 take $12\frac{3}{11}$*

Mental Work

3 *elevenths* from 11 *elevenths* (*i e.* unity) leaves 8 *elevenths.*

12 from 16 leaves 4.

Written Work.

$$17-12\tfrac{3}{11}$$
$$=4\tfrac{8}{11}\ \underline{Ans.}$$

EXAMPLE vi —*Take* $\frac{1881}{8181}$ *from* $\frac{6703}{101}$.

$$\frac{6703}{101}-\frac{1881}{8181}=66\frac{37}{101}-\frac{209}{909}$$
$$=66\frac{333-209}{909}$$
$$=66\frac{124}{909}\ \underline{Ans}$$

$$\begin{array}{r} 101)6703(66 \\ \underline{606} \\ 643 \\ \underline{606} \\ \underline{37} \end{array}$$

When Addition and Subtraction are both involved in the same question the method of procedure is similar to that explained on page 8

EXAMPLE vii —*Simplify* $7\frac{3}{5}-2\frac{1}{2}+5\frac{7}{10}-8\frac{11}{20}$.

$$7\frac{3}{5}-2\frac{1}{2}+5\frac{7}{10}-8\frac{11}{20}=2\frac{12-10+14-11}{20}$$
$$=2\frac{5}{20}$$
$$=2\frac{1}{4}\ \underline{Ans}$$

EXAMPLE viii —*Simplify* $3\frac{1}{4}-7\frac{11}{12}+1\frac{5}{48}-8\frac{15}{32}+16$.

$$3\frac{1}{4}-7\frac{11}{12}+1\frac{5}{48}-8\frac{15}{32}+16=5\frac{24-88+10-45}{96}$$
$$=5\frac{34-133}{96}$$
$$=3\frac{192+34-133}{96}*$$
$$=3\frac{93}{96}$$
$$=3\frac{31}{32}\ \underline{Ans}$$

The work of Addition may be lessened by utilizing Subtraction, if any of the fractions are *very nearly equal to unity*

EXAMPLE ix.—*Add* $7\frac{23}{24}$, $11\frac{5}{48}$ *and* $5\frac{15}{16}$.

Here, as $7\frac{23}{24}$ differs from the integer 8 by only $\frac{1}{24}$, we write $8-\frac{1}{24}$ in place of $7\frac{23}{24}$

Similarly, instead of $5\frac{15}{16}$ we write $6-\frac{1}{16}$

$$7\frac{23}{24}+11\frac{5}{48}+5\frac{15}{16}$$
$$=8-\frac{1}{24}+11\frac{5}{48}+6-\frac{1}{16}$$
$$=25+\frac{5}{48}-\frac{1}{24}-\frac{1}{16}$$
$$=25\frac{5-2-3}{48}$$
$$=25\ \underline{Ans.}$$

* In this Example it was necessary to change 2 units into fractional parts

XVIII. FRACTIONS.

MULTIPLICATION AND DIVISION BY AN INTEGER

From the definition of Multiplication on page 9, it follows that *to multiply a fraction by an* **integer** *is to repeat the fraction as many times as there are units in that integer.*

For instance, 3 times two-*sevenths* is six-*sevenths*, *i e* $\frac{2}{7} \times 3 = \frac{6}{7}$

Thus *a fraction is* **multiplied** *by an* **integer** *if its* **numerator** *be multiplied by that integer*

This multiplication we at first merely *indicate by the sign*, and then, *before completing the operation, remove from the resulting numerator and denominator any factors common to both*, in order that the product may be in its lowest terms

For instance, $\frac{5}{12} \times 9 = \frac{5 \times 9}{12} = \frac{5 \times 3}{4}$ (dividing num^r and den^r by 3).

$$= \frac{15}{4} = 3\tfrac{3}{4}$$

EXAMPLE i.—*Multiply* $\frac{55}{252}$ by 189

As 189 and 252 are evidently divisible by 9, we divide both by 9, and then "cancel" them, writing the quotients 21 and 28 above and below them respectively

Then, as 21 and 28 are both divisible by 7, we cancel them also, and write the quotients 3 and 4 above and below them respectively.

Thus the fraction has been reduced to its lowest terms, and replaced by $\frac{55 \times 3}{4}$, *i e* $\frac{165}{4}$, which we reduce to a mixed number.

Written Work

$$\frac{55}{252} \times 189 = \frac{55 \times \overset{\overset{3}{\cancel{21}}}{\cancel{189}}}{\underset{\underset{4}{\cancel{28}}}{\cancel{252}}}$$

$$= \frac{165}{4}$$

$$= 41\tfrac{1}{4} \quad \underline{Ans}$$

The process of thus removing factors common to both numerator and denominator is called **cancelling**.

A *mixed number* may be multiplied by an *integer* without reducing the mixed number to an improper fraction

EXAMPLE ii —*Multiply* $105\frac{6}{13}$ *by* 4.

$$105\tfrac{6}{13} \times 4 = 105 \times 4 + \tfrac{6}{13} \times 4$$

$$= 420 + \tfrac{24}{13}$$

$$= 420 + 1\tfrac{11}{13} = 421\tfrac{11}{13} \quad \underline{Ans.}$$

Again, *to divide a fraction by an integer is to separate the fraction into an integral number of equal portions; i.e* to reverse the process of multiplication *

For instance, as 2 *sevenths* × 3 = 6 *sevenths*,
∴, conversely, 6 *sevenths* ÷ 3 = 2 *sevenths*, *i e* $\frac{6}{7} \div 3 = \frac{2}{7}$

Thus a fraction is divided by an integer if its numerator be divided by that integer

This method, however, will only apply to the case of a numerator which *exactly contains* that integer

For instance, we cannot thus divide $\frac{1}{8}$ by 3 for 3 is not contained in 1.

But as we know that $\frac{1}{8} = \frac{3}{24}$, $\therefore \frac{1}{8} \div 3 = \frac{3}{24} \div 3 = \frac{1}{24}$

Now $\frac{1}{24} = \frac{1}{8 \times 3}$ Hence the quotient in this case is obtained by *multiplying* the *denominator* of the fraction by 3

It is evident also that the quotient in the former case can be obtained in the same manner, for $\frac{2}{7} = \frac{2 \times 3}{7 \times 3} = \frac{6}{7 \times 3}$

Thus *a fraction* is divided by an integer if its denominator *be multiplied by that integer*

This process we at first *merely indicate by the sign,* and then, before completing the work, remove from the resulting num^r and den^r any factors common to both.

EXAMPLE III —*Divide* $\frac{91}{100}$ *by* 26.

Here we first indicate the multiplication of the denominator by 26 by the sign of multiplication

Then as 91 and 26 are both divisible by 13, we cancel 91 and 26, writing the quotients 7 and 2 above and below them respectively

Written Work

$$\frac{91}{100} \div 26 = \frac{\overset{7}{\cancel{91}}}{100 \times \underset{2}{\cancel{26}}} = \frac{7}{200} \ \underline{Ans}$$

In order to divide a mixed number by an integer we reduce the mixed number to an improper fraction.†

EXAMPLE IV —*Divide* $34\frac{2}{9}$ *by* 352

$$34\frac{2}{9} \div 352 = \frac{308}{9} \div 352 = \frac{\overset{\overset{7}{\cancel{77}}}{\cancel{308}}}{9 \times \underset{\underset{8}{\cancel{88}}}{\cancel{352}}} = \frac{7}{72} \ \underline{Ans.}$$

Here, in cancelling, we divide both num^r and den^r first by 4, then by 11.

* Only one of the two ways of regarding Division (see pages 14 and 43) applies in all cases to the division of a *fraction* by an integer, for, if the fraction be less than the integer it evidently does not contain it *any number of times*

† This is not *always necessary,* for instance, $8\frac{8}{17} \div 4 = 8 \div 4 + \frac{16}{17} \div 4 = 2 + \frac{4}{17} = 2\frac{4}{17}$.

Note —Hence it follows that

(i) a fraction is *doubled* either by *doubling the num*^{r}, or by *halving the den*^{r}, according as the *den*^{r} *is odd, or even*

(ii) a fraction is *halved* either by *halving the num*^{r}, or by *doubling the den*^{r}, according as the *num*^{r} *is even, or odd*

For instance (i) *Twice* $\frac{3}{7}$ is $\frac{6}{7}$, *Twice* $\frac{3}{8}$ is $\frac{3}{4}$

(ii) *Half* of $\frac{4}{7}$ is $\frac{2}{7}$, *Half* of $\frac{5}{7}$ is $\frac{5}{14}$

If we multiply a mixed number by an integer equal to the denominator, the product which results is an integer

For instance, $6\frac{4}{11} \times 11 = 66 + 4 = \mathbf{70}$

$\therefore$, conversely, $70 \div 11 = 6\frac{4}{11}$,

i e $6\frac{4}{11}$ expresses the *complete quotient* resulting from the division.

Hence, in cases which admit of a non-integral quotient, *the remainder in an inexact division may be regarded as part of the quotient*

For instance, "*Divide* 70 *into* 11 *equal parts*," admits of the answer, "The size of each part is $6\frac{4}{11}$"

But the question, "*How* many times *does* 70 *contain* 11?" does not admit of a fractional answer

Or, to take a concrete example, "*Divide* 70 *oranges equally among* 11 *boys*," admits of the answer, "The share of each boy is $6\frac{4}{11}$ oranges"

But to the question, "*How many boys could each receive* 11 *oranges from a heap of* 70 *oranges?*", the answer is "6 boys, and 4 oranges would be left over," for the answer "$6\frac{4}{11}$ boys" would be meaningless

Hence, also, *a fraction may be regarded as indicating the* **division** *of the numerator by the denominator*

For instance, $\frac{70}{11}$ indicates the division of 70 *units* into 11 *equal parts*

Similarly, $\frac{3}{4}$ indicates the division of 3 *units* into 4 *equal parts*

But, by definition of a fraction on page 64, $\frac{3}{4}$ also indicates the division of 1 *unit* into 4 equal parts, of which parts 3 are taken

Hence, 3 *fourths* of 1 *unit* is equivalent to 1 *fourth* of 3 *units*
And so on

Or, to take a concrete example,

as one *tenth* of £1 is 2 *shillings*, ∴ 3 *tenths* of £1 is 6 *shillings*.

But one *tenth* of £3 is also 6 *shillings*,

$$\therefore \frac{3}{10} \text{ of } £1 = \frac{1}{10} \text{ of } £3$$

XIX. MULTIPLICATION OF FRACTIONS.

A fraction as defined on page 64 is called a *Simple* Fraction: *i e.* a simple fraction is a fraction of a *unit*

A Fraction *of a fraction* is called a **Compound Fraction.**

For instance, $\frac{3}{5}$ of $\frac{2}{7}$ is a compound fraction.

Now, one *fifth* of $\frac{2}{7}$ is found by *dividing* $\frac{2}{7}$ into 5 equal parts, and we know by Chapter XVIII., that one of these parts is $\frac{2}{7 \times 5}$

And as three fifths of $\frac{2}{7}$ must be 3 times as great as one fifth of $\frac{2}{7}$,

$\therefore$ $\frac{3}{5}$ of $\frac{2}{7}$ is found by *multiplying* $\frac{2}{7 \times 5}$ by 3, and this product we know is found by multiplying the numerator of the fraction by 3;

$$i\,e.\ \frac{3}{5} \text{ of } \frac{2}{7} = \frac{2 \times 3}{7 \times 5} = \frac{6}{35}$$

Hence a *compound* fraction is converted into a simple fraction by *multiplying the numerators of the fractions together, and the denominators together.*

It follows, from (I) on page 9, that the value of a compound fraction is not altered by reversing the places of the fractions.

For instance, $\frac{3}{5}$ of $\frac{2}{7} = \frac{2 \times 3}{7 \times 5} = \frac{3 \times 2}{5 \times 7} = \frac{2}{7}$ of $\frac{3}{5}$.

The operation of finding the value of *a fraction of a fraction* is also indicated by placing the *sign of multiplication* between the fractions, and the result is then called the *product* of the fractions.

For instance, $\frac{2}{7} \times \frac{3}{5}$ means $\frac{2 \times 3}{7 \times 5}$, or $\frac{6}{35}$.

But it should be noticed that the definition of Multiplication on page 9 only applies to the case of an *integral multiplier*, and that "multiplication" by a fraction includes *division, as well as multiplication, as defined for integers* *

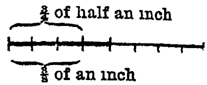
For instance, to multiply, in the fractional sense, $\frac{1}{3}$ by $\frac{1}{4}$, is equivalent to *dividing* $\frac{1}{3}$ by 4

Again, to multiply $\frac{2}{7}$ by $\frac{3}{5}$, is equivalent to *dividing* $\frac{2}{7}$ by 5, and then *multiplying* the result by 3

Consequently the number so "multiplied" is not necessarily *increased*, but is decreased, or increased, according as the fractional *multiplier is a proper, or an improper*, fraction

* For a wider definition of Multiplication, applicable alike to fractions as well as integers, see Appendix

Note.—The accompanying diagram illustrates the rule of this chapter, for each small part is evidently an *eighth* of *a whole inch*, but a *fourth* of *half an inch*



Hence, 3 *fourths* of *half* an inch = 3 *eighths* of a *whole* inch

$$i.e.\ \frac{3}{4} \text{ of } \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8},$$

when an inch is taken as unit

EXAMPLE i.—*Simplify* $\frac{15}{28}$ *of* $\frac{16}{25}$

Here, as before, we do not actually multiply 15 by 16, and 28 by 25, but merely indicate the operation by the sign, until we have, by cancelling, reduced the fraction $\frac{15 \times 16}{28 \times 25}$ to its lowest terms This consists in dividing 15 and 25 both by 5, and 16 and 28 both by 4, when we obtain the final result $\frac{3 \times 4}{7 \times 5}$, *i.e.* $\frac{12}{35}$

$$\frac{15}{28} \text{ of } \frac{16}{25}$$

$$= \frac{\overset{3}{\cancel{15}} \times \overset{4}{\cancel{16}}}{\underset{7}{\cancel{28}} \times \underset{5}{\cancel{25}}}$$

$$= \frac{12}{35} \ \underline{Ans}$$

When mixed numbers occur we convert them to improper fractions.—

EXAMPLE ii.—*Multiply* $6\frac{6}{7}$ *by* $2\frac{13}{18}$

$$6\frac{6}{7} \times 2\frac{13}{18} = \frac{\overset{8}{\cancel{48}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{7}{\cancel{49}}}{\underset{3}{\cancel{18}}} = \frac{56}{3} = 18\frac{2}{3} \ \underline{Ans}$$

Here, having reduced the mixed numbers to improper fractions, the 48 above, and the 18 below, are cancelled and replaced by 8, and 3, respectively. Also the 49 above, and the 7 below, by 7, and 1, respectively

---

Similarly, a *fraction* of *a compound fraction* is reduced to a simple fraction

For instance, $\frac{2}{3}$ of $\frac{2}{5}$ of $\frac{1}{7} = \frac{2}{3}$ of $\frac{2}{35} = \frac{4}{105}$

$$i.e.\ \frac{2}{3} \text{ of } \frac{2}{5} \text{ of } \frac{1}{7} = \frac{2 \times 2 \times 1}{3 \times 5 \times 7} = \frac{4}{105}$$

Here, as before, the operation is also indicated by placing signs of multiplication between the fractions, when the result is called the *continued* product of the fractions.

For instance, $\frac{2}{3} \times \frac{2}{5} \times \frac{1}{7} = \frac{2 \times 2 \times 1}{3 \times 5 \times 7} = \frac{4}{105}$

EXAMPLE iii.—*Simplify* $1\frac{1}{15}$ *of* $3\frac{1}{5}$ *of* $1\frac{7}{8}$ *of* $\frac{25}{36}$.

We first reduce the mixed numbers to improper fractions.

Then the 15 above and the 15 below are cancelled and replaced by 1 and 1, the 16 above and 8 below are replaced by 2 and 1 respectively, the 16 above and 36 below by 4 and 9 respectively, the 25 above and 5 below by 5 and 1 respectively

Thus the required fraction expressed in lowest terms is $\frac{2\times4\times1\times5}{1\times1\times1\times9}$, *i e.* $\frac{40}{9}$ or $4\frac{4}{9}$.

$$1\frac{1}{15} \text{ of } 3\frac{1}{5} \text{ of } 1\frac{7}{8} \text{ of } \frac{25}{36}$$

$$= \frac{\overset{2}{\cancel{16}}}{\underset{1}{\cancel{15}}} \times \frac{\overset{4}{\cancel{16}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{15}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{5}{\cancel{25}}}{\underset{9}{\cancel{36}}}$$

$$= \frac{40}{9}$$

$$= 4\frac{4}{9} \quad \underline{Ans}$$

EXAMPLE iv.—*Find the continued product of* $\frac{7}{19}$, $\frac{2}{27}$, $2\frac{1}{4}$ *and* $5\frac{3}{7}$

$$\frac{7}{19} \times \frac{2}{27} \times 2\frac{1}{4} \times 5\frac{3}{7} = \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{19}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{27}}} \times \frac{\overset{1}{\cancel{9}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{4}}} \times \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{38}}}{\underset{1}{\cancel{7}}} = \frac{1}{3} \quad \underline{Ans.}$$

Note—The beginner is advised not to omit to *write down* "1" when that number replaces a number cancelled, or he may fall into the serious error of omitting it in the answer, and of writing (for instance) "Ans 3," instead of "Ans $\frac{1}{3}$"

EXAMPLE v—*Express* $\frac{171}{187}$ *of* $\frac{682}{868} \times \frac{204}{209}$ *in its simplest form.*

As 682 and 868 are even, we divide them both by 2

As 204 and 434 are also even, we divide them both by 2

On inspection we observe that 341 and 209 are both divisible by 11, we divide both by 11.

Now, as 217 is evidently divisible by 31 we cancel 31 and 217, replacing them by 1 and 7 respectively

We find, on trial, that 171 is divisible by 19

Finally, 102 and 187 are both divisible by 17, and the required fraction is $\frac{9\times1\times6}{11\times7\times1}$ or $\frac{54}{77}$

$$\frac{\overset{9}{\cancel{171}}}{\underset{11}{\cancel{187}}} \times \frac{\overset{\overset{\overset{1}{\cancel{31}}}{\cancel{341}}}{\cancel{682}}}{\underset{\underset{\underset{7}{\cancel{217}}}{\cancel{434}}}{\cancel{868}}} \times \frac{\overset{\overset{6}{\cancel{102}}}{\cancel{204}}}{\underset{\underset{1}{\cancel{19}}}{\cancel{209}}} = \frac{54}{77} \quad \underline{Ans.}$$

XX. DIVISION OF FRACTIONS.

The *undoing* or reversing, of the process called "Multiplication of Fractions" is called *Division of Fractions* *

For instance, to *divide* $\frac{2}{11}$ by $\frac{5}{7}$, is to find a fraction which when multiplied by $\frac{5}{7}$, produces $\frac{2}{11}$

$$i.e.\ \textit{the req}^{d}\textit{ quotient} \times \frac{5}{7} = \frac{2}{11}.$$

But we know, by the last chapter, that $\frac{14}{55} \times \frac{5}{7} = \frac{2}{11}$.

$$\therefore \textit{the req}^{d}\textit{ quotient is } \frac{14}{55}$$

Now $\frac{14}{55} = \frac{2 \times 7}{11 \times 5} = \frac{2}{11} \times \frac{7}{5} =$ *the dividend* $\times$ *the divisor* inverted

Hence, *to divide any number by a fraction*, we **multiply** *the number by the fraction* **inverted.**

EXAMPLE I.—*Divide* $\frac{7}{8}$ *by* $\frac{9}{10}$

$$\frac{7}{8} \div \frac{9}{10} = \frac{7}{\cancel{8}_{4}} \times \frac{\cancel{10}^{5}}{9} = \frac{35}{36}\ \underline{\textit{Ans}}$$

If mixed numbers occur we reduce them to improper fractions †

EXAMPLE II —*Divide* $9\frac{3}{8}$ *by* $2\frac{11}{12}$

$$9\frac{3}{8} \div 2\frac{11}{12} = \frac{75}{8} \div \frac{35}{12} = \frac{\cancel{75}^{15}}{\cancel{8}_{2}} \times \frac{\cancel{12}^{3}}{\cancel{35}_{7}} = \frac{45}{14} = 3\frac{3}{14}\ \underline{\textit{Ans}}$$

If *unity* be divided by any number, the quotient is called the **Reciprocal** of that number

For instance, the *reciprocal* of 7 is $1 \div 7$, *i.e* $\frac{1}{7}$;

the *reciprocal* of $\frac{4}{5}$ is $1 \div \frac{4}{5}$, *i.e.* $\frac{5}{4}$.

Hence the rule for Division of Fractions may be expressed thus

Multiply the dividend by the **reciprocal** *of the divisor,*

in which form it includes the rule of Chapter XVIII.

---

* When we regard Division as the separation of the dividend into *a number of equal parts*, it is evident that we exclude the idea of a *fractional* divisor

† It is not absolutely necessary always to reduce a mixed number in the *dividend* to an improper fraction. for instance, $16\frac{3}{8} \div \frac{4}{5} = 16\frac{3}{8} \times \frac{5}{4} = 16 \times \frac{5}{4} + \frac{3}{8} \times \frac{5}{4} = 20 + \frac{15}{32} = 20\frac{15}{32}$

EXAMPLE iii.—*Divide* $3\frac{1}{4}$ *by* 5

$$3\tfrac{1}{4}\div 5=\tfrac{13}{4}\div 5=\tfrac{13}{4}\times\tfrac{1}{5}=\tfrac{13}{20}\ \underline{Ans}$$

EXAMPLE iv.—*Divide* $2\frac{1}{5}$ *by* $\frac{1}{7}$.

$$2\tfrac{1}{5}\div\tfrac{1}{7}=2\tfrac{1}{5}\times 7=14+1\tfrac{2}{5}=15\tfrac{2}{5}\ \underline{Ans.}$$

A Complex Fraction is a fraction the numerator and denominator of which are, one or both of them, *fractions.*

For instance, $\dfrac{\frac{3}{5}}{\frac{7}{8}}$, $\dfrac{5\frac{7}{8}}{9\frac{1}{3}}$, $\dfrac{2}{\frac{8}{11}}$ and $\dfrac{3\frac{1}{4}}{7}$ are *complex* fractions.

Now we know, by Chapter XVIII, that a fraction expresses the quotient of its numerator divided by its denominator.

Hence $\dfrac{\frac{3}{5}}{\frac{7}{8}}=\frac{3}{5}\div\frac{7}{8}$; $\dfrac{2}{\frac{8}{11}}=2\div\frac{8}{11}$; and $\dfrac{3\frac{1}{4}}{7}=3\frac{1}{4}\div 7$.

EXAMPLE v.—*Simplify* $\dfrac{3\frac{2}{5}}{6\frac{3}{8}}$

$$\frac{3\frac{2}{5}}{6\frac{3}{8}}=\frac{\frac{17}{5}}{\frac{51}{8}}=\frac{\overset{1}{\cancel{17}}}{5}\times\frac{8}{\underset{3}{\cancel{51}}}=\frac{8}{15}\ \underline{Ans}$$

EXAMPLE vi.—*Simplify* $\dfrac{3}{7\frac{1}{5}}$

$$\frac{3}{7\frac{1}{5}}=\frac{3}{\frac{36}{5}}=\overset{1}{\cancel{3}}\times\frac{5}{\underset{12}{\cancel{36}}}=\frac{5}{12}\ \underline{Ans}$$

A complex fraction may also be simplified by *multiplying both its num*^{r} *and den*^{r} *by the* L.C.M *of the den*^{rs} *of the simple fractions,* thus:—

EXAMPLE vii.—*Simplify* $\dfrac{4\frac{1}{5}}{9\frac{2}{3}}$

$$\frac{4\frac{1}{5}}{9\frac{2}{3}}=\frac{4\frac{1}{5}\times 15}{9\frac{2}{3}\times 15}=\frac{60+3}{135+10}=\frac{63}{145}\ \underline{Ans}$$

EXAMPLE viii.—*Simplify* $\dfrac{7\frac{1}{8}}{38}$

$$\frac{7\frac{1}{8}}{38}=\frac{7\frac{1}{8}\times 8}{38\times 8}=\frac{56+1}{38\times 8}=\frac{\overset{3}{\cancel{57}}}{\underset{2}{\cancel{38}}\times 8}=\frac{3}{16}\ \underline{Ans}$$

Note.—In *writing* complex fractions care must be taken to mark the *line of division* clearly, for such a fraction as $\dfrac{\frac{3}{4}}{5}$ is ambiguous: we cannot tell whether $3\div\frac{4}{5}$, or $\frac{3}{4}\div 5$ is intended.

XXI. FRACTIONS.

Simplification of Fractional Expressions

In order to find correctly the value of an expression consisting of fractions connected in various ways, it is necessary clearly to understand, not only the *meaning* of the *symbols* employed, but also the *order* in which the *operations* indicated are intended to be performed. The work, moreover, should be arranged in an orderly manner. The following are points of special importance:—

(I.) *Fractions connected by the word* of *must be regarded as* one single *fraction.*

Example i.—*Simplify* $2\frac{3}{4} \div 1\frac{3}{8}$ *of* $\frac{5}{6}$.

$$2\frac{3}{4} \div 1\frac{3}{8} \text{ of } \frac{5}{6} = \frac{11}{4} \div \frac{11}{8} \text{ of } \frac{5}{6} = \frac{\overset{1}{\cancel{11}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}} \times 6}{\underset{1}{\cancel{11}} \times 5} = \frac{12}{5} = 2\frac{2}{5} \ \underline{Ans.}$$

N.B.—Here we invert *both* $\frac{11}{8}$ and $\frac{5}{6}$, for $\frac{11}{8}$ *of* $\frac{5}{6}$, *i.e.* $\frac{55}{48}$, is *one* fraction.*

Example ii.—*Simplify* $\frac{3}{8} + \frac{5}{6}$ *of* $\frac{3}{10}$.

$$\frac{3}{8} + \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} \text{ of } \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{10}}} = \frac{3}{8} + \frac{1}{4} = \frac{5}{8} \ \underline{Ans.}$$

N.B.—Here we first find the value of $\frac{5}{6}$ *of* $\frac{3}{10}$, namely $\frac{1}{4}$, and *then* add *this result* to $\frac{3}{8}$. We *must not* add $\frac{5}{6}$ to $\frac{3}{8}$.

Example iii.—*Simplify* $2\frac{2}{3} - \frac{3}{5}$ *of* $\frac{5}{6} + \frac{1}{4}$ *of* $2\frac{1}{3}$.

$$2\frac{2}{3} - \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} \text{ of } \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} + \frac{1}{4} \text{ of } \frac{7}{3} = 2\frac{2}{3} - \frac{1}{2} + \frac{7}{12} = 2\frac{8-6+7}{12} = 2\frac{3}{4} \ \underline{Ans.}$$

---

* In such cases a distinction is made between "of" and the sign ×, for, if such an expression as $2\frac{3}{4} \div 1\frac{3}{8} \times \frac{5}{6}$ be met with, the operations are intended to be performed *in order from left to right*, *i.e.* $2\frac{3}{4} \div 1\frac{3}{8} \times \frac{5}{6}$ means that $2\frac{3}{4}$ is to be divided by $1\frac{3}{8}$, and *the result* multiplied by $\frac{5}{6}$.

(II.) *Multiplications and Divisions must be performed* **before** *Additions and Subtractions.*

EXAMPLE iv —*Simplify* $1\frac{1}{3}\times\frac{5}{8}+\frac{3}{4}$.

$$1\frac{1}{3}\times\frac{5}{8}+\frac{3}{4}=\frac{\overset{1}{\cancel{4}}}{3}\times\frac{5}{\underset{2}{\cancel{8}}}+\frac{3}{4}=\frac{5}{6}+\frac{3}{4}=\frac{19}{12}=1\frac{7}{12}\ \underline{Ans}$$

N B —Here we *first multiply* $\frac{4}{3}$ by $\frac{5}{8}$ and *then add* the result to $\frac{3}{4}$ We *must not add* $\frac{5}{8}$ to $\frac{3}{4}$

EXAMPLE v —*Simplify* $5\frac{1}{2}-\frac{3}{16}\div2\frac{1}{4}+\frac{5}{7}\times1\frac{1}{6}$

$$5\frac{1}{2}-\frac{3}{16}\div2\frac{1}{4}+\frac{5}{7}\times1\frac{1}{6}=5\frac{1}{2}-\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{16}}}\times\frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}}+\frac{5}{\underset{1}{\cancel{7}}}\times\frac{\overset{1}{\cancel{7}}}{6}$$

$$=5\frac{1}{2}-\frac{1}{12}+\frac{5}{6}$$

$$=5\frac{6-1+10}{12}$$

$$=5\frac{15}{12}=6\frac{1}{4}\ \underline{Ans.}$$

EXAMPLE vi —*Simplify* $\dfrac{1\frac{1}{2}}{2\frac{1}{3}}-\dfrac{3\frac{1}{5}}{2\frac{2}{3}}+\dfrac{4\frac{1}{2}}{2\frac{1}{4}}$

1st Method.

$$\dfrac{1\frac{1}{2}}{2\frac{1}{3}}-\dfrac{3\frac{1}{5}}{2\frac{2}{3}}+\dfrac{4\frac{1}{2}}{2\frac{1}{4}}$$

$$=\frac{3}{2}\times\frac{3}{7}-\frac{\overset{2}{\cancel{16}}}{5}\times\frac{3}{\underset{1}{\cancel{8}}}+\frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{2}}}\times\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{9}}}$$

$$=\frac{9}{14}-1\frac{1}{5}+2$$

$$=1\frac{45-14}{70}=1\frac{31}{70}\ \underline{Ans}$$

2nd Method.

$$\dfrac{1\frac{1}{2}}{2\frac{1}{3}}-\dfrac{3\frac{1}{5}}{2\frac{2}{3}}+\dfrac{4\frac{1}{2}}{2\frac{1}{4}}$$

$$=\frac{1\frac{1}{2}\times6}{2\frac{1}{3}\times6}-\frac{3\frac{1}{5}\times15}{2\frac{2}{3}\times15}+\frac{4\frac{1}{2}\times4}{2\frac{1}{4}\times4}*$$

$$=\frac{9}{14}-\frac{48}{40}+\frac{18}{9}$$

$$=\frac{9}{14}-1\frac{1}{5}+2$$

$$=1\frac{45-14}{70}=1\frac{31}{70}\ \underline{Ans.}$$

---

* See page 85, Example vii.

(III) **Brackets** *indicate that their contents are to be regarded as* **one single** *number* (See page 18.)

Example vii—*Simplify* $\left(\frac{3}{8}+\frac{5}{6}\right)$ *of* $\frac{3}{10}$.

$$\left(\frac{3}{8}+\frac{5}{6}\right) \text{ of } \frac{3}{10}=\frac{9+20}{24} \text{ of } \frac{3}{10}=\frac{29}{\underset{8}{\cancel{24}}} \text{ of } \frac{\overset{1}{\cancel{3}}}{10}=\frac{29}{80} \quad \underline{\underline{Ans}}$$

Example viii.—*Simplify* $\left(2\frac{1}{4}-1\frac{1}{5}\right)\div\left(5\frac{1}{6}+1\frac{1}{8}\right)$

$$\left(2\frac{1}{4}-1\frac{1}{5}\right)\div\left(5\frac{1}{6}+1\frac{1}{8}\right)=1\frac{1}{20}\div 6\frac{4+3}{24}=\frac{21}{\underset{5}{\cancel{20}}}\times\frac{\overset{6}{\cancel{24}}}{151}=\frac{126}{755} \quad \underline{\underline{Ans}}$$

Example ix—*Simplify* $\frac{117}{285}$ *of* $1\frac{61}{91}-\dfrac{3\frac{2}{7}}{9\frac{1}{5}}\div\left(\frac{2}{3}+\frac{5}{6}\times\frac{8}{15}\right)+\frac{11}{20}$

$$\frac{117}{285} \text{ of } 1\frac{61}{91}-\frac{3\frac{2}{7}}{9\frac{1}{5}}-\left(\frac{2}{3}+\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}}\times\frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{15}}}\right)+\frac{11}{20}$$

$$=\frac{\overset{\overset{3}{\cancel{9}}}{\cancel{117}}}{\underset{\underset{5}{\cancel{95}}}{\cancel{285}}}\times\frac{\overset{8}{\cancel{152}}}{\underset{7}{\cancel{91}}}-\frac{\overset{1}{\cancel{23}}}{7}\times\frac{5}{\underset{2}{\cancel{46}}}\div\left(\frac{2}{3}+\frac{4}{9}\right)+\frac{11}{20}$$

$$=\frac{24}{35}-\frac{5}{14}-\frac{10}{9}+\frac{11}{20}$$

$$=\frac{24}{35}-\frac{\overset{1}{\cancel{5}}}{14}\times\frac{9}{\underset{2}{\cancel{10}}}+\frac{11}{20}$$

$$=\frac{96-45+77}{5\times7\times4}$$

$$=\frac{\overset{32}{\cancel{128}}}{5\times7\times\underset{1}{\cancel{4}}}=\frac{32}{35} \quad \underline{\underline{Ans}}$$

In the case of expressions arranged in the form of complex fractions the thick line serves the purpose of brackets, indicating that the *value* of the expression which forms the *numerator* is to be *divided* by the *value* of that which forms the *denominator*.

EXAMPLE x.—*Simplify* $\dfrac{2\frac{1}{4}-1\frac{1}{5}}{5\frac{1}{6}+1\frac{1}{8}}$

$$\frac{2\frac{1}{4}-1\frac{1}{5}}{5\frac{1}{6}+1\frac{1}{8}}=\frac{1\frac{1}{20}}{6\frac{4+3}{24}}=\frac{\frac{21}{20}}{\frac{151}{24}}=\frac{21}{\cancel{20}_{5}}\times\frac{\cancel{24}^{6}}{151}=\frac{126}{755}\ \underline{Ans.}$$

Note —This question is identical with that of Example viii.

EXAMPLE xi.—*Simplify* $\dfrac{2\frac{1}{2}-1\frac{2}{3}}{3\frac{3}{4}\times1\frac{1}{5}}\div\dfrac{\frac{7}{8}+1\frac{3}{4}}{4\frac{1}{2}\times2\frac{1}{3}}.$

$$\frac{2\frac{1}{2}-1\frac{2}{3}}{3\frac{3}{4}\times1\frac{1}{5}}\div\frac{\frac{7}{8}+1\frac{3}{4}}{4\frac{1}{2}\times2\frac{1}{3}}=\frac{2\frac{1}{2}-1\frac{2}{3}}{3\frac{3}{4}\times1\frac{1}{5}}\times\frac{4\frac{1}{2}\times2\frac{1}{3}}{\frac{7}{8}+1\frac{3}{4}}=\frac{\frac{15-10}{6}}{\frac{15}{4}\times\frac{6}{5}}\times\frac{\frac{9}{2}\times\frac{7}{3}}{\frac{7+14}{8}}.$$

Here we begin by inverting the complex divisor bodily Afterwards, before cancelling, all the three fractions $\frac{15}{4}$, $\frac{6}{5}$, $\frac{21}{8}$, which stand *below the thick lines*, are inverted

$$=\frac{5}{\cancel{6}_{3}}\times\frac{\cancel{4}^{\cancel{2}^{1}}}{\cancel{15}_{3}}\times\frac{\cancel{5}^{1}}{\cancel{6}_{3}}\times\frac{\cancel{9}^{\cancel{3}^{1}}}{\cancel{2}_{1}}\times\frac{\cancel{7}^{1}}{\cancel{3}_{1}}\times\frac{\cancel{8}^{4}}{\cancel{21}_{\cancel{3}_{1}}}=\frac{20}{27}\ \underline{Ans}$$

A fraction of the form $\dfrac{2}{3+\frac{1}{4}}$ is called a **Continued Fraction.**

In order to simplify such fractions, *we begin with the lowest part, and proceed, step by step, upwards.*

EXAMPLE xii —*Simplify* $\dfrac{2}{3+\dfrac{1}{5+\frac{3}{4}}}.$

$$\frac{2}{3+\dfrac{1}{5+\frac{3}{4}}}=\frac{2}{3+\dfrac{1}{\frac{23}{4}}}=\frac{2}{3+\frac{4}{23}}=\frac{2}{\frac{73}{23}}=2\times\frac{23}{73}=\frac{46}{73}\ \underline{Ans}$$

XXII. FRACTIONS.

Value of a Fraction of a Concrete Quantity.

It follows from the definition of a fraction that if we divide the given concrete quantity by the *denominator*, and multiply the result by the *numerator*, we obtain the value of the fraction

For instance, $\frac{2}{5}$ of £10, 15*s* is found if we divide £10, 15*s* into 5 equal parts and multiply one of those parts by 2

In practice, however, it is generally more convenient to reverse the order of these two operations, *i e* to multiply by the numerator first, and afterwards to divide by the denominator *

Example i —*Find the value of $\frac{5}{8}$ of £7, 16s 3d.*

After dividing the *pence* by 8, there remain 7 pence over, which is expressed as part of the quotient (see page 80) by the fraction $\frac{7}{8}$ of a penny

N B —We do not in such cases reduce the pence which are over to *farthings*

	£	*s*	*d*	
	7	16	3	
			5	
8	39	1	3	
	4	17	$7\frac{7}{8}$	*Ans*

Example ii —*Multiply £4, 2s $3\frac{5}{6}$d. by $7\frac{3}{11}$*

Here we may either

(i) find $\frac{3}{11}$ of £4, 2*s* $3\frac{5}{6}$*d* and then *add the result* to 7 *times* £4, 2*s* $3\frac{5}{6}$*d* , or

(ii) reducing $7\frac{3}{11}$ to the improper fraction $\frac{80}{11}$, find $\frac{80}{11}$ of £4, 2*s* $3\frac{5}{6}$*d*

Adopting the latter method, we first multiply $\frac{5}{6}$*d* by 10, obtaining $8\frac{1}{3}$*d* , so we set down $\frac{1}{3}$ and "carry" 8, &c

Also, when we reach the division of the pence by 11, there remain over $2\frac{2}{3}$*d*, and $2\frac{2}{3} \div 11 = \frac{8}{3} \times \frac{1}{11} = \frac{8}{33}$

	£	*s*	*d*	
	4	2	$3\frac{5}{6}$	
			10	
	41 .	3	$2\frac{1}{3}$	
			8	
11	329	5 .	$6\frac{2}{3}$	
	29	18 .	$8\frac{8}{33}$	*Ans*

Note —It must be left to the judgment of the student to decide in any particular case which of the two above-mentioned methods to adopt In this example the 2nd is evidently the better, but in many cases (for instance, in multiplying by $39\frac{1}{7}$) much labour would be wasted if the multiplier were reduced to an improper fraction

* We know, from page 80, that the result is not altered by reversing the order of these operations, since it was there shown that $\frac{1}{4}$ of $3 = \frac{3}{4}$ of 1, &c

EXAMPLE iii.—*Divide £27, 17s. $6\frac{3}{11}$d. by $5\frac{3}{10}$.*

Here we *must* reduce the *divisor* to an improper fraction.

Now (£27, 17s. $6\frac{3}{11}$d.) $\div 5\frac{3}{10}$
= (£27, 17s. $6\frac{3}{11}$d.) $\div \frac{53}{10}$
= (£27, 17s. $6\frac{3}{11}$d.) $\times \frac{10}{53}$

Hence we multiply by 10 and divide the result by 53, obtaining the quotient £5, 5s. 2d., with 16 pence remaining over and $\frac{8}{11}$d. not yet brought down.

Now $16\frac{8}{11}d. \div 53$
$= \frac{184}{11} \times \frac{1}{53} = \frac{184}{583}d.$

∴ the complete quotient is £5, 5s. $2\frac{184}{583}$d.

$$
\begin{array}{rlll}
 & £ & s & d \\
 & 27 & 17\ . & 6\frac{3}{11} \\
 & & & 10 \\
\hline
53) & 278 & 15\ . & 2\frac{8}{11}(\underline{5}\ £ \\
 & \underline{265} & & \\
 & 13 & & \\
 & \underline{20}\ s & & \\
53) & 275(\underline{5} & & \\
 & \underline{265} & & \\
 & 10 & & \\
 & \underline{12}\ d & & \\
53) & 122(\underline{2} & & \\
 & \underline{106} & & \\
 & 16\frac{8}{11} \div 53 & & \\
 & = \frac{184}{11} \times \frac{1}{53} = \frac{184}{583} & &
\end{array}
$$

Ans. £5, 5s. $2\frac{184}{583}$d.

In many cases it is best to avoid *compound* multiplication and division, by expressing the given compound quantity in *one single denomination*.

EXAMPLE iv.—*Find the value of* $\dfrac{142\frac{1}{7}}{83\frac{3}{14}}$ *of* 9s. $8\frac{1}{2}$d.

$$\frac{142\frac{1}{7}}{83\frac{3}{14}} \text{ of } 116\tfrac{1}{2}d. = \frac{\overset{199}{\cancel{995}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{14}}}{\underset{\underset{1}{\cancel{233}}}{\cancel{1165}}} \times \frac{\overset{1}{\cancel{233}}}{\underset{1}{\cancel{2}}}d. = 199d. = \underline{16s.\ 7d.\ Ans.}$$

EXAMPLE v.—*Find the value of* £$235\frac{1}{3} \times \dfrac{3\frac{4\frac{1}{2}}{12}}{20}$.

$$£235\tfrac{1}{3} \times \frac{3\frac{4\frac{1}{2}}{12}}{20} = £\frac{706}{3} \times \frac{3\frac{9}{24}}{20} = £\frac{706}{3} \times \frac{3\frac{3}{8}}{20} = £\frac{\overset{353}{\cancel{706}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{9}{\cancel{27}}}{8 \times \underset{10}{\cancel{20}}}$$

$$= £\frac{317.7}{8.0} = £39\tfrac{57}{80} = \underline{£39,\ 14s.\ 3d.\ Ans.}$$

Note.—The value of £$\frac{57}{80}$ may easily be found *mentally*, for £$\frac{57}{80} = \frac{57}{4}s. = 14\frac{1}{4}s.$

XXIII. FRACTIONS.

TO EXPRESS ONE CONCRETE QUANTITY AS THE FRACTION OF ANOTHER OF THE SAME KIND

The method follows at once from the definition of a fraction

For instance, as 20 *shillings* make £1,

$\therefore$ 1 *shilling* is $\frac{1}{20}$ of £1,

$\therefore$ 3 *shillings* is $\frac{3}{20}$ of £1,

and 17 *shillings* is $\frac{17}{20}$ of £1, and so on

Similarly, as 112 *lbs* make 1 cwt,

1 *lb* is $\frac{1}{112}$ of 1 *cwt.*

5 *lbs* is $\frac{5}{112}$ of 1 *cwt*, and so on

Hence, in order to express the first of two quantities as the fraction of a second quantity of the same kind, we express both quantities in terms of the same unit, *i e* we reduce them to a *common denomination*, then *the number of these units in the first gives the numerator, and the number of these units in the second gives the denominator* of the fraction which, when expressed in its lowest terms, is the fraction required.

EXAMPLE I.—*Reduce 5s 4d to the fraction of £1.*

As 5*s* 4*d* = 16 *fourpences*,
and £1 = 60 *fourpences*,

$\therefore$ the req^d fraction is $\frac{16}{60} = \frac{4}{15}$

Ans 5*s* 4*d* is $\frac{4}{15}$ of £1.

EXAMPLE ii.—*What fraction of 3 cwts is 2 qrs 7 lbs?*

As 2 qrs 7 lbs = $2\frac{1}{4}$ *qrs.*,
and 3 cwts. = 12 *qrs*,

$$\therefore \text{the req}^{\text{d}} \text{ fraction is } \frac{2\frac{1}{4}}{12} = \frac{9}{12 \times 4} = \frac{3}{16}.$$

Ans 2 qrs 7 lbs is $\frac{3}{16}$ of 3 cwts

N B —In this question the *order* in which the quantities are mentioned is *reversed.* The question is identical with the following —

Reduce 2 qrs 7 lbs to the fraction of 3 cwts.

EXAMPLE iii.—*Express $\frac{5}{9}$ of £2, 15s. 9d as the fraction of $\frac{2}{3}$ of $\frac{7}{9}$ of 12s 6d.*

$$\tfrac{5}{9}\text{ of £2, 15s 9d.} = \tfrac{5}{9}\text{ of }55\tfrac{3}{4}s,$$
$$\tfrac{2}{3}\text{ of }\tfrac{7}{9}\text{ of 12s. 6d} = \tfrac{2}{3}\text{ of }\tfrac{7}{9}\text{ of }12\tfrac{1}{2}s.,$$

$$\therefore \text{ the req}^{d}\text{ fraction is } \frac{\frac{5}{9}\times 55\frac{3}{4}}{\frac{2}{3}\times\frac{7}{9}\times 12\frac{1}{2}} = \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{9}}}\times\frac{223}{4}\times\frac{3}{\underset{1}{\cancel{2}}}\times\frac{\overset{1}{\cancel{9}}}{7}\times\frac{\overset{1}{\cancel{2}}}{\underset{5}{\cancel{25}}} = \frac{669}{140}\ \underline{Ans}$$

EXAMPLE iv.—*What fraction of 18 tons is $\frac{3}{5}$ of 5 cwts 3 qrs 12 lbs*

$$\tfrac{3}{5}\text{ of 5 cwts. 3 qrs 12 lbs} = \tfrac{3}{5}\text{ of }23\tfrac{3}{7}\ qrs.,$$
$$18\text{ tons} = 18\times 20\times 4\ qrs,$$

$$\therefore \text{ the req}^{d}\text{ fraction is } \frac{\frac{3}{5}\times 23\frac{3}{7}}{18\times 20\times 4} = \frac{\overset{1}{\cancel{3}}}{5}\times\frac{\overset{41}{\cancel{164}}}{7}\times\frac{1}{\underset{6}{\cancel{18}}\times 20\times\underset{1}{\cancel{4}}}$$
$$= \frac{41}{4200}\ \underline{Ans}$$

EXAMPLE v —*Express 2 tons 13 cwts 3 qrs 12 lbs. 4 ozs. in cwts. and the fraction of a cwt*

$$\text{2 tons 13 cwts. 3 qrs. 12 lbs 4 ozs} = 53\frac{3\frac{12\frac{1}{4}}{28}}{4}\text{ cwts.}$$

$$= 53\frac{3\frac{7}{16}}{4}\text{ cwts.} = \underline{53\tfrac{55}{64}\text{ cwts. } Ans}$$

Here the first step results from the following *mental* process:—

$$\text{As 1 lb is }\tfrac{1}{28}\text{ of 1 qr},\ \therefore 12\tfrac{1}{4}\text{ lbs is }\frac{12\frac{1}{4}}{28}\text{ qrs.;}$$

$$\text{and as 1 qr is }\tfrac{1}{4}\text{ of 1 cwt.},\ \therefore 3\frac{12\frac{1}{4}}{28}\text{ qrs is }\frac{3\frac{12\frac{1}{4}}{28}}{4}\text{ cwts.}$$

XXIV. MISCELLANEOUS EXAMPLES.

In questions which involve both Multiplication and Division much labour may often be saved by *at first merely indicating the operations by the signs and then* cancelling *before completing the work*

A (i) *Divide the continued product of* 54, 55 *and* 56 *by the continued product of* 14, 15 *and* 16.

$$\frac{\overset{9}{\overset{\cancel{18}}{\cancel{54}}} \times \overset{11}{\cancel{55}} \times \overset{1}{\overset{\cancel{4}}{\cancel{56}}}}{\underset{1}{\cancel{14}} \times \underset{1}{\underset{\cancel{3}}{\cancel{15}}} \times \underset{2}{\underset{\cancel{4}}{\cancel{16}}}} = \frac{99}{2} = 49\tfrac{1}{2} \quad \underline{\textit{Ans}}$$

A (ii) *How many miles an hour does a body travel which moves at the rate of* 16 *feet per second?*

As there are 60×60 *seconds* in 1 *hour*,

$\therefore$ the body travels $60 \times 60 \times 16$ *feet* per *hour*

$$\text{And } 60 \times 60 \times 16 \textit{ feet} = \frac{60 \times \overset{1}{\overset{\cancel{2}}{\cancel{60}}} \times \overset{2}{\cancel{16}}}{\underset{11}{\underset{\cancel{88}}{\cancel{1760}}} \times \underset{1}{\cancel{3}}} \textit{ miles} = \frac{60 \times 2}{11} \textit{ miles}$$

$$= 10\tfrac{10}{11} \textit{ miles} \quad \underline{\textit{Ans}}$$

B. *How many bits, each* $3\frac{2}{7}$ *inches long, could be cut from a yard of string, and what length would remain over?*

Expressing both lengths in the *same denomination*, namely, *sevenths* of an inch, we have

1 *yard*, or 36 *inches* = 252 *sevenths of an inch*
and $3\frac{2}{7}$ *inches* = 23 *sevenths of an inch*

Dividing 252 by 23 we find that 23 *sevenths* is contained 10 *times* in 252 *sevenths*, with 22 *sevenths* of an inch, i e $3\frac{1}{7}$ *inches* over

```
23)252(10
   23
   --
    22 sevenths
```

Ans 10 bits, $3\frac{1}{7}$ *in* over.

C. *Multiply* $999\frac{20}{23}$ *by* 173, *shortly*

$$999\tfrac{20}{23} \times 173 = 1000 \times 173 - \tfrac{3}{23} \times 173$$
$$= 173000 - \tfrac{519}{23}$$
$$= 173000 - 22\tfrac{13}{23} = 172977\tfrac{10}{23} \quad \underline{\textit{Ans}}$$

D. *Find the G C.M and L C.M. of $3\frac{3}{5}$, $2\frac{1}{4}$ and $3\frac{3}{8}$.*

Reducing the given fractions to their *least common denominator* we have

$$3\tfrac{3}{5},\ 2\tfrac{1}{4},\ 3\tfrac{3}{8} = \frac{18}{5},\ \frac{9}{4},\ \frac{27}{8} = \frac{144}{40},\ \frac{90}{40},\ \frac{135}{40}$$

Now the G C M of 144, 90 and 135 is 9, and the L C M is 2160

∴ the G C M of 144 *fortieths*, 90 *fortieths* and 135 *fortieths* is 9 *fortieths*, i e $\frac{9}{40}$ *Ans*, and the L C M $\frac{2160}{40}$, i e 54 *Ans.*

E. *A boy spent $\frac{3}{8}$ of his money at one shop, $\frac{4}{5}$ of the remainder at another, and had 1s $0\frac{1}{2}d$ left, how much had he at first?*

After he had spent $\frac{3}{8}$, he had $\frac{5}{8}$, of his money left

He then spent $\frac{4}{5}$ of this remainder, i e $\frac{4}{\cancel{5}}$ of $\frac{\cancel{5}}{8} = \frac{4}{8}$ of the whole sum.

∴ he spent altogether $\frac{3}{8}+\frac{4}{8}$, i.e $\frac{7}{8}$, of his money. After which he must have had $\frac{1}{8}$ of it left. Hence $\frac{1}{8}$ of his money = 1s. $0\frac{1}{2}d$

∴ he had at first 8 times 1s $0\frac{1}{2}d$ = 8s. 4d *Ans*

F. *Divide £14, 15s. 2d. between A and B so that A may have a third as much again as B*

As A's share = $1\frac{1}{3}$ times B's share,

∴ A's share + B's share = $2\frac{1}{3}$ times B's share = £14, 15s 2d

Hence, dividing £14, 15s 2d by $2\frac{1}{3}$ we obtain B's share, £6, 6s 6d
And subtracting this from the whole we obtain A's share, £8, 8s 8d } *Ans*

In order to *compare* fractions we may reduce them to a *common numerator* instead of to a common denominator, and thus sometimes save a considerable amount of labour.

G. *Arrange the fractions $\frac{2}{19}$, $\frac{3}{29}$, $\frac{4}{37}$ and $\frac{5}{47}$ in descending order of magnitude.*

$$\frac{2}{19},\ \frac{3}{29},\ \frac{4}{37},\ \frac{5}{47} = \frac{1}{\frac{19}{2}},\ \frac{1}{\frac{29}{3}},\ \frac{1}{\frac{37}{4}},\ \frac{1}{\frac{47}{5}}$$

$$= \frac{1}{9\frac{1}{2}},\ \frac{1}{9\frac{2}{3}},\ \frac{1}{9\frac{1}{4}},\ \frac{1}{9\frac{2}{5}}$$

Now as the *numerators* of these fractions are all *equal*, the *greatest* fraction is that which has the *least denominator*, i.e. the third fraction; and so on

Hence we have $\frac{4}{37}$, $\frac{5}{47}$, $\frac{2}{19}$, $\frac{3}{29}$ as the *descending* order.

XXV. SIMPLE PRACTICE.

An **Aliquot Part** of any quantity is *exactly* contained in that quantity.

For instance, 5*s* is an *aliquot part* of £1, for 5*s* is contained *exactly four times* in £1

Also, 1*s* 3*d* is an *aliquot* part of 10*s*, for 1*s* 3*d* is contained *exactly eight times* in 10*s*.

Hence, if an Aliquot Part be expressed as a fraction in its lowest terms of the quantity, *the* **numerator** *is always* **1.**

For instance, 5*s* is $\frac{1}{4}$ of £1, and 1*s* 3*d* is $\frac{1}{8}$ of 10*s*

Practice is a method of finding, by aliquot parts, the value (or weight, &c) of a *quantity*, when the value (or weight, &c) of a *unit* of the same is given

It is called **Simple,** or **Compound,** according as the *quantity* is *simple*, or *compound*

For instance, the value of 712 tons at £1, 5*s* 6*d* per ton may be found by *Simple* Practice, the value of 5 tons 11 cwts 1 qr at £1, 5*s* 6*d* per ton by *Compound* Practice.

In **Simple Practice** the general method of procedure is as follows —The *given value* (or weight, &c) is separated into portions such that the first is some unit, or multiple of the unit, and each of the others is an aliquot part of this unit, or of some preceding portion

For instance, to find the cost of 712 tons at £1, 7*s* 6*d* per ton, we separate the *given value* £1, 7*s* 6*d* into the portions £1, 5*s*, and 2*s* 6*d*, where £1 is a unit, 5*s* is an aliquot part ($\frac{1}{4}$) of £1, and 2*s* 6*d* is an aliquot part ($\frac{1}{2}$) of 5*s* We then reason thus

The cost of 712 tons at **£1** per ton is evidently **£712**

And, as 5*s* is $\frac{1}{4}$ of £1, the cost of any number of things at 5*s* each must be $\frac{1}{4}$ of their cost at £1 each

the cost of 712 tons at **5***s*. per ton is $\frac{1}{4}$ of £712, *i e* **£178**

Also, as 2*s* 6*d* is $\frac{1}{2}$ of 5*s*, the cost of any number of things at 2*s* 6*d* each must be $\frac{1}{2}$ of their cost at 5*s* each

the cost of 712 tons at **2***s* **6***d* per ton is $\frac{1}{2}$ of £178, *i e* **£89**

Hence the cost of 712 tons at **£1, 7***s* **6***d* per ton is **£712 + £178 + £89,** *i e* £979.

Note —The principle involved in the last step is (II) on page 10, as employed in Compound Multiplication,

i e 712 times £1, 7*s* 6*d*. = 712 times £1 + 712 times 5*s* + 712 times 2*s* 6*d*

Hence it appears that by the method of Simple Practice the result of any question in Compound Multiplication may be obtained

The following examples illustrate some of the many ways in which the method of Simple Practice may be employed —

EXAMPLE i.—*Find the cost of* 233 *things at* 3*s.* 4*d each.*

Here, as 3*s* 4*d* is an aliquot part of £1, namely $\frac{1}{6}$ of £1, it is only necessary to *divide* the cost of 233 things at £1 each, *by 6,* in order to obtain their cost at 3*s* 4*d* each.

Written Work.

	£	*s*	*d*	
3*s* 4*d* = $\frac{1}{6}$	233 .	0	0	= cost at £1 each.
Ans	38	16	8	= . . 3*s* 4*d*....

EXAMPLE ii —*Find the cost of* 3281 *lemons at* 1½*d each.*

3281 things at 1*s* each would cost 3281*s*, and, as 1½*d* is $\frac{1}{8}$ of 1*s*, ∴ 3281 things at 1½*d* each will cost $\frac{1}{8}$ of 3281*s*

	s	*d*	
1½*d* = $\frac{1}{8}$	3281	0	= cost at 1*s* each.
2̦0	41̦0	1½	= .... 1½*d* . .
Ans	£20, 10*s* 1½*d*		

We therefore divide 3281*s* by 8, and then express the result in £, *s d.*

EXAMPLE iii —*Find the cost of* 1526 *things at* 8*s* 4*d. each.*

We separate 8*s* 4*d* into the aliquot parts 6*s* 8*d* ($\frac{1}{3}$ of £1), and 1*s* 8*d* ($\frac{1}{4}$ of 6*s* 8*d*)

	£	*s*	*d*		
6*s* 8*d* = $\frac{1}{3}$	1526	0 .	0	= cost at £1 each	
1*s* 8*d* = $\frac{1}{4}$	508	13 .	4	= . 6*s* 8*d*	(i)
	127	3 .	4	= . . 1*s* 8*d*	(ii)
Ans	635	16	8	= 8*s* 4*d.*	

Dividing £1526 (the cost at £1) by 3, we obtain the cost at 6*s* 8*d* (i). Dividing the cost at 6*s* 8*d* by 4, we obtain the cost at 1*s* 8*d.* (ii). Finally, adding (i) and (ii) together, we obtain the cost at 8*s* 4*d*

EXAMPLE iv —*Find the cost of* 425 *articles at* £3, 7*s* 9¼*d each.*

We separate £3, 7*s* 9¼*d* into the portions— £3 (a multiple of the unit £1); 5*s* ($\frac{1}{4}$ of £1), 2*s* 6*d* ($\frac{1}{2}$ of 5*s*), 3*d* ($\frac{1}{10}$ of 2*s* 6*d*), and ¼*d* ($\frac{1}{12}$ of 3*d.*). .

	£	*s*	*d*	
5*s.* = $\frac{1}{4}$	425 .	0 .	0	= cost at £1 each.
			3	
	1275 .	0 .	0	= ... £3 .....
2*s* 6*d.* = $\frac{1}{2}$	106	5 .	0	= . 5*s* . .
3*d* = $\frac{1}{10}$	53	2 .	6	= .. . 2*s* 6*d* .
¼*d* = $\frac{1}{12}$	5 .	6 .	3	= . 3*d* ..
		8	10¼	= . ¼*d.* ..
Ans	1440	2	7¼	= £3, 7*s* 9¼*d*

We then write down £425, the cost of 425 things at £1 each, which we multiply by 3, obtaining the cost at £3 each; &c

EXAMPLE V —*Find the cost of* 561 *tons at* £1, 14*s* 10¼*d. per ton*

Here two aliquot parts of £1 are used, namely, 10*s* and 4*s*, so we divide the top line first by 2, and then again by 5 Also, the top line (the cost at £1) is not cut off, but is included in the addition

	£	s	d		
	561	0	0	=	cost at £1 per ton
10*s* = ½	280	10	0	=	10*s*
4*s*. = ⅕	112	4	0	=	4*s*
8*d* = ⅙	18	14	0	=	8*d*
2*d* = ¼	4	13	6	=	2*d*
¼*d* = ⅛		11	8¼	=	¼*d*
Ans	977	13	2¼	=	£1, 14*s* 10¼*d*.

EXAMPLE VI.—*Find the cost of* 216 *lbs at* 3*s* 4½*d per lb*

	s		
	216	=	cost at 1*s*
	3		
	648	=	3*s*
3*d* = ¼	54	=	3*d*
1½*d* = ½	27	=	1½*d*
2,0	72,9	=	3*s* 4½*d*.
Ans	£36, 9*s*		

or thus —

	£	s		
	216	0	=	cost at £1
3*s* 4*d* = ⅙	36	0	=	3*s* 4*d*
½*d*. = $\frac{1}{80}$		9	=	½*d*
Ans	£36, 9*s*		=	3*s* 4½*d*

When the given price falls very little short of an exact number of pounds (or shillings), labour may be saved by first finding *the cost at this higher value,* and then **subtracting from it the excess cost,** as in the following examples —

EXAMPLE VII —*Find the value of* 4125 *sheep at* £2, 18*s each.*

Here we multiply the cost at £1 by **3**, obtaining the cost at **£3** (1) We then find the cost at **2*s***, and **subtract** it from (1)

	£	s	d			
2*s* = $\frac{1}{10}$	4125	0	0	=	value at £1 each	
			3			
	12375	0	0	=	£3	(1)
	412	10	0	=	2*s*	
Ans	11962	10	0	=	£2, 18*s*	

EXAMPLE VIII —*Find the cost of* 1206 *things at* 17*s* 4½*d each.*

Here, as 17*s* 4½*d* falls short of £1 by only 2*s* 7½*d*., we separate 2*s* 7½*d* into aliquot parts, and having obtained the corresponding costs, we *add these and subtract from the top line in one operation.* (See page 39)

	£	s	d		
	1206	0	0	=	cost at £1 each
2*s* = $\frac{1}{10}$	120	12	0	=	2*s*
6*d* = ¼	30	3	0	=	6*d*
1½*d* = ¼	7	10	9	=	1½*d*
Ans	1047	14	3	=	17*s* 4½*d*

It sometimes happens that one of the portions into which we separate the given value is so small an aliquot part of the preceding portion that its value cannot be obtained by ordinary short division in one step, in which case we insert *an extra step, which we cancel* before the final addition, or subtraction

EXAMPLE ix —*Find the cost of* 4763 *tons at* 19*s.* 7*d per ton.*

To obtain the cost at 5*d* from the cost at £1, we insert an extra step (the cost at 5*s*), which we cancel before subtracting

	£	s	d	
5*s* = $\frac{1}{4}$	4763	0 .	0	= cost at £1 per ton
5*d* = $\frac{1}{12}$	~~1190~~	~~15~~ .	~~0~~	= . . ~~5~~*s*. . ...
	99 .	4 .	7	= 5*d* . . ..
Ans	4663	15	5	= . . 19*s*. 7*d*. . .

When the quantity, the cost of which is required, is expressed as a mixed number, and the fractional part is such that the corresponding fraction of £1 can be exactly expressed in current coin, we proceed thus —

EXAMPLE x.—*Find the cost of* $87\frac{3}{4}$ *tons at* £1, 9*s* 3*d. per ton.*

As $\frac{3}{4}$ of a ton, at £1 per ton, would cost 15*s*

∴ $87\frac{3}{4}$ tons, at £1 per ton, will cost £87, 15*s*

	£	s	d	
4*s* = $\frac{1}{5}$	87	15 .	0	= cost at £1 per ton
5*s* = $\frac{1}{4}$	17	11 .	0	= . . .. 4*s* ..
3*d*. = $\frac{1}{20}$	21	18	9	= . .. 5*s* . ...
	1 .	1 .	$11\frac{1}{4}$	= . . 3*d* .. . .
Ans	128 .	6 .	$8\frac{1}{4}$	= .. .. £1, 9*s* 3*d*.

If, however, the fractional part of the quantity is such that the corresponding fraction of £1 cannot be expressed exactly in current coin, we proceed thus —

EXAMPLE xi.—*Find the cost of* $341\frac{2}{7}$ *tons at* £1, 3*s* $1\frac{1}{4}$*d. per ton*

Here as $\frac{2}{7}$ of £1 would yield a fraction of a penny, the labour of the succeeding divisions would be greatly increased if we included this in the top line, we therefore find the cost of $\frac{2}{7}$ of a ton separately, and include the result in the final addition

	£	s.	d
	1 .	3 .	$1\frac{1}{4}$
			2
7)	2 .	6 .	$2\frac{1}{2}$
		6 .	$7\frac{3}{14}$

	£	s	d	
2*s* = $\frac{1}{10}$	341	0 .	0	= cost of 341 tons at £1 per ton.
1*s* = $\frac{1}{2}$	34	2	0	= . 2*s* . .... .
1*d* = $\frac{1}{12}$	17	1	0	= . 1*s* . .. .. .
$\frac{1}{4}$*d* = $\frac{1}{4}$	1	8	5	= 1*d* . .
		7	$1\frac{1}{4}$	= $\frac{1}{4}$*d* ....
		6 .	$7\frac{3}{14}$	= cost of $\frac{2}{7}$ ton
Ans.	394	5 .	$1\frac{13}{28}$	= cost of $341\frac{2}{7}$ tons at £1, 3*s* $1\frac{1}{4}$*d*

EXAMPLE XII —*Find the quantity of coal in* $953\frac{1}{4}$ *truck-loads, averaging* 9 *tons* 13 *cwts* 2 *qrs per truck*

	tons	cwts	qrs		
10 cwts $= \frac{1}{2}$	953 .	5	0	=	quantity at 1 ton per truck
			9		
	8579 .	5 .	0	=	9 tons ..
2 cwts. $= \frac{1}{5}$	476 .	12 .	2	=	. 10 cwts ..
1 cwt. $= \frac{1}{2}$	95 .	6 .	2	=	. 2 cwts
2 qrs. $= \frac{1}{2}$	47 .	13	1	=	1 cwt.
	23	16 .	$2\frac{1}{2}$	=	. 2 qrs
	9222	13	$3\frac{1}{2}$	=	9 tons 13 cwts 2 qrs

Ans. 9222 tons 13 cwts $3\frac{1}{2}$ qrs

Many questions which have the form of a question on Compound Practice may with advantage be treated by the methods of Simple Practice

EXAMPLE XIII —*A bankrupt pays a dividend of* 17*s.* $10\frac{1}{2}$*d. in the pound, find the loss sustained by a creditor to whom he owes* £892, 13*s* 4*d*

The bankrupt pays a *dividend* of 17*s* $10\frac{1}{2}$*d* in the pound
i e he pays 17*s* $10\frac{1}{2}$*d* for each £1 he owes
∴ his creditors lose 2*s* $1\frac{1}{2}$*d* on each £1 due to them

	£	s	d		
2*s.* $= \frac{1}{10}$	892	13 .	4	=	loss at the rate of £1 in the pound
$1\frac{1}{2}$*d.* $= \frac{1}{16}$	89	5	4	=	2*s* ..
	5 .	11 .	7	=	. . $1\frac{1}{2}$*d*
Ans	94 .	16 .	11	=	. . 2*s* $1\frac{1}{2}$*d* ....

EXAMPLE XIV.—*Find the cost of* 17 *cwts.* 3 *qrs.* 21 *lbs at* 4*s.* 2*d. per qr.*

Here, as 17 cwts. 3 qrs 21 lbs. $= 71\frac{3}{4}$ qrs, we proceed thus:—

	£	s	d		
3*s* 4*d* $= \frac{1}{6}$	71	15 .	0	=	cost at £1 per qr
10*d* $= \frac{1}{4}$	11 .	19	2	=	3*s* 4*d* .
	2	19	$9\frac{1}{2}$	= ...	10*d* ..
Ans	14	18	$11\frac{1}{2}$	=	4*s* 2*d.* ...

EXAMPLE XV —*Find the rent of* 893 A 2 R 20 P *at* £1, 7*s* 6*d. per acre.*

893 A 2 R. 20 P. $= 893$ A $2\frac{1}{2}$ R $= 893\frac{5}{8}$ A

	£	s	d		
5*s* $= \frac{1}{4}$	893 .	12	6	=	rent at £1 per acre
2*s.* 6*d* $= \frac{1}{2}$	223	8	$1\frac{1}{2}$	=	5*s*
	111	14	$0\frac{3}{4}$	= ..	2*s* 6*d* .
Ans	1228	14	$8\frac{1}{4}$	=	.... £1, 7*s.* 6*d.*...

XXVI. COMPOUND PRACTICE.

In Compound Practice *the compound quantity* is treated in the same way as *the given value* is treated in Simple Practice.

For instance, to find the cost of 5 tons 11 cwts 2 qrs at £1, 7*s* 6*d* per ton, we separate the *compound quantity* 5 tons 11 cwts 2 qrs into the portions 5 tons, 10 cwts; 1 cwt, and 2 qrs, where 5 tons is a multiple of the unit 1 ton, 10 cwts is an aliquot part (½) of 1 ton; 1 cwt is an aliquot part ($\frac{1}{10}$) of 10 cwts, and 2 qrs is an aliquot part (½) of 1 cwt. We then reason thus

The cost of **5** tons is 5 times £1, 7*s* 6*d*, *i.e* **£6, 17*s* 6*d*.**

And, as 10 cwts is ½ of 1 ton, the cost of 10 cwts must be ½ of the cost of 1 ton

∴ the cost of **10** cwts is ½ of £1, 7*s* 6*d*, *i e* **13*s* 9*d***

Similarly, the cost of **1** cwt. is $\frac{1}{10}$ of 13*s* 9*d*, or **1*s*. 4½*d***

And the cost of **2** qrs is ½ of the cost of 1 cwt, *i e.* **8¼*d***

Hence the cost of **5** tons **11** cwts **2** qrs.

is **£6, 17*s* 6*d* + 13*s* 9*d* + 1*s* 4½*d* + 8¼*d***, *i e.* £7, 13*s* 3¾*d*.

EXAMPLE i.—*Find the cost of* 1 *ton* 2 *cwts* 3 *qrs* 5 *lbs* 8 *ozs. at* £5, 12*s per cwt*

Here, as the cost per *cwt* is given, we multiply £5, 12*s* by 22, obtaining the cost of 22 cwts, *i e.* of 1 ton 2 cwts

We now separate 3 qrs. 5 lbs 8 ozs into the aliquot parts—2 qrs. (½ of 1 cwt); 1 qr (½ of 2 qrs.), 4 lbs. (⅐ of 1 qr.): 1 lb. (¼ of 4 lbs), and 8 ozs (½ of 1 lb), &c

	£	s	d	
2 qrs = ½	5 .	12	0	= cost of 1 cwt.
			2	
	11	4	0	
			11	
	123	4	0	= . . 22 cwts
1 qr = ½	2	16	0	= . 2 qrs
4 lbs = ⅐	1	8	0	= . . 1 qr.
1 lb = ¼		4 .	0	= 4 lbs
8 ozs = ½		1 .	0	= .... 1 lb
			6	= . . 8 ozs.
Ans	127	13 .	6	= . 22 cwt. 3 qr. 5 lb. 8 oz.

EXAMPLE ii —*Find the rent of* 11 *ac.* 3 *ro* 24 *po at* £2, 12*s.* 6*d. per acre.*

Here, as 11 ac 3 ro 24 po differs from 12 ac. by only 16 po, we adopt the "excess method".

	£	s	d	
16 po = $\frac{1}{10}$	2	12 .	6	= rent of 1 ac
			12	
	31 .	10 .	0	= . . 12 ac
		5	3	= . .. 16 po
Ans	31 .	4 .	9	= .. . . 11 ac 3 ro. 24 po.

EXAMPLE III —*Find the value of* 1 *oz* 13 *dwts* 16 *grains of gold at* £3, 17*s* $10\frac{1}{2}$*d per oz*

		£	s	d		
10 dwts	$= \frac{1}{2}$	3	17	$10\frac{1}{2}$	=	value of 1 oz
2 dwts	$= \frac{1}{5}$	1	18	$11\frac{1}{4}$	=	10 dwts
1 dwt	$= \frac{1}{2}$		7	$9\frac{9}{20}$	=	2 dwts
12 grs	$= \frac{1}{2}$		3	$10\frac{29}{40}$	=	1 dwt
4 grs	$= \frac{1}{3}$		1	$11\frac{29}{80}$	=	12 grs
				$7\frac{63}{80}$	=	4 grs
	Ans	6	11	$1\frac{3}{40}$	=	1 oz 13 dwt 16 grs

Here, in dividing £1, 18*s* $11\frac{1}{4}d$ by 5, there remains $2\frac{1}{4}d$ over from the pence, and $2\frac{1}{4}d - 5 = \frac{9}{4} \times \frac{1}{5} = \frac{9}{20}d$

Similarly in dividing 7*s* $9\frac{9}{20}d$ by 2, there remains $1\frac{9}{20}d$ over from the pence, and $1\frac{9}{20}d - 2 = \frac{29}{20} \times \frac{1}{2} = \frac{29}{40}d$, and so on

Again, in the addition, $\frac{1}{2} + \frac{1}{4} + \frac{9}{20} + \frac{29}{40} + \frac{29}{80} + \frac{63}{80} = \frac{40+20+36+58+29+63}{80}$ $= \frac{246}{80} = 3\frac{3}{40}d$, so we set down $\frac{3}{40}d$ and "carry" 3 to the pence column

Note —In actual business, results are often only required *correct to the nearest penny* in such case the fraction $\frac{3}{40}d$ in this result would be neglected It would, however, still be necessary to retain the fractions of a penny in the working, for the sum of these yields 3 *complete pence.*

In some cases we use *both* kinds of Practice, for instance —

EXAMPLE IV —*Find the value of* 451 *tons* 12 *cwts* 1 *qr* 11 *lbs at* £3, 13*s* 4*d per ton*

Here we find the value of 451 tons at £3, 13*s* 4*d* per ton, by *Simple* Practice, and then the value of 12 cwts 1 qr 11 lbs, at £3, 13*s* 4*d* per ton, by *Compound* Practice, adding in the first result

		£	s	d		
6*s* 8*d*	$= \frac{1}{3}$	451	0	0	=	value of 451 tons at £1 per ton
6*s* 8*d*	$= \frac{1}{3}$			3		
		1353	0	0	=	£3
		150	6	8	=	6*s* 8*d*
		150	6	8	=	6*s* 8*d*
		1653	13	4	=	£3, 13*s* 4*d* *

		£	s	d		
10 cwts	$= \frac{1}{2}$	3	13	4	=	value of 1 ton
2 cwts	$= \frac{1}{5}$	1	16	8	=	10 cwts
1 qr	$= \frac{1}{8}$		7	4	=	2 cwts
7 lbs	$= \frac{1}{4}$			11	=	1 qr
4 lbs	$= \frac{1}{7}$			$2\frac{3}{4}$	=	7 lbs
				$1\frac{4}{7}$	=	4 lbs
		1653	13	4	=	451 tons *
	Ans	1655	18	$7\frac{9}{28}$	=	451 tons 12 cwts 1 qr 11 lbs.

XXVII. INVOICES.

An **Invoice**, or **Bill**, is a written statement made by the seller and delivered to the buyer, with the goods, at the time of purchase, showing the *quantity*, *kind*, and *cost* of each article supplied

Specimen of an Invoice, or Bill

REV W JONES LONDON, Jan. 21st, 1892.

Bought of SMITH & Co,
School Stationers,
ALDERSGATE, E C

			£	s	d
	2 gross Penholders	2/3		4	6
	2 „ Pencils	4/6		9	0
	3 doz Copy-books	1/6		4	6
	6 Slates	5*d*		2	6
			1	0	6

Here the column on the left of the £, s d columns gives the price per gross, dozen, or unit as the case may be, while opposite, in the £, s d. columns, is placed the cost of the quantity of each article purchased. The column on the extreme left, intended for dates, is blank in this case, as all the articles were supplied on the date given at the head of the bill

The *quantity* and *cost* of any **one** article mentioned in the Invoice is called an **Item.**

When *credit* is given (*i e* when the buyer does not pay "ready" money), a further statement is sent at intervals (usually of a quarter, or half-year), by the seller to the buyer, showing the dates and amounts of the invoices which then remain unpaid, with the total sum of money owing this is called an **Account.**

Specimen of an Account

REV W JONES LONDON, June, 1892.

Dr to SMITH & Co,
School Stationers,
ALDERSGATE, E C

		£	s	d
Jan 21	To Goods as per Invoice	1	0	6
Feb 2	„ „ „	2	7	2
April 19	„ „ „	1	16	8
May 3	„ „ „	4	12	5
		9	16	9

A Detailed Account is one in which each Invoice, or Bill, is reproduced in full, *i e* in which all the *Items* are mentioned.

Specimen of a Detailed Account

MRS BROWN		STAFFORD, Mids, 1892			
	Bought of ROBINSON BROTHERS, Drapers &c, 85 and 86 Greengate Street				
April 4	19 yds Dutch carpet	2/11	2	15	5
—	Making same			5	
—	2 doz drugget pins				5
May 2	$\frac{1}{2}$ doz table-napkins	16/6		8	3
—	27 yds calico	7*d*		15	9
June 7	1 doz buttons				$2\frac{1}{4}$
—	$\frac{5}{8}$ yd. oil-cloth	1/9		1	$1\frac{1}{4}$
			4	6	$1\frac{1}{2}$

Here the *exact* cost of the last item is 1*s* $1\frac{1}{8}d$ In such cases it is customary to charge the *farthing next above* the exact value

Note —Tradesmen often make an allowance, called "Trade Discount", for ready money, or for prompt payment of an account For instance, suppose the tradesmen mentioned in the above specimen of a Detailed Account make it a rule to allow a deduction of 6*d* in the pound on accounts paid within a month, then 2*s* $1\frac{1}{2}d$ would be the discount subtracted from this particular account, namely, 6*d* on each of the 4 pounds and $1\frac{1}{2}d$ on 5*s* (*i e* $\frac{1}{4}$ of £1), any remainder less than 5*s* being neglected for this purpose

In working the exercises on this chapter the student should endeavour to calculate the items with as little *written* work as possible

Towards this end he should bear in mind

(i) That when the price of one article is given in pence and farthings, the price per dozen is obtained at once by taking a shilling for each *penny*, and three-pence for each *farthing* of the given price

For instance, 1 doz. at $7\frac{3}{4}d$ each cost 7*s* 9*d*,
and 7 doz at $9\frac{1}{4}d$ each cost 7 times 9*s* 3*d*, *i e* £3, 4*s* 9*d*

Hence, when the number of articles, the cost of which is required, but little exceeds, or falls short of, an *exact* number of *dozens*, it is well to calculate the *dozens* separately

For instance, 37 at $2\frac{3}{4}d$ each cost 3 times 2*s* 9*d* + $2\frac{3}{4}d$, *i e* 8*s* $5\frac{3}{4}d$

(ii) The *aliquot parts* of £1 and 1*s*

For instance, 30 at 3*s* 4*d* each cost 30 times £$\frac{1}{6}$, *i e* £5.

(iii) That when the *cost* but little exceeds, or falls short of, an *exact* number of pounds, or shillings, the "excess method" may be used with advantage

For instance, 17 at $11\frac{3}{4}d$ each cost 17*s* − 17 *farthings*, *i e.* 16*s* $7\frac{3}{4}d$

XXVIII. DECIMALS.

NOTATION.

We have seen, in Chapter I., that our system of notation is called the *Decimal* system on account of the *ten*-fold change in local value, a tenfold *increase* in each successive place towards the *left*, and, consequently, a corresponding *decrease* in each successive place towards the *right*

Thus the local value of a figure in any place is **one-tenth** of the value it would possess in *the preceding place on the left*

For instance, in the number 333, representing 3 *hundreds* + 3 *tens* + 3 *units*, the value of the middle 3 is *one-tenth* of that of the left-hand 3, and the value of the right-hand 3 is *one-tenth* of that of the middle 3

If, then, we regard the position of the units' place as fixed, and *extend the operation of this law to places to the right of the units*, the local value of a figure in *the place immediately to the right of the units' place* will also be *one-tenth* of the value it would possess *in* the units' place; *i.e.* a figure standing in this new place will represent tenth-parts of unity, or *tenths*

Similarly, the next figure on the right will represent tenth-parts of one-tenth of unity, or *hundredths*, and so on.

Hence it appears that figures placed *to the right* of the units' place represent *parts* of unity, or fractions, on the same system that figures placed *to the left* of the units' place represent *multiples* of unity, or integers, and such fractions are, consequently, called *Decimal* Fractions, or shortly, "Decimals"*

Thus, a **Decimal** is a number of *tenths, hundredths, thousandths, &c*

i.e. in the case of a *decimal* fraction the *number of equal parts* into which the unit is divided is always some **power**† **of ten.**

Hitherto, when *integers only* have been thus represented, the units' place has always been occupied by *the figure on the extreme right* of the number, but when figures, representing *parts* of unity, stand *to the right* of the units' place, *it becomes necessary to mark the position of the units' place so that it may not be lost sight of.*

This is done by placing a *dot*, called the **Decimal Point,** *immediately to the right of the units' place.*

For instance, 33·3 represents 3 *tens* + 3 units + 3 *tenths*, and 333·33 represents 3 *hundreds* + 3 *tens* + 3 units + 3 *tenths* + 3 *hundredths*.

Again, ·07 represents 0 *tenths* + 7 *hundredths*, *i.e.* 7 *hundredths*; any *vacant decimal place* being occupied by a *cipher*

---

* This development of the Arabic system is due to English mathematicians at the beginning of the 17th century

† See page 12

A decimal may be "read" in the same way as an integer:

For instance, $\cdot 34$ is 3 *tenths* + 4 *hundredths*,
and as we know, by the *place-law*, that ten *hundredths* make one *tenth*,
3 *tenths* = 30 *hundredths*,
and 3 *tenths* + 4 *hundredths* = 34 *hundredths*
i e $\cdot 34$ may be read as "Thirty-four *hundredths*"
Similarly, $\cdot 257$, *i e* 2 *tenths* + 5 *hundredths* + 7 *thousandths*,
may be read "Two-hundred-and-fifty-seven *thousandths*,"

i e we may read the decimal as if it were an integer, and then mention *the local value of the last figure*

In practice, however, decimals are very commonly read figure by figure, the word "decimal" (or "point") being used to indicate the decimal point

For instance, $\cdot 402$ is commonly read thus "Decimal, four, nought, two"
And 25 0087, thus "Twenty-five, decimal, nought, nought, eight, seven."

Ciphers on the extreme right of a decimal do not affect its value

For instance, $\cdot 3$ is 3 *tenths*, and $\cdot 30$ is 3 *tenths* + 0 *hundredths*,
i e $\cdot 30$ is neither greater, nor less, than $\cdot 3$

Hence, **we may append ciphers,** or **strike off ciphers, on the extreme right** of a decimal with impunity

To multiply a Decimal by 10,

Move the decimal point **one** *place to the* **right.**

For, by so doing, the *units* become *tens*, the *tenths* become *units*, the *hundredths* become *tenths*, &c

i e each figure has its value increased *tenfold*, and, consequently, the entire decimal is multiplied by 10

For instance, $3\ 45 \times 10 = 34\cdot 5$, and $\cdot 0057 \times 10 = \cdot 057$

Similarly, to multiply a decimal by 100, 1000, &c, *move the decimal point* **two, three** &c *places to the* **right,** *respectively*

For instance, $12\cdot 503 \times 100 = 1250\ 3$, and $\cdot 08537 \times 1000 = 85\cdot 37$.

Conversely, to divide a Decimal by 10,

Move the decimal point **one** *place to the* **left**

For, by so doing, the *units* become *tenths*, the *tenths* become *hundredths*, &c, and thus the entire decimal is divided by 10

For instance, $3\cdot 45 \div 10 = \cdot 345$, and $\cdot 0057 - 10 = 00057$

Similarly, to divide a decimal by 100, 1000, &c, *move the decimal point* **two, three** &c *places to the* **left,** *respectively*

For instance, $1250\cdot 3 \div 100 = 12\cdot 503$, and $\cdot 08537 - 1000 = \cdot 00008537$.

XXIX. DECIMALS.

ADDITION AND SUBTRACTION.

As Decimals are expressed in the same way as integers, it follows that they can be added, or subtracted, in the same way

For instance, just as 3 *tens* and 4 *tens* make 7 *tens*,
so 3 *tenths* and 4 *tenths* make 7 *tenths*, and so on.

For convenience then we arrange the decimals to be added, or subtracted, so that the *tenths* are all in one vertical column, the *hundredths* all in another, and so on, and this is ensured if *all the decimal points are in one vertical column*, we then proceed exactly as in Simple Addition, or Subtraction, remembering, however, that we may, if desirable, *append ciphers to the extreme right* of any of the given decimals, and may also *reject ciphers on the extreme right* of any *result.*

EXAMPLE I —*Add together* 4·532, 17·06, ·3574 *and* 1·8.

Here, arranging the given decimals so that all the decimal points are in one vertical column, we add each vertical column, beginning with that on the extreme right, and "carry" just as in Simple Addition, not forgetting to insert the decimal point in the result immediately beneath the column of decimal points

```
  4·532
 17·06
   ·3574
  1·8
 -------
 23·7494 Ans
```

EXAMPLE II.—*Find the sum of* 129·546, 1·254 *and* 26.

Here it should be noticed that 26, being an integer, is placed on the *left* of the points' column Also, that in writing down the result, two superfluous ciphers on the extreme right of the result are discarded *Ans* 156·8

```
129·546
  1·254
 26·
-------
156·800
```

EXAMPLE III —*From* 4·573 *take* 2·08751.

Here, having arranged the decimals with the points in column, we append two ciphers to the right of the upper line before subtracting *

```
4·57300
2·08751
-------
2·48549 Ans.
```

EXAMPLE IV —*Simplify* 4·37451 + 190·28 − 72·003 − ·608751 + 241 − 330·0876

Here we proceed as in the example on page 8, *i e* we first *add* 4 37451, 190·28 and 241 together, we then *add* 72 003, ·608751 and 330·0876 together, and finally *subtract* the second result from the first

```
  4·37451        72·003
190 28             ·608751
241             330·0876
---------       ----------
435·65451       402·699351
402·699351
----------
 32·955159 Ans
```

* Such extra ciphers need not necessarily be *actually written* down after a little practice the student will append them *mentally*.

XXX. MULTIPLICATION OF DECIMALS.

The process of multiplying a *decimal* by an **integer** exactly corresponds to that of multiplying one integer by another.

For instance, just as 7 *units* × 2 = 14 *units*, or 1 *ten* + 4 *units*,
so 7 *tenths* × 2 = 14 *tenths*, or 1 *unit* + 4 *tenths*,
i e ·7 × 2 = 1·4

and 7 *hundredths* × 2 = 14 *hundredths*, or 1 *tenth* + 4 *hundredths*,
i e ·07 × 2 = ·14 And so on

Again as, on page 11 it was shown that

243 *units* × 75 = 18225 *units*, *i e* 243 × 75 = 18225,
so 243 *tenths* × 75 = 18225 *tenths*, *i e* 24·3 × 75 = 1822·5,
243 *hundredths* × 75 = 18225 *hundredths*, *i e* 2·43 × 75 = 182·25,
243 *thousandths* × 75 = 18225 *thousandths*, *i e* ·243 × 75 = 18·225, &c.,

the *number of decimal places* in any *product* being the *same* as the number in the corresponding *multiplicand*

Hence to multiply a Decimal by an **Integer**,

We may multiply as in Simple Multiplication, disregarding the decimal point during the operation, and afterwards insert a decimal point in the product, so as to mark off *as many* **decimal places** *as there are in the multiplicand*

EXAMPLE 1.—*Multiply* 2·105 *by* 782

$$\begin{array}{r} 2{\cdot}105 \\ \underline{782} \\ 4\,210 \\ 168\,40 \\ \underline{1473\,5} \\ \underline{1646\,110} \end{array}$$

Multiplying 2105 by 782 we obtain the product 1646110.

Now in 2·105 there are *three* decimal places

We therefore insert a decimal point in the product so as to mark off *three* decimal places

Thus 2·105 × 782 = 1646 110

Finally, *after inserting the decimal point*, we discard the cipher on the extreme right

Ans 1646·11

Again, a method of multiplying a *decimal* by a **decimal** may be deduced from the foregoing case

For instance, we have already seen that ·7 × 2 = 1·4, and we know, by the *place-law*, that ·2 is *one-tenth* of 2

Hence ·7 × ·**2** = *one-tenth* of 1·4, *i e* ·14 (see page 106).

Similarly, as ·02 is *one-hundredth* of 2,
·7 × ·**02** = *one-hundredth* of 1·4, *i e* ·014

And, as ·002 is *one-thousandth* of 2,
·7 × ·**002** = *one-thousandth* of 1·4, *i e* ·0014

And so on, the *number of decimal places* in any product being the *number in the multiplicand* increased by *the number in the multiplier*

Hence, to multiply **Decimals** together,

We may multiply as in Simple Multiplication, disregarding the decimal points during the operation, and afterwards insert a decimal point in the product, so as to mark off in it *as many* **decimal places** *as there are, altogether, in the factors.*

EXAMPLE ii.—*Multiply* 134·25 *by* ·032

Multiplying 3425 by 32 we obtain the product 429600

Now in 134·25 there are *two* decimal places, and in ·032 there are *three* decimal places

We therefore insert a decimal point in the product so as to mark off 2 + 3, or *five* decimal places.

Thus 134·25 × ·032 = 4·29600.
or, discarding the ciphers on the right, 4·296

$$\begin{array}{r} 134{\cdot}25 \\ \underline{{\cdot}032} \\ 268\ 50 \\ \underline{4027\ 5} \\ \underline{4{\cdot}296\ 00} \end{array}$$

Ans. 4·296

EXAMPLE iii.—*Find the product of* ·0673 *and* ·012.

Multiplying 673 by 12 we obtain the product 8076.

Now in ·0673 there are *four* decimal places, and in ·012 there are *three* decimal places.

∴ in the product there are 4 + 3, or *seven* decimal places

So we must *prefix* three ciphers to the *left* of 8076 before we can mark the position of the decimal point

$$\begin{array}{r} {\cdot}0673 \\ \underline{{\cdot}012} \\ \underline{{\cdot}0008076} \end{array}$$

·0008076 *Ans.*

Note.—In long operations, it is convenient to omit decimal points and *left*-hand ciphers in the working, as in the following example:—

EXAMPLE iv.—*Find the continued product of* ·0023, ·00008, 7·1 *and* 600

We find the continued product of 23, 71, 8 and 600 to be 7838400

Now in ·0023 there are *four* dec places; in ·00008 there are *five* dec. places; in 7·1 there is *one* dec place, and in 600 there is *no* dec place

Hence in the req^{d} product there must be 4 + 5 + 1 + 0, or *ten* decimal places.

We therefore *prefix* three ciphers to the *left* of 7838400 before we can mark the decimal point

Finally, we discard the two ciphers on the *right*

$$\begin{array}{r} 23 \\ \underline{71} \\ 23 \\ \underline{161} \\ 1633 \\ \underline{8} \\ 13064 \\ \underline{600} \\ \underline{7838400} \end{array}$$

∴ ·0023 × ·00008 × 7·1 × 600 = ·0007838400
= ·00078384 *Ans.*

XXXI. DECIMALS.

DIVISION BY AN INTEGER

The method is that of Simple Division, applied to decimals

For instance, *to divide* 11·48 *by* 8

1st step

11 *units* ÷ 8 yield quot[t] 1 *unit*, with rem[r] 3 *units*, so we set down **1** in the *units' place* in the quotient, *marking the decimal point* immediately on the right of it We now change the rem[r] 3 *units* into 30 *tenths*

$$8\,\underline{|\,11\ 48}\quad \mathbf{1\cdot}$$

2nd step

30 *tenths* + 4 *tenths* make 34 *tenths*, and 34 *tenths* ÷ 8 yield quot[t] 4 *tenths*, with rem[r] 2 *tenths* So we set down **4** in the *tenths' place* in the quotient and change the rem[r] 2 *tenths* into 20 *hundredths*

$$8\,\underline{|\,11\ 48}\quad 1\cdot\mathbf{4}$$

3rd step.

20 *hundredths* + 8 *hundredths* = 28 *hundredths*, and 28 *hundredths* ÷ 8 yield quot[t] 3 *hundredths*, with rem[r] 4 *hundredths* So we set down **3** in the *hundredths' place* in the quotient

$$8\,\underline{|\,11\ 48}\quad 1\ 4\mathbf{3}$$

Here, as a rem[r] occurs, we can still continue the division, although the last figure in the given dividend has been dealt with, for the 4 *hundredths* which form the remainder can be changed to 40 *thousandths*

We, therefore, append a cipher to the right of the dividend and proceed thus —

4th step

40 *thousandths* ÷ 8 yield quot[t] 5 *thousandths*, with *no* rem[r], so we write **5** in the *thousandths' place* in the quotient

$$8\,\underline{|\,11\cdot480}\quad 1\cdot43\mathbf{5}$$

Thus the complete quotient is 1·435

i e if 11 48 be divided into 8 equal parts, each of these parts is 1 435; or, 1·435 is the decimal which, when multiplied by 8, gives the product 11·48

Hence, we divide as in Simple Division, marking the *decimal point in the quotient as soon as we reach the figure in the* **tenths'** *place in the dividend*

N B —When *short* division is used this amounts to simply *copying down the decimal point in a vertical line*

Also, if, after the last of the given figures in the dividend has been dealt with, a remainder occurs, we may append ciphers, one by one as required, to the right of the dividend and continue the division

Note.—Such appended ciphers need not always be *written*, they may be appended *mentally*

EXAMPLE i.—*Divide* 2·8455 *by* 35.

Here we divide, as in Simple Division, by the factors of 35, copying down the decimal point in a vertical line, and writing a cipher in the vacant *tenths'* place of the result

$$35\left\{\begin{array}{r|l} 5 & \underline{2\cdot8455} \\ 7 & \underline{\cdot5691} \\ & \cdot0813 \ \textit{Ans.} \end{array}\right.$$

EXAMPLE ii —*Express the quotient of* 5 ÷ 8 *as a decimal.*

As there are no *units* in the quotient, we mark the decimal point on the right of the 5, change the 5 *units* into *tenths*, and so on

$$\begin{array}{r|l} 8 & \underline{5\cdot000} \\ & \cdot625 \ \textit{Ans.} \end{array}$$

EXAMPLE iii —*Divide* ·03702 *by* 600

Dividing ·03702 by 6, we obtain quotient ·00617

We now divide this result by 100, *i e* we move the decimal point *two places* to the left (See p 106)

$$\begin{array}{r|l} 6 & \underline{\cdot03702} \\ & \cdot00617 \end{array}$$

$$\therefore \cdot03702 \div 600 = \cdot0000617 \ \textit{Ans.}$$

EXAMPLE iv.—*Divide* 69·3 *by* 45000.

Dividing 69·3 by factors of 45 we obtain the quotient 1·54

We now divide this result by 1000, *i e* we move the dec point *three places* to the *left*

$$45\left\{\begin{array}{r|l} 5 & \underline{69\cdot3} \\ 9 & \underline{13\cdot86} \\ & 1\cdot54 \end{array}\right.$$

$$\therefore 69\cdot3 \div 45000 = \cdot00154 \ \textit{Ans.}$$

EXAMPLE v.—*Divide* 2·5696 *by* 73

2 ÷ 73 yields *no units*, so we *at once mark the dec point in quotient*

25 *tenths* — 73 yields no *tenths*, so we place a cipher in the *tenths'* place in the quotient, &c

```
73)2·5696(·0352 Ans.
   2 19
    379
    365
     146
     146
```

EXAMPLE vi —*Divide* 540·8 *by* 325.

540 — 325 yields 1 *unit*, so we write 1 in the quotient, and then, before dealing with the *tenths*, mark the decimal point *to the right* of the 1

2158 — 325 yields 6 *tenths* and 208 *tenths* rem[r]

We now append ciphers one by one as required to the dividend.

```
325)540·800(1·664 Ans.
    325
    2158
    1950
     2080
     1950
      1300
      1300
```

XXXII. DIVISION OF DECIMALS.

We know (see page 17) that *the* **quotient is not altered** *if we multiply both divisor and dividend by the* **same** *number*

For instance the quotient of 36 ÷ 12 is the same as that of 10 *times* 36 ÷ 10 *times* 12, *i e*, of 360 ÷ 120

Similarly, the quotient of ·36 ÷ 1·2 is the same as that of 10 *times* ·36 ÷ 10 *times* 1·2, *i e*, of **3·6 ÷ 12**

Also, the quotient of ·475 ÷ ·05 is the same as that of 100 *times* ·475 ÷ 100 *times* ·05, *i e*, of **47·5 ÷ 5**

And the quotient of 5·3 ÷ ·625 is the same as that of 1000 *times* 5·3 ÷ 1000 *times* ·625, *i e*, of **5300 ÷ 625**

Thus, the case of a *decimal* divisor can always, by multiplying both divisor and dividend by some power of ten, be reduced to that of an *integral* divisor

Hence, when the divisor is a *decimal*,

we multiply both divisor and dividend by that power of ten which makes the divisor an integer, and then proceed as in Chapter XXXI

EXAMPLE i.—*Divide* ·35 *by* ·8.

Multiplying both divisor and dividend by 10 we have

·35 ÷ ·8 = 3·5 ÷ **8**.

```
8 | 3·5
  ·4375 Ans
```

EXAMPLE ii.—*Divide* ·7 *by* ·0064.

Multiplying both divisor and dividend by 10000, we have

·7 ÷ ·0064 = 7000 ÷ **64**

```
64 { 8 | 7000
   { 8 |  875
        109·375 Ans.
```

EXAMPLE iii —*Divide* 3 *by* 31·25.

Multiplying both divisor and dividend by 100, we have

3 ÷ 31·25 = 300 ÷ **3125**

```
3125)300·000(·096 Ans
     281 25
      18 750
      18 750
```

EXAMPLE iv.—*Divide* 7·31 *by* ·0017.

Multiplying both divisor and dividend by 10000, we have

7·31 ÷ ·0017 = 73100 ÷ **17**

Here the division is that of an *integer* by an *integer*, and the resulting quotient is the *integer* 4300

```
17)73100(4300 Ans.
   68
    51
    51
     00
```

EXAMPLE v.—*Divide* ·013 *by* 2·9, *to five places of decimals.*

Multiplying both divisor and dividend by 10, we have ·013 ÷ 2·9 = ·13 ÷ 29

Here, as there is no integer in the dividend, there can be none in the quotient, so we begin by marking the decimal point in the quotient

And as 29 is not contained in 1, nor in 13, we next place *ciphers* in the *tenths'* and *hundredths'* places in the quotient.

We now carry on the division for three steps, and no further, as there are then *five places of decimals* in the quotient

```
29)·13000(·00448
    116
     140
     116
      240
      232
        8
```

Ans ·00448 +

The sign + in the result indicates that the division has not terminated.

Note—It is evident that in such a case the division can never terminate however many ciphers we append to the dividend, for there is no digit by which 9 can be multiplied so as to yield a product ending in 0.

EXAMPLE vi.—*Find the integral part of the quotient of* 38·7563 ÷ ·41, *and the decimal remainder.*

Multiplying both divisor and dividend by 100, we have 38·7563 ÷ ·41 = 3875·63 ÷ 41.

In this case we cease dividing after dealing with the *units'* figure in the dividend.

```
41)3875·63(94·
   369
    185
    164
     21·63
```

Hence, 3875·63 ÷ 41 yields quot^t 94, rem^r 21·63;

∴ 38·7563 ÷ ·41 yields quot^t 94, rem^r ·21 63,

for although the *quotient* is not altered by multiplying both divisor and dividend by 100, the *remainder* 21·63 *which is a part of the dividend*, is 100 times the required rem^r, and must consequently be now *divided* by 100.

Ans. 94; rem^r ·2163.

EXAMPLE vii—*Divide* ·416 *by* 3·25, ·416 *by* ·0325; *and* 4·16 *by* 32500.

(i) ·416 ÷ 3·25 = 41·6 ÷ 325.

Now, as the *same figures* occur in all three cases the operation need only be performed *once.*

```
325)41·600(·128 Ans.(i).
    32 5
     9 10
     6 50
     2 600
     2 600
```

(ii) ·416 ÷ ·0325 = 4160 ÷ 325

Here the *working* is the same as in (i), but the decimal point in the dividend is moved *two places* to the *right* ∴ the decimal point in the quotient will also be moved *two places* to the *right.*

Hence in this case the result is 12·8 *Ans* (ii)

(iii) 4·16 ÷ 32500 = ·0416 ÷ 325 (*dividing* both dividend and divisor by 100).

Here also the *working* is the same as in (i), but the decimal point in the dividend is moved *three places* to the *left,* ∴ the decimal point in the quotient must also be moved *three places* to the *left*

Hence in this case the result is ·000128 *Ans* (iii).

XXXIII. DECIMALS.

To Express a Decimal as a Vulgar Fraction, and a Vulgar Fraction as a Decimal

If we "read" a decimal, in the way indicated on page 106, the local value of the right-hand figure gives the *denominator* of an equivalent *vulgar fraction*

For instance, $\cdot 37$ is 37 *hundredths*, *i e*, $\frac{37}{100}$,

And $\cdot 0319$ is 319 *ten-thousandths*, *i e*, $\frac{319}{10000}$,

Also $\cdot 000001$ is 1 *millionth*, *i e*, $\frac{1}{1000000}$

Hence we see that in the *denominator* of the vulgar fraction thus obtained there must be as many ciphers as there are decimal places in the decimal

If this vulgar fraction is not in lowest terms we reduce it

Example i.—*Convert* $\cdot 025$ *to a vulgar fraction in its lowest terms*

$$\cdot 025 = 25 \textit{ thousandths} = \frac{25}{1000} = \frac{1}{40} \quad \underline{\textit{Ans}}$$

Example ii. —*Reduce* $2\cdot 108$ *to a mixed number.*

$$2\cdot 108 = 2\frac{108}{1000} = 2\frac{27}{250} \quad \underline{\textit{Ans}}$$

Example iii —*Express* $70\cdot 04$ *as an improper fraction.*

$$70\cdot 04 = \frac{7004}{100} = \frac{1751}{25} \quad \underline{\textit{Ans}}$$

Conversely, a vulgar fraction whose denominator is some *power of ten* (*i e.* 10, 100, 1000, &c) can at once be written in the form of a decimal.

For instance, $\frac{37}{100}$ is 37 *hundredths*, *i e*, $\cdot 37$,

And $\frac{9}{10000}$ is 9 *ten-thousandths*, *i e*, $\cdot 0009$,

Also $\frac{7261}{1000}$ is 7261 *thousandths*, *i e*, $7\cdot 261$

Thus a fraction having some *power of ten* as denominator is expressed as a decimal by taking the figures which form the numerator and inserting a decimal point so as to mark off as many decimal places as there are ciphers in the *denominator*.

Example iv —*Write* $\frac{217}{1000}$ *as a decimal*

$$\frac{217}{1000} = \cdot 217 \quad \textit{Ans}$$

EXAMPLE v.—*Express* $31\frac{17}{10000}$ *as a decimal.*

$$31\frac{17}{10000} = 31{\cdot}0017 \quad \underline{Ans.}$$

EXAMPLE vi.—*Express* $\frac{53467}{100}$ *as a decimal.*

$$\frac{53467}{100} = 534{\cdot}67 \quad \underline{Ans}$$

In some few cases a vulgar fraction whose denominator is **not** a *power of ten*, may also at **sight** be expressed as a decimal, for we know (see page 68) that the value of a fraction is not altered by multiplying both its numerator and denominator by the same number, and, consequently, *if the denominator of the given fraction is evidently a* **factor of 10**, or 100, or 1000, &c., the equivalent decimal can be at once written down

For instance, $\frac{3}{5} = \frac{6}{10} = {\cdot}6;$

$\frac{7}{25} = \frac{28}{100} = {\cdot}28;$

$\frac{3}{20} = \frac{15}{100} = {\cdot}15;$

and $\frac{1}{8} = \frac{125}{1000} = {\cdot}125.$

Again, we know (see page 80) that a vulgar fraction represents the *quotient resulting from the division of its numerator by its denominator*

In the case, then, of a vulgar fraction which cannot easily by inspection be expressed as a decimal, we **perform this division.**

EXAMPLE vii.—*Express* $\frac{13}{64}$ *as a decimal.*

Dividing 13 by factors of 64 we obtain the quotient ·203125 as the req^d decimal

$$64\left\{\begin{array}{l} 8\,\underline{|\,13{\cdot}\quad} \\ 8\,\underline{|\,1{\cdot}625} \\ \quad\underline{{\cdot}203125}\ \ \underline{Ans.} \end{array}\right.$$

EXAMPLE viii.—*Convert* $\frac{11}{625}$ *into a decimal.*

Dividing 11 by 625 we obtain the quotient ·0176 as the req^d decimal

Note.—In all work in Decimals involving Long Division, the "abridged" method (see p. 15) is, of course, available, if preferred.

$$\begin{array}{r l} 625)\,11{\cdot}0000 & (\underline{{\cdot}0176\ Ans.} \\ \underline{6\ 25} & \\ 4\ 750 & \\ \underline{4\ 375} & \\ 3750 & \\ \underline{3750} & \end{array}$$

XXXIV. RECURRING DECIMALS.

We have already observed (see note on page 113) that there are cases in Division of Decimals in which *a remainder must always occur*, no matter how far we proceed with the work, while in other cases the division, sooner or later, *terminates*

Hitherto we have confined our attention almost entirely to divisions which terminate, we now proceed to consider some examples of non-terminating division.

For instance, if we divide ·71 by 11, we obtain the quotient ·06454545 &c &c. where the pair of figures 45 appear over and over again in the quotient, as we continue the division

$$11\,\underline{|\cdot 71\qquad\quad}$$
$$\cdot 06454545$$

Also, if we divide 8 by 37, we obtain the quotient ·216216 &c &c where the set of figures 216 recur over and over again in the quotient.

$$
\begin{array}{r}
37)\,8\cdot 0000(\ 216216\ .\\
\underline{7\,4}\\
60\\
\underline{37}\\
230\\
\underline{222}\\
8
\end{array}
$$

Now in each of the above instances we observe that, after a certain stage is reached the remainders reappear, and consequently, in the quotients, the figures which correspond to these remainders also reappear

And this must always happen when the division does not terminate

For instance, if we divide 1 by 7 it is evident that the division can never terminate, for there is no digit by which we can multiply 7 and obtain a product ending in 0

$$7\,\underline{|1\cdot 0000000000}$$
$$\cdot 1428571428\ .$$

And there are but *six* possible remainders, namely, 1, 2, 3, 4, 5 and 6 Hence, after *six* steps at most (if not before) some one of these remainders, say 3, which has already appeared, must *re*appear, and then all succeeding stages of the division are repetitions in regular order of the earlier ones

Hence, in every non-terminating division of decimals some figure, or set of figures must, sooner or later, reappear over and over again in the quotient as the division is continued

A decimal in which some *figure*, or *set of figures arranged in the same order*, is repeated over and over again without limit, is called a **Recurring, or Circulating,** Decimal

For instance, ·47777 and ·216216216 are *recurring* decimals

Decimals which, as distinguished from recurring decimals, contain but a *limited* number of figures are called **terminating** decimals

The figure, or set of figures, which is repeated in a recurring decimal is called the recurring **Period.**

A recurring period is marked by placing a **dot** above the *first* and *last* figures in the period

For instance, $\cdot47777$ .. is written shortly thus, $\cdot4\dot{7}$,
$\cdot06454545$ .. , thus, $\cdot06\dot{4}\dot{5}$,
and $\cdot216216216$, thus, $\cdot\dot{2}1\dot{6}$

A recurring decimal is called *pure*, or *mixed*, according as **all** the figures on the right of the decimal point *do*, or *do not*, recur.

For instance, $6\cdot\dot{4}\dot{5}$, $\cdot\dot{2}1\dot{6}$ and $\cdot\dot{1}4285\dot{7}$ are *pure* recurring decimals
But $\cdot4\dot{7}$, $\cdot06\dot{4}\dot{5}$ and $\cdot9\dot{1}4285\dot{7}$ are *mixed* recurring decimals

EXAMPLE i.—*Divide* $8\cdot9$ *by* 120

$8\cdot9 \div 120 = \cdot89 \div 12.$

$$\begin{array}{r|l} 12 & \cdot89 \\ \hline & \cdot074166\ldots \end{array}$$

Ans. $\cdot0741\dot{6}$

EXAMPLE ii —*Divide* 23 *by* $\cdot011$.

$23 \div \cdot011 = 23000 \div 11.$

Note —In such cases as this the beginner sometimes falls into the error of placing a recurring dot upon an *integer*!

$$\begin{array}{r|l} 11 & 23000\cdot \\ \hline & 2090\cdot9090\ldots \end{array}$$

Ans. $2090\cdot\dot{9}\dot{0}$.

EXAMPLE iii —*Divide* $\cdot5$ *by* 28

Here, in dividing by the factors of 28, we *first* divide by 4, which we foresee will yield a *terminating* quotient, and *afterwards* by the factor 7, which we expect to yield a recurring quotient. It will be found that labour is generally saved by adopting this order.

$$28\left\{\begin{array}{r|l} 4 & \cdot5 \\ \hline 7 & \cdot125 \\ \hline & \cdot017857142 85\ldots \end{array}\right.$$

Ans $\cdot017\dot{8}5714\dot{2}$.

EXAMPLE iv.—*Divide* $6\cdot76$ *by* $8\cdot08$.

$6\cdot76 \div 8\cdot08 = 676 \div 808$

Here, after six steps, we obtain the same figures, 296, in the rem^r which had before appeared four steps earlier Thus the recurring period is 3663

Note —The appearance of the same figure twice over in the *quotient* is no sure sign that the quotient has begun to recur, *unless the corresponding remainders are alike.*

$$\begin{array}{r} 808)676\cdot0(\cdot8\dot{3}66\dot{3}\ \textit{Ans.} \\ 646\ 4 \\ \hline 29\ 60 \\ 24\ 24 \\ \hline 5\ 360 \\ 4\ 848 \\ \hline 5120 \\ 4848 \\ \hline 2720 \\ 2424 \\ \hline 296 \end{array}$$

XXXV. RECURRING DECIMALS.

VULGAR FRACTIONS EXPRESSED AS DECIMALS.

We have seen, on page 115, that a vulgar fraction is converted into a decimal by dividing its numerator by its denominator, but the examples there considered were such as produced *terminating* decimals only. It follows from the last chapter that, in many cases, the decimal which results from this division will be a *recurring* decimal

EXAMPLE 1 —*Express* $\frac{13}{88}$ *in the form of a decimal.*

$$88\left\{\begin{array}{r|l} 8 & 13\cdot \\ \hline 11 & 1\cdot 625 \\ \hline & \cdot 1477272 \end{array}\right. \qquad \underline{Ans\ \cdot 147\dot{7}\dot{2}}$$

Thus we have two classes of vulgar fractions, namely —

(1) those which produce *terminating* decimals,
and (2) those which produce *recurring* decimals

We shall now show how these may be distinguished

We have seen on page 115 that a vulgar fraction may be converted to a decimal, *without dividing*, if we can find a number by which to multiply both numerator and denominator such that the resulting new denominator is a *power of ten*

For instance, $\frac{7}{25} = \frac{7}{5\times5} = \frac{7\times4}{10\times10} = \frac{28}{100} = \cdot 28$

Here, as the given den^r was 25, i e 5×5, it was evident that by multiplying both num^r and den^r by 4, i e by 2×2, the new den^r obtained, i e $5 \times 2 \times 5 \times 2$, would be a power of ten

Similarly, $\frac{11}{400} = \frac{11}{100\times2\times2} = \frac{11\times25}{100\times10\times10} = \frac{275}{10000} = \cdot 0275$

Now as the only prime factors of 10 are 2 and 5, it follows that if *the denominator of a fraction*, in **its lowest terms,** *contains* **no other prime factors than 2 and 5,** the fraction will produce a *terminating* decimal, but that if it contain *any other* prime factor it *cannot* produce a *terminating* decimal, but must produce a *recurring* decimal, for there are no multipliers of 3, 7, 11, 13 &c. which yield a *power of ten* as product

For instance, $\frac{1}{600}$ will produce a *recurring* decimal, since the denominator contains the factor **3**

Note —The condition that the fraction shall be in *lowest terms* before applying this test is important.

Hence we can, by inspection, or by resolving the den^r into factors, predict the *number of places of non-recurring decimals* which any given fraction, expressed in its lowest terms, will, if converted into a decimal, yield, for each 10 in the den^r will yield *one* place, and each additional 2, or 5, will also yield *one* place

For instance, $\frac{7}{16}$ will produce a *terminating* decimal of four places, for $\frac{7}{16} = \frac{7}{2\times2\times2\times2} = \frac{7\times5\times5\times5\times5}{10\times10\times10\times10} = \frac{4375}{10000} = \cdot4375$,

And $\frac{1}{12500}$ will produce a *terminating* decimal of five places, for $\frac{1}{12500} = \frac{1}{100\times5\times5\times5} = \frac{1\times2\times2\times2}{100\times10\times10\times10} = \cdot00008$

Again, $\frac{3}{11}$ will produce a *recurring* decimal with no places of *non*-recurring figures, for the den^r contains neither 10, nor 2, nor 5

And $\frac{1}{14}$ will produce a *recurring* decimal, since its den^r contains the factor 7, and there will be one *non*-recurring place, since the den^r contains one factor 2,

for $\frac{1}{14} = \frac{1}{7\times2} = \frac{1\times5}{7\times10} = \frac{5}{7}\div10 = \cdot\dot{7}1428\dot{5} - 10 = \cdot0\dot{7}1428\dot{5}$

Similarly $\frac{19}{8800}$ will produce a *mixed recurring* decimal with five *non*-recurring places

EXAMPLE ii —*Which of the fractions* $\frac{1}{32250}$, $\frac{93}{128}$ and $\frac{17}{2222}$ *will produce terminating decimals?*

As 32250 is divisible by 3, $\frac{1}{32250}$ will produce a *recurring* decimal.

As $128 = 8 \times 16$, and thus contains no other prime factor than 2,

$\therefore$ $\frac{93}{128}$ will produce a *terminating* decimal

As 2222 is divisible by 11, $\frac{17}{2222}$ will produce a *recurring* decimal.

EXAMPLE iii —*Find the number of places of non-recurring decimal figures in the decimals equivalent to* $\frac{493}{62500}$, $\frac{13}{224}$ and $\frac{1437}{2750}$.

$$62500 = 5\times5\times5\times5\times10\times10,$$

$\therefore$ $\frac{493}{62500}$ will produce a *terminating* decimal of six places.

$$224 = 2\times2\times2\times2\times2\times7,$$

$\therefore$ $\frac{13}{224}$ will produce a *mixed recurring* decimal with five places of *non*-recurring decimal figures

$2750 = 10\times5\times5\times11$, (and the num^r 1437 is *not* divisible by 11),

$\therefore$ $\frac{1437}{2750}$ will produce a *mixed recurring* decimal, with three places of *non*-recurring decimal figures.

XXXVI. RECURRING DECIMALS.

To Express a Recurring Decimal as a Vulgar Fraction

A recurring decimal may be multiplied, or divided, by a power of ten by moving the decimal point to right, or left, in the same way as was shown in the case of a terminating decimal

For instance, as $\cdot\dot{4}$ signifies 44444 &c &c without limit, it is evident that, if we remove to the left of the point a *limited number* of this *unlimited number* of figures there will still remain, on the *right* of the point an *unlimited number* of figures

Hence $4 \times 10 = 4\cdot44444$ &c &c without limit,

i e $\cdot\dot{4} \times 10 = 4 \cdot\dot{4}$

Similarly $62\dot{3} \times 1000 = 623\ 623623$ &c &c $= 623\cdot623$

Also $\cdot4\dot{7} \times 10 = 4\cdot\dot{7}$, $\cdot\dot{4}\dot{7} \times 100 = 47\cdot\dot{4}\dot{7}$, and so on

We shall now show how the vulgar fraction equivalent to any *pure* recurring decimal may be obtained

For instance, *to find the vulgar fraction equivalent to* $\cdot\dot{4}$

$$\begin{aligned} \text{As } \mathbf{10} \textit{ times } \cdot\dot{4} &= 4\cdot44444 \\ \text{and } \cdot\dot{4} &= \cdot44444 \ \ldots \\ \therefore \text{(subtracting) } \mathbf{9} \textit{ times } \cdot\dot{4} &= 4 \\ \text{Hence } \cdot\dot{4} &= \frac{4}{9} \end{aligned}$$

Again, *to find the vulgar fraction equivalent to* $\cdot\dot{6}2\dot{3}$

$$\begin{aligned} \text{As } \mathbf{1000} \textit{ times } \cdot623 &= 623\cdot623623623 \ \ldots\ldots \\ \text{and } \cdot\dot{6}23 &= \cdot623623623. \ \ldots \\ \therefore \text{(subtracting) } \mathbf{999} \textit{ times } \cdot\dot{6}2\dot{3} &= 623. \\ \text{Hence } \cdot\dot{6}2\dot{3} &= \frac{623}{999} \end{aligned}$$

N B.—We multiply the given decimal by 10, or 100, or 1000, &c., according as there are *one*, or *two*, or *three*, &c , *figures in the period*

From such *results* as those above we obtain the following rule.—

(I) To express a pure recurring decimal as a vulgar fraction

For the *numerator*, write **the figures of the recurring period,**

for the *denominator*, write **as many nines as there are figures in the period.**

Example i.—*Convert* $\cdot\dot{7}\dot{2}$ *and* $\cdot\dot{0}19\dot{8}$ *into vulgar fractions.*

$\cdot\dot{7}\dot{2} = \frac{72}{99} = \frac{8}{11}$ Ans. $\cdot\dot{0}19\dot{8} = \frac{198}{9999} = \frac{22}{1111} = \frac{2}{101}$ Ans.

Next, in the case of any *mixed* recurring decimal.

For instance, *to find the vulgar fraction equivalent to* $\cdot 4\dot{7}$.

$$\text{As } 100 \textit{ times } \cdot 4\dot{7} = 47\cdot 77777\ldots\ldots$$
$$\text{and } 10 \textit{ times } \cdot 4\dot{7} = 4\cdot 77777\ldots\ldots\ldots$$
$$\therefore \text{ subtracting, } 90 \textit{ times } \cdot 4\dot{7} = 47 - 4.$$
$$\text{Hence } \cdot 4\dot{7} = \frac{47-4}{90}.$$

Again, *to find the vulgar fraction equivalent to* $\cdot 63\dot{1}2\dot{5}$.

$$\text{As } 100000 \textit{ times } \cdot 63\dot{1}2\dot{5} = 63125\cdot 125125125\ldots$$
$$\text{and } 100 \textit{ times } \cdot 63\dot{1}2\dot{5} = 63\cdot 125125125\ldots$$
$$\therefore \text{ subtracting, } 99900 \textit{ times } \cdot 63\dot{1}2\dot{5} = 63125 - 63$$
$$\text{Hence } \cdot 63\dot{1}2\dot{5} = \frac{63125-63}{99900}.$$

N B—We multiply the given decimal first by that power of ten which moves the decimal point to the *end* of the period, and next by that power of ten which moves the point up to the *beginning* of the period

From such *results* as those above we obtain the following rule.—

(II.) To express a **mixed** recurring decimal as a vulgar fraction.

For the *numerator*, **subtract the number formed by the non-recurring figures from the number formed by all the figures as far as the end of the period;**

for the *denominator*, **write as many nines as there are figures in the period, followed by as many ciphers as there are non-recurring decimal figures.**

EXAMPLE ii —*Reduce* $\cdot 23\dot{4}\dot{5}$ *to a vulgar fraction*

$$\cdot 23\dot{4}\dot{5} = \frac{2345-23}{9900} = \frac{2322}{9900} = \frac{258}{1100} = \frac{129}{550} \textit{ Ans}$$

EXAMPLE iii.—*Express* $8\cdot 708\dot{3}$ *in the form of a vulgar fraction.*

$$8\cdot 708\dot{3} = 8\frac{7083-708}{9000} = 8\frac{6375}{9000} = 8\frac{2125}{3000} = 8\frac{17}{24} \textit{ Ans.}$$

$$\text{or thus, } 8\cdot 708\dot{3} = \frac{87083-8708}{9000} = \frac{78375}{9000} = \frac{209}{24}.$$

The value of a *mixed* recurring decimal may also be deduced from that of a *pure* one

For instance, as $\cdot 4\dot{7} \times 10 = 4\cdot\dot{7} = 4\frac{7}{9}$, $\quad \cdot 4\dot{7} = 4\frac{7}{9} \div 10 = \frac{43}{90}$

N B—It is important to notice that $\cdot\dot{9} = \frac{9}{9} = 1$, and, consequently, $\cdot 0\dot{9}$, $\cdot 4\dot{9}$, $\cdot 23\dot{9}$ &c are equivalent to the *terminating* decimals $\cdot 1$, $\cdot 5$, $\cdot 24$ &c. respectively

We have seen that every *pure* recurring decimal can be expressed as a vulgar fraction which has a den^r consisting of as many *nines* as there are figures in the period If, therefore, we can find a number, by which to multiply both num^r and den^r, such that the resulting new den^r is all *nines*, we may, *without dividing*, express the fraction as a recurring decimal

For instance, $\frac{2}{3} = \frac{2\times3}{3\times3} = \frac{6}{9} = \cdot\dot{6}$; $\frac{5}{11} = \frac{5\times9}{11\times9} = \frac{45}{99} = \cdot\dot{4}\dot{5}$,

also $\frac{2}{37} = \frac{6}{111} = \frac{54}{999} = \cdot\dot{0}5\dot{4}$, $\frac{3}{37} = \frac{81}{999} = \cdot\dot{0}8\dot{1}$, $\frac{4}{37} = \frac{108}{999} = \cdot\dot{1}0\dot{8}$, &c.

Hence all fractions in lowest terms which have *the same den^r* yield decimals having the same *number of figures in the period*

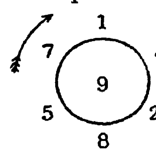
Also, by resolving 9, 99, 999 &c. into their prime factors, we can discover all the possible den^{rs} of fractions in lowest terms which yield periods of 1, 2, 3 &c figures respectively —

Thus the *only* den^{rs} of fractions yielding pure recurring decimals with periods of

one figure are	3 and 9,
two figures	11, 33 and 99,
three	27, 37, 111, 333 and 999,
four	101, 303, 909, 1111, 3333 and 9999,
five	41, 123, 369, 271, 813, 2439, 11111, 33333 and 99999,
six	7, 13, and either of these numbers, or their product, multiplied by any one or more of the nos. 3, 9, 27, 11 or 37.

Note —The number 7, the only den^r less than 17 which yields a decimal with the *maximum* period, is noteworthy For, as all the possible rem^{rs} are exhausted during the division, it is evident that, whatever be the *numerator* of the fraction, the *same figures* must appear in the quotient.

Thus $\frac{1}{7} = \cdot\dot{1}4285\dot{7}$, $\frac{2}{7} = \cdot\dot{2}8571\dot{4}$, $\frac{3}{7} = \cdot\dot{4}2857\dot{1}$,
$\frac{4}{7} = \cdot\dot{5}7142\dot{8}$, $\frac{5}{7} = \cdot\dot{7}1428\dot{5}$, $\frac{6}{7} = \cdot\dot{8}5714\dot{2}$



Now, if we arrange the figures 1, 4, 2, 8, 5, 7 in a circle and read them in the direction of the arrow, beginning in turn with 1, 2, 4 &c, we obtain the periods of $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$ &c respectively

Moreover, as the sum of any opposite pair of figures thus arranged is 9, if we *commit to memory the first three*, namely, "One, Four, Two," the other three figures are at once obtained by subtracting these in turn from 9, and thus the decimal value of $\frac{1}{7}$, $\frac{2}{7}$ &c, or the fractional value of any one of these periods, can be written down at once

EXAMPLE IV —*Reduce* $3\cdot43\dot{5}71428\dot{}$ *to a vulgar fraction.*

$$3\cdot43\dot{5}7142\dot{8} = 3\cdot43\tfrac{4}{7}{}^{*} = 3\frac{43\frac{4}{7}}{100} = 3\tfrac{305}{700} = 3\tfrac{61}{140}\ \underline{Ans.}$$

EXAMPLE V —*Reduce* $\frac{3}{35}$ *to a decimal*

$$\frac{3}{35} = \frac{6}{70} = \frac{6}{7} \div 10 = \dot{8}5714\dot{2} \div 10 = \underline{\cdot0\dot{8}5714\dot{2}\ Ans.}$$

* See the above Note, also the last paragraph on page 121, also Appendix

XXXVII. RECURRING DECIMALS.

Addition and Subtraction

In order to add, or subtract, recurring decimals, we arrange them with the decimal points in a vertical line and repeat the recurring periods a sufficient number of times to enable us to see the recurring period in the sum, or difference, we then proceed as in the case of terminating decimals, *except that we omit to write down* the sum, or difference, of *the two columns on the extreme right*, from which we obtain the figure to be "carried"

We can determine beforehand to what extent to repeat the periods, for the *first column which contains none but recurring figures* must appear again after an interval not greater (though it may be less) than the L C.M of *the number of figures in each of the recurring periods*, and, consequently, the *sum*, or *difference*, of the figures in this column must also reappear.

EXAMPLE I.—*Add together* $\cdot 7$, $2\cdot 0\dot{3}\dot{1}$, $\cdot 1\dot{4}7\dot{5}$ *and* $26\cdot 0\dot{8}$

Here we have recurring periods of 2, 3 and 1 figures respectively; and the L C M of 2, 3 and 1 is 6. Also the first column containing none but recurring figures is the 2nd to the right of the decimal points Hence this, and the five succeeding columns (as far as the vertical line) will yield the recurring period in the result We, however, extend the periods *two* columns further, in order to obtain the figure to be "carried" to the column on the left of the vertical line.

·7	
2·0313131	31
·1475475	47
26·0888888	88
28·9677495	

Ans $28\cdot 9\dot{6}7749\dot{5}$.

EXAMPLE II.—*Subtract* $9\cdot 32\dot{1}5\dot{7}$ *from* $16\cdot\dot{4}80\dot{3}$

Here we have periods of 3 and 4 figures respectively, and the L.C M. of 3 and 4 is 12. Also the column containing 0 and 1 is the first recurring column, so we draw the vertical line to the left of this on its reappearance 12 columns further on

16·480348034803348	03
9·321571571571 57	15
7·15877646323190	

Ans. $7\cdot 15\dot{8}7764632319\dot{0}$

EXAMPLE III —*Find the sum of* $3\cdot 20\dot{7}\dot{4}$, $\cdot 0\dot{1}$ *and* $21\cdot 486\dot{4}\dot{1}$.

Here we expect a *two*-figure period in the result, but as both figures in this period are *alike*, we have really a period of *one* figure, namely 24·7049

But we know (see page 121) that, as $\dot{9} = 1$, $\cdot 4\dot{9} = \cdot 5$ Hence in this case the result becomes the terminating decimal 24·705

3·20747	47
01111	11
21·48641	41
24·70499	

Ans 24·705

XXXVIII. RECURRING DECIMALS.

MULTIPLICATION AND DIVISION

Similarly a recurring decimal may be multiplied, or divided, by either an *integer* or a *terminating* decimal, if we repeat the recurring period a sufficient number of times to enable us to see the recurring period in the product, or quotient

EXAMPLE I —*Multiply* $6{\cdot}248\dot{3}$ *by* $\cdot 05$

Here the two cancelled figures on the right are only multiplied for the sake of the figure to be "carried," and are not counted in marking the position of the decimal point

$$\begin{array}{r} 6{\cdot}24833\cancel{3}\cancel{3} \\ \cdot 05 \\ \hline \cdot 31\ 24166 \\ \\ \underline{Ans\ \ \cdot 31241\dot{6}} \end{array}$$

EXAMPLE II —*Multiply* $1{\cdot}25\dot{7}$ *by* 64

Here, multiplying $1{\cdot}2577$ by 4, and obtaining the carried figure as before, the product is $5{\cdot}0311$; also, multiplying $1{\cdot}2577$ by 60, we obtain the product $75{\cdot}466$

We now add $5{\cdot}03\dot{1}$ and $75{\cdot}4\dot{6}$ by the method of the last chapter, obtaining the result $80{\cdot}497$

$$\begin{array}{r} 1{\cdot}25777\cancel{7} \\ 64 \\ \hline 5{\cdot}03111. \\ 75{\cdot}4666 \\ \hline 80{\cdot}4977. \\ \\ \underline{Ans\ \ 80{\cdot}49\dot{7}} \end{array}$$

Note—It is only necessary to repeat the period in the multiplicand *once* in order to obtain the figure to be "carried," for it follows, from page 122, that the *number of figures* in the *recurring period* of the various products will be the *same* as in the multiplicand.

EXAMPLE III —*Multiply* $4{\cdot}06\dot{8}\dot{5}$ *by* $1{\cdot}23$

Multiplying by $\cdot 03$ the product is $\cdot 12205\dot{7}$. Similarly, multiplying by $\cdot 2$, the product is $\cdot 8137\dot{1}$, and multiplying by 1, the product is $4{\cdot}0685$

We now, *after the multiplication is finished*, repeat the periods of these products, in order to add them by the method of the previous chapter *Note*—The figures in thin type are those appended *after* the *multiplication* was finished in order to add the results

$$\begin{array}{r|l} 4{\cdot}06\ 85\cancel{8}\cancel{5} & \\ 1{\cdot}23 & \\ \hline \cdot 12\ 20\ 57 & 57 \\ \cdot 81\ 37\ 17 & 17 \\ 4{\cdot}06\ 85\ 85 & 85 \\ \hline 5{\cdot}00\ 43\ 60 & \\ \multicolumn{2}{r}{} \\ \multicolumn{2}{r}{\underline{Ans\ \ 5{\cdot}0043\dot{6}\dot{0}.}} \end{array}$$

EXAMPLE IV.—*Divide* $37{\cdot}48\dot{3}$ *by* $\cdot 05$

$$37{\cdot}48\dot{3} \div \cdot 05$$
$$= 3748{\cdot}\dot{3} \div 5$$

$$\begin{array}{r|l} 5 & \underline{3748{\cdot}333\ldots} \\ & 749{\cdot}66\ldots. \\ & \underline{Ans\ \ 749{\cdot}\dot{6}.} \end{array}$$

When the multiplier, or divisor, is itself a *recurring* decimal, we reduce the decimals to vulgar fractions in order to perform the operation This method also is to be preferred in certain cases of a *non*-recurring multiplier, or divisor For instance.—

EXAMPLE v.—*Multiply* $1\cdot\dot{5}7142\dot{8}$ *by* $2\cdot17$.

$$1\cdot\dot{5}7142\dot{8} \times 2\cdot17 = 1\tfrac{4}{7}^{*} \times \tfrac{217}{100} = \frac{11}{\cancel{7}} \times \frac{\overset{31}{\cancel{217}}}{100} = \tfrac{341}{100} = \underline{3\cdot41\ \textit{Ans.}}$$

EXAMPLE vi —*Multiply* $3\cdot\dot{2}\dot{7}$ *by* $2\cdot8\dot{3}$

$$3\cdot\dot{2}\dot{7} \times 2\cdot8\dot{3} = 3\tfrac{27}{99} \times 2\tfrac{83-8}{90} = 3\tfrac{3}{11} \times 2\tfrac{5}{6} = \frac{\overset{6}{\cancel{36}}}{11} \times \frac{17}{\cancel{6}}$$
$$= \tfrac{102}{11} = 9\tfrac{3}{11} = 9\tfrac{27}{99} = \underline{9\cdot\dot{2}\dot{7}\ \textit{Ans.}}$$

EXAMPLE vii —*Divide* $6\cdot\dot{7}$ *by* $2\cdot\dot{6}$.

$$6\cdot\dot{7} \div 2\cdot\dot{6} = 6\tfrac{7}{9} \div 2\tfrac{6}{9} = \frac{61}{\underset{1}{\cancel{9}}} \times \frac{\overset{1}{\cancel{9}}}{24}$$
$$= \tfrac{61}{24} = \underline{2\cdot541\dot{6}\ \textit{Ans}}$$

$$24\left\{\begin{array}{l} 8\,\underline{|\,61\cdot\quad} \\ \;\;3\,\underline{|\,7\cdot625\quad} \\ \qquad 2\cdot54166.. \end{array}\right.$$

EXAMPLE viii.—*Simplify* $\dfrac{\cdot7\dot{0}\dot{9}}{\cdot\dot{8}\dot{1}} + \dfrac{2\cdot01\dot{6}}{1\cdot\dot{6}} \times \cdot\dot{0}\dot{1}$.

$$\frac{\cdot7\dot{0}\dot{9}}{\cdot\dot{8}\dot{1}} + \frac{2\cdot01\dot{6}}{1\cdot\dot{6}} \times \cdot\dot{0}\dot{1} = \frac{\;\frac{709-7}{990}\;}{\frac{81}{99}} + \frac{\;\frac{2016-201}{900}\;}{1\frac{6}{9}} \times \tfrac{1}{99}$$

$$= \frac{\overset{78}{\cancel{702}}}{\underset{10}{\cancel{990}}} \times \frac{\overset{1}{\cancel{99}}}{\underset{9}{\cancel{81}}} + \frac{\overset{\overset{11}{\cancel{121}}}{\cancel{1815}}}{900} \times \frac{\cancel{9}}{\underset{1}{\cancel{15}}} \times \frac{1}{\underset{\underset{1}{\cancel{11}}}{\cancel{99}}} = \tfrac{78}{90} + \tfrac{11}{900} = \tfrac{791}{900}$$
$$= \tfrac{7\cdot91}{9} = \underline{\cdot87\dot{8}\ \textit{Ans.}}$$

---

* See note on page 122

XXXIX. DECIMALS.

Value of a Decimal of a Concrete Quantity

In the case of a *terminating* decimal the method corresponds to that of ordinary Reduction

Example i —*Find the value of* ·6875 *of* £1

As ·6875 of £1 = ·6875 of 20*s*, we multiply 6875 by 20 and mark off *four* places of decimals in the result, thus obtaining 13·7500 *s* We now *reserve the integral part*, 13*s*, and reduce the remaining ·7500*s* to pence Multiplying 7500 by 12, and again marking off *four* places of decimals, we obtain the integer 9*d* Thus the complete result is 13*s* 9*d*

```
     6875
       20
s 13 7500
       12
d  9 0000
Ans 13s 9d
```

Example ii —*Find the value of* 2·55625 *of* 2 *tons*

Multiplying 2·55625 by 2, we obtain 5·11250 as the number of tons, so we reserve the integer 5 as part of the answer, and reduce the remaining ·11250 of a ton to cwts &c

Note —Ciphers on the right may of course be discarded at each step, if we then mark off at each step a number of decimal places correspondingly less

```
       2·55625
             2
tons 5 11250
            20
cwts 2 25000
Ans 5 tons 2 cwts 1 qr
```

Example iii.—*Find the value of* ·2896 *of* £15, 12*s* 6*d*

As ·2896 of £15, 12*s*. 6*d* = ·2896 of 312½*s*
= ·2896 of 312·5*s*,

we multiply 2896 and 312·5 together, and mark off *five* places of decimals in the result, thus obtaining 90·5*s*, *i e* £4, 10*s* 6*d*

Ans £4, 10*s* 6*d*

```
    ·2896
    312·5
    14480
    5792
   2896
  8688
s 90·50000
```

Example iv —*Find the value of* ·3425 *of* £1 — ·2375 *of a guinea* + ·875 *of* 7*s* 6*d*.

Here we first express all the quantities in the same denomination, in this case, in *shillings*;

```
                  ·2375           ·875
·3425                21            7·5
   20              2375           4375
6·8500 . (i)      4750           6125
                4·9875 . (ii)   6·5625...(iii)
```

We now add (i) to (iii) and subtract (ii) from the result, thus obtaining 8·425*s* Then reducing ·425*s* to pence, we obtain 5·1*d*. Thus the complete result is 8*s* 5·1*d*

$$\begin{array}{rr} & 6{\cdot}5625 \\ & \underline{6{\cdot}8500} \\ & 13\ 4125 \\ & \underline{4\ 9875} \\ s & 8{\cdot}425 \\ & \underline{12} \\ d & 5{\cdot}100 \end{array}$$

Ans 8*s* 5·1*d*

Similarly we may proceed in the case of a recurring decimal, or we may convert the recurring decimal into a vulgar fraction.

EXAMPLE v —*Find the value of* $\cdot 3958\dot{3}$ *of* £1.

Here, in multiplying by 20 we multiply by 2, as in the last chapter, and then move the decimal point one place to the right in the product Finally, we obtain the result, 7*s* 10 $\dot{9}$*d*, and, as $\cdot\dot{9} = 1$, this is equivalent to 7*s* 11*d*

$$\begin{array}{rl} & \cdot 3958333\cancel{3}\cancel{3}\ .. \\ & \underline{\qquad 20} \\ s. & 7{\cdot}91666\cancel{6}\cancel{6}\ . \\ & \underline{\qquad 12} \\ d. & 10{\cdot}9999.\ \ldots \end{array}$$

Ans 7*s* 11*d*

EXAMPLE vi —*Find the value of* $\cdot 3\dot{8}\dot{4}$ *of* £1, 7*s* 6*d*. + $\cdot 38\dot{4}$ *of* £1, 17*s*. 6*d*

$\cdot 3\dot{8}\dot{4}$ of £1, 7*s*. 6*d* + $\cdot 38\dot{4}$ of £1, 17*s*. 6*d*.

$$= \frac{384-3}{990} \times 27\tfrac{1}{2}s. \quad + \quad \frac{384-38}{900} \times 37\tfrac{1}{2}s$$

$$= \frac{\overset{127}{\cancel{381}}}{\underset{\underset{\underset{6}{\cancel{18}}}{\cancel{90}}}{\cancel{990}}} \times \frac{\overset{\overset{1}{\cancel{5}}}{\cancel{55}}}{2}s \quad + \quad \frac{\overset{173}{\cancel{346}}}{\underset{\underset{12}{\cancel{36}}}{\cancel{900}}} \times \frac{\overset{\overset{1}{\cancel{3}}}{\cancel{75}}}{\underset{1}{\cancel{2}}}s$$

$$= \quad 10\tfrac{7}{12}s \quad + \quad 14\tfrac{5}{12}s = 25s. = \underline{£1,\ 5s\ \ Ans.}$$

EXAMPLE vii.—*Find the value of* $\cdot 3\dot{4}2857\dot{1}$ *of* 2 *tons* 13 *cwts* 3 *qrs*.

$\cdot 3\dot{4}2857\dot{1}$ of 2 tons 13 cwts 3 qrs

$$= \cdot 3\tfrac{3}{7}^{*} \text{ of } 53\tfrac{3}{4} \text{ cwts} = \frac{\overset{\overset{3}{\cancel{6}}}{\cancel{24}}}{\underset{\underset{7}{\cancel{14}}}{\cancel{70}}} \times \frac{\overset{43}{\cancel{215}}}{\cancel{4}} \text{ cwts} = \frac{129}{7} \text{ cwts.}$$

$$= 18\tfrac{3}{7} \text{ cwts.} = \underline{18 \text{ cwts } 1 \text{ qr } 20 \text{ lbs. } Ans}$$

---

* See note on page 121.

XL. DECIMALS.

To Express one Concrete Quantity as the Decimal of another of the same kind

This may always be done by expressing the given quantity as a *vulgar fraction* of the other (see Chap XXIII), and then reducing this vulgar fraction thus obtained to a decimal

Example i —*Express 2 qrs 21 lbs as the decimal of 2 cwts 2 qrs*

2 qrs 21 lbs $= 2\frac{3}{4}$ qrs , 2 cwts. 2 qrs $=$ 10 qrs ;

$$\text{and } \frac{2\frac{3}{4}}{10} = \frac{11}{40} = \frac{1\cdot 1}{4} = \underline{\cdot 275 \; Ans}$$

Example ii —*What decimal of 18 yds 2 ft 3 in is 2 yds. 1 ft. 4 in.?*

2 yds 1 ft 4 in $= 7\frac{1}{3}$ ft , 18 yds 2 ft. 3 in. $= 56\frac{1}{4}$ ft ;

$$\text{and } \frac{7\frac{1}{3}}{56\frac{1}{4}} = \frac{22}{3} \times \frac{4}{225} = \frac{88}{675}$$

$$= \frac{88 \times 4^{*}}{675 \times 4} = \frac{352}{2700} = \frac{3\cdot 52}{27} = \underline{\cdot 13\dot{0}3\dot{7} \; Ans}$$

$$27 \begin{cases} 3 \,|\, 3\cdot 52 \\ 9 \,|\, 1\cdot 1733333 \,. \end{cases}$$
$$\cdot 1303703 \,. \,.$$

If, however, the given quantity is to be expressed as the decimal of some *simple* quantity, we may, instead of the above method, reverse the "Reduction" method of the last chapter

Example iii —*Reduce 15s. $8\frac{1}{4}d$ to the decimal of £1*

$8\frac{1}{4}d$ = 8 25d

Now to reduce 8·25d to *shillings* we divide by 12, obtaining ·6875s.

Hence 15s $8\frac{1}{4}d$ = 15·6875s, so we set 15 to the left of ·6875

```
12  | 8 25
2,0 | 15·6875
      7 84375
Ans ·784375.
```

And to express this in *pounds* we divide by 20 (*i e* we divide by 2, and then move the decimal point *one* place to the left), obtaining £·784375 as the req^{d} decimal

---

* We multiply both numerator and denominator of the fraction by 4 at this stage, foreseeing that by so doing a more convenient denominator is obtained as the divisor in the final division

EXAMPLE iv —*Reduce £2, 17s 10¾d. to the decimal of £5.*

10¾*d* = 10·75*d*

Now to reduce 10·75*d* to *shillings* we divide by 12, obtaining ·895833 *s*

∴ £2, 17*s* 10¾*d* = 57·895833 *s*

And, to reduce 57·895833 *s* to the decimal of £5, we divide by 100 (the number of shillings in £5), *i e* we merely move the decimal point *two* places to the left

$$\begin{array}{r|l} 12 & 10\ 75 \\ 1\,00 & 57{\cdot}895833\ldots \\ \hline & {\cdot}5789583 \end{array}$$

Ans ·578958$\dot{3}$

EXAMPLE v —*Reduce* 1·53 *of* £2, 17*s.* 3½*d* *to pence and the decimal of a penny*

Reducing £2, 17*s.* 3½*d.* to pence, we obtain 687·5*d*

We now multiply this by 1·53

$$\begin{array}{rrr} £ & s & d \\ 2 & 17 & 3\tfrac{1}{2} \\ 20 & & \\ \hline 57 & & \\ 12 & & \\ \hline 687\tfrac{1}{2} & = & 687{\cdot}5d. \end{array}$$

$$\begin{array}{r} 68\,7{\cdot}5 \\ 1{\cdot}5\,3 \\ \hline 20\,6\,2\,5 \\ 343\,7\,5\ \ \\ 687\,5\ \ \ \ \\ \hline 1051\,8\,7\,5 \end{array}$$

Ans 1051·875*d.*

EXAMPLE vi —*Subtract* ·48 *of* 3 *weeks* 2 *days from* 2·47 *of* 5 *days* 1 *hour, and express the result in minutes*

Here, in order to perform the subtraction, we first express both quantities in the common denomination *hours.*

We now subtract (i) from (ii), obtaining 33·91 *hours.*

Finally, to reduce 33·91 *hours* to *minutes*, we multiply it by 60

$$\begin{array}{r} {\cdot}48 \\ 23 \\ \hline 144 \\ 96\ \ \\ \hline 1104 \\ 24 \\ \hline 264{\cdot}96\ \text{(i)} \end{array} \qquad \begin{array}{r} 2{\cdot}47 \\ 1\,21 \\ \hline 2\,47 \\ 49\,4\ \ \\ 247\ \ \ \ \\ \hline 298{\cdot}87\ \text{(ii)} \\ 264{\cdot}96 \\ \hline 33{\cdot}91 \\ 60 \\ \hline 2034{\cdot}60 \end{array}$$

Ans 2034·6 mins

XLI. MISCELLANEOUS EXAMPLES

The devices illustrated in Chapter III with integers may, of course, be employed in similar cases of multiplication and division of decimals For instance —

A. *Divide* 3·12 *by* ·0125

$$3{\cdot}12 \div {\cdot}0125 = 31200 - 125 = 31{\cdot}2 \times 8 = \underline{249{\cdot}6 \ \textit{Ans}}$$

It is often convenient to express a division of decimals in the following form —

B. *Divide* 9·1 × ·005 × 3·7 *by* ·074 × 5·2

$$\frac{9{\cdot}1 \times {\cdot}005 \times 3{\cdot}7}{{\cdot}074 \times 5{\cdot}2} = \frac{\overset{7}{\cancel{91}} \times 5 \times \overset{1}{\cancel{37}}}{\underset{2}{\cancel{74}} \times \underset{4}{\cancel{52}} \times 10} = \frac{35}{80} = \frac{3{\cdot}5}{8} = \underline{{\cdot}4375 \ \textit{Ans}}$$

Here we multiply both num^r^ and den^r^ by 10000, thus replacing all the decimals by integers, *i e* we multiply 9·1 by 10, ·005 by 1000, and 3 7 by 10, and therefore the entire num^r^ by 10000 Similarly we deal with the den^r^ We then "cancel" before performing the division

---

C. *Show that* $\frac{2}{17} = {\cdot}117647\frac{1}{17}$, *and then, without further division, obtain the value of* $\frac{2}{17}$ *to twelve places of decimals*

Dividing 2 by 17 to six places of decimals, we obtain the quotient ·117647 and rem^r^ ·000001

$$\tfrac{2}{17} = {\cdot}117647\tfrac{1}{17} \ \textit{Ans} \ \text{(i)}$$

Now $\frac{1}{17} = \frac{1}{2}$ of $\frac{2}{17} = \frac{1}{2}$ of $\cdot117647\frac{1}{17} = \cdot058823\frac{9}{17}$,

∴ $\frac{1}{17}$ part of *one-millionth* = 000000058823$\frac{9}{17}$.

Hence $\frac{2}{17} = \cdot117647058823\frac{9}{17}$ *Ans.* (ii)

D. *Find the least integer which contains both* 4·15 *and* 49·8 *each an integral number of times.*

4·15 is 415 *hundredths*, and 49·8 is 4980 *hundredths*

The L C.M of 415 and 4980 is found to be 5 × 12 × 83

The L C M of 415 *hundredths* and 4980 *hundredths* is 5 × 12 × 83 *hundredths.*

i e. $\frac{5 \times 12 \times 83}{100}$, or $\frac{3 \times 83}{5}$, contains both the given decimals an integral number of times, but is not itself an integer

Hence the required least *integer* is evidently $\frac{3 \times 83}{5} \times 5 = \underline{249 \ \textit{Ans}}$

XLII. THE UNITARY METHOD,

OR SINGLE RULE OF THREE.

We shall now consider a class of questions involving both multiplication and division.

The first eight examples illustrate the "Unitary" method of solution, the rest the "Fractional" method.

EXAMPLE I.—*If* 8 *lbs. of tea cost* 14*s what do* 3 *lbs of the tea cost?*

Here the cost of 8 lbs is given; from which we obtain the cost of the unit* (i e of 1 lb) *by dividing the given cost by* 8

The cost of 3 lbs is then found *by multiplying the cost of* 1 *lb by* 3

Written work

8 *lbs.* cost 14*s.*

$\therefore$ 1 *lb* costs $\frac{1}{8}$ of 14*s.*

Hence 3 *lbs* cost $\frac{3}{8}$ of 14*s* $= \frac{21}{4}$*s*

$=$ 5*s* 3*d* *Ans.*

Note—We do not actually perform the operations at each stage, but merely indicate them until the final stage of the reasoning is reached

---

EXAMPLE ii—*If* 5 *lbs. of tea cost* 9*s* 2*d*, *how many lbs. of the same kind of tea cost* £10, 10*s.* 10*d*?

Here we first reduce the two sums of money to the same denomination, viz *twopences* Then, observing that the answer required is *weight*, not *money*, we re-state the fact given in the question, arranging it so that "lbs" shall occupy the right-hand column.

Written work

s	*d*		£	*s*	*d*
9 .	2		10 .	10 .	10
6			20		
55			210		
			6		
			1265		

For 55 *twopences* we buy 5 *lbs*

$\therefore$. 1 *twopence* . . $\frac{1}{55}$ of 5 *lbs*

Hence 1265 *twopences* $\frac{1265}{55}$ of 5 *lbs.* $= \frac{1265}{11}$ *lbs.*

$=$ 115 *lbs. Ans.*

---

* Hence the name "Unitary" method

In the solution of the last example we expressed the compound quantities in *one* denomination, as is generally more convenient Sometimes, however, it is better to use Compound Multiplication and Division, for instance —

EXAMPLE III —*If 7 articles cost £6, 10s. $2\frac{3}{4}d$, what is the cost of 3 of them?*

7 *articles* cost £6, 10*s* $2\frac{3}{4}d$

$\therefore$ 1 *article* costs $\dfrac{£6,\ 10s\ 2\frac{3}{4}d}{7}$

Hence 3 *articles* cost $\dfrac{£6,\ 10s\ 2\frac{3}{4}d}{7} \times 3 = \dfrac{£19,\ 10s\ 8\frac{1}{4}d}{7}$

$= \underline{£2,\ 15s\ 9\frac{3}{4}d\ \ Ans}$

EXAMPLE IV —*If 6 men could hoe a field in 10 days, how long should 15 men take to do it?*

Written work

6 *men* can do the work in 10 *days*

$\therefore$ 1 *man* 6 times 10 *days*

Hence 15 *men* $\frac{1}{15}$ of 60 *days* = 4 *days Ans*

Note —Here we *multiply* in the second line, for 1 man would take *longer* than 6 men

---

EXAMPLE V —*How far should 40 lbs be carried by rail for the money that pays the carriage of 1 cwt for 130 miles?*

Written work

	lbs		miles
	112	is carried	130
$\therefore$	1		130×112
Hence	40		$\frac{1}{40}$ of 130×112
			$= 13 \times 28 = \underline{364\ \text{miles}\ Ans}$

Note —Here, also, we *multiply* in the second line, for the money that pays the carriage of 112 lbs for 130 miles will pay the carriage of 1 lb for a *greater* distance

---

In the foregoing examples the work is arranged with the whole of the reasoning written down The reasoning may, however,

be easily performed *mentally* and the result alone be written. For instance —

EXAMPLE VI —*Find the cost of 7 articles at the rate of 4s 9d. per dozen*

1st step

As "*cost*" is wanted, we first set down **57***d*, the given cost of 12 things

2nd step

As the cost of 1 thing is found by dividing the cost of 12 things by 12, we now set **12** *under* the 57*d*

3rd step.

As the cost of 7 things is 7 times the cost of 1 thing, we next append the factor **7**

Finally, we simplify this result

Written work.

$$\overset{d}{\frac{57 \times 7}{12}} = \frac{133}{4}$$

$$= 33\tfrac{1}{4} = \underline{2s\ 9\tfrac{1}{4}d\ \textit{Ans.}}$$

EXAMPLE VII.—*How far will a train, travelling at the rate of 40 miles per hour, go in 18 minutes?*

1st step

As "*distance*" is wanted, we first set down **40** *miles*, the given distance travelled in 60 minutes

2nd step

And, as the distance travelled in 1 minute is found by dividing the distance travelled in 1 hour by 60, we next set **60** *under* the 40 miles

3rd step

Then, as the train goes 18 times as far in 18 minutes as it goes in 1 minute, we append the factor **18**, and simplify the result

Written work.

$$\overset{miles}{\frac{40 \times 18}{60}}$$

$$= \underline{12\ \textit{miles Ans.}}$$

EXAMPLE VIII.—*If 14 men can mow 95 acres of grass in 5 days, how many men would do as much in 7 days?*

Here it should be noticed that the *number of acres* in no way affects the reasoning, and may be ignored.

1st step

As the *number of "men"* is wanted, we first write down **14** *men*, the given number who do the work in 5 days

2nd step

Now to do the work in 1 day we require 5 times as many men, so we *multiply* by **5**

3rd step.

And to do the work in 7 days we require *fewer* men than do it in 1 day, in fact $\frac{1}{7}$ of this number, so we now *divide* by **7**.

Written work.

$$\overset{men.}{\frac{14 \times 5}{7}}$$

$$= \underline{10\ \textit{men Ans.}}$$

After some practice on the last plan, the student will, without difficulty, be able to dispense with the "unitary" process by *combining the two last steps*, thus —

EXAMPLE IX —*If a labourer earn £6, 6s. in 7 weeks, how much will he earn in 10 weeks?*

Written work

	weeks			s	
In	7	he	earns	126	
∴ „	10	„	„	$126 \times \frac{10}{7}$	$\overset{s}{=} 180 =$ £9 *Ans*

Explanation

The top line of the above written work is the *fact* given in the question, not, however, in this case, quoted just as it stands, but transposed so that *money* (the name of the required answer) heads the *right-hand* column

The lower line may be supposed to be arrived at by the following mental process —

In 1 week he earns $\frac{1}{7}$ of what he earns in seven,
∴ in 2 weeks „ $\frac{2}{7}$ „ „ „
„ 3 „ „ $\frac{3}{7}$ „ „ „ , and so on
∴ in 10 weeks he earns $\frac{10}{7}$ of what he earns in seven

Or we may reason thus —

"10 weeks is *more* than 7 weeks, so *more* would be earned, and we therefore must multiply 126*s* by the *improper* fraction $\frac{10}{7}$ (for to multiply by an *improper* fraction *increases*, but to multiply by a *proper* fraction *decreases*, the quantity so treated)"

The student is advised to adopt the above plan of arranging the written work in solving the Rule of Three questions which occur in most of the higher rules, *e g* Stocks, &c

Sometimes, however (*e g* in the Chain Rule), it is better to omit all "wording", and to write down only the result, as is done in the following example —

EXAMPLE X —*If I lend a man £40 for 3 months, for how long ought he to lend me £60 in return?*

1st step

"*Time*" is wanted, so we set down 3 months

2nd step

As £60 is greater than £40, the favour is requited by lending £60 for *less* time than £40 was lent, in fact for $\frac{40}{60}$ of that time, so we multiply 3 months by the *proper* fraction $\frac{40}{60}$

Written work

months

$3 \times \frac{40}{60}$

= 2 *months Ans.*

XLIII. THE UNITARY METHOD (*continued*), OR DOUBLE RULE OF THREE

The questions we are now about to consider differ only from those of the preceding chapter in requiring a *repeated application* of the same process of reasoning

EXAMPLE I.—*If* 14 *men in* 7 *days earn* £9, 16s., *what would* 20 *men earn in* 4 *days at the same rate?*

14 *men*	in	7 *days*	earn	$£9\frac{4}{5}$
∴ 1 *man*	in	7 *days*	earns	$\frac{1}{14}$ of $£9\frac{4}{5}$
∴ 1 *man*	in	1 *day*	earns	$\frac{1}{7}$ of $\frac{1}{14}$ of $£9\frac{4}{5}$
∴ 20 *men*	in	1 *day*	earn	$\frac{1}{7}$ of $\frac{1}{14}$ of $£9\frac{4}{5} \times 20$
∴ 20 *men*	in	4 *days*	earn	$\frac{1}{7}$ of $\frac{1}{14}$ of $£9\frac{4}{5} \times 20 \times 4$
				= £8 *Ans.**

In questions which involve many quantities the above process is very cumbrous when the whole of the reasoning is *written down* The reasoning, however, may easily be performed *mentally*, thus,

EXAMPLE II —*If* 6 *labourers, working* 8 *hours a day, mow* 21 *acres in* 9 *days, in how many days will* 5 *labourers, working* 9 *hours a day, mow* 35 *acres?*

Written work

days

$$\frac{9 \times 6 \times 8 \times 35}{21 \times 5 \times 9}$$

= 16 *days Ans.**

1st step As "time" is wanted, we first set down **9** *days*

2nd step As 1 man would take 6 times as long to do the same work as 6 men take, we next *multiply* by **6**

3rd step As a man who works 1 hour a day would take 8 times as long to do the the same work as a man who works 8 hours a day, we next *multiply* by 8

4th step As 1 acre takes but $\frac{1}{21}$ of the time that 21 acres take, we *divide* by **21**

[We have now obtained the time, namely, $\frac{9 \times 6 \times 8}{21}$ days, in which 1 man, working 1 hour a day, mows 1 acre]

5th step As 5 men would take but $\frac{1}{5}$ of the time of 1 man, we next *divide* by **5**

6th step As 5 men, working 9 hours a day, would take but $\frac{1}{9}$ of the time they take when they work 1 hour a day, we next *divide* by **9**

7th step As 35 acres will take them 35 times as long as 1 acre, we *multiply* by **35.**

*In order to save space the "cancelling" is omitted.

"Fractional" method To the student familiar with Fractions this method is as easy to apply as the Unitary method, while it has the advantages of greater rapidity and conciseness

EXAMPLE III —*If* 90 *men can dig a ditch* 50 *yards long in* $4\frac{1}{2}$ *days, how many men can dig a ditch* 360 *feet long in* 27 *days?*

Written work

	yards		days		men
A ditch	50	long is dug in	$4\frac{1}{2}$	by	90,
∴ „	120	„ „ „	27	„	$90 \times \frac{120}{50} \times \frac{4\frac{1}{2}}{27}$
					$= $ 36 *men Ans*

Explanation

1st step

Set down as the top line the fact given in the question, so arranging that "men" (the name of the answer) heads the *right-hand* column

2nd step (change in *length of trench* alone considered)

Set 120 under the 50 yards, and consider what effect lengthening the trench will have on the number of men required As a *longer trench* would require *more men*, we multiply 90 by the *improper* fraction $\frac{120}{50}$

3rd step (change in *number of days* alone considered)

Set 27 under the $4\frac{1}{2}$ days, and consider what effect more time will have on the number of men required As *more time* enables the work to be done by *fewer men*, we now append as a second multiplier the *proper* fraction $\frac{4\frac{1}{2}}{27}$

In questions involving many changes the "wording" may be omitted and the result alone written, thus —

EXAMPLE IV.—*If* 3 *compositors, in* 12 *days of* 10 *hours each, can set in type* 360 *pages averaging* 60 *lines per page and* 40 *letters per line, in how many days of* 8 *hours each would* 9 *compositors set up* 480 *pages averaging* 45 *lines per page and* 50 *letters per line?*

1st step Set down 12 *days*

2nd step (Change in *number of men* alone considered)

9 men take *less* time than 3 men to do the same work, so we multiply by the *proper* fraction $\frac{3}{9}$

Written work

days

$$12 \times \frac{3}{9} \times \frac{10}{8} \times \frac{480}{360} \times \frac{45}{60} \times \frac{50}{40}$$

$$= \frac{25}{4} = 6\frac{1}{4} \textit{ days Ans}$$

3rd step (Change in *length of day* alone considered)

Men who work fewer hours per day take *more* days to do the same work, so we multiply by the *improper* fraction $\frac{10}{8}$ And so on.

BOOKWORK.—PART II.

XLIV. VARIOUS APPLICATIONS OF THE FOREGOING METHODS.

A. AGENTS OF DIFFERENT POWERS, &c.

A (i). *If* 4 *men can do as much as* 7 *boys, how long would* 12 *men and* 5 *boys take to do the work which* 8 *men alone can do in* 39 *days?*

[Here we first express the question in terms of *one* agent, namely boys]

As 4 men = 7 boys $\therefore$ **8 men = 14 boys.**

also 12 men = 21 boys $\therefore$ **12 men and 5 boys = 26 boys.**

[Now in place of the given question we solve the following:—
"*If* **14 boys** *can do the work in* 39 *days, how long will* **26 boys** *take?*"]

$$\text{Hence the req}^{\text{d}}\text{ time} = \overset{\text{days}}{39} \times \frac{14}{26} = \underline{21 \textit{ days Ans}}$$

Note—In this, and some of the following examples, for the sake of clearness the cancelling is omitted

---

A (ii). *If a man can do three times, and if a woman can do twice, as much work as a boy, how long would* 7 *men,* 5 *women and* 3 *boys together take to do the work which* 3 *men,* 4 *women and* 5 *boys can accomplish in* 51 *days?*

[**We first substitute the equivalent number of boys for each set of men and women**]

Here 1 man = **3** boys; and 1 woman = 2 boys

$\therefore$ **3 men + 4 women + 5 boys** = 9 + 8 + 5 boys
= **22** boys.

Also **7 men + 5 women + 3 boys** = 21 + 10 + 3 boys
= **34** boys.

[Thus we obtain, in place of the given question, the following:—
"*If* 22 *boys do the work in* 51 *days, how long would* 34 *boys take?*"]

$$\text{Hence the req}^{\text{d}}\text{ time} = \overset{\text{days}}{51} \times \frac{22}{34} = \underline{33 \textit{ days Ans.}}$$

A (iii) *If* 10 *sheep, or* 15 *lambs, eat* 40 *bushels of turnips in* 7 *days, how long will* 36 *bushels last* 6 *sheep and* 18 *lambs?*

It is evident from the question that

10 sheep eat as much as 15 lambs,

$\therefore$ 6 sheep $\qquad 15 \times \frac{6}{10} = 9$ lambs

$\therefore$ 6 sheep + 18 lambs $\qquad 9 + 18 = 27$ lambs.

[Thus instead of the given question we have the following —

"*If* 15 *lambs eat* 40 *bushels in* 7 *days, how long will* 27 *lambs take to eat* 36 *bushels?*"]

$$\text{Hence the req}^{d}\text{ time} = \overset{\text{days}}{7} \times \frac{15}{27} \times \frac{36}{40} = \underline{3\tfrac{1}{2}\ \textit{days Ans}}$$

A (iv) *Thirty-three masons could build a certain wall in* 47 *days, but, after working for* 11 *days, fifteen of them "strike", in how many days after this will the wall be finished?*

In the 11 days that 33 men work they do $\frac{11}{47}$ of the work; we have now to find how long the 18 men left after the strike will take to do the remaining $\frac{36}{47}$ of the work

$$\text{Hence the req}^{d}\text{ time} = \overset{\text{days}}{11} \times \frac{33}{18} \times \frac{36}{11} = \underline{66\ \textit{days Ans}}$$

B. Assets, Dividends, &c

A person is *insolvent* when he owes more than the value of his possessions, *i e.* when his *debts* are greater than his **assets.**

The persons to whom he owes money are called his **creditors**

When the debtor is made a **bankrupt** his assets are fairly distributed among his creditors, *i e* in place of each £1 he owes, some fraction of a pound, called a **dividend**, is paid.

B (i) *A bankrupt pays a dividend of* 5*s* 8*d in the pound what was due to a creditor who received* £61, 1*s* 2*d* ?

$$5s\ 8d = 5\tfrac{2}{3}s\ ,\quad £61,\ 1s\ 2d = 1221\tfrac{1}{6}s$$

The creditor received $\overset{s}{5\tfrac{2}{3}}$ in place of $\overset{£}{1}$

$\therefore$ . . . . $1221\tfrac{1}{6}$ $\qquad 1 \times \dfrac{1221\frac{1}{6}}{5\frac{2}{3}}$

$$= \frac{431}{2} = \underline{£215,\ 10s.\ \textit{Ans.}}$$

B (ii) *A bankrupt's debts amount to £4563; his assets to £1872, the legal expenses connected with his bankruptcy amount to £105. what loss will a creditor sustain whose claim is for £507?*

The sum available for distribution among creditors = £1872 — £105 = £1767.

∴ the loss to creditors on debts of £4563 = £4563 — £1767 = £2796.

∴ the loss to a creditor whose claim is £507 $= \overset{£}{2796} \times \frac{507}{4563} = \overset{£}{\frac{932}{3}}$

= £310, 13s 4d. *Ans.*

C. Rates, Taxes, &c.

Taxes are charges on property which are fixed by Parliament, and are payable to the Government of the country

Income-tax is assessed (*i.e* calculated) as a charge (subject to certain conditions and exceptions) on each *pound* of a person's *income*

Rates differ from taxes in being fixed by, and payable to, the Council or Corporation (*i e.* the local governing body) of a county, town, or parish Rates are assessed as a charge on each *pound* of the *rental* value of the land, house, &c, occupied by a person

The total sum of money (*e g* salary, profits on trade, &c) which comes to a person *annually** is called his *gross** income

After fixed "outgoings" (*e g* rates, taxes, &c) have been deducted, the remainder is called his *net** income

C (i). *The rates on a certain property, assessed at 1s 10d. in the pound, amounted to £26, 5s 3d. what was the rateable value of the property?*

£26, 5s. 3d = $525\frac{1}{4}$s.

Now $\overset{s}{1\frac{5}{6}}$ was paid on $\overset{£}{1}$

∴ $525\frac{1}{4}$. $1 \times \frac{525\frac{1}{4}}{1\frac{5}{6}} = \frac{\overset{191}{\cancel{2101}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{6}}}{\cancel{11}} = \overset{£}{\frac{573}{2}}$

= £286, 10s *Ans*

**Annual*, from Latin *annus*, a year *Gross*=thick, *i e* whole *Net*=neat, *i.e* trimmed.

C (ii) *A man's gross income was £540, find his net income after paying income-tax at the rate of 5d. in the pound*

£1, or 240d, is reduced by the tax to 235d

∴ Each £1 *gross* becomes £$\frac{235}{240}$ *net*

∴ £540. . . £$540 \times \frac{235^*}{240} = \frac{2115}{4} =$ £528, 15*s* *Ans*

N B—**Net* income is obtained from *gross* by multiplying by the *proper* fraction $\frac{240 - tax}{240}$, the tax being expressed in *pence per* £1

C (iii) *A man's net income after deducting income-tax at 6d in the pound is £819, find his gross income*

£1, or 240d, is reduced by the tax to 234d

∴ £$\frac{234}{240}$ *net* results from £1 *gross*

∴ 1 . $1 \times \frac{240}{234}$

∴ 819 . ... $819 \times \frac{240^*}{234} = 7 \times$ £120 = £840 *Ans.*

N.B—* *Gross* income is obtained from *net* by multiplying by the *improper* fraction $\frac{240}{240 - tax}$, the tax being expressed in *pence per* £1

D. Fractions

D (i). *Three-eighths of a certain number is 174, find the number.*

$\frac{3}{8}$ of the req^d number $= 174$

∴ $\frac{1}{8}$ $= \frac{174}{3}$

∴ the whole, or $\frac{8}{8}$, $= \frac{174}{3} \times 8 = 464$ *Ans*

D (ii) *What number exceeds its seventh part by 78?*

By the question, 6 *sevenths* of the req^d number $= 78$

∴ the whole, or 7 *sevenths* $= 78 \times \frac{7}{6} = 91$ *Ans*

D (iii) *What number is that the sum of whose fifth and eighth parts is 39?*

As $\frac{1}{5} + \frac{1}{8} = \frac{13}{40}$, ∴ $\frac{13}{40}$ of the req^d number $= 39$

∴ $\frac{40}{40}$ .... $= 39 \times \frac{40}{13} = 120$ *Ans*

D (iv) *After paying $\frac{1}{3}$ of the money in my purse to one person, and then $\frac{1}{6}$ of what remained to another, I had 10d. left. How much money was there in my purse at first?*

To the first person I paid $\frac{1}{3}$ of my money, and then had left $\frac{2}{3}$ of it.

To the second person I then paid $\frac{1}{6}$ of $\frac{2}{3} = \frac{1}{9}$ of my money.

$\therefore$ altogether I paid away $\frac{1}{3} + \frac{1}{9} = \frac{4}{9}$ of my money,

$\therefore$ I had left at last $\frac{5}{9}$ of my money.

Hence 5 *ninths* of the req^d sum = 10 d.

$\therefore$ the whole, or 9 *ninths* . . . . $= 10 \times \frac{9}{5} = \underline{18d.\ Ans}$

D (v) *The liquid in a cask filled $\frac{2}{3}$ of the cask; after 24 gallons had been drawn out the remaining liquid filled $\frac{2}{7}$ of the cask: how many gallons would the cask hold?*

As $\frac{2}{3} - \frac{2}{7} = \frac{8}{21}$, $\therefore \frac{8}{21}$ of the cask contained 24 gallons

$\therefore \frac{21}{21}$ . . . . . $24 \times \frac{21}{8} = \underline{63\ gal.\ Ans.}$

E. Time and Work.

E (i) *A man, A, alone could mow a field in 6 days; another, B, alone could mow it in 8 days: how long would they take together?*

N B—In questions of this class we first find the *fractional part of the work* done by each agent in a unit of time

Here A mows $\frac{1}{6}$, and B $\frac{1}{8}$, of the field in 1 day.

$\therefore$ together they mow $\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$ of the field in 1 day

$\therefore$ . . . . $\frac{1}{24}$ . . . . $\frac{1}{7}$

$\therefore$ . . . . $\frac{24}{24}$ . . . . $\frac{24}{7} = \underline{3\frac{3}{7}\ days\ Ans.}$

E (ii) *A cistern can be filled from one tap in 24 minutes or be emptied by a valve in 44 minutes. If the cistern is empty and both tap and valve are open, how long will it take to fill?*

The tap *fills* $\frac{1}{24}$, the valve *empties* $\frac{1}{44}$, of the cistern in 1 min.

$\therefore$ with both open, $\frac{1}{24} - \frac{1}{44} = \frac{5}{264}$ is filled in 1 min

$\therefore$ the whole, or $\frac{264}{264}$, ... $1 \times \frac{264}{5}$

$= \underline{52\frac{4}{5}\ min\ \ Ans.}$

E (iii) *A and B working together can mow 7 acres in 3 days, B and C, 9 acres in 5 days, A and C, 8 acres in 4 days How long would A, B and C together take to mow 23 acres?*

$$A+B \text{ mow } \overset{\text{acres}}{\tfrac{7}{3}},\ B+C \text{ mow } \overset{\text{acres}}{\tfrac{9}{5}},\ A+C \text{ mow } \overset{\text{acres}}{2} \text{ in } 1 \text{ day.}$$

$$\therefore, \text{ by addition, } 2A+2B+2C \text{ mow } \tfrac{92}{15} \text{ acres in } \overset{\text{day}}{1}$$

$$\therefore A+B+C \quad \tfrac{46}{15} \quad 1$$

$$\therefore A+B+C \quad 23 \quad 1\times\frac{23}{\frac{46}{15}}$$

$$= 23\times\tfrac{15}{46} = 7\tfrac{1}{2} \textit{ days Ans}$$

E (iv) *A and B can do a piece of work alone in 12 and 16 days respectively, they work together for 3 days when A leaves, and 2 days later B is joined by C, after which the work is finished in 3 more days. How long would C take to do the work alone?*

A does $\frac{1}{12}$, and B $\frac{1}{16}$, of the work per day

$\therefore$, in the 3 days A works he does $\frac{3}{12}$, or $\frac{1}{4}$, of the work,

and in the 8 days B works he does $\frac{8}{16}$, or $\frac{1}{2}$, of the work

$\therefore$, without C's help, $\frac{1}{4}+\frac{1}{2}=\frac{3}{4}$ of the work is done

Hence C does $\frac{1}{4}$ of the work in 3 days

$\therefore$ C could do the whole in 12 *days Ans*

F. Time and Distance.

The *rate*, or *velocity*, of a body which moves uniformly, is expressed by the *distance* it travels in some *unit of time*

For instance, when we say that a train is travelling at the rate of 30 miles an hour, we mean that if it continued to move on steadily for 1 hour it would go 30 miles, if for four hours, it would go 4 times 30 miles, if for $\frac{1}{2}$ hour, it would go $\frac{1}{2}$ of 30 miles, and so on

F (i) *How many feet per second does a body travel which moves uniformly at the rate of 50 miles an hour?*

In 60×60 *seconds* it goes $\overset{\text{miles}}{50}$

$$\therefore \ldots \ 1 \textit{ second} \ldots\ldots \frac{50}{60\times60} = \frac{\overset{\text{feet}}{50\times1760\times3}}{60\times60} = 50\times\overset{\text{feet}}{\frac{22^{*}}{15}}$$

$$= 73\tfrac{1}{3} \textit{ ft per sec Ans}$$

F (ii). *How many miles per hour correspond to 44 feet per second?*

	sec		feet		miles
In	1	the distance is	44		
∴ in	60×60	. .		$44 \times 60 \times 60 =$	$\dfrac{44 \times 60 \times 60}{3 \times 1760} = 44 \times \dfrac{15}{22}$*

30 *mi per hour Ans*

N B—From the results marked thus * in Exs F (i) and (ii) we see that the multiplier $\frac{22}{15}$ converts *mi per hr* into *ft per sec*, and conversely, the multiplier $\frac{15}{22}$ converts *ft. per sec* into *mi per hr*

F (iii) *How long will a bicyclist whose rate is* 12 *miles an hour take to go* 60 *miles?*

miles		hour
12	is the distance in	1
∴ 60	. . . .	$1 \times \dfrac{60}{12}$† = 5 *hours Ans.*

F (iv) *Two persons, A and B, start at the same time from places* 35 *miles apart, A at the rate of* 3 *miles an hour, and B at the rate of* 4 *miles an hour, and walk to meet each other. how long do they take?*

miles		hour
3 + 4, or 7	is the distance gained in	1
∴ 35	. . . .	$1 \times \dfrac{35}{7}$† = 5 *hours Ans*

F (v) *A and B start at the same time from places* 9 *miles apart, B at the rate of* 6 *miles an hour, and A following him at the rate of* 8 *miles an hour; at what time will A overtake B?*

miles		hour
8 − 6, or 2	is gained by A in	1
∴ 9	. . . .	$1 \times \dfrac{9}{2}$† = $4\frac{1}{2}$ *hours Ans.*

N B—The results† in Exs (iii), (iv) and (v) may be generalized:—

(I) In the case of a single body moving uniformly, $\textit{time} = \dfrac{\textit{distance}}{\textit{rate}}$.

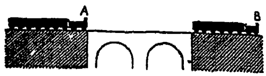
i e the *number of units of length* in the distance to be travelled, when divided by the rate (*expressed in terms of the* same unit of length and *some unit of time*), gives the *number of such units of time* occupied.

(II) When two bodies move in *opposite* directions, the *time* taken to gain a given distance $= \dfrac{\textit{distance}}{\text{sum } \textit{of rates}}$.

(III). When two bodies move in the *same* direction, the *time* taken to gain a given distance $= \dfrac{\textit{distance}}{\text{difference } \textit{of rates}}$.

F (vi). *How long will a train 90 yards long, going at the rate of 20 miles per hour, take to pass completely over a bridge 130 yards long?*

The train begins to cross the bridge when the *engine* is at A, it has not *completely* crosed it until the *guard's van* is over and the *engine* has reached B



i e the engine must travel a distance = length of bridge + length of train $= 130 + 90 = 220$ yds

But $\overset{\text{yds}}{20 \times 1760}$ is traversed by the engine in $\overset{\text{seconds}}{60 \times 60}$

$\therefore$ 220 .. . $60 \times 60 \times \dfrac{220}{20 \times 1760}$

$= \underline{22\frac{1}{2} \textit{ secs} \quad \textit{Ans}}$

F (vii) *A train, travelling at the rate of 40 miles an hour, whilst inside a tunnel, meets another train of half its length, travelling at 60 miles an hour, and passes it completely in $4\frac{1}{2}$ seconds Find the length of the tunnel, if the first train passes completely through it in 4 minutes $37\frac{1}{2}$ seconds*

In order to pass each other completely the engines must separate by a distance = the sum of the lengths of the trains = $1\frac{1}{2}$ times the length of the 1st train,
and the rate at which they separate = the sum of their rates $= 40 + 60$ $= 100$ mi per hour $= 100 \times \frac{22}{15}$ *ft per sec* (See p 143

$\therefore$ in $4\frac{1}{2}$ sec they separate by $100 \times \frac{22}{15} \times 4\frac{1}{2}$ ft

$\therefore$ **the length of the 1st train** $= 100 \times \frac{22}{15} \times 4\frac{1}{2} \div 1\frac{1}{2}$ **ft = 440 ft.**

Again, in order to pass completely through the tunnel, the 1st train must travel a distance = length of tunnel + its own length

But in 1 sec it goes $40 \times \frac{22}{15}$ feet

$\therefore$ in 4 min $37\frac{1}{2}$ secs, or $277\frac{1}{2}$ secs $\quad 40 \times \frac{22}{15} \times \frac{555}{2} = \frac{440 \times 111}{3}$ ft

Hence, the length of the tunnel $= \dfrac{440 \times 111}{3} - 440$ feet $= 440 \times \left(\dfrac{111}{3} - 1\right)$ ft

$= 440 \times \dfrac{108}{3}$ feet $= \frac{1}{12} \times \dfrac{108}{3}$ miles $= \underline{3 \textit{ mi} \quad \textit{Ans}}$

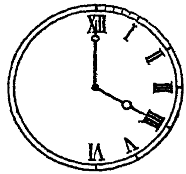
To this section belong questions relating to the motion of the hands of a clock As the long hand makes a complete circuit of of the face in an hour it travels past 60 of the *minute-spaces*

marked on the dial in 1 hour, while the short hand travels past 5 of these *spaces* only.

$\therefore$ *the long hand gains* **55** *minute-spaces in* 1 *hour*.

i e the difference of their rates is 55 minute-spaces per hour.

F (viii) *At what time between* 4 *and* 5 *o'clock are the hands of a watch pointing in the same direction?*



At 4 o'clock the long hand points to XII., and the short hand to IIII, *i e* they are 20 minute-spaces apart

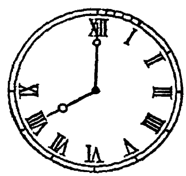
So the long hand will overtake the short hand and they will point in the same direction when it has gained 20 minute-spaces

But the long hand gains 55 *min -spaces* in 1 hour

$$\therefore \ldots\ldots\ldots\ldots\ 20\ \textit{min.-spaces} \ldots\ 1 \times \frac{20}{55} = \frac{20 \times 60}{55} = 21\frac{9}{11}\ \text{min.}$$

Ans. $21\frac{9}{11}$ *minutes past* 4.

F (ix) *At what time between* 8 *and* 9 *o'clock are the hands pointing in opposite directions?*



They will point in opposite directions when the long hand is 30 *minutes-spaces* behind the short hand But at 8 o'clock the long hand is 40 min -spaces behind

$\therefore$ the long hand must gain 10 min -spaces

But it gains 55 min -spaces in 1 hour

$$\therefore \ldots\ldots\ 10 \ \ldots\ 1 \times \frac{10}{55} = \frac{10 \times 60}{55}\ \text{minutes} = \frac{120}{11}\ \text{min.}$$

Ans $10\frac{10}{11}$ *min. past* 8.

F (x). *At what times between* 5 *and* 6 *o'clock are the hands at right angles?*

The hands are at right angles when they are 15 minute-spaces apart, and this happens twice in the hour

1st when the long hand is 15 min -spaces *behind* the short hand
2nd *in front* of

Now at 5 o'clock the hands are 25 min -spaces apart

Hence they will be at right angles, 1st when the long hand has gained 10 min -spaces, and again when it has gained 40 min -spaces,

i e. in $\frac{10}{55}$, and $\frac{40}{55}$, of an hour, or at $10\frac{10}{11}$, and $43\frac{7}{11}$, *min. past* 8. *Ans.*

G. Races

G (i). *A can beat B by 5 yards in 100 yards, and B can beat C by 5 yards in 100 yards by what distance would A beat C in $\frac{1}{4}$ mile?*

	yds			yds
A runs	**100**	while	B runs	**95**
But B	100		C	95
∴ B	**95**		C . .	$95 \times \frac{95}{100} = \frac{361}{4}$
i e A	100		C . .	$\frac{361}{4}$
∴ A .	440	.	C ..	$\frac{361}{4} \times \frac{440}{100} = \frac{361 \times 11}{10} = 397\frac{1}{10}$

∴ A can give C $440 - 397\frac{1}{10} = 42\frac{9}{10}$ yds *Ans*

G (ii) *A can give B 10 points in 50 at billiards, B can give C 25 in 100, and C can give D 50 in 500 how many points should A give D in 1000?*

	points			points
A scores	**50**	while	B scores	**40**
But B	100		C	75
∴ B .	**40**		C	$75 \times \frac{40}{100} = 30$
And C	500		D	450
∴ C .	**30** .		D	$450 \times \frac{30}{500} = 27$
i e A .	50		D	27
∴ A	1000		D	$27 \times 20 = 540$

Hence A should give D $1000 - 540 = 460$ *Ans*

H. Chains

H (i) *If 3 oranges are worth 7 apples, and 9 apples are worth 5 pears, and 2 pears are worth 15 strawberries, and 80 strawberries cost 6d, what is the value of 4 doz oranges?*

1 *orange* is worth $\frac{7}{3}$ of an *apple*

$= \frac{7}{3}$ of $\frac{5}{9}$ of a *pear*

$= \frac{7}{3}$ of $\frac{5}{9}$ of $\frac{15}{2}$ of a *strawberry*

$= \frac{7}{3}$ of $\frac{5}{9}$ of $\frac{15}{2}$ of $\frac{6}{80}$ of a *penny*

∴ 48 *oranges* are worth $\frac{7}{3} \times \frac{5}{9} \times \frac{15}{2} \times \frac{6}{80} \times 48$ *pence*,

$= 35d = 2s\ 11d$ *Ans*

XLV. RATIO AND PROPORTION.

RATIO.

The ratio of one number to another, or of one quantity to another of *like* kind, is the relation which the first bears to the second with respect to *magnitude*, expressed by the number of times the first contains the second, or by the fraction which the first is of the second

For instance, the ratio of 8 to 2 is expressed by the number 4,
the ratio of 7 to 3 is expressed by the fraction $\frac{7}{3}$,
the ratio of 3 cwts to 1 ton 2 cwts is expressed by the fraction $\frac{3}{22}$.

The two numbers, or quantities, thus compared are called the **terms** of the ratio, the first being sometimes called the *antecedent*, and the second the *consequent*

A ratio is indicated by two dots, one over the other, placed between the terms, or by placing the first term over the second with a line between them.

For instance, the ratio of 5*s* to 6*s* 6*d*, thus, 5*s* : 6*s*. 6*d*, or thus, $\frac{5s.}{6s.\ 6d.}$.

Note—The *continuous* notation 2 : 3 : 4 indicates *three* ratios, namely, the ratio of 2 to 3, of 3 to 4, and of 2 to 4.

The **terms** of a ratio must always be of *like* kind.

For instance, we *can* compare the magnitude of 3 *cwts* with that of 4 *tons*, but we *cannot* compare the magnitude of 3 *cwts* with that of 4 *miles*.

A ratio whose *first* term is *greater* than its *second* is sometimes called a *ratio of greater inequality*, and a ratio whose first term is *less* than its second, a *ratio of less inequality*.

EXAMPLE i—*Express in simplest form the ratio £2, 5s : £3, 7s. 6d.*

$$\text{The ratio} = \frac{2\frac{1}{4}}{3\frac{3}{8}} = \frac{18}{27} = \frac{2}{3}. \qquad \underline{Ans.\ 2:3.}$$

EXAMPLE ii.—*What length bears to a mile the ratio 5 : 11?*

$$\frac{5}{11} \text{ of a mile} = \frac{5}{11} \times 1760 \text{ yds.} = \underline{800 \text{ yds } Ans.}$$

EXAMPLE iii—*If 4 be added to the numerator of the fraction $\frac{16}{23}$, what number must be added to the denominator in order that the value of the fraction may be unchanged?*

The new den^r must be greater than 16 + 4, or 20, in the ratio of 23 to 16 *i e* the new den^r is $\frac{23}{16}$ of $20 = \frac{115}{4} = 28\frac{3}{4}$

$\therefore$ the req^d number is $28\frac{3}{4} - 23 = \underline{5\frac{3}{4}}$ *Ans.*

PROPORTION

Four numbers, or quantities, are called **proportionals** when the *ratio* of the *first* to the *second* is equal to the *ratio* of the *third* to the *fourth*

For instance, 8, 12, 14*s* and 21*s* are proportionals,

for the ratio of 8 to 12, or $\frac{8}{12} = \frac{2}{3}$,

and the ratio of 14*s* to 21*s* or $\frac{14s}{21s} = \frac{14}{21} = \frac{2}{3}$ also,

and thus the ratio of 8 (the first) to 12 (the second) is equal to the ratio of 14*s* (the third) to 21*s* (the fourth)

Also, 9*s*, 7*s* 6*d*., 3 tons, and 2 tons 10 cwts are proportionals,

for the ratio of 9*s*. to 7*s* 6*d*, or $\frac{9s}{7\frac{1}{2}s} = \frac{18}{15} = \frac{6}{5}$,

and the ratio of 3 tons to 2 tons 10 cwts, or $\frac{3 \text{ tons}}{2\frac{1}{2} \text{ tons}} = \frac{6}{5}$ also

The statement of the fact that two ratios are equal is called a *proportion*, and is often written thus —

$$8 : 12 :: 14s : 21s \qquad \text{(I)}$$

and read thus "8 *is to* 12 *as* 14*s* *is to* 21*s*"

or written thus $8 : 12 = 14s : 21s$ (II)

or thus. $$\frac{8}{12} = \frac{14s}{21s} \qquad \text{(III)}$$

It follows from (III) that any two equal fractions form a proportion, which may be read in four ways

For instance, as $\frac{2}{3} = \frac{4}{6}$, $\therefore$ 2 is to 3 as 4 is to 6,

also as $\frac{4}{6} = \frac{2}{3}$, $\therefore$ 4 is to 6 as 2 is to 3,

also as $\frac{3}{2} = \frac{6}{4}$, $\therefore$ 3 is to 2 as 6 is to 4,

also as $\frac{6}{4} = \frac{3}{2}$, $\therefore$ 6 is to 4 as 3 is to 2

Since the two terms of a ratio must always be *alike*, it follows that the 1st and 2nd terms of a proportion must always be alike, and the 3rd and 4th terms also alike

For instance, 2 tons : 3 tons :: 4 acres : 6 acres is a *correct* statement, but 2 tons : 3 acres :: 4 tons : 6 acres is *incorrect*

EXAMPLE IV —*The first, third and fourth terms of a proportion are 7s, 8 and 12, find the second term.*

The req[d] quantity is *greater* than 7*s* *in the ratio of* 12 *to* 8;

i e the req[d] quantity $= \frac{12}{8}$ of 7*s*. $= 10\frac{1}{2}s$ = 10*s* 6*d* *Ans.*

EXAMPLE v.—*Find a fourth proportional to 2s., 2s. 6d., and 1 yd. 2 ft.*

The req^d^ quantity is *greater than* $1\frac{2}{3}$ yds in the ratio of $2\frac{1}{2}s$ to $2s$.

$$i.e. \text{ the req}^{d} \text{ quantity} = \frac{2\frac{1}{2}}{2} \text{ of } 1\frac{2}{3}\text{ yds} = \frac{5}{4} \times \frac{5}{3} \text{ yds.} = \underline{2 \text{ yds } 3 \text{ ins. } Ans.}$$

Note.—All the Exercises on Chapter XLII, it should be noticed, consist in finding a *fourth proportional* to three given terms, and the "Fractional Factor" (see pages 134, 136) expresses *the ratio in which the 3rd term must be increased or diminished,* to produce the req^d^ 4th term.

The *first* and *fourth* terms of a proportion are called the **extremes**; and the *second* and *third* terms, the **means**.

When the terms of a proportion are *abstract* numbers the **product** *of the extremes is equal to the* **product** *of the means.*

For instance, in the proportion $3 : 5 :: 15 : 25$
the product of the extremes is $3 \times 25 = 75$
and the product of the means is $5 \times 15 = 75$ also.

Three numbers are said to be in **Continued Proportion** when the ratio of the *first* to the *second* is equal to the ratio of the *second* to the *third*,
and the *second* is called a **mean proportional** between the 1st and 3rd.

For instance, 4, 6, 9 are in continued proportion, for $\frac{4}{6} = \frac{6}{9}$,
and 6 is a mean proportional between 4 and 9.
Also 9 is a *third* proportional to 4 and 6.

EXAMPLE vi.—*Find a mean proportional between* 11 *and* 99.

Here 11 : the req^d^ number :: the req^d^ number : 99

[But product of means = product of extremes.]

$$\text{Hence, the } \textit{square} \text{ of the req}^{d} \text{ number} = 11 \times 99$$
$$= 11 \times 11 \times 3 \times 3$$
$$= 33 \times 33$$
$$\therefore \text{ the req}^{d} \text{ number} = \underline{33 \quad Ans}$$

Note.—It is shown in Geometry that in similar figures (*i.e.* figures which are of *exactly the same shape* though of different *size*) corresponding lines are proportionals.

For instance, if two triangles are of the *same shape*, and the longest side of the first is 5 *times* the length of the longest side of the second, then the shortest side of the first will also be 5 *times* the length of the shortest side of the second, and so on.

XLVI. PROPORTIONAL PARTS
AND PARTNERSHIP

To divide a given number, or quantity, into parts proportional to given numbers, i e into parts which shall be in given ratios, we proceed thus —

EXAMPLE i —*Divide £21 into two parts proportional to 2 and 5.*

As $2+5=7$, and $\frac{1}{7}$ of £21 = £3

∴ the req^d parts are £3 × 2 and £3 × 5, i e *£6 and £15 Ans*

EXAMPLE ii —*Divide 375 into parts proportional to 3, 8 and 14*

$3+8+14=25$, and $\frac{1}{25}$ of $375=15$

Hence the req^d parts are 15×3, 15×8 and 15×14

i e *45, 120, and 210 Ans*

EXAMPLE iii —*Divide 3 cwts in the ratio 42 : 53 : 17.*

$42+53+17=112$ Hence the req^d parts are

$\frac{42}{112}$ of 3 cwts , $\frac{53}{112}$ of 3 cwts , and $\frac{17}{112}$ of 3 cwts

or, *1 cwt 14 lbs , 1 cwt 47 lbs , and 51 lbs Ans*

Note —In this, and succeeding Examples, in order to save space part of the "working" is omitted

---

EXAMPLE iv —*A and B are partners in a business, A's capital is £1200 and B's is £950 Divide profits amounting to £344 fairly between them*

[Here we have to divide £344 into parts proportional to 1200 and 950]

$1200+950=2150$

Hence A's share of the profits is $\frac{1200}{2150}$ of £344 = £192

and B's $\frac{950}{2150}$ of £344 = £152 } *Ans*

From the above examples we obtain the following rule :—

To divide a given quantity into parts proportional to given numbers.

Add together the given numbers, divide the given quantity by the result, and multiply the quotient by each of the given numbers in succession

Note —It is generally best to *merely indicate* the above division and

multiplication by signs, and not actually to perform them until any possible cancelling has been performed (See Ex. III and Ex. IV)

EXAMPLE V.—*Divide £65, 2s. into parts proportional to* $\frac{1}{3}, \frac{2}{5}, \frac{3}{10}$.

$$\tfrac{1}{3}+\tfrac{2}{5}+\tfrac{3}{10}=\frac{10}{30}+\frac{12}{30}+\frac{9}{30}=\frac{31}{30}.$$

Hence the req^d parts are

$$\frac{10}{31}^{*} \text{ of } £65\tfrac{1}{10};\ \frac{12}{31} \text{ of } £65\tfrac{1}{10};\ \text{and } \frac{9}{31} \text{ of } £65\tfrac{1}{10};$$

i.e. £21; £25, 4s, and £18, 18s *Ans.*

EXAMPLE VI.—*Divide* 2·18 *into parts proportional to* ·8, 1·7, ·95 *and* 2

$$·8+1·7+·95+2=5·45.$$

Hence the req^d parts are

$$\frac{·8}{5·45} \text{ of } 2·18,\ \frac{1·7}{5·45} \text{ of } 2·18;\ \frac{·95}{5·45} \text{ of } 2·18;\ \text{and } \frac{2}{5·45} \text{ of } 2·18$$

$$\text{or, } \frac{80}{545}\times 2·18,\ \frac{170}{545}\times 2·18,\ \frac{95}{545}\times 2·18;\ \text{and } \frac{200}{545}\times 2·18,$$

i.e. ·32; ·68; ·38; and ·8 *Ans.*

We shall now give some examples of a rather more complex nature.—

EXAMPLE VII.—*Divide £252 between A, B and C, so that for every £2 A receives B receives £5, and for every £3 B receives C receives £7.*

[Here, in order to obtain the *continuous* ratio of the shares, we take the L.C.M. of 5 and 3, the two numbers which relate to B's share and replace the given ratios by equivalent ones having this common number, 15, instead of 5 and 3 respectively.]

$$\frac{\text{A's share}}{\text{B's share}}=\frac{2}{5}=\frac{6}{15}, \text{ and } \frac{\text{B's share}}{\text{C's share}}=\frac{3}{7}=\frac{15}{35};$$

∴ A's share : B's share : C's share :: 6 : 15 : 35.

[We now divide £252 into parts proportional to 6, 15 and 35.]

$$6+15+35=56.$$

Hence the shares are $\frac{6}{56}$ of £252, $\frac{15}{56}$ of £252; $\frac{35}{56}$ of £252.

i.e. £27, £67, 10s, £157, 10s *Ans.*

---

* For $\frac{10 \text{ thirtieths}}{31 \text{ thirtieths}}=\frac{10}{31}$, and so on.

EXAMPLE viii —*Divide £125 between A, B and C so that A may have four times as much as B, and C one-fourth of the sum of A's and B's shares*

B's share $= \frac{1}{4}$ of A's, and C's share $= \frac{1}{4}$ of A's $+ \frac{1}{4}$ of B's,

$\therefore$ C's share $= \frac{1}{4}$ of A's $+ \frac{1}{16}$ of A's $= \frac{5}{16}$ of A's

Hence A's share . B's C's . A's $\frac{1}{4}$ of A's $\frac{5}{16}$ of A's

$:: 1 \cdot \frac{1}{4} \quad \frac{5}{16}$

Now $1 + \frac{1}{4} + \frac{5}{16} = \frac{16}{16} + \frac{4}{16} + \frac{5}{16} = \frac{25}{16}$

$\therefore$ the shares are $\frac{16}{25}$ of £125, $\frac{4}{25}$ of £125, $\frac{5}{25}$ of £125.

i e. £80, £20, £25 *Ans*

N B.—In questions on Partnership in which the capitals of the partners are employed for **unequal** intervals of time, we *substitute for the given capitals* their **equivalents** *referred to* **the same unit of time.**

EXAMPLE ix —*A begins to trade with a capital of £600. At the end of 4 months he takes a partner, B, with a capital of £500. How must they divide the profits, £224, at the end of the year?*

A has £600 employed for 12 months,
which is equivalent to £600 × 12 for 1 month.
B has £500 employed for 8 months,
which is equivalent to £500 × 8 for 1 month
Hence the profits must be divided in the ratio

$600 \times 12 \;.\; 500 \times 8$

i e in the ratio $7200 \cdot 4000$, &c

EXAMPLE x —*Three men, A, B and C, together hire the grazing of a pasture for £50, 5s A puts on 70 sheep for 6 months, B puts on 90 sheep for 5 months, and C puts on 45 sheep for 3 months What share of the rent should each pay?*

	A must pay for the food of	70 sheep for	6 months
i e.	A .	70 × 6	1 month
	B .	90	5 months
i.e	B .	90 × 5	1 month
	C	45	3 months
i e	C	45 × 3	1 month.

Hence we must divide £50, 5*s* into parts proportional to
70×6, 90×5, and 45×3, *i e.* to 420, 450, and 135, &c

EXAMPLE XI —*A and B enter into partnership. A puts in £2000, but at the end of 3 months withdraws £500 of this, and again at the end of 8 months withdraws £300. At the end of the year B's share of profits : A's : : 4 : 5. Find B's capital.*

A's working capital is

£2000 for 3 mo + £1500 for 5 mo + £1200 for 4 mo,

which is equivalent to (£6000 + £7500 + £4800) for 1 month.

= £18300 for 1 month = £1525 for a year.

Hence, as B's share of profits is $\frac{4}{5}$ of A's,

∴ B's capital is $\frac{4}{5}$ of £1525 = £1220 *Ans.*

EXAMPLE XII —*A and B are partners, with capitals in the ratio 4 : 5. After 3 months they withdraw $\frac{1}{5}$ and $\frac{1}{3}$ of their capitals respectively, and 4 months later B again withdraws $\frac{1}{3}$ of his original capital How must the profits, which at the end of the year amount to £581, be divided?*

A's capital is proportional to £4 for 3 mo. + $\frac{4}{5}$ of £4 for 9 mo.

= £12 + £$\frac{144}{5}$, for 1 month = £$\frac{204}{5}$ for 1 month.

B's capital is proportional to

£5 for 3 mo + $\frac{2}{3}$ of £5 for 4 mo. + $\frac{1}{3}$ of £5 for 5 mo

= £15 + £$\frac{40}{3}$ + $\frac{25}{3}$, for 1 month = £$\frac{110}{3}$ for 1 month

Hence the profits must be divided in the ratio $\frac{204}{5} : \frac{110}{3}$; &c.

EXAMPLE XIII —*Volumes proportional to the numbers 3, 4 and 7 of three different substances are mingled together The weights of equal volumes of the substances are as 5 : 2 : 6 respectively. Find the weight of the amount of the 3rd substance contained in 52 lbs. of the mixture.*

The weight of	1 volume*	of the 1st is proportional to	5
∴	3 volumes		15
	1 volume	2nd	2
∴	4 volumes		8
	1 volume	3rd	6
∴	7 volumes		42

Now 15 + 8 + 42 = 65.

∴ weight of 3rd substance in mixture = $\frac{42}{65}$ of 52 lbs = $33\frac{3}{5}$ lbs *Ans*

* It is immaterial to the reasoning what this particular "volume" is—whether pint, cubic foot, &c —so long as by "1 volume" we always indicate the same bulk.

XLVII. AREAS.

Rectangular Surfaces

A rectangle is a four-sided plane figure whose angles are *right* angles.

Its opposite sides are equal, and parallel

The length and breadth of a rectangle are called its *dimensions*

A square is a four-sided, right-angled plane figure whose sides are *all* equal

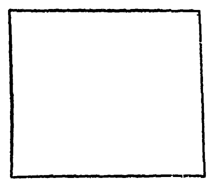
Thus a square is one special kind of rectangle

Rectangles, other than squares, are sometimes called *oblongs*

The **perimeter** of a plane figure is the measure round it, *i e.* the *sum* of the lengths of its sides

For instance, the perimeter of a rectangle 5 inches long and 4 inches wide is $5 + 4 + 5 + 4$, or 18, inches

The **area** of a plane figure is the amount of *surface* enclosed within its boundary

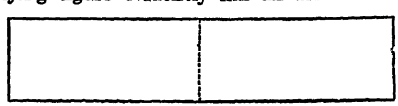


The English units of measurement for areas are the square inch, square foot, &c (See page 30)

A *square inch of surface* is an amount of surface equal to that enclosed within the boundary of a *square* each side of which is *one inch* long

N B—As figures of the same *size* are not necessarily of the same *shape*, it follows that a surface which has *an area of one square inch* (or foot, &c) *is not necessarily in shape a square*

For instance, the accompanying figure evidently has an area of *one square inch*, for by dividing it along the dotted line the two parts would together exactly cover the inch square represented above



Thus a surface **an inch square** is *square in shape*, but a surface whose area is **a square inch** may be of *any shape*

So also a surface **3 feet square** is *in shape a square*, whose sides are each 3 feet long, and its *area* is (as we know from page 30) 9 square feet, but a surface **3 square feet** in area may be of *any shape*

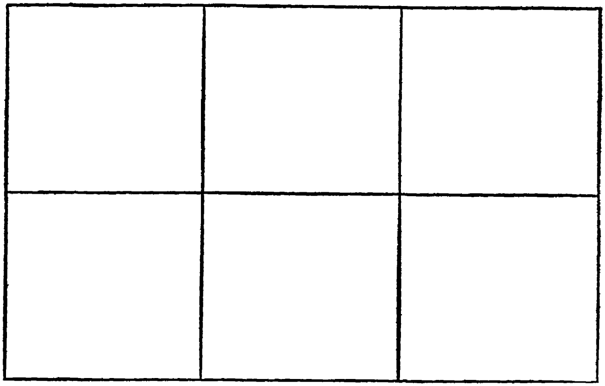
We shall now show how the area of any **rectangular*** surface is calculated

---

*The measurement of the surfaces of triangles, circles, &c , and of figures of irregular shape belongs to Mensuration and Trigonometry

CASE I. *When the dimensions are expressed by* **integers.**

If we take (for example) six squares, each having sides *one inch* long, and place them together thus—



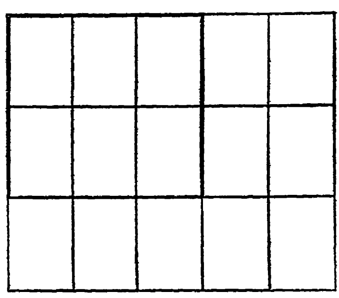
they form a rectangle 3 inches long and 2 inches wide

∴ a rectangle 3 *inches* long and 2 *inches* wide has an area of $\underline{3 \times 2}$, or 6, *square inches.*

In the same way we might show that the area of a rectangle, say, 5 *feet* long and 4 *feet* wide, is $\underline{5 \times 4}$ *square feet* And so on

CASE II *When the dimensions are expressed by* **fractions.**

If we take (for example) 15 little rectangles, each *half an inch* long and *one-third of an inch* wide, and arrange them thus—



they form a larger rectangle 5 *thirds of an inch* long and 3 *halves of an inch* wide.

Now it is evident from the diagram that **one square inch** contains 6, while the larger rectangle contains 15, *of the small rectangles*

Thus the area of the larger rectangle is $\frac{15}{6}$ of the area of the square inch,

i e a rectangle $\frac{5}{3}$ of an *inch* long and $\frac{3}{2}$ of an *inch* wide has an area of $\underline{\frac{5}{3} \times \frac{3}{2}}$, or $\frac{15}{6}$, *square inches.*

In the same way we might show that the area of a rectangle, say,

$3\frac{1}{2}$ *feet* (*i.e* 7 *half-feet*) long and $2\frac{3}{5}$ *feet* (*i e* 13 *fifths of a foot*) wide, is $3\frac{1}{2} \times 2\frac{3}{5}$ *square feet* And so on

Hence the following rule —

To find the area of a rectangular surface:—

Multiply *the* **number** (whole or fractional) *of inches* (or feet, &c.) *in the* **length** *by the* **number** *of inches* (or feet, &c) *in the* **breadth**, *and the result is the* **number** *of square inches* (or square feet, &c) *in the* **area.**

The above rule is often represented shortly by the formula.—

$$\textbf{length} \times \textbf{breadth} = \textbf{area} \qquad \text{(I)}$$

Or, conversely,

$$\left.\begin{aligned}\textbf{breadth} &= \textbf{area} \div \textbf{length}\\ \text{and}\quad \textbf{length} &= \textbf{area} \div \textbf{breadth}\end{aligned}\right\} \qquad \text{(II)}$$

N.B.—The shortened form (I) must not be misunderstood Areas form no exception to the rule that the multiplier can never be *concrete* (see page 40) (I) is merely an abbreviated statement, and should be understood to mean that if we multiply the number (abstract) of units in the length by the number of the *same* kind of units in the breadth, we obtain (as was shown in the diagrams) the number of corresponding square units in the area

We do *not* multiply "feet" by "feet" to obtain "square feet", but *number* (of feet) by *number* (of feet) to obtain *number* (of square feet)

This may strike the student at first as "a distinction without a difference", but a little consideration will convince him that it is not so

(In answer to the question "If a man earn a shilling an hour what will 3 men earn in 4 hours?" we do *not* multiply 3 *men* by 4 *hours* to obtain the result, 12 *shillings*!)

It is very important to notice that the length and breadth must both be expressed in terms of the **same** unit of length before we multiply their numbers together

For instance, if a rectangle is 3 *feet* long and 2 *inches* wide, its area is neither 6 sq *feet* nor 6 sq *inches*

EXAMPLE i—*Find the area of a rectangle 4 ft 3 in long and 2 ft 4 in broad*

$$\text{Length} = 4\tfrac{1}{4}\text{ feet, and breadth} = 2\tfrac{1}{3}\text{ feet}$$

$$\therefore \text{area} = 4\tfrac{1}{4} \times 2\tfrac{1}{3}\text{, sq feet} = 9\tfrac{11}{12}\text{ sq ft} = \underline{1\textit{ sq. yd } 132\textit{ sq in Ans}}$$

EXAMPLE ii.—*Find the area of a rectangular field 33 yards wide and 15 chains long*

$$15\textit{ chains} = 15 \times 22\textit{ yards}$$

$$\text{Hence, by (I), area} = 15 \times 22 \times 33\text{, sq yds} = \frac{15 \times 22 \times 33}{4840}\text{ acres}$$

$$= 2\tfrac{1}{4}\text{ ac.} = \underline{2\textit{ ac } 1\textit{ ro Ans}}$$

EXAMPLE iii.—*Find the length of a floor* 15 ft. wide, the area of which is 30 sq. yds. 5 sq. ft.*

Area $= 30$ sq. yds. 5 sq. ft. $= 275$ *sq. feet*; width $= 15$ *ft.*

Hence, by (II), length $= \frac{275}{15}$ *feet* $= \frac{55}{3}$ ft. $= \underline{18 \text{ ft. } 4 \text{ in. Ans.}}$

EXAMPLE iv.—*If the price is 3d. per square foot, what length must be cut off a nine-inch board that the value of the piece cut off may be sixpence?*

For 6*d.* we obtain an *area* of 2 sq. ft. $= 2 \times 144$ *sq. inches*, and the *width* of the board is 9 *inches*.

$\therefore$, by (II), the *length* $= \frac{2 \times 144}{9}$ *inches* $= 32$ in. $= \underline{2 \text{ ft. } 8 \text{ in. Ans.}}$

EXAMPLE v.—*A roll of matting which is 40 yards long would just cover a floor 20 ft. long and 16 ft. wide. Find the width of the matting.*

[As the matting would just cover the floor, the area of the matting is the same as that of the floor.]

Area of matting $= 20 \times 16$ *sq. feet*, *length* of it $= 40 \times 3$ *feet*;

$\therefore$, by (II), *width* of matting $= \frac{20 \times 16}{40 \times 3}$ *feet* $= \underline{2 \text{ ft. } 8 \text{ in. Ans.}}$

EXAMPLE vi.—*How many tickets, each $2\frac{1}{4}$ in. long and $1\frac{1}{2}$ in. wide, could be cut from a sheet of cardboard $16\frac{1}{2}$ in. long and $13\frac{1}{2}$ in. wide?*

[This is merely an example of Comp. Division, Class II. (see p. 46), *i.e.* we have to find *how many* of the smaller areas are contained in the larger.]

Area of card $= 16\frac{1}{2} \times 13\frac{1}{2}$ *sq. in.*, area of a ticket $= 2\frac{1}{4} \times 1\frac{1}{2}$ *sq. in.*;

$\therefore$ the req^d number $= \frac{16\frac{1}{2} \times 13\frac{1}{2}}{2\frac{1}{4} \times 1\frac{1}{2}} = \frac{33 \times 27 \times 2}{9 \times 3} = \underline{66 \text{ Ans.}}$

EXAMPLE vii.—*How many bricks measuring 9 in. by 4 in. would pave a kitchen measuring 18 ft. by 15 ft.?*

Area of floor $= 18 \times 15$ *sq. ft.*, area of face of a brick $= \frac{3}{4} \times \frac{1}{3}$ *sq. ft.*

$\therefore$ the req^d number $= \frac{18 \times 15}{\frac{3}{4} \times \frac{1}{3}} = 18 \times 60 = \underline{1080 \text{ Ans.}}$

---

* Unless the contrary is stated, all *floors* are supposed to be *rectangular*.

EXAMPLE VIII —*How many yards of drugget 3 ft 6 in wide will cover a floor 21 ft long by 15 ft wide?*

1st Method (that of Example VI)

[Here a "*yard*" of drugget is a *rectangle 3 ft long by $3\frac{1}{2}$ ft wide*, and we have to find *how many* such rectangles are contained in the area of the floor]

Area of floor $= 21 \times 15$ sq ft, area of a "yard" $= 3 \times 3\frac{1}{2}$ sq ft

$\therefore$ the *number* of such "yards" req$^{d} = \dfrac{21 \times 15}{3 \times 3\frac{1}{2}} = \underline{30 \; Ans}$

2nd Method (that of Example V)

Area of drugget $= 21 \times 15$ *sq feet*, *width* of drugget $= 3\frac{1}{2}$ *feet*,

$\therefore$, by (II), *length* of drugget $= \dfrac{21 \times 15}{3\frac{1}{2}}$ *feet* $= 90$ ft $= \underline{30 \; yds \;\; Ans}$

EXAMPLE IX —*Find the cost of carpet, $\frac{3}{4}$ yard wide, at 3s 6d per yard for a floor 18 ft 9 in long and 11 ft 3 in wide*

[$\frac{3}{4}$ yard $= 2\frac{1}{4}$ feet, and adopting, as preferable, the 1st method of Ex. VIII, we regard a "yard" of carpet as a rectangle 3 ft long by $2\frac{1}{4}$ ft wide]

Area of floor $= 18\frac{3}{4} \times 11\frac{1}{4}$ sq ft, area of a "yard" $= 3 \times 2\frac{1}{4}$ sq ft

$\therefore$ the *number* of "yards" req$^{d} = \dfrac{18\frac{3}{4} \times 11\frac{1}{4}}{3 \times 2\frac{1}{4}} = \dfrac{125}{4}$

Hence the *cost* $= \overset{s}{3\frac{1}{2}} \times \dfrac{125}{4} = \dfrac{875}{8}s = \underline{£5, \; 9s \;\; 4\frac{1}{2}d \;\; Ans}$

Note —Such questions as this are not of a very *practical* nature, for allowance has to be made, in most cases, for waste in matching the pattern, &c

---

EXAMPLE X —*Find the cost of staining a margin 2 ft wide all round a floor 21 ft long and 16 ft wide, at 6d per sq yard.*

1st Method

Area of margin = Area of floor − area of unstained portion

$= 21 \times 16$, sq ft $- 17 \times 12$, sq ft

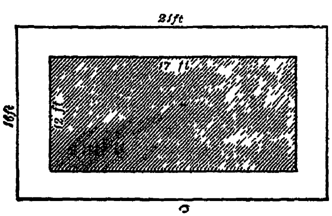
$= 336$ sq ft $- 204$ sq ft

$= 132$ sq ft

$= \dfrac{132}{9}$ sq yds

$\therefore$ cost $= \overset{d}{6} \times \dfrac{132}{9} = 88d$

$= \underline{7s \;\; 4d \;\; Ans}$



2nd Method

[Dividing the margin in the way shown in the diagram, we see that it consists of two equal rectangles each 19 ft long by 2 ft wide + two other equal rectangles each 14 ft long by 2 ft wide]

Area of margin

$= \text{twice}(19 \times 2 + 14 \times 2)$ sq ft

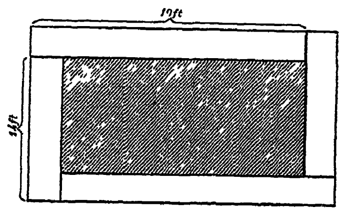
$= 4 \times (19 + 14)$ sq ft

$= 4 \times 33$, sq ft

$= \frac{4 \times 33}{9}$ sq yds

$\therefore \text{cost} = \overset{d}{6} \times \frac{4 \times 33}{9}$

$= $ *7s 4d Ans*



EXAMPLE XI —*If the value of matting 2 ft 3 in wide, at 1s 9d per yard, which covers a floor 16 ft wide is £3, 14s 8d, how long is the floor?*

[This question is one of a class converse to that of Ex IX. We consequently *reverse the order* of the operations]

As £3, 14*s* 8*d* $= 74\frac{2}{3}s$, and 1*s* 9*d* $= 1\frac{3}{4}s$

$\therefore$ the *number of yards* of matting used $= \frac{74\frac{2}{3}}{1\frac{3}{4}} = \frac{128}{3}$.

And the area of a "yard" of it $= 3 \times 2\frac{1}{4}$, sq ft

[But the area of a "yard" multiplied by the *number* of yards = the area of the matting, *i e* of the floor]

Hence, *area* of floor $= \overset{\text{sq ft}}{3 \times 2\frac{1}{4}} \times \frac{128}{3} = 9 \times 32$ sq ft

$\therefore$, by (II), *length* of floor $= \frac{9 \times 32}{16}$ *feet* = 18 *feet Ans*

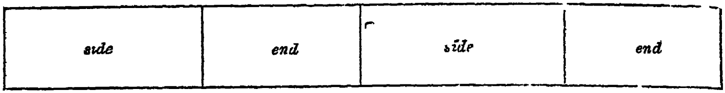
Note—In this and the preceding examples we have adopted the 1st method of Ex VIII, in preference to the 2nd method, as being more general in its application, *i e* the number of equal small rectangles (whether slabs of stone, sheets of paper, or "yards" of carpet) which will cover a larger rectangle is found *by dividing the area of the larger by the area of one of the smaller rectangles expressed in terms of the same unit*

WALLS OF A ROOM

In an ordinary room the walls consist of two equal rectangles which form the sides, and two other equal rectangles which form the ends

Now if we suppose these four rectangles to be opened out so

as to form one long rectangle, as shown in the diagram, it is



evident that the area of the walls = area of this long rectangle
= (length + breadth + length + breadth) × height, of room
= twice (length + breadth) × height

i e **area of walls = perimeter** of floor × **height** of room

EXAMPLE xii —*Find the area of the walls of a room* 18 *ft long,* 15 *ft broad, and* 12 *ft high*

$$\text{Area of walls} = \text{perimeter} \times \text{height}$$
$$= \text{twice } (18 + 15) \times 12, \text{ sq ft}$$
$$= 2 \times 33 \times 12, \text{ sq ft} = \underline{792 \textit{ sq ft Ans}}$$

EXAMPLE xiii.—*How many yards of paper* 2 *ft wide would just cover the walls of a room* 22 *ft long,* 17 *ft wide, and* 11 *ft high?*

$$\text{Area of walls} = \text{perimeter} \times \text{height} = 2 \times (22 + 17) \times 11, \text{ sq ft}$$
$$= 2 \times 39 \times 11, \text{ sq ft}$$
$$\text{Area of a "yard" of paper} = 3 \times 2, \text{ sq ft}$$
$$\therefore \text{ the } \textit{number} \text{ of yards req}^{d} = \frac{2 \times 39 \times 11}{3 \times 2} = \underline{143 \textit{ Ans}}$$

EXAMPLE xiv —*Find the cost of paper* 27 *inches wide, at* $1\frac{1}{2}d$ *per yard, sufficient to just cover the walls of a room* 17 *ft long,* 14 *ft* 6 *in wide, and* 12 *ft* 4 *in high*

$$\text{Area of walls} = \text{perimeter} \times \text{height} = 2 \times (17 + 14\tfrac{1}{2}) \times 12\tfrac{1}{3}, \text{ sq ft}$$
$$= 2 \times 31\tfrac{1}{2} \times 12\tfrac{1}{3}, \textit{ sq feet}$$
$$\text{Area of a "yard" of paper} = 3 \times 2\tfrac{1}{4}, \textit{ sq feet}$$
$$\therefore \text{ the } \textit{number} \text{ of yards req}^{d} = \frac{2 \times 31\frac{1}{2} \times 12\frac{1}{3}}{3 \times 2\frac{1}{4}} = \frac{28 \times 37}{9}$$
$$\text{Hence the } \textit{cost} = \overset{d}{1\tfrac{1}{2}} \times \frac{28 \times 37}{9} = 172\tfrac{2}{3}d = \underline{14s\ 4\tfrac{2}{3}d \textit{ Ans}}$$

Note —If the dimensions of doors, windows, &c , are given, we subtract their areas from the total area of the walls in order to obtain the area to be covered with paper It should, however, be observed that the results obtained are of little *practical* value, inasmuch as no allowance is made for waste in cutting, or in matching the pattern on, the paper. Moreover,

wall-paper is not sold by the yard but by the "piece", usually 12 yards long and 21 inches wide, so that an *exact* number of "pieces" must always be bought, *e g* if a fraction over 13 pieces was wanted, 14 complete pieces at least must be bought The following is a more *practical* question.

EXAMPLE XV — *Find the cost of paper 21 in. wide, at 1s 6d per piece of 12 yards, for the walls of a room 18 ft 6 in long, 16 ft 6 in wide, and 11 ft 3 in high, supposing that the amount of paper saved from windows, &c, together with two complete extra "pieces", is a sufficient allowance for waste*

Area of walls $= 2 \times (18\frac{1}{2} + 16\frac{1}{2}) \times 11\frac{1}{4}$ sq ft $= 2 \times 35 \times \frac{45}{4}$, sq ft.

Area of a "piece" of paper $= (12 \times 3) \times \frac{21}{12}$, sq ft $= 3 \times 21$, sq ft.

$\therefore$ number of pieces which would just cover the walls $\left.\right\} = \dfrac{2 \times 35 \times \frac{45}{4}}{3 \times 21} = 12\frac{1}{2}$

Hence number of pieces to be bought $= 13 + 2 = 15$.

$\therefore$ Cost $= 1\frac{1}{2}s \times 15 =$ £1, 2s 6d *Ans*

EXAMPLE XVI —*How many square feet of sheet-lead would line an open tank measuring inside 4 ft in length, 3 ft 6 in. in width, and 1 ft 10 in in depth?*

Area of *sides and ends* $=$ perimeter $\times$ depth

$= 2 \times (4 + 3\frac{1}{2}) \times 1\frac{5}{6}$, sq feet $= 2 \times 7\frac{1}{2} \times 1\frac{5}{6}$, sq. ft

area of *bottom* $= 4 \times 3\frac{1}{2}$, sq. ft.

$\therefore$ total area of lead req^d $= 2 \times 7\frac{1}{2} \times 1\frac{5}{6} + 4 \times 3\frac{1}{2} = 41\frac{1}{2}$ sq. ft *Ans*

EXAMPLE XVII —*The cost of lining an open rectangular reservoir 25 yards long and 5 ft 6 in. deep, with tiles six inches square, at 1s. 6d. per dozen, was £67, 18s 6d Find the width of the reservoir.*

£67, 18s 6d $= 2717$ *sixpences*, 1s 6d $= 3$ *sixpences*:

$\therefore$ number of *dozens* of tiles used $= \dfrac{2717}{3}$

$\therefore$ no of tiles used $= \dfrac{2717}{3} \times 12 = 2717 \times 4$. But area of a tile is $\frac{1}{4}$ sq ft

$\therefore$ the total area covered with tiles $= \frac{1}{4} \times 2717 \times 4 = 2717$ sq ft

But area of the two *sides* of reservoir $= 75 \times 5\frac{1}{2} \times 2 = 825$ sq. ft

$\therefore$ area of the *ends* + area of *bottom* $= 2717 - 825 = 1892$ sq. ft

i.e the *width* $\times 5\frac{1}{2} \times 2 +$ the *width* $\times 75 = 1892$ sq ft.

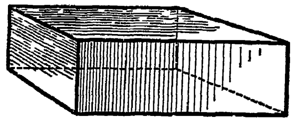
or, the *width* $\times (11 + 75) = 1892$ sq ft

$\therefore$ the *width* $= \dfrac{1892}{86}$ ft. $=$ 22 ft. *Ans.*

XLVIII. VOLUMES.

Rectangular Solids

A **rectangular solid** is a body bounded by *six rectangular* surfaces



These are called its *faces*

Its opposite pairs of faces are equal and parallel

It has three *dimensions*, namely, *length*, *breadth* (or width), and *thickness* (or height)

A **cube** is a body bounded by *six equal square* surfaces

Thus a cube is one special kind of rectangular solid Rectangular solids, other than cubes, are sometimes called *rectangular parallelepipeds*

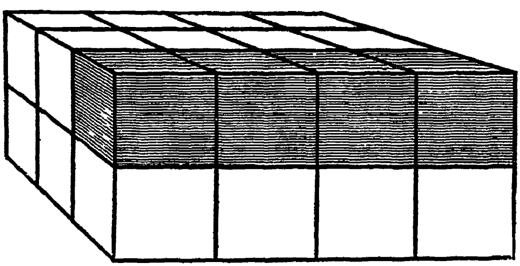
The **volume**, or capacity, or content, of a body is the amount of *space* enclosed within its bounding surfaces

The English units of measurement for volumes are the cubic inch, cubic foot, &c (See pages 30, 31)

A *cubic inch of volume* is an amount of space equal to that enclosed by the faces of a *cube*, each edge of which is *one inch* long

We shall now show how the volume of a **rectangular** solid is calculated

If we take (for example) 24 cubes each having edges *one foot* long, and arrange them thus—



they form a rectangular solid 4 feet long, 3 feet wide and 2 feet thick for there are evidently **three** *rows of* **four** *cubes* in the upper layer, and the same number in the lower layer ∴ the volume of a rectangular solid, 4 *feet* long, 3 *feet* wide and 2 *feet* thick, is $\underline{4 \times 3 \times 2}$, or 24, *cubic feet*

In the same way we might show that the volume of a rectangular solid, say, 7 *inches* long, 5 *inches* wide and 3 *inches* thick, is $7 \times 5 \times 3$ *cub in*

Also, by a method similar to that exemplified for areas, we might show that the volume of a rectangular solid, say, $\frac{7}{4}$ *in* long, $\frac{5}{3}$ *in* wide and $\frac{3}{2}$ *in.* thick, is $\frac{7}{4} \times \frac{5}{3} \times \frac{3}{2}$ *cub in* And so on

Hence the following rule —

To obtain the **number** of *cubic inches* (or feet, &c.) in the **volume of a rectangular solid.**

Find the **continued product** of the **numbers** (integral or fractional) which measure its **length, breadth** and **height** (or thickness), in *inches* (or feet, &c), the result is the **number** of *cubic inches* (or cub ft, &c.) in its **volume.**

This rule is often expressed shortly by the formula—

$$\left.\begin{array}{r}\textbf{length} \times \textbf{breadth} \times \textbf{height} = \textbf{volume}\\ \text{or, } \textbf{area of a face} \times \textbf{3rd dimension}^* = \textbf{volume}\end{array}\right\} \quad \text{(I).}$$

$$\left.\begin{array}{r}\text{Hence, conversely, } \textbf{length} = \dfrac{\textbf{volume}}{\textbf{breadth} \times \textbf{height}}\\ \&c,\\ \text{or, } \textbf{3rd dimension} = \dfrac{\textbf{volume}}{\textbf{area of a face}}\end{array}\right\} \quad \text{(II).}$$

Note—For the sense in which the words volume, length, &c, are used in these formulæ, see note on page 156

It is very important to observe that the length, breadth, and height must all be expressed in terms of the **same unit.**

EXAMPLE I.—*Find the volume of a rectangular solid* 5 *ft* 3 *in long,* 4 *ft* 8 *in wide and* 3 *ft.* 6 *in thick*

The volume $= 5\frac{1}{4} \times 4\frac{2}{3} \times 3\frac{1}{2}$, cub ft $= \frac{343}{4}$ cub ft $= 85\frac{3}{4}$ cub ft

$= 3$ *cub yd* 4 *ft* 1296 *in Ans.*

EXAMPLE II—*A beam* 24 *feet long and* 9 *inches wide contains* $11\frac{1}{4}$ *cubic feet of timber; how thick is the beam?*

By (II), the *thickness* of beam $= \dfrac{11\frac{1}{4}}{24 \times \frac{3}{4}}$ *ft* $= \frac{5}{8}$ ft. $= 7\frac{1}{2}$ *in. Ans*

EXAMPLE III—*A uniform iron bar,* $\frac{3}{4}$ *of a sq in* in section,† *contains a cubic foot of metal how long is it?*

By (II), *length* $= \dfrac{1728}{\frac{3}{4}}$ *inches* $= \dfrac{1728 \times 4}{3 \times 12}$ *feet* $= 192$ *ft. Ans.*

---

* i.e the dimension not involved in the area of the particular face considered.

† i e the *face*, to which the length of the bar is perpendicular, is $\frac{3}{4}$ sq in in *area.*

EXAMPLE IV.—*How many bricks each 9 in. long, $4\frac{1}{2}$ in. wide and 3 in. thick, would (making no allowance for mortar) be required for a wall 17 yards long, 6 ft. 6 in. high and $13\frac{1}{2}$ in. thick?*

$$\text{Volume of wall} = 51 \times 6\tfrac{1}{2} \times \frac{13\frac{1}{2}}{12}, \text{ cubic feet}$$

$$\text{Volume of a brick} = \tfrac{3}{4} \times \tfrac{3}{8} \times \tfrac{1}{4}, \text{ cubic feet}$$

$$\therefore \textit{number} \text{ of bricks req}^{d} = \frac{51 \times 6\frac{1}{2} \times \frac{13\frac{1}{2}}{12}}{\frac{3}{4} \times \frac{3}{8} \times \frac{1}{4}} = \underline{5304 \textit{ Ans.}}$$

EXAMPLE V.—*How many gallons of water will a tank 5 feet long, 2 ft. 6 in. wide and 3 ft. deep hold?*

[Remembering that "*a pint of water weighs a pound and a quarter*", *i.e.* that a gallon of water weighs 10 lbs., and that a *cubic foot of water weighs* 1000 *ozs.*, we proceed thus :—]

$$\textit{Content} \text{ of tank} = 5 \times 2\tfrac{1}{2} \times 3, \textit{ cub. feet}$$

$$\therefore \textit{weight} \text{ of water it will hold} = 5 \times \tfrac{5}{2} \times 3 \times 1000, \textit{ ozs.}$$

$$= 5 \times \tfrac{5}{2} \times 3 \times \tfrac{1000}{16}, \textit{ lbs.}$$

$$\therefore \text{the quantity of water} = 5 \times \tfrac{5}{2} \times 3 \times \tfrac{1000}{16} \times \tfrac{1}{10}, \textit{ gallons}$$

$$= \underline{234\tfrac{3}{8} \textit{ gallons Ans.}}$$

EXAMPLE VI.—*To what depth would 25 gallons of water fill a tank 10 feet long and 2 ft. 8 in. wide?*

As a gallon of water weighs 10 lbs., *i.e.* 160 ozs.

$\therefore$ the *volume* of a gallon of water is $\frac{160}{1000}$ of a cubic foot

$\therefore$ the volume of 25 gallons is $\frac{16}{100} \times 25$, cub. ft. $= 4$ cub. ft.

$$\text{Hence, by (II), } \textit{depth} = \frac{4}{10 \times 2\frac{2}{3}} \text{ feet} = \tfrac{3}{20} \text{ ft.} = \underline{1\tfrac{4}{5} \textit{ in. Ans.}}$$

EXAMPLE VII.—*How many cubic feet of brickwork are there in a wall 10 feet high and 18 inches thick, surrounding a rectangular plot which measures outside the wall, 36 feet in length and 28 feet in breadth?*

[From the accompanying ground-plan it is evident that *two* lengths of wall each $34\frac{1}{2}$ ft. long + *two* lengths each $26\frac{1}{2}$ ft. long, would surround the plot. Hence—]

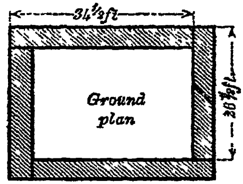
Number of cubic ft of brickwork

$= \text{total length of wall} \times \text{thickness} \times \text{height}$

$= twice \left(34\frac{1}{2} + 26\frac{1}{2}\right) \times 1\frac{1}{2} \times 10$, cub. ft

$= 2 \times 61 \times \frac{3}{2} \times 10$, cub ft

$= 1830$ *cub ft Ans*



EXAMPLE viii.—*How many cubic feet of wood are there in a box, with lid, 4 feet long, 3 feet wide and 2 feet high, the board of which it is made being* $1\frac{1}{2}$ *in thick?*

1st method.

[Here we find the *internal* dimensions of the box by subtracting *twice* the thickness of the wood from each of the given *external* dimensions, then]

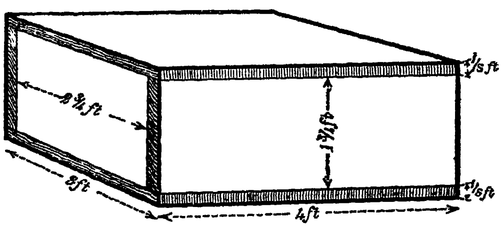
Volume of *wood* = vol of box, *outside* measure — vol *inside* measure,

$= 4 \times 3 \times 2$, cub ft $- 3\frac{3}{4} \times 2\frac{3}{4} \times 1\frac{3}{4}$, cub ft

$= 24$ cub. ft $- 18\frac{3}{64}$ cub ft. $= 5\frac{61}{64}$ *cub ft Ans.*

2nd method

[Here we dissect the box in the way shown in the diagram, and find the volume of lid, side and end as separate pieces]



Vol of *lid* and *bottom* = twice (*area of lid*) × (*thickness of wood*)

$= \text{twice } (4 \times 3) \times \frac{1}{8}$, cub ft.

Vol of the two *sides* = twice (*area of side*) × (*thickness of wood*)

$= \text{twice } \left(4 \times 1\frac{3}{4}\right) \times \frac{1}{8}$, cub ft.

Vol of the two *ends* = twice (*area of end*) × (*thickness of wood*)

$= \text{twice } \left(2\frac{3}{4} \times 1\frac{3}{4}\right) \times \frac{1}{8}$, cub ft

$\therefore$ total vol of wood $= \frac{2}{8} \times \left(4 \times 3 + 4 \times 1\frac{3}{4} + 2\frac{3}{4} \times 1\frac{3}{4}\right)$, cub ft

$= \frac{1}{4} \times \left(12 + 7 + 4\frac{13}{16} \right)$, cub ft

$= \frac{1}{4} \times 23\frac{13}{16}$, cub ft $= 5\frac{61}{64}$ *cub. ft Ans*

XLIX. DUODECIMALS.

In the calculation of rectangular areas and volumes, builders, glaziers, &c, use a *duodecimal** method

A *foot*, whether of length, area, or volume, is divided into 12 equal parts called **primes**, a *prime* into 12 equal parts called **seconds**, a *second* into 12 equal parts called **thirds**, and so on.

A *linear* **prime** $= \frac{1}{12}$ of a *foot* (*i e* 1 inch),

a *linear* **second** $= \frac{1}{12}$ of a *prime* $= \frac{1}{(12)^2}$ of a *foot*, or $\frac{1}{144}$ *ft*,

a *linear* **third** $= \frac{1}{12}$ of a *second* $= \frac{1}{(12)^3}$ of a *ft*, or $\frac{1}{1728}$ *ft*,

and so on

A *superficial* **prime** $= \frac{1}{12}$ of a *square foot* (*i e* 12 *sq in*),

a *superficial* **second** $= \frac{1}{(12)^2}$ of a *sq ft*, or $\frac{1}{144}$ *sq ft* (*i e* 1 sq in),

a *superficial* **third** $= \frac{1}{(12)^3}$ of a *square foot*, or $\frac{1}{1728}$ *sq foot*,

and so on.

A *cubic* **prime** $= \frac{1}{12}$ of a *cubic foot* (*i e* 144 *cub ins*),

a *cubic* **second** $= \frac{1}{(12)^2}$ of a *cu ft*, or $\frac{1}{144}$ *cu ft* (*i e* 12 *cu in*),

a *cubic* **third** $= \frac{1}{(12)^3}$, or $\frac{1}{1728}$, of a *cu ft* (*i e* 1 cu in),

and so on

The notation commonly used,† whether for length, area, or volume, is the following —

primes are indicated thus, $'$, *seconds*, thus, $''$; *thirds*, thus, $'''$ *fourths*, thus, $^{\text{IV}}$, *fifths*, thus, $^{\text{V}}$, and so on

For instance, the length, 3 ft $4'$ $6''$,
the area, 5 sq ft $7'$ $8''$ $9'''$,
the volume, 8 cub ft $2'$ $0''$ $4'''$ 5^{IV}

EXAMPLE 1.—*Express the length* 3 *yds* 2 *ft* $7\frac{3}{8}$ *in in duodecimals*

$$\tfrac{3}{8}\text{ in} = \tfrac{3}{8}\text{ of a } prime = \tfrac{3}{8} \times 12,\ seconds = 4\tfrac{1}{2}''$$

Hence 3 yds 2 ft $7\frac{3}{8}$ in $=$ 11 *ft* $7'$ $4''$ $6'''$ *Ans*

---

* Latin, *duodecim*, twelve

† Sometimes, however, the words "*feet*", "*inches*" and "*parts*" are used *e g* 3 ft 5 in 6 pts for 3 ft 5 primes 6 seconds Sometimes also 1 *foot* is indicated thus, 1′, and *then* 1 *inch is written thus*, 1″

EXAMPLE ii.—*Express in duodecimals 5 sq ft* $27\frac{3}{4}$ *sq. in.*

[As *sq in* are superficial (*i e* surface) seconds we reduce *sq in* to primes by *dividing* by 12 Hence]

5 sq ft $27\frac{3}{4}$ sq in = 5 sq. ft. $2'\ 3\frac{9}{12}''$ = 5 *sq ft* $2'.3''.9'''$ *Ans*

EXAMPLE iii —*Reduce 7 cub ft* $635\frac{1}{4}$ *cub in to duodecimals.*

[As *cub. in* are cubic thirds, we reduce *cub in* to cub seconds by *dividing* by 12, and then *cub seconds* to *cub primes* by again dividing by 12. Hence]

$$
\begin{aligned}
7\text{ cub ft. } 635\tfrac{1}{4}\text{ cub in.} &= 7\text{ cub. ft } 52''\ 11\tfrac{1}{4}''' \\
&= 7\text{ cub ft } 4'\ 4''\ 11\tfrac{3}{12}''' \\
&= \underline{7\textit{ cub ft } 4'\ 4''\ 11'''\ 3^{\text{iv}}\ \textit{Ans.}}
\end{aligned}
$$

EXAMPLE iv —*Express 8 ft* $5'.10''$ *in yards, feet and inches.*

8 ft $5'\ 10''$ = 2 yds 2 ft $5\frac{10}{12}$ in = 2 *yds.* 2 *ft* $5\frac{5}{6}$ *in Ans*

EXAMPLE v —*Reduce 13 sq ft.* $7'\ 4''\ 8'''$ *to sq. ft and sq in.*

[As surface *seconds* are *sq in* we reduce surface *primes* to sq. in by multiplying by 12. Hence]

13 sq ft. $7'\ 4''\ 8'''$ = 13 sq ft $88\frac{8}{12}$ sq in = 13 *sq ft.* $88\frac{2}{3}$ *sq. in Ans*

EXAMPLE vi —*Express 17 cub ft* $6'.10''\ 5'''.3^{\text{iv}}\ 9^{\text{v}}$ *in cub ft and in.*

[As cubic *thirds* are *cub in.* we reduce cub *seconds* to *cub in* by *multiplying* by 12, cub *primes* to *cub in* by *multiplying* by 144, cub *fourths* to *cub in* by *dividing* by 12, &c Hence]

$$
\begin{aligned}
&17\text{ cub. ft } \quad 6'.\ 10''.\ 5'''.\ 3^{\text{iv}}.\ 9^{\text{v}} \\
&\qquad\qquad 120\tfrac{3}{12} \\
&\qquad\qquad 864\tfrac{9}{144} \\
&= 17\text{ cub ft } \quad 989\tfrac{45}{144}''' = \underline{17\textit{ cub ft } 989\tfrac{5}{16}\textit{ cub in. Ans}}
\end{aligned}
$$

N.B —To *multiply*, or *divide*, by **12**, in Duodecimals, we have merely to *shift all the figures one place to the left*, or *right*, respectively (just as in *Decimals* we multiply or divide by *10*)

For instance, $(3\text{ ft } 7'\ 5'') \div 12 = 3'\ 7''.5'''$;
and $(5'.8''\ 4''') \times 60 = (5\text{ ft. } 8'\ 4'') \times 5 = 28\text{ ft } 5'\ 8''$.

AREAS

We shall now show how, when the dimensions of a rectangle are expressed in duodecimals, its area may be conveniently found

The method depends upon the following considerations —

(I) As a *prime* is $\frac{1}{12}$ of a *foot*, a second is $\frac{1}{144}$ of a *foot*, &c, *whether the foot be linear or superficial*, it follows that—

Linear *primes* × **feet** give area in *superficial* **primes**,*

For instance, a rectangle whose dimensions are 5′, and 2 *ft*, has area $\frac{5}{12} \times 2$, or $\frac{10}{12}$, *sq ft* = 10′ (superficial)

linear *seconds* × **feet** give area in superficial **seconds**,

For instance, a rectangle whose dimensions are 5″, and 2 *ft*, has area $\frac{5}{144} \times 2$, or $\frac{10}{144}$, *sq ft* = 10″ (superficial)

linear *thirds* × **feet** give area in superficial **thirds**,

For instance, a rectangle whose dimensions are 5‴, and 2 *ft*, has area $\frac{5}{1728} \times 2$, or $\frac{10}{1728}$, *sq ft* = 10‴ (superficial)

and so on.

Linear *primes* × **primes** give area in superficial **seconds**,

e g a rect 5′ by 2′ has area $\frac{5}{12} \times \frac{2}{12}$, or $\frac{10}{144}$, sq ft = 10″

linear *seconds* × **primes** give area in superficial **thirds**,

e g a rect 5″ by 2′ has area $\frac{5}{144} \times \frac{2}{12}$, or $\frac{10}{1728}$, sq ft = 10‴

linear *thirds* × **primes** give area in superficial **fourths**,

e g a rect 5‴ by 2′ has area $\frac{5}{(12)^3} \times \frac{2}{12}$, or $\frac{10}{(12)^4}$, sq ft = 10^{iv}.

and so on

Similarly, linear *primes, seconds, thirds*, &c, multiplied by **seconds**, give area in superficial **thirds, fourths, fifths**, &c, respectively. And so on

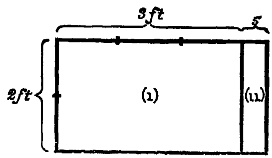
The above results are summarized in the statement that in Duodecimals—

The *order* of a product is the **sum** of the *orders* of the factors

e g seconds × *thirds* give *fifths*

(II) The area of any rectangle = the *sum* of the areas of all the rectangles into which it is divided

For instance, the area of a rectangle 3 ft 5′ long by 2 ft wide
= area of rect 3 ft by 2 ft (i)
+ area of rect 5′ by 2 ft (ii)



* *i e* a *number* of linear primes multiplied by a *number* of linear feet gives the *number* of superficial primes in the area of a rectangle of those dimensions See note on p 156

EXAMPLE vii —*Find the area of a rectangle 3 ft 5′. 4″ long and 2 ft wide.*

We set 2, the number of *feet* in the width, under 3, the number of *feet* in the length, and then, beginning on the right, we proceed as in compound mult^n

It follows, from (i) and (ii) above, that the result thus obtained is the area of the rectangle in superficial feet, primes, and seconds

ft	′	″
3 .	5	4
2		
6 .	10 .	8

Ans. 6 *sq ft* 10′. 8″.

EXAMPLE viii —*Find the area of a rectangle 7 ft 3′ 4″ by 2′.*

We set 2, the No of *primes* in the width, under 3, the No of *primes* in the length, and then, multiplying as before, we set *each partial result* one place to the right *of the column from which it is obtained*, for by (i), when we multiply a number of linear *seconds* by a number of linear *primes* we obtain area in superficial *thirds*, and so on.

ft.	′	″	‴
7 .	3 .	4 .	
	2		
	14 .	6 .	8

Ans 1 *sq ft.* 2′. 6″. 8‴.

EXAMPLE ix —*Find the area of a rectangle 5 ft 2′ long and 4 ft. 3′ wide.*

Setting 4 ft 3′ under 5 ft 2′ we first multiply by 4 in the manner shown in Ex vii, then by 3 in the manner shown in Ex viii Finally we add these results

	ft	′	″	
	5	2 .		
	4	3		
(ii)	20 .	8		(i)
(iv)	.	15 .	6	(iii)
	21	11 .	6	

Ans 21 *sq ft* 11′. 6″.

Note —The accompanying diagram illustrates the above process

Here each of the *larger squares* represents a *square foot*,

each very thin *rectangle*, a *superficial prime* (*i e.* 12 sq in., or a rect 1 *ft* long and 1′ wide);

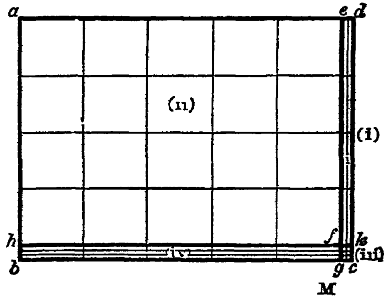
each very *little square*, a *superficial second* (*i e.* a sq in.)

And rectangle *abcd* = rectangles *efkd* + *ahfe* + *fgck* + *hbgf*

(i) (ii) (iii) (iv)

= 8′ + 20 sq ft + 6″ + 15′

= 21 sq ft 11′ 6″, as above.



EXAMPLE x —*Find the area of a rectangle the dimensions of which are* 3 *yds* 2 *ft* 5′ 9″ *and* 5 *yds* 1 *ft* 7′ 6″.

1st step (Multiply by 11)
$6 \times 11 = 66$, set down **6″**, carry *5*
$7 \times 11 = 77, 77 + 5 = 82$, set down **10′**, carry *6*
$16 \times 11 = 176, 176 + 6 =$ **181**

2nd step (Multiply by 5)
$6 \times 5 = 30$, set down **6‴**, carry *2*
$7 \times 5 = 35$, $35 + 2 = 37$, set down ″, carry *3*
$16 \times 5 = 80$, $80 + 3 = 83$, set down **11′**, and carry *6*, i e set **6** in the column headed *ft.*

3rd step (Multiply by 9)
$6 \times 9 = 54$, set down **6ⁱᵛ**, carry *4*
$7 \times 9 = 63$, $63 + 4 = 67$, set down **7‴**, carry *5*
$16 \times 9 = 144$, $144 + 5 = 149$, set down **5″**, and carry 12′, i e **1** *sq ft.*

4th step Add the results, as in Comp Addⁿ

ft	′	″	‴	iv
16	7	6		
11	5	9		
181	10	6		
6	11	1	6	
1	0	5	7 .	6
189 .	10	1	1	6

Ans 189 *sq ft* 10′ 1″ 1‴ 6ⁱᵛ

Note —We may extend the duodecimal arrangement to the *left* of the column headed "*feet*", should the number of feet in the given dimensions be inconveniently large Then the two next columns on the left of the "feet" column would contain *dozens* and *gross* of feet, respectively

For instance, to find *by the duodecimal method*, the area measuring 17 yards 2 ft 6 in by 16 yards 1 ft 4 in, we might proceed thus —

17 yds 2 ft 6 in
= 53 ft 6 in = 4 doz 5 ft 6′

16 yds 1 ft 3 in
= 49 ft 3 in = 4 doz 1 ft 3′

Now, when we multiply by 4 *dozens* of feet, we set each partial result *one step to the left* of the column treated, &c

gross	doz	ft	′	″
	4 .	5 .	6	
	4	1	3	
17 .	10	0		
	4 .	5	6	
	1	1	4	6
17	15	6	10	6

Ans 2634 *sq ft* 10′ 6″

VOLUMES

In the same way as for *areas* it may be shown that for volumes

(I) The *order* of a product is the sum of the *orders* of the factors

For instance, a rectʳ solid one face of which is 5 *superficial* primes in area and its 3rd dimension 2 *linear* seconds, has volume $\frac{5}{12} \times \frac{2}{144}$ *cub, ft* $= \frac{10}{1728}$ *cub ft* = 10 *cubic* thirds

(II) The volume of a rectangular solid = the sum of the volumes of all the rectʳ solids into which it is divided.

EXAMPLE XI —*Find the volume of a rectangular solid, the dimensions of which are 3 ft 4' 6", 2 ft 7' and 2 ft 1'. 6".*

Here we first find the *area* of a *face* (i), and then obtain the *volume* (ii) by multiplying the area of the face by the 3rd dimension of the solid, the process, in both cases, being exactly similar to that shown in Ex. x.

ft	'	"	'''	iv	
3 .	4 .	6			
2 .	1	6			
6	9 .	0			
	3 .	4 .	6		
	1	8	3	0	
7 .	2	0 .	9		(i)
2 .	7				
14 .	4 .	1 .	6		
4 .	2	2	5	3	
18 .	6 .	3 .	11 .	3	(ii)

Ans 18 *cub ft* 6' 3". 11''' 3iv.

The converse of the above operations, *i e.* Division, may be performed in Duodecimals, but is not of much practical value

EXAMPLE XII —*Find, by duodecimals, the length of a rectangle 8 ft. 3'. 6" wide and 194 sq. ft. 3' 4". 1''' in area*

ft ' "	sq ft	'	"	'''	iv	ft. ' "
8 . 3 . 6)	194 .	3 .	4 .	1 .		(20+3 . 5 . 2
	165	. 10 .	0			
	28	5 .	4			
	24	. 10 .	6			
	3 .	6 .	10 .	1		
	3	5	5 .	6		
		1 .	4 .	7 .	0	
		1 .	4 .	7 .	0	

When we consider how many times 8 is contained in 19 (the first two figures of 194) we must remember that 2, the quotient figure, is 2 *tens*, and so we multiply the *entire divisor* by **20**, and not by 2

Ans 23 *ft* 5' 2"

Note—If we adopt the method suggested in the Note on page 170, the work would stand thus.—

ft ' "	doz.	sq ft.	'	"	'''	iv	doz. ft ' "
8 . 3 . 6)	16 .	2	3 .	4 .	1 .		(1 . 11 . 5 . 2
	8	3 .	6				
	7	. 10 .	9 .	4			
	7 .	7 .	2 .	6			
		3	6 .	10 .	1		
		3 .	5 .	5 .	6		
			1 .	4 .	7 .	0	
			1 .	4 .	7 .	0	

Ans 23 *ft.* 5' 2".

L. APPROXIMATION.

Decimals

In calculations made for commercial, and other practical, purposes it would often happen that, if the *exact* result were obtained, some small part of it would be practically useless

For instance, a very small fraction of a *penny* in a *money* result, for it could not be paid in our coinage

Yet the labour involved in obtaining this small part of the exact result would often be considerable.

The methods we are now about to consider enable us, if we operate in Decimals, to **approximate** to an exact result with any degree of nearness that may be desired, without such waste of labour as would often occur in obtaining the *complete* result

Note—*All results based upon measurement*, no matter how completely the calculations themselves are carried out, must of necessity be but *approximate*, since in the measurements themselves, which depend upon our imperfect powers of observation, *absolute accuracy* is unattainable

The degree of nearness to the exact result to which an approximation should be carried depends, of course, upon the particular purpose for which the calculation is made

In the case of *money*, a result "correct to the nearest penny" is sufficiently accurate for most *practical* purposes

Within the limit fixed upon, the *closest possible* approximation will sometimes be a little *greater*, and sometimes a little *less*, than the *exact* result, *i e* the **error** may be either one of defect or of excess.

For instance, taking *pence* as the limit in money,

4*d* would be a closer approximation to $3\frac{5}{8}d$ than 3*d*. would be

So, also, if we fix the limit at *two* places of decimals,

5·64 is nearer to 5·6375 than 5·63 is,

for 5·64 exceeds 5·6375 by only ·0025, but 5·63 falls short of it by ·0075.

N.B.—Similarly, the closest approximation to **·5986173**

in *six* places of decimals is	·59861**7**
five	·5986**2**
four	·598**6**
three	·59**9**
two	**·60**

Hence the following general rule:—

If the first of the decimal figures discarded be less than 5, *the last figure retained is unaltered, but if the first of the figures discarded be* 5, *or greater than* 5, *the last figure retained is* **increased by one.**

For, by so doing we ensure that the *error* can never be more

than *half* of what it otherwise might be without this correction; that is, can never be more than half *of the value of* unity *standing in the place of the last figure retained*

For instance, the error in taking ·38 as a *two*-place approximation for ·3751 is ·0049, *i e* is *less* than ·005, or less than *half* of ·01; whereas if we took ·37 as a *two*-place approximation for ·3799, the error would be ·0099, or very nearly ·01.

EXAMPLE i —*Find the nearest approximation in two places of decimals to the sum of* 4·563825, 120·468539, ·054938 *and* 30·5.

Here, and in all similar cases, we retain *two* columns of decimal places beyond the number required in the result, in order to obtain the figures to be "carried".

$$\begin{array}{r} 4{\cdot}5638 \\ 120{\cdot}4685 \\ {\cdot}0549 \\ 30{\cdot}5 \\ \hline 155{\cdot}587 \end{array} \qquad \textit{Ans } 155{\cdot}59$$

EXAMPLE ii —*Find, to the nearest thousandth of unity, the difference between* (i) $3{\cdot}41\dot{6}$ *and* 1·253075; (ii) 12·75 *and* 3·216675.

Here we require the results to three places of decimals.

$$\text{(i)}\quad \begin{array}{r} 3{\cdot}41666 \\ 1{\cdot}25307 \\ \hline 2{\cdot}1635 \end{array} \quad \textit{Ans.}\ 2{\cdot}164. \qquad\qquad \text{(ii)}\quad \begin{array}{r} 12{\cdot}75 \\ 3{\cdot}21667 \\ \hline 9{\cdot}5333 \end{array} \quad \textit{Ans.}\ 9{\cdot}533.$$

To obtain a result "*correct within one-thousandth of the* whole", we find the first four *significant** figures of the result.

For, if we divide any number by 1000, the first figure in the quotient has the same *local* value as the *fourth* figure in the original number.

For instance, the number 17·4625. One-thousandth of it is ·0174625 (i) The four-figure approx^{n} to it is 17·46, which differs from it by ·0025, *i e* by *less* than (i), or one-thousandth of the whole.

Similarly, to obtain a result correct within one-millionth of the whole, we approximate to the first seven figures. And so on.

EXAMPLE iii.—*Find, within one-thousandth of the whole, the sum of* 423·56285, $81{\cdot}28\dot{3}$, 17·46025 *and* $120{\cdot}\dot{8}\dot{2}$.

Here, as we require only a four-figure result, we shall need but *one* place of *decimals* in the *result*, so we retain *three* in the working.

$$\begin{array}{r} 423{\cdot}562 \\ 81{\cdot}283 \\ 17{\cdot}460 \\ 120{\cdot}828 \\ \hline 643{\cdot}13 \end{array} \qquad \textit{Ans.}\ 643{\cdot}1.$$

* In such a decimal as ·008057, 8 is the first *significant* figure, and 8, 0, 5, are the first *three* significant figures

Approximate results in Multiplication and Division, *when the multipliers and divisors contain only* one *or* two *figures*,* may be easily obtained by the methods of Chap XXXVIII For instance

EXAMPLE IV —*Find, to three places of decimals*, $1\cdot73425876 \times 4\frac{1}{4}$

We multiply (1) by 4, retaining five places and adding in the carried figure from four times 8

We then divide (1) by 4 to five places, and add the two results

$$\begin{array}{r|ll} 4 & 1\cdot734258 & (1) \\ & \quad\quad\;\; 4 & \\ \hline & 6\;93703 & \\ & \cdot43356 & \\ \hline & 7\;37059 & \quad \textit{Ans}\ 7\cdot371 \end{array}$$

To *Decimalize* a sum of English money approximately (*i e* to express it as the decimal of £1)—

As a *florin* (or 2*s*) = $\frac{1}{10}$ of £1 = £·1,
and a *shilling* = $\frac{1}{2}$ of a florin = $\frac{1}{2}$ of £·1 = £·05,
any number of shillings mentioned in the sum of money can be at once decimalized, by *writing the number of* florins *it contains in the* first *place, and then representing the* odd shilling (if there be one) *by a* 5 *in the* second *place, in decimals*

For instance, 13*s* (*i e* 6 florins + 1*s*) = £·65,
18*s* = £ 9, and 17*s* = £ 85

And then any number of pence and farthings mentioned in the sum of money may be treated by the method of *aliquot parts*, thus

EXAMPLE V —*Decimalize* £2, 13*s* $3\frac{1}{2}d$ *to five places*

Explanation	*Written Work*
	£
1st step £2, 13*s* = £2 + 6 fl + 1*s* = £2·65	2·65
2nd step 3*d* = $\frac{1}{4}$ of 1*s* = $\frac{1}{4}$ of £ 05 = £ 0125	·0125
3rd step $\frac{1}{2}d$ = $\frac{1}{6}$ of 3*d* = $\frac{1}{6}$ of £·0125 = £·002083	·002083
	2·664583
	Ans £2 66458

EXAMPLE VI —*Decimalize* £17, 18*s* $8\frac{3}{4}d$ *to five places*

Explanation	£
Remembering that a florin = £·1, a shilling = £·05, and sixpence = £·025	
1st step £17, 18*s* = £17·9	17·9
2nd step 8*d* = $\frac{1}{3}$ of a fl = $\frac{1}{3}$ of £·1 = £·0333	·033333
3rd step. $\frac{3}{4}d$ = $\frac{1}{8}$ of 6*d* = $\frac{1}{8}$ of £·025 = £ 003125	·003125
	17·936458
	£17·93646 *Ans*

* For methods of dealing with larger multiplications and divisions, see pages 177, &c.

Similarly we may treat other concrete quantities For instance.

EXAMPLE VII —*Express 3 tons 13 cwts 2 qrs 15 lbs as a decimal of a ton to four places.*

Explanation

Here cwts are dealt with in exactly the same way as shillings in Ex. v

1st step
3 tons 13 cwts $= 3\cdot65$ ton.

2nd step
2 qrs $= \frac{1}{2}$ of 1 cwt $= \frac{1}{2}$ of $\cdot05$ ton

3rd step
8 lbs $= \frac{1}{7}$ of 2 qrs $= \frac{1}{7}$ of $\cdot025$ ton

4th step
7 lbs $= \frac{1}{8}$ of 2 qrs $= \frac{1}{8}$ of $\cdot025$ ton.

Written Work.

Tons
3·65
·025
·003571 .
·003125 ..
3·681696 ..

Ans. 3·6817 tons.

Note —To decimalize money to *three* places *mentally*

As $6d = 25$ *thousandths* of £1, and the number of *farthings* in $6d$. is 24,

(i) The *number of farthings* in any sum *less* than $6d$ gives all the *thousandths* of £1 in that sum

For instance, $3\frac{3}{4}d. = £\cdot015$, and $14s.\ 5\frac{1}{4}d. = £\cdot721$, to three places.

(ii) The *number of farthings* + 1 in any sum between $6d$ and $9d$ gives all the *thousandths* of £1 in that sum.

For instance, $7\frac{1}{2}d = £\cdot031$, and $13s\ 8d = £\cdot683$, to three places.

(iii) The *number of farthings* + 2 in any sum between $9d$ and $1s$ gives the nearest approximation in *thousandths* of £1 to its value

For instance, $16s\ 10\frac{3}{4}d = £\cdot845$, and $17s.\ 11\frac{1}{4}d. = £\cdot897$, to three places.

In the converse operation, that of reducing a decimal of £1 to £, *s. d* ; if the result be required *to the nearest penny*, we may ignore figures beyond the *fourth* place of decimals, if to the nearest farthing, beyond the *fifth* place.

EXAMPLE VIII —*Find, to the nearest penny, the value of* £13·578675

Here, in multiplying by 20, we first multiply ·578 by 2, *carrying* from the fourth place, and then move the dec point in the result one place to the right (*i.e* multiply by 10)

$$\begin{array}{r} £13\cdot578675 \\ 2 \\ \hline s\ 11\cdot57 \\ 12 \\ \hline d\ 6\cdot8 \end{array}$$

Ans £13, 11*s.* 7*d.*

Note —Such results may also be obtained *mentally* by the converse of the rules given in the last note

For instance, $£\cdot684 = 13s\ 8d$ to the nearest penny

[For the 6 *tenths* represents 6 *fl* or 12*s*, a 5, taken from the 8, gives 1*s* ; and the remaining 34 *thousandths* $= (34 - 1)$ *farthings*, *i.e.* 8*d.* nearly.]

Decimalized Practice

Approximate results may often be very conveniently obtained by this method

Example ix.—*Find, to the nearest penny, the value of* 3 *cwts* 1 *qr* 22 *lbs at* £2, 13*s* 6*d per cwt*

Here we decimalize the money, and then carry each successive division to *five* places of decimals, so as to ensure the result being correct to *three* places

	£		
1 qr $= \frac{1}{4}$	2·675	= value of	1 cwt
	3		
	8·025	= .	3 cwts
14 lbs $= \frac{1}{2}$	·66875	=	1 qr
7 lbs $= \frac{1}{2}$	·33437	=	14 lbs
1 lb $= \frac{1}{7}$	·16718	=	7 lbs
	·02388	=	1 lb
	£9·21918	=	3 cwts 1 qr 22 lbs

Ans £9, 4*s* 5*d*

Example x.—*Find, within a penny, the rent of* 423 *ac* 2 *ro.* 31½ *po at* £1, 14*s* 6*d per acre.*

	£		
	423 5		
	·125		
	·0625		
	·00625		
	·00312		
10*s*. $= \frac{1}{2}$	423·69687	=	rent at £1 per ac
4*s* $= \frac{1}{5}$	211·84843	=	10*s*
6*d* $= \frac{1}{8}$	84·73937	=	4*s*
	10·59242	=	6*d*
	730·87710	=	£1, 14*s* 6*d.*

Ans £730, 17*s*. 6*d*.

Explanation

To *five* places of decimals,

423 ac 2 ro = 423·5 ac, of which the rent, at £1 an ac = £423·5
20 po = $\frac{1}{4}$ of 2 ro = $\frac{1}{4}$ of ·5 ac = ·125 ac, = £ ·125
10 po = $\frac{1}{2}$ of 20 po = $\frac{1}{2}$ of ·125 ac = ·0625 ac, = £ ·0625
1 po = $\frac{1}{10}$ of 10 po = $\frac{1}{10}$ of ·0625 ac = ·00625 ac, = £ 00625
$\frac{1}{2}$ po = $\frac{1}{2}$ of 1 po = $\frac{1}{2}$ of ·00625 ac = ·00312 ac, = £ ·00312

Hence, adding these results, we obtain £423 69687 as the rent of the whole at £1 per acre, from which we obtain by Practice the rent at £1, 14*s* 6*d* per ac

The approximate result, £730·8771, we then convert to £, *s* *d*.

Contracted Multiplication.

We shall now show how the product of *any* two decimals may be approximately obtained *

(I) We noticed, on page 11, that, in any multiplication sum, we might equally well *begin* with the *left-hand* figure instead of with the right-hand figure. This plan we shall now adopt

For instance, in finding the product of 423 and 651, instead of the ordinary arrangement (i), we may multiply by the 6 *first*, when the work stands as in (ii).

(i)		(ii)	
	423		423
	651		651
	423		2538
	2115		2115
	2538		423
	275373		275373

Again, if we multiply a decimal by an *integer* figure, the decimal point in the result will stand *vertically below* the decimal point in the multiplicand

For instance, in the product of 7·423 by 6, the decimal point in the result is vertically below that in the multiplicand.

$$\begin{array}{r} 7{\cdot}423 \\ 6 \\ \hline 44{\cdot}538 \end{array}$$

(II) But *every* multiplication of decimals can be performed with a multiplier consisting of *one single* integer figure followed by decimal figures, if we make use of the following principle —

The product *is unaltered if we* multiply *one factor, and* divide *the other, by the* same *number.*

For instance, $4 \times 6 =$ (*ten* times 4) $\times$ (6 divided by *ten*)
also, $4 \times 6 =$ (a *hundred* times 4) $\times$ (6 divided by a *hundred*)

Hence we may, *without affecting the result, move the decimal point in the multiplier any number of places either to right or left, if we also move the decimal point in the multiplicand the same number of places in the* opposite *direction.*

For instance, 23·47 × 21·38 gives the same product as 234·7 × 2·138;
·1425 × ·4362 . . . ·01425 × 4·362,
8105·75 × ·05637 .. . .. 81·0575 × 5·637.

As an illustration of the arrangements (I) and (II), we will now find the product of 261·4326 and ·2354.

Multiplying the multiplier by 10, and *dividing* the multiplicand by 10, we see that 261·4326 × ·2354 is equivalent to 26·14326 × 2·354 Then, arranging the work in the way suggested above, we obtain the product 61·54123404, with the decimal point in the result vertically below that in the multiplicand

$$\begin{array}{l} 26{\cdot}14326 \\ \phantom{26{\cdot}14}2{\cdot}354 \\ \hline 52{\cdot}28652 \\ 7\;842978 \\ 1\;3071630 \\ 10457304 \\ \hline 61{\cdot}54123404 \end{array}$$

* The method which follows is derived from that given in De Morgan's Arithmetic

But, if we only required to know this result approximately, say to *two* places of decimals, then all the figures which here stand to the right of the vertical line might be omitted altogether, the two columns next on the left of this line only being retained for the sake of "carrying" in the addition

```
26·1432 6
      2·254
-----------
52 2865|2
 7 8429|78
 1 3071|630
   1045|7304
-------|----
61·5412|3404
```

All the work necessary might then be performed as follows —

To find 26·14326 × 2·354 to *two* places of decimals

As the result is required correct to *two* places of decimals only, we set 2, the first figure of the multiplier, under the figure which stands in the *fourth* place in decimals in the multiplicand, and begin there, adding in, however, mentally anything there would have been to carry from the figure on the right in the multiplicand (i e twice 2, and 1 carried, 5)

```
26 14326
   2 354
--------
52 2865|
 7 8429|
 1 3071|
   1045|
--------
61·541
```

In the second line, when we multiply by 3, we begin at the figure which stands in the *third* place in decimals in the multiplicand

And so on, in each successive line beginning to multiply at a figure of the multiplicand one place further to the left than in the preceding line, adding in mentally anything there would have been to carry from its right-hand neighbour, setting the first figure obtained of each line of partial results close to the vertical line, and *marking the decimal point in the final result* vertically below *the decimal point in the multiplicand*

Here it will be observed that a little difficulty is caused to the eye, by reason of the distance of some of the figures of the multiplier from the part of the multiplicand treated

This difficulty is avoided by the device of (III) reversing the multiplier, when each figure of the multiplier will stand immediately under that figure of the multiplicand at which its work begins

Finally, then, the work will stand thus —

```
26·14326
   4532
--------
52 2865|
 7 8429|
 1 3071|
   1045|
--------
61·541        Ans 61·54
```

EXAMPLE XI —*Find* 2·530625 × 47·306547 *to one place of decimals*

Here we take 2 530625 as multiplier, since it already has one, and only one, integer figure Then, as we require to have *one* place of decimals correct in the result, we retain *three* places in the working, and so set 2, the first figure of the multiplier, under the 6 in the multiplicand

```
  47·30656
5260 352
----------
   94·613
   23 653
    1 419
       28
----------
  119·71      Ans 119·7
```

EXAMPLE xii —*Find the nearest approximation, in four places of decimals, to* ·1320566 × $\cdot 41\dot{2}$

First ·1320566 × ·412412

= ·01320566 × 4·124124

Now, as the result is req^{d} to *four* places, we set 4, the integer figure of our multiplier, under 5, the figure in the *sixth* place of the multiplicand.

```
·0132056
 214214
·052822
  1320
   264
    52
     1
·05446      Ans ·0545.
```

Note —Here the sum of the figures in the right-hand "carrying" column is **9**, so we carry **1** to the next column (since 9 is nearer to 10 than it is to 0) Similarly, in other "carrying" *columns*, for *sums* from **5** to **15** we may carry one; from **15** to **25**, carry two, and so on

---

EXAMPLE xiii —*Find, within a millionth of the whole, the product of* 7834·25 *and* ·9362825

Here we require the first *seven* figures of the result (see page 173)

Now 7834·25 × ·9362825

= **783**·425 × **9**·362825

And as 9 times 700 is more than 6000, we see that there will be *four integer* figures in the result

Hence we require *three dec* figures

We therefore retain *five* places of decimals in the working

```
 783·42500
  52 82639
7050·82500
 235 02750
  47 00550
   1 56685
     62673
      1566
       391
7335·0712     Ans 7335·071.
```

EXAMPLE xiv —*Find the integral part of the continued product of* 171·421534, ·842056, *and* 246·327057.

We rearrange the decimal points so that *both* multipliers have *one integer* figure, thus—

246·327057 × 171·421534 × ·842056

= 246·327057 × 17·1421534 × 8·42056

= 2463·27057 × **1**·71421534 × **8**·42056

Then, as we require the result correct so far as the integral part only is concerned, we retain *two* places of decimals in the working

N B —Here, in adding the first column of the first result (i), we obtain **9**, which we count as **10**, thus making some allowance for a probable carried figure from the *unwritten* column on its right.

```
     2463·270
      6502 48
    19706·16
      985 30
       49 26
        1 23
          14
    20742·10.. ...(i)
   351241 71
    20742·10
    14519 46
      207 42
       82 96
        4 14
          20
          10
Ans 35556
```

In finding any *power** of a decimal higher than the cube, much labour may be saved by making use of the following principle —

The **index of the power** *of the product of two powers of any number is the* **sum** *of the indices of those two powers*

For instance, $3^4 \times 3^2 = 3^6$,
for $3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3) = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

Hence, in raising a number to a higher power, by utilizing some of the lower powers first obtained, we can reduce the number of steps in the multiplication

For instance, to find 2^{12} we proceed thus —As $2^2 = \mathbf{4}$
∴ 2^4 (*i e* $2^2 \times 2^2$, or 4^2) $= \mathbf{16}$
∴ 2^8 (*i e* $2^4 \times 2^4$, or 16×16) $= \mathbf{256}$
Hence $2^{12} = 2^8 \times 2^4 = 256 \times 16 = \mathbf{4096}$.

Thus we obtain the 12th power of a number in *four* steps instead of *eleven*, if we multiplied successively by the number itself

---

EXAMPLE XV.—*Find* $(1 \cdot 04)^{10}$ *to three places of decimals.*

1st step
Here we first obtain the square, or 2nd power, of 1 04, by adding 1·04 to $\frac{4}{100}$ of 1·04

2nd step
We now square 1·0816 (retaining *five* places in the working), and obtain thus the 4th power of 1·04

3rd step
We next square 1·16985, thus obtaining the 8th power of 1·04

4th step
Finally, we multiply 1·36852 (the 8th power of 1·04) by 1·0816 (the 2nd power of 1·04), thus obtaining the 10th power of 1·04

* In both the 2nd and 3rd stages we increase the sum of the right-hand column by 1 (See Note on Ex xiv)

	1·04	
	0416	
	1·08160	2nd power
	61801	
	1·08160	
	8652	
	108	
	64	
	1·16985*	4th power
	5 89611	
	1·16985	
	11698	
	7018	
	1052	
	92	
	5	
	1·36851*	8th power
	61801	
	1·36851	
	10948	
	136	
	81	
Ans	1·480	.. 10th power

EXAMPLE XVI —*Find* $236 \cdot 75 \times (1 \cdot 05)^6$ *correct to three places*

Here as $(1 \cdot 05)^6$, when obtained, has to be multiplied by *hundreds* (*i e.* 236) we must, in calculating $(1\ 05)^6$, retain *seven* dec places in the *working.*

---

* See page 12

CONTRACTED DIVISION.

We shall now show how in Long Division the labour involved in obtaining the last few figures in the quotient may be curtailed.

Suppose, for example, that we require the quotient of $436 \cdot 563875 \div 17 \cdot 6824$ correct to *three* places of decimals

If we performed the division in the ordinary way (adopting the integral divisor method of Chapter XXXII), the work would stand thus —

$$
\begin{array}{r|l}
176824)436563 & 8\cdot750(24\cdot689 \\
353648 & \\
\hline
82915 & 8 \\
70729 & 6 \\
\hline
12186 & 27 \\
10609 & 44 \\
\hline
1576 & 835 \\
1414 & 592 \\
\hline
162 & 2430 \\
159 & 1416 \\
\hline
3 & 1014
\end{array}
$$

Hence the required quotient is $24 \cdot 689$, for the figure in the *fourth* decimal place in the quotient, obtained by *inspecting* the remainder, would evidently be *less* than 5 (See Rule on p 172)

Now if all the figures on the right of the vertical line are omitted, the same *quotient* can be obtained as follows:—

Having obtained the first quotient figure in the usual way, we then, instead of bringing down 8, strike off the last figure, 4, in the divisor, and obtain the second figure in the quotient by considering how many times the divisor 17682 is contained in 82915

Then, when we multiply this divisor by the new figure, 4, in the quotient, we add in mentally anything there would have been to carry from the figure cancelled (*i e* 4 times 2 + 1 carried = 9*)

And so on, *cancelling one fresh figure in the divisor at each stage.*

The work then stands thus.—

$$
\begin{array}{r}
17\cancel{6}\cancel{8}\cancel{2}\cancel{4})4365638\cdot75(24\cdot689 \\
353648 \\
\hline
82915 \\
70729^{*} \\
\hline
12186 \\
10609 \\
\hline
1576 \\
1414 \\
\hline
162 \\
158 \\
\hline
4
\end{array}
$$

Here the chief difficulty consists in deciding *when to begin the cancelling process.* Now it will have been observed that

(i) The number of quotient figures which can be so obtained is **two less than the number of figures in the divisor.**

Also, before beginning the work, *we can discover by inspection*

(ii) **The local value of the first figure in the quotient.**

And (i) and (ii) will together enable us to decide how many quotient figures (if any) must be obtained by the *ordinary* process.

For instance, $536 \cdot 214575 \div 132 \cdot 654$.

Here it is evident, without shifting the dec. points, that there will be *one integral* figure in the quotient.

If then we require the result to *six* places of decimals we must continue the work until the quotient contains *seven* figures

But *four* of these only can be obtained by the cancelling process

Hence the first *three* must be obtained by the ordinary process

If, however, the result were required to *two* places of decimals only, then the quotient need only contain *three* figures, and so one figure in the divisor might be cancelled before starting, and we should only have to consider the division of 536·21 by 132·65

N B—In order to avoid possible serious error, the student should *invariably, before any cancelling is done*, decide *by inspection* **what the local value of the first significant figure in the quotient** will be, *i e* whether it will represent *tens, units, tenths*, or *hundredths*, &c The quotient may then be *planned out and the decimal point marked* before beginning the actual work

EXAMPLE xvii.—*Divide* ·0632815 by 146·52065 *to six places*

Here it is evident that the first *significant* figure in the quotient will represent *ten-thousandths*, *i e* there will be *three* ciphers after the decimal point, so we have to perform but *three* steps in division in order to obtain the quotient to *six* places of decimals (The *seventh* figure we obtain by *inspection*) We therefore at first retain but *five* figures of the divisor

$$
\begin{array}{r}
14\cancel{6}\ \cancel{5}\cancel{2})\ 063281\ (\cdot 0004319 \\
58608 \qquad\qquad\quad \\
\hline
4673 \qquad\qquad\quad \\
4395 \qquad\qquad\quad \\
\hline
278 \qquad\qquad\quad \\
146 \qquad\qquad\quad \\
\hline
132 \qquad\qquad\quad
\end{array}
$$

Ans ·000432

EXAMPLE xviii —*Find within a millionth of the whole the quotient of* 6930·5 ÷ 91·5024

Here we have to find the first *seven* figures of the quotient, and as *four* can, with this *six* figure divisor, be obtained by the cancelling process, we see that the first *three* must be obtained by the ordinary process

Now in 6930 5 — 91·5024 it is evident (comparing 6000 with 90) that the first two figures in the quotient will be *integers*

Here the *eighth* figure in the quotient (obtained by *inspecting* the rem^r) is not written, as, being less than 5, it will not affect the req^d result See Rule on p 172

$$
\begin{array}{r}
91\cancel{5}\cancel{0}\cancel{2}\cancel{4})\,69305000\,(75\cdot 74118 \\
6405168 \qquad\qquad\quad \\
\hline
5253320 \qquad\qquad \\
4575120 \qquad\qquad \\
\hline
6782000 \qquad\quad \\
6405168 \qquad\quad \\
\hline
376832 \qquad\quad \\
366009 \qquad\quad \\
\hline
10823 \qquad\quad \\
9150 \qquad\quad \\
\hline
1673 \qquad\quad \\
915 \qquad\quad \\
\hline
758 \qquad\quad \\
732 \qquad\quad \\
\hline
26 \qquad\quad
\end{array}
$$

Ans 75·74118

In the two preceding examples, the divisors given consisted of *some integral* figures followed by decimal figures, and there was nothing to be gained by adopting the device utilised in Chap. XXXII In fact it is easier to plan out the quotient when only *one*, or *two*, of the divisor figures are integral than when all are

N B.—Hence, it is often convenient to *multiply (or divide) both divisor and dividend by such a power of ten as will convert the given divisor to this standard form, namely, to a number consisting of either* **one, or two, integral** *figures followed by decimal figures.*

For instance, 61·5638 ÷ ·0425 may be replaced by 6156·38 ÷ 4·25. Here we *multiply* the given dividend and divisor both by 100

Again, 584·63 ÷ 93860 = 058463 ÷ 9·386, *dividing* both by 10000

Also, instead of 7·12456 ÷ ·01368 we work with 7124·56 ÷ 13·68, having *multiplied* both by 1000, for, in this case, the first significant figure of the divisor is the figure 1, so it is well to have *two* integral figures in the divisor we work with

EXAMPLE XIX.—*Find* $1\cdot28\dot{3} \div \cdot4\dot{3}\dot{7}$ *correct to three decimals.*

Multiplying both by 10, we have

12·833 ÷ 4·3737

Now it is evident that there will be *one integral figure* in the quotient; so altogether we must obtain a quotient of *five* figures Therefore, we must repeat the recurring figures in the divisor until our divisor consists of at least five figures, in order to allow of four being successively cancelled.

$$
\begin{array}{r}
4\cdot\cancel{3}\cancel{7}\cancel{3}\cancel{7})\,12\cdot8333\,(2\cdot9342 \\
8\ 7475 \\
\hline
4\ 0858 \\
3\ 9363 \\
\hline
1495 \\
1312 \\
\hline
183 \\
175 \\
\hline
8
\end{array}
$$

Ans 2·934

EXAMPLE XX.—*Find the value of* £294, 17*s* 3*d*. $\times \dfrac{87}{36500}$ *to the nearest penny.*

Here we require the quotient approximately correct to *three* places, and one figure may be obtained by cancelling.

We need not, therefore, retain more than *three* places of decimals in the working of the multiplication

$$£294\cdot8625 \times \frac{8\cdot7}{3650}$$

$$= \frac{£29\cdot48625 \times 8\cdot7}{365}$$

$= £\cdot703$

$= \underline{14s.\ 1d\ \ Ans.}$

$$
\begin{array}{r}
£ \\
29\cdot48625 \\
78 \\
\hline
235\cdot889 \\
20\ 640 \\
\hline
36\cancel{5})\,256\ 53\,(\cdot7028\ £ \\
255\ 5 \\
\hline
1\ 03 \\
73 \\
\hline
30
\end{array}
$$

EXAMPLE xxi.—*Simplify* $\dfrac{428\cdot75 \times \cdot6425}{(3\cdot1416)^2}$, *to two places of decimals*

In cases of this kind, in order to decide how many figures to retain in the working, we make a rough *mental* calculation of the result before beginning the work thus —

The product in the dividend will evidently be rather less than $400 \times \frac{7}{10}$, i e rather less than 280, and the divisor will evidently be rather less than 10

Hence we expect a result nearly equal to 28

$$\begin{array}{r} 3\ 141600 \\ 61413 \\ \hline 9\ 424800 \\ 314160 \\ 125664 \\ 3141 \\ 1884 \\ \hline 9\cdot869649 \end{array} \qquad \begin{array}{r} 42\cdot8750 \\ 5246 \\ \hline 257\ 2500 \\ 17\ 1500 \\ 8575 \\ 2143 \\ \hline 275\cdot4718 \end{array}$$

Now, as we require the result to *two* places of decs, we must, with the two integer figures expected, have *four* figures in the quotient Also, as the complete divisor would, if worked out in full, have 8 decimal places in it, all the quotient figures *must* be obtained by the cancelling process So we must retain *six* figures in the divisor and not *less* than *six* in the dividend

$$98\cancel{6}\cancel{9}\cancel{6}\cancel{5})\begin{array}{r} 275472 \\ 197393 \\ \hline 78079 \\ 69087 \\ \hline 8992 \\ 8882 \\ \hline 110 \\ 98 \\ \hline 12 \end{array}(27\cdot91$$

Ans 27 91.

We therefore calculate the product $428\cdot75 \times \cdot6425$ to three *places of decimals*, and $(3\ 1416)^2$ to five places

Note —In such cases *one* extra place only need be retained in the working, as any small error thus resulting in the dividend will be counterbalanced by a corresponding one in the divisor

---

EXAMPLE xxii —*Find, to four places of decimals, the sum of the unlimited series* $\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \cdots$

Here each successive fraction is $\frac{1}{4}$ of the preceding one, so, having written the first as a decimal, we find the value of succeeding ones by dividing again and again by 4, retaining *6* places, and continuing the work until we reach a stage when *no significant figure* occurs in the first *six* places We then add the results obtained

$$\begin{array}{l} \cdot25 \\ \cdot0625 \\ \cdot015612 \\ \cdot003903 \\ \cdot000975 \\ \cdot000243 \\ \cdot000060 \\ \cdot000015 \\ \cdot000003 \\ \cdot000000 \\ \hline \cdot3333 \end{array}$$

Ans ·3333

LI. PERCENTAGES.

The words "*per cent*"* signify "*for each hundred*"

For instance, 7 per cent of 2000 is 7 for each hundred in 2000, i e 7×20, or 140 But $\frac{7}{100}$ of 2000 is also 140

Hence, 7 *per cent* is the same as $\frac{7}{100}$.

And so 7 per cent of $50 = \frac{7}{100}$ of $50 = 3\frac{1}{2}$

A percentage of a number, or quantity, is a number of *hundredths* of it

The *number* of hundredths is called the **rate** per cent

Now any fraction can be expressed with den^r 100; for if its den^r is not a factor of 100, we can put it in the form of a complex fraction having den^r 1, and then multiply both num^r and den^r of this complex fraction by 100.

For instance, $\frac{2}{3} = \frac{\frac{2}{3}}{1} = \frac{\frac{2}{3} \times 100}{1 \times 100} = \frac{66\frac{2}{3}}{100} = 66\frac{2}{3}$ *hundredths.*

Hence any ratio can be expressed as a percentage

For instance, the ratio of 3*s* 6*d* to £1 is expressed by the fraction $\frac{3\frac{1}{2}}{20}$, and $\frac{3\frac{1}{2}}{20} = \frac{3\frac{1}{2} \times 5}{20 \times 5} = \frac{17\frac{1}{2}}{100} = 17\frac{1}{2}$ per cent, i e 3*s* 6*d*. is $17\frac{1}{2}$ per cent of £1

EXAMPLE I —*Find the percentage equivalent to the fraction* $\frac{11}{24}$.

[Here we express $\frac{11}{24}$ with den^r 100, and then the *numerator* is the req^d number]

$$\frac{11}{24} = \frac{\frac{11}{24}}{1} = \frac{\frac{11}{24} \times 100}{1 \times 100} = \frac{45\frac{5}{6}}{100} = 45\frac{5}{6} \text{ per cent } \textit{Ans.}$$

EXAMPLE II.—*Find, in lowest terms, the fraction equivalent to* $21\frac{7}{8}$ *per cent.*

$$21\frac{7}{8} \text{ per cent} = \frac{21\frac{7}{8}}{100} = \frac{175}{800} = \frac{7}{32} \quad \textit{Ans}$$

EXAMPLE III.—*Find* 3·75 *per cent of* 3920.

$$3{\cdot}75 \text{ per cent of } 3920 = \frac{3{\cdot}75}{100} \text{ of } 3920 = {\cdot}375 \times 392$$
$$= 147 \quad \textit{Ans.}$$

* Latin, *centum*, a hundred

EXAMPLE IV —*Find 3 per cent of £235, 8s 4d.*

$$3 \text{ per cent of } £235,\ 8s\ 4d = \frac{3}{100} \text{ of } £235,\ 8s\ 4d.$$

$$= \frac{£706,\ 5s.\ 0d}{100}$$

$$= \underline{£7,\ 1s\ 3d\ \ Ans.}$$

£	s	d
7,06	5	0
20		
1,25		
12		
3,00		

EXAMPLE V —*How much per cent is £3, 10s of £140?*

1st Method

[Here we reduce £3, 10s to the fraction of £140, and then express this fraction with den^r 100, when the numerator is the req^d percentage]

$$\frac{£3,\ 10s}{£140} = \frac{3\frac{1}{2}}{140} = \frac{1}{40} = \frac{\frac{1}{40} \times 100}{1 \times 100} = \frac{2\frac{1}{2}}{100} = \underline{2\tfrac{1}{2} \text{ per cent } \ Ans}$$

2nd Method

[Or we may regard the question as one in Proportion, namely—*to find a number which bears to* 100 *the ratio of* £3, 10s *to* £140]

$$i\,e \quad \frac{\text{the req}^d \text{ no}}{100} = \frac{3\frac{1}{2}}{140}$$

$$\therefore \text{ the req}^d \text{ no } = \frac{3\frac{1}{2}}{140} \times 100 = \underline{2\tfrac{1}{2}\ \ Ans.}$$

EXAMPLE VI —$5\frac{1}{2}$ *per cent of a certain number is* $41\frac{1}{4}$; *find the number*

$$\frac{5\frac{1}{2}}{100},\ i\,e\ \frac{11}{200}, \text{ of the req}^d \text{ no } = 41\tfrac{1}{4}$$

$$\therefore \text{ the req}^d \text{ no } = 41\tfrac{1}{4} \div \frac{11}{200} = \underline{750\ \ Ans.}$$

To find, *mentally*, 5 per cent of any sum of money, neglecting fractions of a penny

As 5 per cent $= \frac{5}{100} = \frac{1}{20}$, and 1*s* is $\frac{1}{20}$ of £1, we have but to take **1s for each £1** in the sum, then 6*d* for any 10*s.* that remains, and 1*d* for each *complete* 20*d* still remaining

For instance, 5 per cent of £157, 14*s* 3*d* = 157*s.* + 6*d* + 2*d*
= £7, 17*s* 8*d.*

Hence, to find $2\frac{1}{2}$ per cent of any sum, first find 5 per cent of it, and then halve the result

Note —"Trade Discount" (see page 104) is often calculated as a *percentage* in this way.

To increase, or decrease, a given quantity by a given percentage.

This may, of course, be done by *first* finding the percentage and *then* adding it to, or subtracting it from, the given quantity.

For instance, *to decrease £191, 9s 2d by 4 per cent*

$$\frac{4}{100} \text{ of } £191,\ 9s.\ 2d = \frac{1}{25} \text{ of } £191,\ 9s\ 2d.$$
$$= £7,\ 13s\ 2d.$$

and £191, 9s. 2d − £7, 13s 2d = £183, 16s.

	£	s.	d
25 { 5)	191	9 .	2
5)	38 .	5 .	10
	7 .	13 .	2

It is, however, often much more convenient to combine the two operations.

For instance, to *increase* (say) £8 by (say) 3 per cent, we must to £8 add $\frac{3}{100}$ of £8

Now $£8 + \frac{3}{100}$ of $£8 = \left(1 + \frac{3}{100}\right)$ of $£8 = £8 \times \frac{100+3}{100}$. Hence

(I) *To* increase *by a given percentage*—

Multiply by the factor $\frac{100 + rate}{100}$.

Again, to *decrease* (say) £13 by (say) 7 per cent, we must from £13 take $\frac{7}{100}$ of £13.

Now $£13 - \frac{7}{100}$ of $£13 = \left(1 - \frac{7}{100}\right)$ of $£13 = £13 \times \frac{100-7}{100}$. Hence

(II) *To* decrease *by a given percentage*—

Multiply by the factor $\frac{100 - rate}{100}$.

EXAMPLE vii.—*The population of a town decreased* 4·3 *per cent during a certain period At the end of the period the population was* 73520; *what was it at the beginning?*

The req^d pop when decreased by 4·3 per cent = 73520

i e., the req^d pop. $\times \frac{100 - 4{\cdot}3}{100} = 73520$

or, the req^d pop. $\times \frac{95{\cdot}7}{100} = 73520$

∴ the req^d pop $= 73520 \div \frac{95{\cdot}7}{100} = 76823$ *Ans*

Cent per cent, or 100 per cent, of any number, or quantity, is the *whole*, since 100 per cent is $\frac{100}{100}$, or 1.

Hence, for example, if from any number, or quantity, 37 per cent be taken away, 100 − 37, or 63, per cent of it is left.

EXAMPLE VIII.—*In an examination 45 per cent of the total number of candidates were under 15 years of age; of these, 65 per cent were boys, and 441 were girls. Find the total number of candidates.*

$$\frac{45}{100} \text{ of the total no. were under 15,}$$

$$\text{and } \frac{65}{100} \text{ of } \frac{45}{100} \text{ of the total no. were } \textit{boys} \text{ under 15,}$$

$$\therefore \frac{35}{100} \text{ of } \frac{45}{100} \text{ of the total no. were } \textit{girls} \text{ under 15,}$$

$$\textit{i.e.} \ \frac{35}{100} \text{ of } \frac{45}{100} \text{ of the total no.} = 441,$$

$$\therefore \text{the total no.} = 441 \div \frac{35 \times 45}{100 \times 100} = \underline{2800 \ \textit{Ans.}}$$

EXAMPLE IX.—*What change per cent is made in the area of a rectangle by decreasing its length, and increasing its breadth, each 5 per cent?*

The original area = original length × original breadth.

$$\text{The new area} = \frac{95}{100} \text{ of original length} \times \frac{105}{100} \text{ of original breadth}$$

$$= \frac{9975}{10000} \text{ of original area.}$$

$$\therefore \text{the area is } \textit{decreased} \text{ by } \frac{25}{10000} \text{ of its original size}$$

$$= \frac{25}{100} \text{ of its original size} = \underline{\cdot 25 \text{ per cent } \textit{Ans.}}$$

EXAMPLE X.—*One year the revenue of a country was £3480560, the next year it was £3513200. Find, correct to two places of decimals, the increase per cent.*

The increase on £3480560 = £3513200 − £3480560 = £32640.

[We have now, as in Ex. I, to reduce £32640 to the fraction of £3480560, and express this fraction with den^{r} 100; then its num^{r} gives the req^{d} number.

Now $\frac{£32640}{£3480560} = \frac{3264}{348056}$, and this fraction, when multiplied by 100, gives the percentage num^{r}. As, however, we require but 2 places of decimals, instead of appending two ciphers to 3264, we drop two figures from 348056, and then use contracted division.]

$$\frac{£32640}{£3480560} = \frac{3264}{348056} = \frac{\cdot 94}{100}$$

$$= \underline{\cdot 94 \text{ per cent } \textit{Ans.}}$$

```
3480) 3264 ( 94 nearly
      3132
       132
```

LII. PROFIT AND LOSS.

If anything is sold for more than it cost it is said to be sold *at a profit*, if for less, *at a loss* Profit, or loss, is often expressed as **a percentage of the cost price.**

Money employed in any business or other undertaking (by means of which the profit, if any, is gained) is called **capital.**

EXAMPLE i —*Find the gain per cent of the cost price when an article which cost* 12*s* *is sold for* 12*s* 9*d*

Here the *actual* profit is 9*d*, and the capital employed is 12*s* We have, therefore, to express 9*d.* as a percentage of 12*s*

$$\text{Now } \frac{9d}{12s} = \frac{\frac{3}{4}}{12} = \frac{1}{16} = \frac{\frac{1}{16} \times 100}{100} = \frac{6\frac{1}{4}}{100} = \underline{6\tfrac{1}{4} \text{ per cent } Ans.}$$

EXAMPLE ii.—*Find the loss per cent of the cost price when articles which cost* £10 *per dozen are sold at* 16*s.* 6*d. each*

Here the cost price of one article is £$\frac{5}{6}$ = 16*s.* 8*d.* Hence on 16*s* 8*d.* capital employed the actual loss is 2*d.*

$$\text{Now } \frac{2d}{16s\ 8d.} = \frac{2}{200} = \frac{1}{100} = \underline{1 \text{ per cent } Ans.}$$

EXAMPLE iii.—*If an article which costs* 5*s* *is sold at a loss of* $7\frac{1}{2}$ *per cent, what does it sell for?*

[Here we must *decrease* the cost by $7\frac{1}{2}$ per cent]

$$\therefore \text{ the article sells for } \overset{s}{5} \times \frac{100 - 7\frac{1}{2}*}{100} = \overset{s}{5} \times \frac{92\frac{1}{2}}{100} = \underline{4s\ 7\tfrac{1}{2}d.\ Ans.}$$

EXAMPLE iv —*Find the cost price of an article which sold, at a profit of* 8 *per cent, for* £9

[Here the cost price *increased* by 8 per cent = the selling price.]

$$\begin{aligned} i\,e \text{ the cost price} \times \frac{100+8}{100}* &= £9 \\ \text{or, the cost price} \times \frac{108}{100} &= £9 \\ \therefore \text{ the cost price} &= £9 \div \frac{108}{100} \\ &= \underline{£8,\ 6s\ 8d.\ Ans.} \end{aligned}$$

* See (I) and (II) on page 187

EXAMPLE v —*Find the cost price of an article which sold for* 7*s* 6*d* *at a loss of* 10 *per cent*

[Here the cost price *decreased* by 10 per cent = the selling price]

$$\text{i e the cost price} \times \frac{100-10}{100} = 7\tfrac{1}{2}^{s}$$

$$\therefore \text{ the cost price} = 7\tfrac{1}{2} \div \frac{90}{100} = \underline{8s\ 4d\ \ Ans.}$$

EXAMPLE vi —*Find the gain per cent in buying eggs at* 10*d* *per dozen and selling them at ten for a shilling*

One egg *costs* $\frac{5}{6}d$, and sells for $\frac{6}{5}d$

$\therefore$ the *actual* gain is $\frac{6}{5} - \frac{5}{6} = \frac{11}{30}d$, when the capital employed is $\frac{5}{6}d$

[We have, therefore, to express $\frac{11}{30}d$ as a percentage of $\frac{5}{6}d$]

$$\text{Now } \frac{\frac{11}{30}}{\frac{5}{6}} = \frac{11}{30} \times \frac{6}{5} = \frac{11}{25} = \frac{11 \times 4}{25 \times 4} = \frac{44}{100} = \underline{44 \text{ per cent } Ans}$$

EXAMPLE vii —*If by selling goods for* £14, 6*s.* 10 *per cent of their cost is gained, at what price should they have been sold so as to gain* 20 *per cent?*

[Here the cost price *increased* by 10 per cent = the selling price]

$$\text{i e the cost price} \times \frac{110}{100} = £14\tfrac{3}{10},$$

$$\therefore \text{ the cost price} = £14\tfrac{3}{10} \times \frac{100}{110},$$

and this must be *increased* by 20 per cent

$$\therefore \text{ the req}^{d} \text{ selling price} = £14\tfrac{3}{10} \times \frac{100}{110} \times \frac{120}{100} = \underline{£15,\ 12s\ \ Ans}$$

EXAMPLE viii —*If, by selling a horse for* 19 *guineas,* 5 *per cent of its cost is lost, what would have been gained or lost per cent by selling it for* 21 *guineas?*

$$\text{The cost } \textit{decreased} \text{ by 5 per cent} = £19 \times \frac{21}{20}$$

$$\text{i e the cost} \times \frac{95}{100} = £19 \times \frac{21}{20}$$

$$\therefore \text{ the cost} = £19 \times \frac{21}{20} \times \frac{100}{95} = £21$$

Now in selling for 21 guineas what *cost* £21, the *actual gain* is 21*s*, and this we have to express as a percentage of the *cost*

$$\text{Hence } \frac{21s}{£21} = \frac{1}{20} = \frac{5}{100} = \underline{5 \text{ per cent gain } Ans}$$

EXAMPLE ix.—*If 24 lbs of tea costing 1s 9d per lb be mixed with 30 lbs costing 2s 4d per lb, at what price per lb must the mixture be sold so as to gain $12\frac{1}{2}$ per cent?*

The cost price of the 54 lbs of mixture is $\overset{s}{1\frac{3}{4}} \times 24 + \overset{s}{2\frac{1}{3}} \times 30 = 112s.$

$\therefore$ . . . 1 lb . $= \frac{112}{54}s = \frac{56}{27}s.$

Hence the reqd selling price $= \frac{56}{27}s$ *increased* by $12\frac{1}{2}$ per cent

$$= \frac{56}{27}s. \times \frac{112\frac{1}{2}}{100} = \frac{56}{27}s \times \frac{225}{200} = \underline{2s\ 4d\ Ans}$$

EXAMPLE x.—*If a merchant buys 360 gallons of spirit for £540. and makes a profit of 12 per cent by retailing it at 28s per gallon, how much water does he add to it?*

The cost of 1 gal of *mixture increased* by 12 per cent $= 28s.$

i e the cost of 1 gal of *mixture* $\times \frac{112}{100} = 28s$

$\therefore$ the cost of 1 gal of *mixture* $= \overset{s}{28} \times \frac{100}{112} = 25s.$

But the cost of 1 gal. of *pure* spirit $= \frac{540 \times 20}{360}s = 30s$

Hence $\frac{25}{30}$, or $\frac{5}{6}$, of each gal of mixture is pure spirit, and $\frac{1}{6}$, consequently, is water, *i e* the ratio of spirit to water $= 5 : 1.$

$\therefore$ to 360 gals of spirit he adds $\frac{1}{5}$ of 360 gals of water $=$ <u>72 gals *Ans*</u>

EXAMPLE xi —*A makes an article and sells it to B at a profit of 20 per cent of what it cost him to make it B sells it to C at a loss of 12 per cent of what B gave for it C sells it for £63, 16s, making a profit of 16 per cent on what it cost him. What did it cost A to make?*

The cost to C *increased* by 16 per cent $= £63\frac{4}{5}$,

i.e. the cost to C $\times \frac{116}{100} = £63\frac{4}{5}$, $\therefore$ the cost to C $= £63\frac{4}{5} \times \frac{100}{116}.$

The cost to B *decreased* by 12 p c = what he sold it to C for,

i e cost to B $\times \frac{88}{100} = £63\frac{4}{5} \times \frac{100}{116}$, $\therefore$ cost to B $= £63\frac{4}{5} \times \frac{100}{116} \times \frac{100}{88}$

The cost to A *increased* by 20 p c = what he sold it to B for;

i e. the cost to A $\times \frac{120}{100} = £63\frac{4}{5} \times \frac{100}{116} \times \frac{100}{88}$,

$\therefore$ the cost to A $= £63\frac{4}{5} \times \frac{100}{116} \times \frac{100}{88} \times \frac{100}{120} = \underline{£52,\ 1s\ 8d\ Ans}$

LIII. PERCENTAGES.

COMMISSION, BROKERAGE, &c

An *agent* is a person who is employed by another to buy or sell, or to collect rents, &c, for him The agent usually receives for his trouble a *percentage* of the price of the goods he buys or sells, or of the amount of rents, &c, he collects This percentage is called his **commission.** Architects, auctioneers, &c, are often paid for their services in the same way Some agents are called *brokers*, and the percentage they receive is called **brokerage**

If a person insures his goods against fire, he pays annually to the insurance company a certain *percentage* of their supposed value, this percentage is called a **premium.**

These, being *percentages*, are all calculated in exactly the same way.

For instance, 3 per cent *commission* on £750 is $\frac{3}{100}$ of £750,

and $\frac{1}{2}$ per cent *brokerage* on £3000 is $\frac{1}{200}$ of £3000

Hence the rule—*Multiply the sum* (on which the commission is to be calculated) *by the* **rate** *per cent, and divide the result by* 100.

Note —In this connection such an expression as "2*s* 6*d* per cent" is sometimes used, and must be understood to mean 2*s* 6*d* per £100 Thus, 2*s* 6*d* per cent = £$\frac{1}{8}$ per £100 = $\frac{1}{8}$ per cent

EXAMPLE i —*Find, to a penny, the commission on* £853, 16*s* 9*d* *at* 3 *per cent.*

	£	s	d
	853	16	9
			3
	£25,61	10	3
	20		
s	12,30		
	12		
d.	3,63		

Ans £25, 12*s* 3*d*

EXAMPLE ii —*Find, to the nearest penny, the brokerage on* £4566, 10*s* *at* 2*s* 6*d* *per cent*

Brokerage = $\frac{1}{800}$ of £4566, 10*s*.

	£	s	d
8	4566	10	0
	5,70	16	3
	20		
s	14,16		
	12		
d	1,95		

Ans £5, 14*s* 2*d*

EXAMPLE iii —*After the deduction of commission at* $3\frac{1}{2}$ *per cent a landlord's net rental was* £2123, *find his agent's commission*

The gross rental *decreased* by $3\frac{1}{2}$ per cent = £2123,

i e gross rent[l] $\times \frac{96\frac{1}{2}*}{100} =$ £2123, $\therefore$ gross rent[l] = £2123 $\times \frac{100}{96\frac{1}{2}} =$ £2200.

Hence agent's commission = $3\frac{1}{2}$ per cent of £2200 = £77 *Ans.*

* See (II) on page 187

LIV. SIMPLE INTEREST.

Interest is the payment made for the use of money

The money, for the use of which the interest is paid, is called the **Principal**

The interest is always *a percentage* of the principal, and this percentage is generally charged for *each year* the principal is used.

For instance, if money be borrowed at 5 "per cent per annum", then for *each* £100 borrowed, £5 is due yearly as interest

If then, £800 be borrowed, £40 is due yearly as interest at 5 per cent.

The percentage *number* is called the **Rate** per cent.

For instance, if 4 be the rate per cent per annum, then a year's interest on £100 is £4, on £50 is £2, on £700 is £28, and so on

Note—The words "per annum" are often omitted, but, unless the contrary is stated, must always be understood.

The symbol % or the abbreviation p c, is used for "*per cent*"

The *sum* of the principal and interest is called the **Amount.**

For instance, at 4 per cent, £104 is the *amount* of £100 in a year

When the interest is paid year by year as it becomes due, or when, if unpaid, *no interest is charged on overdue interest*, the principal is said to be lent at **Simple** interest.

Thus, in the case of simple interest, the principal remains unchanged year by year, and consequently the interest for 2, or 3, &c years is simply *double*, or *treble*, &c of the interest for *one* year.

If, however, interest is charged on overdue interest as well as on the principal, the principal is said to be lent at *compound* interest (See Chap LV)

As interest is *a percentage*, the process of finding the interest on any sum at any rate for **one** year exactly corresponds to that of finding any other percentage—such as Commission, for instance, *i e the principal multiplied by the rate per cent and divided by* 100 gives the interest for **one** year

And the *simple* interest for 2, 3, &c. years is, as we have already seen, merely *double, treble,* &c, respectively, of the interest for *one* year Similarly the interest for $\frac{1}{2}$, $\frac{2}{3}$, &c of a year is *half, two-thirds*, &c, respectively, of the interest for *one* year

Hence the following rule —

To find the Simple Interest on a given sum of money at a given rate per cent for a given time

Multiply *the principal by the rate per cent, and by the number* (whole, or fractional) *of years, and* **divide** *the result by* 100.

The examples which follow illustrate various ways of performing the multiplication and division involved in the application of the foregoing Rule

Note —Before proceeding to apply the "rule" it should be noticed that, when once the definitions are known, the interest in any given case may be found independently by the "Unitary" method

For instance, *to find the simple interest on £650, at 3 per cent per annum, for 2 years*

	£		£	
On	100	the interest is	3	for a year
∴	1		$\frac{3}{100}$	
∴	650		$\frac{3}{100} \times 650$	

Hence the interest for 2 years is $£\frac{3 \times 650}{100} \times 2$, *i e* $£\frac{650 \times 3 \times 2}{100}$ (1).

It would be well for the beginner to work a few exercises at full length thus, before using the rule enabling him to write down the required interest at once in the form (1).

EXAMPLE i —*Find the simple interest on £1757, 18s 4d for 2 years at 3 per cent per annum*

$$\text{The req}^{d} \text{ int} = \frac{(£1757,\ 18s\ 4d) \times 3 \times 2}{100}$$

$$= \underline{£105,\ 9s\ 6d\ \ \textit{Ans.}}$$

$$\begin{array}{rrrr} & £ & s & d \\ & 1757\ . & 18 & 4 \\ & & & 6 \\ \hline & £105{,}47 & 10 & 0 \\ & 20 & & \\ \hline s & 9{,}50 & & \\ & 12 & & \\ \hline d & 6{,}00 & & \end{array}$$

[In such a case as this the work is easily performed by *compound* mult^{n} and div^{n}]

EXAMPLE ii —*Find the simple interest, for $2\frac{1}{2}$ years at $3\frac{3}{4}$ per cent, on £370.*

$$\text{The req}^{d} \text{ int} = \frac{£370 \times 3\frac{3}{4} \times 2\frac{1}{2}}{100}$$

$$= £37\cancel{0} \times \frac{15}{4} \times \frac{\overset{1}{\cancel{5}}}{2} \times \frac{1}{\underset{2\cancel{0}}{\cancel{100}}}$$

$$= £\frac{555}{16} = £34\tfrac{11}{16}$$

$$= \underline{£34,\ 13s\ 9d\ \ \textit{Ans.}}$$

[Here it is convenient to perform the mult^{n} and div^{n} in *vulgar fractions.*]

When the interest is required for a number of *months* they must be considered as *twelfths of a year.*

EXAMPLE III — *Find the amount of £514 in 7 months at $2\frac{1}{2}$ per cent per annum.*

$$\text{The int} = \frac{£514 \times 2\frac{1}{2} \times \frac{7}{12}}{100}$$

$$= £\overset{257}{\cancel{514}} \times \frac{\cancel{5}}{\underset{1}{\cancel{2}}} \times \frac{7}{12} \times \frac{1}{\underset{20}{\cancel{100}}}$$

$$= £\frac{1799}{240} = 1799d. \qquad \begin{array}{r} 12\,)\,1799 \\ 2,0\,)\,14,9 + 11d. \\ £7 + 9s \end{array}$$

$$= £7,\ 9s\ 11d$$

Hence the req[d] amount $= £514 + £7,\ 9s\ 11d = \underline{£521,\ 9s\ 11d}$ *Ans.*

When the interest is required for a number of *days* they must be considered as 365ths of a year.*

If the day of the month of the beginning, and that of the end, of the period be given, it must be remembered that, in adding the days in the calendar months and parts of months involved, *the first of the days mentioned must* not *be included*

For instance, from June 3 to June 4 is but *one* day
Hence, from April 6 to July 13 = 24 + 31 + 30 + 13 = 98, days

Note —It is well to remember that 73 is a factor of 365, and hence that the fractions $\frac{73}{365}$, $\frac{146}{365}$, $\frac{219}{365}$, $\frac{292}{365}$ reduce to $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$ of a year respectively.

EXAMPLE IV — *Find the amount of £207, 1s. 8d. in 219 days at 5 per cent.*

$$\text{Int} = (£207,\ 1s.\ 8d) \times \frac{\overset{1}{\cancel{5}}}{100} \times \frac{\overset{3}{\cancel{219}}}{\underset{5}{\cancel{365}}} \qquad \begin{array}{r} £\ \ s\ \ d. \\ 207\ .\ 1\ .\ 8 \\ 3 \\ \hline £6,21\ .\ 5\ .\ 0 \\ 20 \\ \hline s.\ 4,25 \\ 12 \\ \hline d\ 3,00 \end{array}$$

$$= (£207,\ 1s.\ 8d) \times \frac{3}{100}$$

$$= £6,\ 4s.\ 3d$$

Hence, req[d] amt. $= £207,\ 1s\ 8d. + £6,\ 4s\ 3d = \underline{£213,\ 5s\ 11d.}$ *Ans.*

---

* Even if the year be a *leap* year, and the extra day be counted in the num[r], it is customary to retain 365 in the den[r]

Bankers, &c, have no dealings in any coin less than a penny; hence, in calculating interest, fractions of a penny in a result are either neglected altogether, or the result is given to the *nearest* penny

Note —Interest at 5 per cent, or $2\frac{1}{2}$ per cent, for one year may be found mentally in the same way as was shown for these percentages on p 186

(Notice, however, that a result so obtained, if multiplied by 2, 3, &c, cannot be relied upon to a penny as the interest for 2, 3, &c years)

As int for *a year* at 5 p c is found by taking 1*s* for each £1 of principal, and as a month is $\frac{1}{12}$ of a year, it follows that to find *mentally*

Int at 5 p c for one month —Take 1*d* for each £1 of principal

For instance, the int on £413, 15*s* 9*d* for *one month* at 5 per cent is 413*d* = £1, 14*s* 7*d*, or, to the *nearest* penny, £1, 14*s* 8*d*

EXAMPLE V.—*Find, neglecting fractions of a penny, the interest on* £521 *from Aug* 17 *to Dec* 31 *at* 2 *per cent*

From Aug 17 to Dec 31 = 14 + 30 + 31 + 30 + 31 = 136 days

$$\text{Hence req}^{d}\text{ int} = £521 \times \frac{2}{100} \times \frac{136}{365}$$

$$= £\frac{521 \times 2 \times 136 \times 2^{*}}{100 \times 365 \times 2}$$

$$= £\frac{283424}{73000}$$

$$= £3 \cdot 882$$

$$= \underline{£3,\ 17s\ 7d\ \ Ans}$$

```
      521
      544
     2084
    2084
   2605
              £
73 )283·424(3·882
    219
     644
     584
      602
      584
       184
```

[* Here, instead of cancelling, we multiply both num^r and den^r by 2, thus obtaining the divisor 73000 instead of a less convenient one We then perform the division in decimals as far as 3 places]

EXAMPLE VI —*Find, within a penny, the interest on* £1874, 17*s* 2*d* *from Jan* 23 *to June* 30, 1896, *at* $4\frac{1}{2}$%

8 + 29 + 31 + 30 + 31 + 30 = 159, days

$$\therefore \text{req}^{d}\text{ int} = £1874\ 86 \times \frac{9}{2} \times \frac{159}{365} \times \frac{1}{100}$$

$$= \frac{£1874 \cdot 86 \times 1\ 431}{73}$$

$$= \underline{£36,\ 15s\ \ Ans.}$$

```
   1874·86
      13 41
   1874·86
    749 94
     56 24
      1 87
                  £
73 )2682 92(36·752
    492
     549
      382
       17
```

[Here we have applied Contracted methods in decimals (see Chap L), and abridged division (see page 15)]

N B —The division by 73000, generally involved in calculating interest for a number of days, may be avoided altogether by using what is called the "*Third, tenth* and *tenth*" rule, as follows —

(i) For the num^r of the interest-fraction multiply the decimalized principal by the rate p c and *double* the number of days (thus making the den^r 73000)

(ii) To the num^r thus obtained add $\frac{1}{3}$ of itself, $\frac{1}{10}$ of this third, and again $\frac{1}{10}$ of this latter.

(iii) Divide the sum by 100,000, *i e move the dec point* 5 *places to the left.* The result, decreased by a farthing for each £10 in it, is the req^d interest.

For instance, *to find the interest on* £817, 13*s for* 67 *days at* $2\frac{1}{2}$ *per cent*

$$\text{The int} = £\frac{817\cdot6 \times 2\frac{1}{2} \times 67 \times 2}{100 \times 365 \times 2}$$

$$= £\frac{273896\cdot}{73000} \quad \ldots \quad \text{(i)}$$

$$= £3,\ 15s$$

$$\begin{array}{r|r} 3 & 273896\cdot \\ 10 & 91298\cdot \\ 10 & 9129\cdot \\ & 913\cdot \\ \hline & £3\cdot75236 \end{array} \quad \begin{array}{l} \text{(ii)}^* \\ \\ \text{(iii)} \end{array}$$

[* It is unnecessary to retain any decimals during this part of the work]

The reason of the rule is due to the fact that

$$73000 + \tfrac{1}{3} \text{ of } 73000 + \tfrac{1}{10} \text{ of } \tfrac{1}{3} \text{ of } 73000 + \tfrac{1}{10} \text{ of } \tfrac{1}{10} \text{ of } \tfrac{1}{3} \text{ of } 73000$$
$$= 73000 \times (1 + \tfrac{1}{3} + \tfrac{1}{30} + \tfrac{1}{300}) = 73000 \times \tfrac{137}{100}$$
$$= 100010, \text{ which is very nearly } 100000.$$

Now if we, in this way, mult both num^r and den^r of the interest-fraction by $\frac{137}{100}$ we do not alter its value, while we change the den^r from 73000 to 100010 Then, if we divide this new num^r by 100000 instead of by 100010, our divisor being too small by about $\frac{1}{10000}$ of itself, our quotient will be too great by about $\frac{1}{10000}$ of itself, *i e.* by about $\frac{1}{4}d$ in each £10.

We have seen, in a variety of examples, how when principal, rate, and time are given, the interest (or amount) can be found. We shall now consider the three *converse* cases

CASE I —To find the Rate when principal, time, and interest (or amount) are given.

EXAMPLE vii —*At what rate per cent would* £47. 5*s. be the simple interest of* £350 *for* 3 *years?*

[We first find the interest at one per cent.]

$$\text{Int at 1 per cent} = \frac{£350 \times 1 \times 3}{100} = £\frac{21}{2}$$

[Now as the rate is a *factor* of the interest, it is evident that the interest at 2, 3, &c. per cent is *double, treble,* &c the int at *one* per cent. We therefore obtain the req^d rate by dividing *the given interest by the interest at* one *per cent*]

$$\text{Hence req}^d \text{ rate per cent} = \frac{£47\frac{1}{4}}{£\frac{21}{2}} = \frac{189}{4} \times \frac{2}{21} = 4\tfrac{1}{2}\ \textit{Ans}$$

EXAMPLE VIII —*At what rate per cent would £520 amount to £551, 4s. in 2 years at simple interest?*

[By subtracting the given principal from the given amount we obtain the *interest* at the req^d rate, and thus reduce this question to the form of the preceding one]

The *int* at req^d rate $= £551\frac{1}{5} - £520 = £31\frac{1}{5}$

But int at 1 per cent $= \dfrac{£520 \times 1 \times 2}{100} = £\dfrac{52}{5}$

$$\text{Hence req}^d \text{ rate} = \frac{£31\frac{1}{5}}{£\frac{52}{5}} = \frac{156}{52} = \underline{3\ \textit{Ans.}}$$

Note —The beginner must be careful not to mistake the *amount* for the *principal* when he finds the interest at 1 per cent

---

CASE II —To find the Time when principal, rate, and interest (or amount) are given

The method of procedure in this case is exactly similar to that of Case I.

EXAMPLE IX —*In what time would the simple interest on £92, 10s. be £12, 19s at 4 per cent?*

The int. for req^d no of years $= £12\frac{19}{20}$

But int for one year $= \dfrac{£92\frac{1}{2} \times 4 \times 1}{100} = £\dfrac{37}{10}$

$$\text{Hence the req}^d \text{ no of years} = \frac{£12\frac{19}{20}}{£\frac{37}{10}} = \frac{259}{37 \times 2} = \frac{7}{2} = \underline{3\tfrac{1}{2}\ \textit{Ans}}$$

EXAMPLE X —*In what time would £650 amount to £661, 7s 6d. at 3 per cent?*

The *int* for the req^d time $= £661\frac{3}{8} - £650 = £11\frac{3}{8}$;

And int for *one* year $= \dfrac{£650 \times 3 \times 1}{100} = £\dfrac{39}{2}$

$$\text{Hence req}^d \text{ no of years} = \frac{£11\frac{3}{8}}{£\frac{39}{2}} = \frac{91}{8} \times \frac{2}{39} = \frac{7}{12}.$$

Ans 7 months.

Note.—All the preceding examples might have been regarded as questions in Double Rule of Three, and have been treated by the "ratio" method of page 136

For instance, *Find the simple interest on £370 for $2\frac{1}{2}$ yrs at $3\frac{3}{4}$ per cent per annum*

This is equivalent to the following question —

"*If £$3\frac{3}{4}$ be the interest on £100 for 1 year,* what is the interest on £370 for $2\frac{1}{2}$ years?"

$$\text{Hence req}^{\text{d}} \text{ int.} = £3\tfrac{3}{4} \times \frac{370}{100} \times \frac{2\frac{1}{2}}{1} = £34,\ 13s\ 9d$$

Again, *At what rate per cent per annum would £47, 5s. be the simple interest on £350 for 3 years?*

This is equivalent to the following question.—

"If £$47\frac{1}{4}$ be the interest on £350 for 3 years, *what is the interest on £100 for 1 year?*"

$$\text{Hence req}^{\text{d}} \text{ rate} = 47\tfrac{1}{4} \times \frac{100}{350} \times \frac{1}{3} = 4\tfrac{1}{2}$$

CASE III.—To find the Principal when rate, time, and interest (or amount) are given.

EXAMPLE xi.—*On what principal is £17 the simple interest at 3 per cent for 4 years?*

[We first find the *interest* on £100]

Int on £100 at 3% for 4 yrs. = £12.

$$\begin{array}{llll} & \overset{£}{} & & \overset{£}{} \\ \text{Hence, as} & 12 & \text{is the interest on} & 100 \\ \therefore & 1 & \ldots\ldots\ldots & \dfrac{100}{12} \\ \text{and} & 17 & \ldots\ldots\ldots & \dfrac{100 \times 17}{12} = \underline{£141,\ 13s\ 4d.\ \textit{Ans}} \end{array}$$

Note—We have inserted here the full reasoning of the "Unitary" method The student who has acquired the "ratio" method would omit the 2nd line of reasoning

---

EXAMPLE xii.—*What sum would amount to £466, 11s at $2\frac{1}{2}$ per cent, simple interest, in 3 years?*

[We first find the *amount* of £100.]

$$\textit{Interest} \text{ on } £100 \text{ at } 2\tfrac{1}{2}\% \text{ for 3 yrs} = £3 \times 2\tfrac{1}{2} = £7\tfrac{1}{2}.$$

$$\therefore \textit{amount} \text{ of } £100 \ldots\ldots = £107\tfrac{1}{2}.$$

$$\begin{array}{llll} & \overset{£}{} & & \overset{£}{} \\ \text{Hence, as} & 107\frac{1}{2} & \text{is the } \textit{amount} \text{ of} & 100 \\ \therefore & 466\frac{11}{20} & \ldots\ldots\ldots & 100 \times \dfrac{466\frac{11}{20}}{107\frac{1}{2}} = \underline{£434\ \textit{Ans.}} \end{array}$$

The following are examples of rather more difficult questions —

EXAMPLE XIII —*A man lent £1600 at a certain rate per cent, and £850 at $\frac{1}{2}$ per cent less His annual income from both sources was £118, 5s Find the rates*

If he had lent both sums at the higher rate his income would have been $£118\frac{1}{4}$ + the extra int at $\frac{1}{2}$ per cent on £850

$$= £118\tfrac{1}{4} + £850 \times \tfrac{1}{2} \times \tfrac{1}{100} = £118\tfrac{1}{4} + £4\tfrac{1}{4} = £122\tfrac{1}{2}.$$

But income at one per cent on both sums $= £\frac{2450 \times 1}{100} = £\frac{49}{2}$.

Hence the higher of the req^d rates $= 122\frac{1}{2} \div \frac{49}{2} = 5$
and ∴ the lower . . .. $= 4\frac{1}{2}$ } *Ans*

EXAMPLE XIV —*Divide £2400 into two parts such that the simple interest on one part, at 3 per cent for $2\frac{1}{3}$ years, shall be equal to that on the other at $4\frac{1}{2}$ per cent for 2 years*

$$\text{The 1st part} \times \frac{3 \times 2\frac{1}{3}}{100} = \text{the 2nd part} \times \frac{4\frac{1}{2} \times 2}{100}.$$

$$\therefore \frac{\text{the 1st part}}{\text{the 2nd part}} = \frac{4\frac{1}{2} \times 2}{3 \times 2\frac{1}{3}} = \frac{9}{7}$$

[We must, therefore, divide £2400 into parts proportional to 9 and 7]

Hence the req^d parts are $£2400 \times \frac{9}{16} = £1350$
and $£2400 \times \frac{7}{16} = £1050$ } *Ans*

EXAMPLE XV —*A man opens a banking account on Jan 31, 1895, by depositing £120, on Feb 14 he pays in £28, on Mar 23 he draws £41, 10s, and on May 7 he draws £32 Find the interest at 2 per cent due to him from the bank on June 30*

There is due to him int from

Jan 31 to Feb 14 (i e for 14 days) on £120 ..		3360	(i)
Feb 14 . Mar 23 (i e for 37 days)	£148	10952	
Mar 23 . May 7 (i e for 45 days)	£106 5	9585	
May 7 . June 30 (i e for 54 days) . .	£74·5	8046	
		31943	(ii)
[We multiply each of the four sums by *double* the no of days for which int. is due on it (i)		2	(iii)
		63886	
Add these products (ii)		21295·	(iv)
Multiply the sum by the rate p c (iii)		2129	
And then apply the "third, tenth and tenth" rule (iv)]		213·	
		·87522	

Ans 17*s* 6*d*

LV. COMPOUND INTEREST.

When the *interest* as it becomes due *is added to the principal*, the money is said to be put out at **Compound Interest.**

For instance, if £200 be put out at *compound* interest at 5 per cent per annum for 3 years, at the end of the 1st year there is due
the interest on £200 at 5 p c for a year, *i e* £10.

At the end of the 2nd year there is due
the interest on £210 at 5 p c for a year, *i e* £10, 10*s*.

At the end of the 3rd year there is due
the interest on £220½ at 5 p c for a year, *i e* £11, 0*s* 6*d*.

Thus the *compound* interest for the 3 years is £31, 10*s* 6*d*.
whereas the *simple* interest for the 3 years is but £10 × 3 = £30

Hence in the case of Compound Interest, the interest for any one year (or other period) is the interest on the **amount** at the end of the previous year (or period).

Thus the original principal + a year's interest on it
= the 2nd principal, *i e* the *amount* at the end of *one* year.

The 2nd principal + a year's interest on it
= the 3rd principal, *i.e* the *amount* at the end of *two* years And so on.

And the difference between the final *amount* and the original principal is the compound interest for the whole time.

Note —Unless it is otherwise expressly stated, the interest is always understood to be due *annually*. Also, fractions of a penny in final results may be neglected, unless the result is required to the *nearest* penny.

EXAMPLE 1 —*Find the amount at compound interest of* £843, 15*s* *in* 3 *years at* 4 *per cent per annum.*

1st year.	*2nd year.*	*3rd year.*
£ *s* *d*	£ *s* *d*	£ *s* *d*
843 . 15 . 0	877 . 10 . 0	912 . 12 . 0
4	4	4
33,75 0 . 0	35,10 . 0 . 0	36,50 . 8 . 0
20	20	20
15,00	2,00	10,08
		12
		,96

£ *s* *d*		£ *s* *d*		£ *s* *d*	
843 . 15 . 0	1st P	877 10 . 0	2nd P	912 . 12 0	3rd P
33 . 15 0	1st I	35 2 . 0	2nd I	36 . 10 . 0	3rd I.
877 . 10 0	2nd P.	912 . 12 . 0	3rd P	949 . 2 0	4th P.

Ans £949, 2*s*

Here we have used comp mult^{n} and div^{n} in the working in order to show how one ignorant of decimals might obtain the result

But, as we shall now show, there is great gain in conciseness when decimals are properly used.

In the following examples, worked in decimals, we *multiply by the rate per cent and divide by* 100 *simultaneously*, and we do *not write down the multiplier*

For instance, in multiplying 123·64 by 3 and dividing the result by 100, instead of setting down the multiplier 3 and, after the multiplication is done, moving the decimal point *two* places to the *left*, thus —

123·64
3
3·7092

we multiply by an *unwritten* 3, setting *each figure of the result* as soon as it is obtained *two* places to the *right*, and mark the decimal point *vertically below* that in the multiplicand, thus —

123·64
3·7092

In this way the interest for any year is obtained in a position suitable for adding it to the principal

We need not retain more than 5 places of decimals in the working

EXAMPLE II —*Find the amount at compound interest of* £520 *in* 4 *years at* 3 *per cent*

£	
520	1st Prin
15·60	1st yr's Int
535·6	2nd Prin
16·068	2nd yr's Int.
551·668	3rd Prin
16·55004	3rd yr's Int
568·21804	4th Prin
17·04654~~12~~	4th yr's Int
585·26458	5th Prin = am^t in 4 yrs
20	
5·29160	
12	
3·49920	*Ans* £585, 5*s* 3*d*

Note —The final multiplication by 20 and 12 will be omitted in future, as the shillings and pence are easily obtained at sight (See p 175)

EXAMPLE III —*Find the compound interest for* 3 *years at* 4 *per cent on* £724, 6*s* 8*d*

Here, as we require but 5 places of decs in the working at each stage, we *begin to multiply at the figure in thick type*, not forgetting to *carry* from its right-hand neighbour Having obtained the *amount* for 3 years we subtract the 1st principal, thus obtaining the *interest* for 3 years (See p 201)

£	
724·33333	1st Prin
28·97333	1st yr's Int.
753·30666	2nd Prin
30·13226	2nd yr's Int
783·43892	3rd Prin
31·33755	3rd yr's Int.
814·77647	4th Prin = am^t in 3 yrs
724·33333	1st Prin
90·44314	Int for 3 yrs
	Ans £90, 8*s* 10*d*

The method of Aliquot Parts may often be used with advantage, especially when the rate per cent is a fractional number.

For instance, for the rate per cent 5

As $\frac{5}{100} = \frac{1}{20}$, instead of multiplying the principal by 5 and setting the figures *two* places to the right, we may divide the principal by 20, *i e* divide by 2 and set all the figures one place to the right

Again, for the rate per cent $2\frac{1}{2}$

As $\frac{2\frac{1}{2}}{100} = \frac{5}{200} = \frac{1}{40}$, we divide the principal by 4 and set the figure one place to the right

Also, for the rate per cent $4\frac{3}{4}$

As $\frac{4\frac{3}{4}}{100} = \frac{4}{100} + \frac{3}{400} = \frac{4}{100} + \frac{2+1}{400} = \frac{4}{100} + \frac{1}{200} + \frac{1}{400}$, we first *multiply* the principal by 4, setting the figures *two* places to the right (i), then *divide* the principal by 2, setting the figures *two* places to the right (ii), and then divide the latter result simply by 2 (iii)

(i), (ii), and (iii) together make up the year's interest at $4\frac{3}{4}$ per cent

And so on.

EXAMPLE iv.—*Find, to the nearest penny, the amount at compound interest of £1273, 13s 9d. in 2 years at $2\frac{1}{2}$ per cent.*

		£	
£1273, 13s. =		1273·65	
6d. =		·025	
3d. =		·0125	
	$\frac{1}{40}$	1273·6875	...1st Prin.
		31·84218	..1st yr.'s Int.
	$\frac{1}{40}$	1305·52968	.2nd Prin
		32·63824	2nd yr.'s Int
Ans. £1338, 3s 4d		1338·16792	3rd Prin. = amt in 2 yrs.

EXAMPLE v.—*Find the compound interest on £261, 10s. for 2 years at $2\frac{3}{4}$ per cent*

As $\frac{2\frac{3}{4}}{100} = \frac{2}{100} + \frac{1}{200} + \frac{1}{400}$, we first multiply by an unwritten 2 and set the figures *two* places to the right (i), then divide the principal by 200 (ii), and then divide the last result by 2 (iii) (i), (ii), and (iii) together make up the 1st year's interest.

	£	
$\frac{1}{200}$	261·5	.... 1st Prin.
	5·230 . (i)	1st yr.'s Int.
$\frac{1}{2}$	1·3075 . (ii)	
	·65375 . (iii)	
$\frac{1}{200}$	268·69125	.... 2nd Prin.
	5·37382	.... 2nd yr.'s Int.
$\frac{1}{2}$	1·34345	
	·67172	
	276·08024	.... amt in 2 yrs
	261·5	..... 1st Prin.
	14·58024	..... Int. for 2 yrs

Ans £14, 11s 7d

* See page 174

The next example illustrates the case in which the interest, due yearly, is required for a *part of a year*

EXAMPLE VI —*Find the amount at compound interest of* £1025 *in* $2\frac{1}{2}$ *years at* 3%

Having obtained the 3rd Principal as before, we now require the interest on it at 3 per cent per ann for *half* a year

Now 3 p c per year = $1\frac{1}{2}$ p c per *half* year

We ∴ (i) divide the 3rd prin by 100, and then (ii) divide this result by 2

(i) and (ii) together make the half-year's int

£	
1025·	1st Prin
30 75	1st yr's Int
1055·75	2nd Prin.
31·6725	2nd yr's Int
1087·4225	3rd Prin
10·87422 5·43711	$\frac{1}{2}$ yr's Int
1103·73383	req^d am^t

Ans £1103, 14*s* 8*d*

The next two examples illustrate cases in which *the interest is added to the principal at intervals of less than a year*

EXAMPLE VII —*Find to the nearest penny the amount, at compound interest payable half-yearly, of* £433, 13*s* 4*d* *in* $1\frac{1}{2}$ *years at* 5 *per cent*

As 5 per cent means 5 per cent per *year, i e* $2\frac{1}{2}$ per cent per *half*-year, and as $1\frac{1}{2}$ years = 3 half-years, we must calculate interest for three periods at $2\frac{1}{2}$ per cent per period

Ans £467, 0*s*. 3*d*

	£	
$\frac{1}{40}$	433 66666	1st P
	10 84166	1st I.
$\frac{1}{40}$	444 50832	2nd P
	11·11270	2nd I
$\frac{1}{40}$	455·62102	3rd P
	11 39052	3rd I
	467 01154	req^d am^t

EXAMPLE VIII —*Find the compound interest on* £825 *for a year at* $2\frac{1}{2}$% *payable quarterly*

$2\frac{1}{2}$% per ann $= \frac{2\frac{1}{2}}{4}$, or $\frac{5}{8}$, per cent per quarter,

and 1 year = 4 quarters,

so we calculate interest for 4 periods at $\frac{5}{8}$ per cent per period

Now $\frac{5}{8} \div 100 = \frac{4+1}{800} = \frac{1}{200} + \frac{1}{800}$,

so *we first divide the princ by* **200** (i), *and then this result by* **4** (ii)

(i) and (ii) together make up the quarter's int

Ans £20, 16*s* 4*d*

£	
825·	1st P
4 125 1·03125	1st I
830·15625	2nd P
4·15078 1·03769	2nd I
835·34472	3rd P
4·17672 1·04418	3rd I
840 56562	4th P
4·20282 1 05070	4th I
845 81914	am^t in 1 yr
825·	1st P
20 819	Int req^d.

EXAMPLE ix —*Find the difference between the simple and compound interest on £231, 12s 6d for 3 years at 4 per cent*

£	
231·625	1st P.
9·26500	1st I
240·89	2nd P.
9·6356	2nd I
250·5256	3rd P.
10·02102	3rd I.
260·54662	amt in 3 yrs.
231·625	1st P
28·92162	comp int for 3 yrs.
27·795	= 1st int × 3 = simp. int for 3 yrs
1·1266	= reqd diff.

Ans. £1, 2s. 6d.

In the preceding examples we have seen how the compound interest, or amount, may easily be obtained for a small number of years (or periods) If, however, the number of years (or periods) exceeds *seven*, a different method of treatment is preferable

Compound interest for a large number of periods.

As £100, at (say) 4% amounts in a year to £104

∴ 1, ... $\frac{104}{100} = £1·04$

∴ any prin. (say) 630, ... 630 × 1·04

Thus at 4 per cent
the **2nd Prin.** = **1st Prin.** × 1·04
. **3rd Prin.** = 2nd Prin × 1·04 = **1st Prin.** × 1·04 × 1·04
... **4th Prin.** = 3rd Prin × 1·04 = **1st Prin.** × 1·04 × 1·04 × 1·04
and so on.

But the 3rd, 4th, &c., Prins. are the *amounts* in 2, 3, &c, years respectively.

∴ at 4 per cent, amount in 1 year = 1st Prin. × 1·04
.... 2 years = 1st Prin. × $(1·04)^2$
.... 3 ...... = 1st Prin. × $(1·04)^3$,
.... = ...
... . 10 ...... = 1st Prin. × $(1·04)^{10}$. And so on.

Similarly, at 3%, amount in 2 years = 1st Prin. × $(1·03)^2$,
... . 7 .... = 1st Prin × $(1·03)^7$. And so on.

Hence, if (i) *we decimalize the fraction* $\frac{100 + \text{rate}}{100}$; (ii) *raise this decimal to the power whose index is the given number of years* (or periods); and (iii) *multiply the principal by this power*, we obtain an expression for the final **amount**.

For instance, the amount of £253 in 8 years at 2½% is £253 × $(1·025)^8$

EXAMPLE x —*Find the amount of* £2500 *in* 10 *years at* 3%

Here, in order to save space, we have omitted the work of calculating $(1\ 03)^{10}$ For this part of the work compare Exs xv, xvi on page 180 We must calculate $(1\cdot03)^{10}$ to **7** places of decimals here in order to have 3 places of decs in the product $2500 \times (1\cdot03)^{10}$

Note —The amount of labour in calculating Comp. Int for a large no of periods may be greatly reduced by the use of Logarithms

$$
\begin{aligned}
\text{Req}^{d}\ \text{am}^{t} &= £2500 \times (1\cdot03)^{10} \\
&= £2500 \times 1\ 3439164^{*} \\
&= £\frac{10000}{4} \times 1\ 3439164 \\
&= £\frac{13439\ 164}{4} \\
&= £3359\cdot791 \\
&= \underline{£3359,\ 15s\ 10d\ \mathit{Ans}}
\end{aligned}
$$

The preceding examples are all varieties of the same general question—to find the compound interest (or amount) when principal, rate, and time are known

There remain the three converse cases to be considered

CASE I —To find the Rate when principal, compound interest (or amount), and time are given

This case is considered in the chapter on Square Root

CASE II —To find the Time when principal, rate, and interest (or amount) are given

EXAMPLE xi.—*In what time would* £600 *amount to* £694, 11*s* 6*d at* 5 *per cent compound interest?*

Here the given am^{t} is £694·575

[The req^{d} time may be found by trial, *i e* we find the amount in 1, 2, 3, &c years until we obtain either the given amount, or the first amount which exceeds it]

Ans 3 years

£	
600	1st P
30	1st I
630	2nd P
31·5	2nd I
661·5	3rd P
33 075	3rd I
694·575	= am^{t} in 3 years

EXAMPLE xii —*In what time would* £423, 13*s* 9*d. be the compound interest, to the nearest penny, on* £3715 *at* 4 *per cent?*

Here £3715 + £423 6875 = **£4138·6875**, the *amount* in req^{d} time

After obtaining the 4th P. we observe that the given amount in the req^{d} time lies between the amounts in 2 and 3 years

Hence the req^{d} time is 2 yrs + some fraction of a year

3715.	1st P	
148·60	1st I	
3863·6	2nd P	
154 544	2nd I	
4018·144	3rd P	or am^{t} in 2 yrs.
160 72576.	3rd I	
4178·86976 .	4th P	or am^{t} in 3 yrs

In order to find this fraction we proceed thus —

$$\text{Am}^t \text{ in req}^d \text{ time} - \text{am}^t \text{ in 2 yrs} = £4138{\cdot}6875 - £4018{\cdot}144$$
$$= £120{\cdot}5435$$
$$\text{Am}^t \text{ in 3 yrs} - \text{am}^t \text{ in 2 yrs} = £160{\cdot}72576$$

$$\therefore \frac{\text{req}^d \text{ fraction}}{1 \text{ year}} = \frac{£120{\cdot}5435}{£160{\cdot}72576} = \frac{3}{4}$$

160·72576) 120·5435 (·75
112 5
80

Note —As the given int was approximate, we obtain the req^d fraction of a year by Approximation.

$\therefore$ req^d time $= 2\frac{3}{4}$ yrs *Ans*

CASE III.—To find the Principal when the rate, time, and compound interest (or amount) are given.

EXAMPLE xiii —*What sum would amount at compound interest to £813, 16s. 2d. (nearly) in 3 years at 5 per cent?*

[In this case the method of page 205 is convenient]

£1 in 3 yrs at 5 per cent amounts to $£1 \times (1{\cdot}05)^3$

Hence as $\overset{£}{1} \times (1{\cdot}05)^3$ is the amount of $\overset{£}{1}$

$\therefore$ 1 .... $\dfrac{1}{(1{\cdot}05)^3}$

and $\therefore$ 813·8083 .... $\dfrac{813{\cdot}8083}{(1{\cdot}05)^3} = £703$ *Ans.*

Note —To save space the approximate calculation of $\dfrac{£813{\cdot}8083}{(1{\cdot}05)^3}$ is omitted. For the method see next Ex

EXAMPLE xiv —*What sum put out at compound interest for 3 years at 4 per cent would gain £64, 12s 4d ?*

£1 in 3 yrs. at 4% comp int amounts to $£(1{\cdot}04)^3$

$\therefore$ 1 . . . . . gains *int* $£(1{\cdot}04)^3 - £1$
$= £{\cdot}124864.$*

Hence as $\overset{£}{{\cdot}124864}$ is the comp int on $\overset{£}{1}$

$\therefore$ 64·6166 .... $\dfrac{64{\cdot}6166}{{\cdot}124864} = £517, 10s.$ *Ans.*

*1·04	·124864) 64·6166 (517·5 nearly. (£)
416	62 4320
1·0816	2 1846
43264	1 2486
1·124864	9360
	8740
	620

* Here we square 1·04 by adding to 1·04, $\frac{4}{100}$ of 1·04, *i.e* we mult. by 4, setting the figures two places to the right; and so on. See p. 202.

The following are examples of other problems involving the methods of Compound Interest

EXAMPLE XV — *The population of a city is* 765240, *and its annual increase is at the rate of* 2·7 *per cent, what will be the population at the end of* 3 *years?*

$\left.\begin{matrix}\frac{1}{40}\\ \frac{1}{500}\end{matrix}\right\}$	765240 .	1st pop.
	19131 1530 48	1st yr 's inc
	785901·48	2nd pop
	19647·53 1571·80	2nd yr.'s inc
	807120·81	3rd pop
	20178·02 1614·24	3rd yr 's inc
	828913·07	pop at end of 3 yrs **Ans 828913**

Here as we only require the result correct to the nearest *integer*, we retain but 2 places of decimals in the working

Now $\frac{2\ 7}{100} = \frac{27}{1000} = \frac{25+2}{1000} = \frac{1}{40} + \frac{1}{500}$ We, therefore, (i) divide the original pop by 40, then (ii) divide the original pop by 500, (i) and (ii) together make up the 1st year's increase And so on

Note —If for the word "increase" we substitute "decrease", then, instead of adding, we *subtract* at each stage, to obtain the final pop

---

EXAMPLE XVI — *The difference between the simple and compound interest on a certain sum for* 2 *years at* 3 *per cent is* £1, 10*s* *Find the sum.*

[We first find the difference between the simple and comp int on £100 for the time]

Simple int. on £100 for 2 yrs at 3% is £6

Comp . . .. $£3 + £\frac{103 \times 3}{100} = £\frac{609}{100}$

Hence the difference is $\overset{£}{\frac{9}{100}}$ on a principal of $\overset{£}{100}$

. . . . $1\frac{1}{2}$. $100 \times \dfrac{1\frac{1}{2}}{\frac{9}{100}}$

$= £\frac{5000}{3} = \underline{£1666,\ 13s\ 4d\ \text{Ans}}$

EXAMPLE xvii —*£1030 is borrowed for 2 years, at 6 per cent compound interest, to be repaid, principal and interest, in two equal annual instalments Find the amount of an instalment.*

It is evident that the two instalments, together with the int on the *first* instal for *one* year, must be equal to the *amount* of £1030 in 2 years.

$$\textit{i e.}\ \text{req}^{d}\ \text{instal.} + \text{req}^{d}\ \text{instal} \times 1\cdot06 = £1030 \times (1\cdot06)^{2*}$$

$$\text{or, req}^{d}\ \text{instal.} \times (1 + 1\cdot06) = £1030 \times (1\cdot06)^{2}$$

$$\therefore\ \text{the req}^{d}\ \text{instal.} = £\frac{1030 \times 1\cdot06 \times 1\cdot06}{2\cdot06}$$

[* See p 205] $= \underline{£561,\ 16s\ \ \textit{Ans}}$

EXAMPLE xviii —*If £400 amounts to £532, 8s in 3 years at compound interest, what would £5000 amount to in 6 years at the same rate per cent?*

[We know, from page 205, that

$$£1 \text{ amounts in 3 yrs to } £1 \times \left(\frac{100 + \text{rate}}{100}\right)^{3}$$

$$\text{and } £1 \quad . \quad 6 \quad £1 \times \left(\frac{100 + \text{rate}}{100}\right)^{6}$$

i e when the number of years is doubled the Amount of £1 is squared.
Similarly, . *trebled* *cubed* And so on]

$$\text{Hence, as } \overset{£}{400} \text{ amounts in 3 yrs to } \overset{£}{532\cdot4}$$

$$\text{and, consequently, } 1 \quad\quad 3\ . \quad \frac{532\cdot4}{400}$$

$$\therefore\ 1 \quad . \quad .. \quad 6\ .\ .\ .. \quad \left(\frac{532\cdot4}{400}\right)^{2}$$

$$\text{and } \therefore\ 5000 \quad . \quad 6\ ..\ .. \quad \left(\frac{532\cdot4}{400}\right)^{2} \times 5000$$

$$= \frac{5324 \times 5324 \times 50}{400 \times 400}$$

$$= \underline{£8857,\ 16s.\ 1d\ \ \textit{Ans.}}$$

Note —If a quantity be subject to a periodical *decrease* at a given rate per cent, the remainder at the end of a given number of periods may be found by multiplying the original quantity by $\frac{100 - \text{rate}}{100}$ *raised to the power whose index is the given number of periods* (Cf. p 205.)

LVI. DISCOUNT AND PRESENT WORTH.

PRACTICAL, OR BANKERS', DISCOUNT

When one merchant purchases goods of another he does not usually pay for the goods in ready money, but by giving a *Bill of Exchange*

A **Bill of Exchange** is a document authorizing the person to whom it is given to receive a specified sum of money *at the end of a stated time*

The person to whom the "Bill" is given need not, however, wait until the end of the stated time for his money, he can *sell the Bill for cash** to a Banker, or Bill-broker

But the amount of *cash* he receives for it will be *rather less* than the *sum named in the Bill*, (for, as interest can always be obtained for money, it is evident that a sum of *ready* money is more valuable than the *same sum at some future time*)

When a Bill of Exchange is sold for cash it is said to be **Discounted.**

The sum of money named in the Bill is called its **Face Value.**

The amount of *cash* obtained for the Bill at the time it is discounted is called its (Commercial) **Present Worth** (or Value), and the difference between the *Face Value* and the *cash* value of the Bill is called the **Bankers'** (or Commercial) **Discount**

The Bankers' **Discount** is *always* **the Interest** (at some rate per cent agreed upon) **on the Face Value** of the Bill, reckoned *for the time the Bill has still to run* before it is *due*

Hence, the **Commercial Present Worth** of a Bill
= **Face Value − Bankers' Discount**
= **Face Value − Int. on Face Value.**

Note —In this, as in any other commercial, Rule fractions of a penny need not be given in results

EXAMPLE 1 —*Find the commercial present worth of a bill for £400, discounted, at 4 per cent, 58 days before it was due*

Banker's Disc = *int* on £400 for 58 days at 4%
= £2, 10*s* 10*d* †

Hence Commercial Present Worth = £400 − £2, 10*s* 10*d*
= £397, 9*s* 2*d* *Ans*

* This sale is *conditional* on the bill not being "dishonoured", *i e* the banker, or bill broker, can claim the return of the money he advanced should the bill not be paid at maturity

† The work involved in calculating this *interest* is omitted here, and in succeeding examples, as methods of calculation have been fully exemplified in Chap LIV

A Bill of Exchange which is *nominally* due on a certain day is not *legally* due until *three days later*; these 3 days are called **Days of Grace.**

For instance, a three-months' bill dated June 3, would be *nominally* due on Sep 3, but not *legally* due until Sep 6

The Banker, or Bill-broker, always includes the days of grace in reckoning the time for which he charges discount

Note —In questions in which the *actual dates are mentioned* the days of grace should always be taken account of, but in questions in which the actual dates are *not* mentioned the days of grace may be supposed to be already included in the interval named.

SPECIMEN OF A BILL OF EXCHANGE.

[Embossed Stamp 1s per £100] £100. London, Sep 7, 1896

Three months after date pay to my order one hundred pounds for value received.

William Jones

To Mr. John Smith,
Birmingham.

Here we may suppose that Mr Jones has supplied Mr. Smith with goods to the value of £100, and that Mr. Jones then writes a Bill of Exchange of the above form.

Mr Jones is called the "*drawer*", and Mr Smith the "*drawee*".

Mr Smith next writes the word "accepted", and signs his name, across the face of the bill, returning it to Mr Jones

Mr Smith having now "*accepted*" the three-months' bill, "*drawn*" (*i.e.* dated) Sep 7, has made himself liable for the £100 on Dec. 10th

Mr. Jones has now choice of the following three alternatives —

(i) He may keep the bill until Dec 10 and then obtain the whole sum of £100, or

(ii) He may at any time before Dec 10 (say on Sep 23), having signed his own name at the back of the bill (which constitutes his "order"), discount it for cash, obtaining from a banker, or bill-discounter, £100 *minus* the int. on £100 at, say 5, % for 78 days (*i.e.* the no. of days from Sep 23 to Dec 10); or

(iii) He may, having signed his name on the back, pass the bill on to a third person, Mr. Brown, to whom he owes money—in which case Mr Brown has choice of the same three alternatives

The person to whose order the bill is made payable is called the "*payee*". In the above specimen Mr. Jones is both drawer and payee, he might, however, have inserted the name of a third person as payee

The above is but one of many forms of a bill of exchange

A *Promissory Note* differs from a bill of exchange mainly in being *drawn by the debtor* instead of by the creditor It can be *used* in the same way as a Bill. (See the Exercises, No. 29, for a specimen.)

EXAMPLE II —*Find the banker's charge for discounting on June 5, at 5%, a bill for £450, drawn May 21 at 3 months*

[Here the days of grace must be taken account of
As the bill was dated May 21, the money was *nominally* due on Aug 21, and therefore *legally* due on Aug 24]

On June 5 the bill had 25 + 31 + 24 = 80, days still to run

Hence Bankers' disc $= £450 \times \frac{5}{100} \times \frac{80}{365} = \underline{£4,\ 18s.\ 8d\ \ Ans}$

Note —Here, and in similar cases, we calculate the *discount* to the *nearest* penny

---

EXAMPLE III —*Find the present worth of a bill for £1386, 15s at 90 days' sight, presented for acceptance on Sep 17, and discounted at 4% on Oct 3*

The bill was legally due on Dec 19 (*i e* 93 days from Sep 17),
∴ on Oct 3 it had still to run $28 + 30 + 19 = 77$, days
Hence disc, *i e* *int* on £1386, 15*s* for 77 days at 4%,
= £11, 14*s*
∴ req^d P W = £1386, 15*s* − £11, 14*s* = $\underline{£1375,\ 1s\ \ Ans}$

EXAMPLE IV —*A three-months' bill drawn April 15, and discounted May 6 at 4%, realized £217 find its face value*

No of days for which disc was charged $= 25 + 30 + 18 = 73$
Now Face val − Int on Face val = P W

$$\therefore \text{Face val.} - \text{Face val} \times \frac{4}{100} \times \frac{73}{365} = \overset{£}{217}$$

$$\text{i e}\ \ \text{Face val} \times \left(1 - \frac{1}{25} \times \frac{1}{5}\right) = 217$$

$$\therefore \text{Face val} = \overset{£}{217} \div \frac{124}{125} = \underline{£218,\ 15s\ \ Ans}$$

EXAMPLE V —*Show that a banker who discounts a bill at 5% when it has still 73 days to run, obtains interest at the rate of $5\frac{5}{99}$ per cent for his money*

Here Banker's disc on a bill for £100 would be $£5 \times \frac{1}{5} = £1$,
∴ the cash he advances on it = £100 − £1 = £99

Hence on $\overset{£}{99}$ lent for $\frac{1}{5}$ yr he gets $\overset{£}{1}$ int

∴ 100 1 $1 \times \frac{100}{99} \times 5 = £5\frac{5}{99}$;

i e. he obtains *int at the rate of* $5\frac{5}{99}$ *per cent per annum on his capital.*

Note—Hence when a banker discounts a bill at a certain rate per cent he obtains rather more than that rate per cent on the money he advances.

We shall now consider *what sum, if charged as discount, would give the discounter interest at the discounting rate per cent, and no more, on the money he advances*

It should, however, be clearly understood that this is purely a matter of theory, and of *no practical* importance, as Bills of Exchange, &c, which only run for *short* periods, are *always* discounted in the manner already exemplified

THEORETICAL (OFTEN CALLED "TRUE") DISCOUNT.

As £100 at, say, 4 per cent amounts in a year to £104, it follows that, at 4 per cent,

£100 is the *theoretical pres worth* of a debt of **£104** due at end of a year,
and £4 is the *theoretical discount* on **£104**

i e the Theoretical *Present Worth* of a debt due at some future time is that sum which, with interest, amounts in the given time to the debt,

and the Theoretical *Discount* is the *interest on the theoretical present worth of the debt* (not on the debt itself)

Hence questions on Theoretical Discount and Present Worth, for *short* periods, are but varieties of Case III in Simple Interest

Ex vi—*Find the Theoretical Present Worth of a debt of £567, due in 3 months, supposing money to be worth 5%*

£100 amounts in 3 mo at 5% to £101¼,

∴ Theo Pres Worth of a debt of £ $101\frac{1}{4}$ is £ 100

∴ . . . . 567 $100 \times \dfrac{567}{101\frac{1}{4}} = \underline{£560\ Ans.}$

Ex vii—*Find the "true" discount at 3 per cent on £282, 16s. due at the end of 4 months*

£100 amounts in 4 mo at 3% to £101,

∴ on £ 101 the "true" discount is £ 1

∴ $282\frac{4}{5}$. $1 \times \dfrac{282\frac{4}{5}}{101} = \underline{£2,\ 16s\ Ans}$

Ex viii—*If the "true" discount on £756 due in 73 days is £6, what is the rate per cent?*

As £6 is the "true" disc. on £756 for $\frac{1}{5}$ year,

∴ £6 . interest **£750**

Hence (see Case I, Simple Int) the rate per cent is $\underline{4\ Ans}$

Ex. ix—*On what sum, due in 6 months, is £8 the "true" discount at 5%?*

£100 in 6 mo. at 5% amounts to £102½,

∴ £ $2\frac{1}{2}$ is the theoretical discount on £ $102\frac{1}{2}$

∴ 8 .... $102\frac{1}{2} \times \dfrac{8}{2\frac{1}{2}} = \underline{£328\ Ans.}$

Note —We have already noticed (see pages 104, 186) that an allowance often made by a tradesman to a customer who pays ready money, is called **Trade Discount** This is merely *a percentage* on the price of the goods; *i e. time* is not considered in the calculation

LONG PERIOD PRESENT VALUE

Here practice agrees with theory

The **Present Value** (or Worth) of a given sum of money *due at the end* of a *number of years* is *that sum which, at compound interest,* at a given rate per cent, *amounts* in the time to the given sum

EXAMPLE x —*Find, to the nearest penny, the Present Worth of £800 due in 3 years, allowing compound interest at 4 per cent*

£1 amounts in 3 years at 4%, comp int, to £$(1\cdot04)^3$*

$$\therefore\ \overset{\pounds}{(1\cdot04)^3} \text{ has Pres Val } \overset{\pounds}{1}$$

$$\therefore\ 1 \quad \ldots \quad \frac{1}{(1\cdot04)^3}$$

$$\text{and } 800 \quad \ldots \quad \frac{800}{(1\cdot04)^3} = \frac{\overset{\pounds}{800}}{1\cdot124864}$$

$$= 711\cdot188 = \underline{\pounds711,\ 3s\ 9d\ \ Ans}$$

$$\begin{array}{ll}
1\cdot04 & \\
\quad\ 416 & \\
\hline
1\cdot0816 & \text{2nd power} \\
\quad\ 43264 & \\
\hline
1\cdot124864 & \text{3rd power}
\end{array}
\qquad
\begin{array}{r}
1\cdot1\cancel{2}\cancel{4}\cancel{8}\cancel{6}\cancel{4})\,800\cdot0000\,(711\cdot188 \\
787\ 4048 \\
\hline
12\ 5952 \\
11\ 2486 \\
\hline
1\ 3466 \\
1\ 1249 \\
\hline
2217 \\
1225 \\
\hline
992 \\
899 \\
\hline
93
\end{array}$$

Note —We square 1·04 by *adding* to 1·04 $\frac{4}{100}$ of 1·04 (See the method on p 202)

EXAMPLE xi —*How many pounds must a man invest, at 3 per cent compound interest, on his daughter's 16th birthday in order that she may come into £1000 on her 21st birthday?*

£1 amounts in 5 yrs at 3%, comp int, to £$(1\cdot03)^5$,

$$\therefore\ \overset{\pounds}{(1\cdot03)^5} \text{ has Pres Val. } \overset{\pounds}{1}$$

$$\therefore\ 1000 \qquad \frac{1000}{(1\cdot03)^5} = \underline{\pounds863\ \ Ans}$$

* See page 187

If by the payment of a sum of money down a person acquires the right to receive a fixed annual income, he is said to purchase an Annuity

An annuity is called (i) *terminable*, or (ii) *perpetual*, according as it is agreed that it shall continue for (i) a certain number of years and then cease, or (ii) for an unlimited number of years.

The following examples show how simple cases of terminable annuities may be treated by the ordinary methods of Arithmetic.

EXAMPLE xii.—*Find, to the nearest pound, the present value of an annuity of £80, to continue for 5 years, the first payment to be made a year hence, allowing compound interest at 3 per cent.*

If an annuity of £1 for 5 years were *left unpaid*, there would be due at the *end* of the time (allowing comp int at 3%)

£1 (i e the last instalment)
+ £1·03 (i e amt in 1 yr at 3% of the last instal. but one)
+ £$(1·03)^2$ (. ... . . 2 yrs ... . two)
+ £$(1·03)^3$ (.. .. .. 3 .. .. three)
+ £$(1·03)^4$ (.. . ... . 4 .... first instalment)
= £1 + 1·03 + 1·0609 + 1·0927 + 1·1254 = £5·309.

Hence as an annuity of £1 amounts to £5·309
∴ . . £80 . .. £5·309 × 80

But £1 amounts in 5 yrs at 3% to £$(1·03)^5$ = £1·1591

Hence, as £1·1591 has Pres Val. £1

$$\therefore\ 5{\cdot}309 \times 80 \ldots\ \ldots\ 1 \times \frac{5{\cdot}309 \times 80}{1{\cdot}1591} = \underline{\pounds 366\ \textit{Ans.}}$$

EXAMPLE xiii.—*What annuity, to continue for 6 years, the first instalment being due a year hence, could be purchased for £1000, when money is worth 4 per cent?*

Amount of an annuity of £1, left unpaid, for 6 yrs. at 4% comp. int.
= £1 + 1·04 + $(1·04)^2$ + $(1·04)^3$ + $(1·04)^4$ + $(1·04)^5$ = £6·632974

But *amount* of £1 in 6 yrs at 4% is £$(1·04)^6$ = £1·265318.

Hence $\dfrac{\pounds 6{\cdot}632974}{1{\cdot}265318}$ is Pres Val of £6·632974 due in 6 yrs.

i.e £$\dfrac{6{\cdot}632974}{1{\cdot}265318}$ will purchase *an annuity of* £1 *for 6 yrs.*

$$\therefore\ 1000 \ldots\ldots\ \ldots\ \frac{1{\cdot}265318}{6{\cdot}632974} \times 1000 = \underline{\pounds 190,\ 15s\ 3d.}$$

LVII. STOCKS AND SHARES.

Stocks

When one person borrows money from another he borrows on the understanding (i) that he must pay interest at some rate per cent agreed upon, (ii) that he must, eventually, *repay the principal*.

The lender's *security* for his money is, usually, a **mortgage** on some property possessed by the borrower

When, however, the Government of a country, or the Corporation of a town, borrows money (to meet some extraordinary expenditure not provided for by the ordinary revenue from taxes, or rates), it borrows on the understanding (i) that a fixed rate of interest will be paid continuously, but (ii) that it *can never be called upon to repay the principal*

A lender, in this case, who wishes to recover his principal **sells his right to the fixed interest.**

The *security*, in this case, is the obligation of the Government, or Corporation, to pay the interest out of the taxes, or rates

A person who has acquired *the right to the interest* of, say, £500 from a Government is said to "hold £500 *stock*", and a person who buys, or sells, *this right to the interest* of £500 is said to buy, or sell, "£500 stock"

A person who buys, or sells, £100 *stock* does not, as a rule, pay, or receive, exactly £100 *cash* for it Stock, like other things bought and sold, has a fluctuating market price—the price *rising*, or *falling*, according as the number of persons who wish to *buy* tends to exceed or fall short of the number who wish to *sell*

The **market price** of stock is always expressed by the *number of pounds*, cash, which will purchase £100 *stock*

For instance, when some particular stock is said to be "at 108", we must understand that £108 *cash* is then the price of £**100** stock of that kind

Any quantity of **Stock** may be bought, or sold

For instance, a person might buy £321, 16*s* 4*d* *stock*, and, if the rate of interest on this stock were 3 per cent, he would then have *bought the right* to £9, 13*s* 1*d*. a year, *i e* he would have *bought an* annuity of £9, 13*s* 1*d*

On some stocks the interest is paid *half-yearly*, on others *quarterly*, these payments are called **Dividends.**

Dividends are always calculated on the amount of **Stock** *held*

Note —The beginner must be very careful not to mistake *pounds stock* for *pounds cash*, or vice versa For instance, in the statement "a man *invests* £1000 in the 3 per cents", the £1000 is *cash*, but in the statement "a man *holds* £1000 in the 3 per cents", the £1000 is *stock*

The following examples* illustrate various cases which occur.

EXAMPLE I.—*Find the cost of £825 stock, at 123.*

[In other words —"If £100 *stock* cost £123 *cash*, what will £825 *stock* cost?"]

$$\begin{array}{llll} & \text{Stock.} & & \pounds \\ \text{As} & 100 & \text{costs} & 123 \\ \therefore & 825 & \ldots\ldots & 123 \times \frac{825}{100} = \underline{\pounds 1014,\ 15s.\ \textit{Ans.}} \end{array}$$

EXAMPLE II.—*How much stock, at 93, can be bought for £217?*

[In other words —"If £93 cash will pay for £100 *stock*, how much stock will £217 cash pay for?"]

$$\begin{array}{llll} & \pounds & & \text{Stock.} \\ \text{As} & 93 & \text{pays for} & 100 \\ \therefore & 217 & \ldots\ \ldots & 100 \times \frac{217}{93} = \underline{\pounds 233,\ 6s\ 8d\ \textit{stock Ans.}} \end{array}$$

EXAMPLE III.—*What annual income is obtained from £8750 stock paying 3 per cent?*

[In other words —"If £100 *stock* yields an income of £3, what income will £8750 *stock* yield?"]

$$\begin{array}{lllll} & \text{Stock.} & & \pounds & \\ \text{As} & 100 & \text{yields} & 3 & \text{income} \\ \therefore & 8750 & \ldots\ldots & 3 \times \frac{8750}{100} = \underline{\pounds 262,\ 10s.\ \textit{Ans.}} & \end{array}$$

Note —As the income is the interest, calculated on the *nominal* principal, *i e* on the amount of *stock*, Income = Stock $\times \frac{\text{rate p c.}}{100}$ (I).

Hence, conversely, the amount of *stock* which yields a *given income* is found from the formula· Stock = Income $\times \frac{100}{\text{rate p. c.}}$ (II).

EXAMPLE IV.—*Find the annual income obtained by investing £5800 in a 5 per cent stock at 116*

[In other words —"If £116 *cash* will buy the right to £5 a year, how much a year will £5800 *cash* buy the right to?"]

$$\begin{array}{llll} & \pounds & & \pounds \\ \text{As} & 116 & \text{cash purchases an income of} & 5 \\ \therefore & 5800 & \ldots\ \ldots\ \ldots\ \ldots & \frac{5 \times 5800}{116} = \underline{\pounds 250\ \textit{Ans.}} \end{array}$$

N B —From this result we see that the income may be found by multiplying the *sum invested* by the rate per cent, and dividing the result by the *market price of the stock*

i e. Income = $\frac{\text{cash invested} \times \text{rate p c.}}{\text{market price of stock}}$. (III).

* In order to save space, the merely mechanical part of the working is omitted, various methods of performing this having already been fully exemplified The student, however, is reminded that *the whole* of *his* work must be shown in an examination.

There is a special market, called the Stock Exchange, where stocks and shares are bought and sold

The agents employed by the public to buy, or sell, for them are called *Stock-brokers* The stockbroker's charge, called **Brokerage**, is *a percentage on the quantity of* stock bought, or sold

Hence when stock is *bought*, the brokerage is added to the *market price* of the stock, when *sold*, it is subtracted from the *market price*

Note —The brokerage on Government stock is 2*s* 6*d* per £100 stock, *i e* $\frac{1}{8}$ per cent on the amount of stock dealt with On certain other stocks the brokerage is $\frac{1}{4}$ per cent

For instance, when a certain Government stock is at 112, the *cost* (through a broker) *to a buyer* would be £112$\frac{1}{8}$ cash per £100 stock bought, the *sum realized by a seller* would be £111$\frac{7}{8}$ cash per £100 stock sold

[When brokerage is not specially mentioned in a question it need *not* be considered, but may be supposed to be already included in the stated price]

EXAMPLE v —*Find the cost of* £733, 6*s* 8*d*. *stock at* 108, *allowing for brokerage at* $\frac{1}{8}$ *per cent*

$$\begin{array}{llll} & \text{Stock} & & \pounds \\ \text{As} & 100 & \text{costs} & 108\frac{1}{8} \\ \therefore & 733\frac{1}{3} & & 108\frac{1}{8} \times \dfrac{733\frac{1}{3}}{100} = \underline{\pounds 792,\ 18s\ 4d\ \textit{Ans}} \end{array}$$

EXAMPLE vi —*How much would be realized by the sale of* £2450 *stock at* 134? (*Brokerage* $\frac{1}{4}$ *per cent*)

$$\begin{array}{llll} & \text{Stock} & & \pounds \\ \text{As} & 100 & \text{realizes} & 133\frac{3}{4} \\ \therefore & 2450 & . & 133\frac{3}{4} \times \dfrac{2450}{100} = \underline{\pounds 3276,\ 17s\ 6d\ \textit{Ans}} \end{array}$$

EXAMPLE vii —*How much stock at* 95$\frac{1}{2}$ *could be bought for* £1020, *allowing* 2*s* 6*d* *per cent for brokerage?*

$$\begin{array}{llll} & \pounds & & \text{Stock} \\ \text{As} & 95\frac{5}{8} & \text{is the cost of} & 100 \\ \therefore & 1020 & . \quad . & 100 \times \dfrac{1020}{95\frac{5}{8}} = \underline{\pounds 1066,\ 13s\ 4d\ \text{stock}\ \textit{Ans.}} \end{array}$$

EXAMPLE viii —*How much stock at* 95$\frac{1}{2}$ *would be sold for* £1526? (*Brokerage* $\frac{1}{8}$ *p c*)

$$\begin{array}{llll} & \pounds & & \text{Stock} \\ \text{As} & 95\frac{3}{8} & \text{is realized by the sale of} & 100 \\ \therefore & 1526 & & 100 \times \dfrac{1526}{95\frac{3}{8}} = \underline{\pounds 1600\ \text{stock}\ \textit{Ans}} \end{array}$$

EXAMPLE ix.—*Find the net quarterly dividend, after deducting income-tax at 8d. in the pound, obtained by investing £1152 in $4\frac{1}{2}$ per cents at $107\frac{7}{8}$, the brokerage being $\frac{1}{8}$ per cent.*

The *gross* quarterly dividend on each £100 stock is $\frac{1}{4}$ of $£4\frac{1}{2} = £\frac{9}{8}$,

$\therefore$ the *net* quarterly dividend, less income-tax, $= £\frac{9}{8} \times \frac{232^*}{240} = £\frac{87}{80}$.

Hence, as £108, invested, yields div^d^ $£\frac{87}{80}$,

$\therefore$ 1152 .. .. $\frac{87}{80} \times \frac{1152}{108} =$ £11, 12s *Ans.*

EXAMPLE x.—*What rate per cent of interest on capital is obtained by investing in a 4 per cent stock at 85?*

As £4 is the interest obtained by investing £85, we have but to express the ratio of £4 to £85 *as a percentage.*

$$\text{Now } \frac{4}{85} = \frac{\frac{4}{85} \times 100}{100} = \frac{4\frac{12}{17}}{100} = 4\tfrac{12}{17} \text{ per cent } Ans.$$

EXAMPLE xi—*Which investment pays better—3 per cent stock at 97, or 5 per cent stock at 161?*

[Here we compare the cost of equal incomes obtained from the two stocks.]

In the 1st stock an income of £3 is bought for £97,

$\therefore$ .. . £1 . . . $£\frac{97}{3} = £32\frac{1}{3}$.

In the 2nd stock an income of £5 . . . . £161,

$\therefore$ . ... £1 . .. $£\frac{161}{5} = £32\frac{1}{5}$.

Now $32\frac{1}{5}$ is *less* than $32\frac{1}{3}$, $\therefore$ in the 2nd stock *the same income* is obtained *by investing* less *capital*, *Ans.* 5 per cents at 161.

EXAMPLE xii.—*Find the change in income caused by selling out £2400 4 per cent stock at 135, and investing the proceeds in $2\frac{1}{2}$ per cent stock at 108.*

The income from £2400 stock at 4% is £24 × 4 = £96.

Again, £2400 stock sells for £135 × 24 cash.

And as £108 invested in the 2nd stock yields $£2\frac{1}{2}$ income,

$\therefore$ 135 × 24 ... . $2\frac{1}{2} \times \frac{135 \times 24}{108} =$ £75.

Hence the loss in income = £96—£75 = £21 *Ans.*

* See page 140

Note —The following terms are used in connection with stocks :—

THE FUNDS —The bulk of the National Debt of England (incurred in time of war) called the Funded Debt, or the *Funds*, now consists of two parts on one of which interest at the rate of $2\frac{1}{2}$ per cent per annum is paid, on the other, $2\frac{3}{4}$ per cent per ann is paid, and will continue to be paid until 1903, when the int on this part also will be reduced to $2\frac{1}{2}$ p c

CONSOLS —The above stocks are called *Consols* (i e. consolidated annuities) The dividends on Consols are paid through the Bank of England, *quarterly*, on April 5, July 5, Oct 5, and Jan 5 (See page 27)

The $2\frac{3}{4}$ per cent Consols are sometimes spoken of as "Goschens", for it was when Mr Goschen was Chancellor of the Exchequer in 1888 that the rate of interest, formerly 3%, was reduced to $2\frac{3}{4}$% It was then further enacted that, in order to reduce the National Debt, the Government may in 1923 *redeem* (i e pay off) any amount of Consols at the rate of £100 cash for each £100 stock

BONDS.—Foreign Government Stocks are so called from the documents given to the holders of such stocks From these Bonds, certificates, called *coupons*, are cut which entitle the holder to his dividend each half-year until the Bond is redeemed

PAR —When the market price of £100 stock is £100 cash, the stock is said to be "*at par*" (Latin, *par*, equal) And the stock is said to be *above*, or *below*, *par*, according as the market price of £100 stock is greater, or less, than £100 cash

PREMIUM —Also, when the market price of £100 stock is *greater* than £100 cash, the stock is said to be "*at a premium*". Thus, if £100 stock is worth £104 cash, that stock is "at 4 p c *premium*"

DISCOUNT.—And, when the market price of £100 stock is *less* than £100 cash, the stock is said to be "*at a discount*" Thus, if £100 stock is worth but £95 cash, that stock is "at 5 p c *discount*"

A *stock-jobber* is a person who deals in stocks and shares A stock-broker the agent of the public, buys from, or sells to, a stock-jobber

When the market price of stock on any day is quoted in the newspapers *two* prices are given, the higher is the price the broker pays for stock he buys from, and the lower the price he receives for stock he sells to, the stock-jobber. Thus, when a stock is quoted at "$108\frac{3}{4}$–$108\frac{7}{8}$", the broker *buys* at £$108\frac{7}{8}$, but *sells* at £$108\frac{3}{4}$, per £100 stock

The market price of a stock is not necessarily high when the rate of interest paid is high, a more important consideration being the degree of *safety* the investment offers. Thus *English* Government stocks pay a low rate of interest, yet their market price is high compared with that of other stocks

SHARES

When it is proposed to carry on some business on a large scale, and, consequently, a large capital is required, a few persons specially interested in the undertaking (called Directors) draw up a statement (called a Prospectus) of what they propose and the amount of capital needed for the purpose This capital they divide into a large number of equal parts, called **Shares**,* and invite the public to become partners in the business

* In some undertakings (e g Railways) the capital is *not* divided up into *shares*, but is treated as *stock*

The whole body of partners thus formed is called a **Company,** and the partners are called **Share-holders.**

The net profits are divided periodically among the shareholders, the **Dividend** being declared as a *percentage* of the capital

A shareholder cannot require a company to refund the money he has paid for shares, but he may *sell his shares*

The market price of shares rises, or falls, according as the company is more or less prosperous.

The market price of shares is generally expressed by the *number of pounds* cash (whole or fractional) which *one share* costs

For instance, when £10 shares in a company are "at $12\frac{1}{2}$", then £$12\frac{1}{2}$ *cash* will purchase *the right to dividends on* £10 *of the company's capital*

Dividends are calculated on the *nominal* value of shares

The chief distinction between stocks and shares is that, while *any quantity of stock* may be bought, or sold, only **whole** *numbers of shares* are, as a rule, dealt with.

Shares are said to be "at a premium", "at par", or "at a discount", according as their *cash* value is greater than, equal to, or less than, their *nominal* value.

For instance, if a £5 share in a company costs £6 cash, the shares of that company are at 1 premium, if a £5 share costs £5 cash, the shares are at par, if a £5 share costs £$4\frac{1}{2}$ cash, the shares are at $\frac{1}{2}$ discount.

N B.—2 premium is here very different from 2 *per cent* premium,
for if a £10 share is at 2 *premium,* its cash value is £12;
whereas if it is at 2 *per cent premium,* its cash value is £$10\frac{1}{5}$.

Brokerage is calculated usually either at the rate of $\frac{1}{2}$ per cent on the *nominal* value of shares, or else at so much *per share.*

For instance, the brokerage, at $\frac{1}{2}$ p c, on 7 £10 shares is £70 ÷ 200 = 7*s.*

Note —The word "Limited" now forms part of the title of most companies; it signifies that the liability of the shareholders is *limited to the number of their shares* Formerly shareholders were fully responsible (like partners in a *firm*) for all the debts of a bankrupt company

Some companies (*e g* Banks and Insurance Companies) do not require their whole capital for immediate use, in such cases only part of the nominal value of each share is *paid up,* and the company then may, in case of need, *call* upon its shareholders for all, or part, of what is not paid up *Dividends* are then calculated on the *paid up capital.*

Companies often divide their shares, or stock, into different classes—Debenture, Preference, and Ordinary Debenture holders have the security of a mortgage on the company's property, *e g* buildings, land, machinery, &c, and receive interest at a *fixed* rate Holders of Preference shares, or stock, have the right to interest at a *fixed* rate to be paid out of profits *before* the claims of *ordinary* shareholders are considered; and then Ordinary shareholders are entitled to the whole of any remaining profits

Hence Debentures offer *good security* with a *moderate* rate of interest, Ordinary Shares, *no security* with the *possibility* of a *high* rate of interest.

EXAMPLE XIII.—*Find the cost of* 27 £10 *shares in a company, at* $12\frac{1}{2}$.

$$\text{As } 1 \text{ share costs } \overset{\pounds}{12\tfrac{1}{2}},$$
$$\therefore\ 27 \text{ shares cost } 12\tfrac{1}{2} \times 27 = \underline{\pounds 337,\ 10s\ \textit{Ans}}$$

EXAMPLE XIV.—*Find the income from* 175 £2 *shares in a company paying a dividend of* 6 *per cent*

$$\text{Income} = \pounds 2 \times 175 \times \frac{6}{100} = \underline{\pounds 21\ \textit{Ans}}$$

EXAMPLE XV.—*How many* £5 *shares at* $6\frac{1}{4}$ *could be bought for* £300?

$$\text{As } \overset{\pounds}{6\tfrac{1}{4}} \text{ will pay for } 1 \text{ share,}$$
$$\therefore\ 300 \ \ldots\ \ldots\ \ 1 \times \frac{300}{6\frac{1}{4}} = \underline{48 \text{ shares } \textit{Ans}}$$

EXAMPLE XVI.—*Find the cost of* 108 £1 *shares at* $\frac{1}{4}$ *premium*

$$\text{As } 1 \text{ share costs } \overset{\pounds}{1\tfrac{1}{4}},$$
$$\therefore\ 108 \text{ shares cost } 1\tfrac{1}{4} \times 108 = \underline{\pounds 135\ \textit{Ans}}$$

EXAMPLE XVII.—*Find the amount of cash realized by the sale of* 75 £20 *shares, at* $2\frac{1}{2}$ *discount* (*Brokerage* $\frac{1}{2}$ *p c*)

The broker sells the shares for $\pounds 17\frac{1}{2} \times 75 = \pounds 1312\frac{1}{2}$

The brokerage ($\frac{1}{2}$ p. c on the nominal value of the shares)

$$= \pounds 20 \times 75 \times \frac{1}{200} = \pounds 7\tfrac{1}{2}$$

$$\therefore\ \text{amount of cash realized} = \pounds 1312\tfrac{1}{2} - \pounds 7\tfrac{1}{2} = \underline{\pounds 1305\ \textit{Ans}}$$

EXAMPLE XVIII.—*A man buys fifty* £50 *shares in a company,* £20 *paid up, at* $22\frac{1}{2}$, *and receives a dividend at the rate of* 4 *per cent. After paying a call of* £5 *per share, he receives a dividend at the rate of* $3\frac{1}{2}$ *per cent, and then sells his shares at* $26\frac{1}{4}$. *What amounts did he receive in dividends, and what capital did he sacrifice by the sale?*

$$\text{1st dividend} = \pounds 20 \times 50 \times \frac{4}{100} = \underline{\pounds 40\ \textit{Ans}\ (\text{i})}$$

$$\text{2nd dividend} = \pounds 25 \times 50 \times \frac{3\frac{1}{2}}{100} = \underline{\pounds 43,\ 15s\ \textit{Ans}\ (\text{ii})}$$

$$\text{Total cost of shares} = £22\tfrac{1}{2} \times 50 + £5 \times 50 = £27\tfrac{1}{2} \times 50.$$

$$\text{Result of sale} = £26\tfrac{1}{4} \times 50,$$

$$\therefore \text{ loss of capital} = £27\tfrac{1}{2} \times 50 - £26\tfrac{1}{4} \times 50$$

$$= £1\tfrac{1}{4} \times 50 = \underline{£62,\ 10s\ \textit{Ans}\ \text{(iii)}.}$$

The following examples in stocks are more complex than those previously given —

EXAMPLE XIX.—*If when* $2\frac{3}{4}$ *per cent Consols were quoted at* $111 - 111\frac{1}{2}$ *a person invested £2000, and sold out when they were quoted at* $109\frac{3}{8} - 109\frac{5}{8}$, *what capital would he sacrifice? (Brokerage* $\frac{1}{8}$ *p c.)*

He buys at $111\frac{1}{2}$* $+ \frac{1}{8} = 111\frac{5}{8}$, and sells at $109\frac{3}{8} - \frac{1}{8} = 109\frac{2}{8}$;

$$\therefore \text{ he reduces his capital in the ratio } \frac{109\frac{2}{8}}{111\frac{5}{8}} = \frac{874}{893}.$$

$$\text{Hence his loss} = £2000 - £2000 \times \frac{874}{893} = £2000 \times \left(1 - \frac{874}{893}\right)$$

$$= £2000 \times \frac{19}{893} = £2000 \times \frac{1}{47} = \underline{£42,\ 11s.\ \text{nearly}\ \textit{Ans.}}$$

EXAMPLE XX —*What is the market price of 4 per cent Railway Debenture stock, if by buying this stock with the proceeds of the sale of £3200* $2\frac{1}{2}$ *per cent Consols at 102, an increase of £1, 10s in annual income results? (Brokerage on Consols* $\frac{1}{8}$; *on Railway stock,* $\frac{1}{4}$ *per cent)*

$$\text{Original income} = £32 \times 2\tfrac{1}{2} = £80.$$

$$\text{New income} = £80 + £1\tfrac{1}{2} = £81\tfrac{1}{2},$$

and the amount of Ry. stock which yields this inc. $= £81\frac{1}{2} \times \frac{100}{4}$ stock †

But the Consols sold for $£101\frac{7}{8} \times 32$ cash $= £3260$ cash.

Hence, as $\overset{\text{Stock}}{81\frac{1}{2} \times \frac{100}{4}}$ is bought for $\overset{£}{3260}$,

$$\therefore \quad 100 \quad \ldots \quad 3260 \times \frac{100}{81\frac{1}{2} \times \frac{100}{4}} = £160.$$

But this cost includes the brokerage,

$$\therefore \text{ the market price of the Ry. stock} = 160 - \tfrac{1}{4} = \underline{159\tfrac{3}{4}\ \textit{Ans}}$$

* See Note, p 220 † See Note, p. 217 (II).

EXAMPLE XXI —*A person invests £5500 partly in a 4 per cent stock at 130, and partly in a 3 per cent stock at 117, and receives the same income from each Find the amount he invests in each stock, and his total income*

An income of £4 results from investing £130 in the 1st stock,

∴ .. . 1 $\frac{130}{4} = £\frac{65}{2}$.

Also . . 3 117 2nd ..

∴ . 1 ... , $\frac{117}{3} = £39$ ..

Hence the sums invested must be in the ratio $\frac{65}{2}$ 39

i e 5 6

[We therefore divide £5500 into parts proportional to 5 and 6]

∴ the req[d] parts are $\frac{5}{11}$ of £5500, $\frac{6}{11}$ of £5500,

i e £2500 must be invested in the 4 p c's
and £3000 .. . 3 } *Ans* (i)

Again, total income = twice $£2500 \times \frac{4}{130}$ * $= £\frac{2000}{13}$

= £153, 16*s* 11*d* *Ans* (ii)

EXAMPLE XXII —*If a person invests £1000 in* $2\frac{3}{4}$ *per cents at 99, how much must he also invest in 4 per cents at* $112\frac{1}{2}$ *so as to obtain interest at the rate of 3 per cent on his whole investment?*

On £100 invested in the 1st stock he obtains $£100 \times \frac{2\frac{3}{4}}{99} = £2\frac{7}{9}$ inc

... £100 . . 2nd $100 \times \frac{4}{112\frac{1}{2}} = £3\frac{5}{9}$ ...

But the *average* income on £100 invested is £3

Thus, on the 1st investment he obtains $\frac{2}{9}$ per cent *below* the average.

and ..... 2nd . . . . $\frac{5}{9}$ *above* .

Hence the sum invested in the 1st stock must bear to that invested in the 2nd the ratio of 5 to 2

∴ the req[d] sum = $\frac{2}{5}$ of £1000 = £400 *Ans*

---

* See note on page 217 (III)

LVIII. FOREIGN MONEY.

Decimal Coinages

With the exception of England and India, all important countries have *decimal* money systems.

In all such systems there are two *principal** coins, *the smaller of which is always some* **decimal part** (generally $\frac{1}{100}$) *of the larger*

For instance, in France these coins are the *franc*, and the *centime* $= \frac{1}{100}$ of a franc.
In Germany they are the *mark*, and the *pfennig* $= \frac{1}{100}$ of a mark.

Compared with a non-decimal system a decimal system of money has the advantage of greatly simplifying calculations.

Reduction from one denomination to another involves *no labour*, being performed *at sight*.

For instance, as centimes are *hundredths* of francs, 6 francs 15 centimes is at once written either as 6·15 *francs*, or as 615 *centimes*, and 17 marks 8 pfennigs either as 17·08 *marks*, or as 1708 *pfennigs*.

Consequently, the complex processes of our Compound Rules are replaced by the Simple Rules in decimals.

$$\text{For instance, 17 francs 5 centimes} \times 30 = \overset{\text{francs.}}{17{\cdot}05} \times 30 = \overset{\text{francs.}}{511{\cdot}50}$$
$$= 511 \text{ francs } 50 \text{ centimes.}$$

Note—On the other hand, our system is not altogether without its own advantages The use of *three* principal coins enables us to express large sums with fewer figures. Also, the number of exact divisors in 240 is much greater than in 100

The following table gives the names of the principal coins of the chief countries using a decimal system of money —

Country	Principal Coins		Approximate Value in English money.†
France … Belgium Switzerland	100 centimes (*c.* or *cts.*)	= 1 franc (*fr.* or *F.*) …	$9\frac{1}{2}d.$
Italy	100 centessimi	= 1 lira (*l.* or *£*).	$9\frac{1}{2}d.$
Greece	100 lepta	= 1 drachma…	$9\frac{1}{2}d.$
Spain	100 centimos	= 1 peseta (*pes.*).	$9\frac{1}{2}d.$
Germany	100 pfennige (*pf*)	= 1 mark (*M.*) …	$11\frac{3}{4}d.$
Denmark Sweden Norway	100 ore	= 1 krona …	13*d.*

* *i e* those coins in terms of which a sum of money is expressed, or in which accounts are kept In our non-decimal system we use three principal coins, namely, £, *s* *d*

† The values in English money will give the student an idea of the purchasing power of the coins (regarded as *tokens*) in their own country. But, as will be seen in Chap LX., their *exchange* values fluctuate.

Country	Principal Coins		Approximate Value in English money
Holland	100 cents (c)	= 1 florin (fl) or guilder	20d
Austria	100 kreuzer (kr)	= 1 florin (fl) or gulden (g)	20d.
Russia	100 kopeks (kop)	= 1 rouble (R)	37½d
Turkey	100 piastres	= 1 Turkish pound (£T)	18s 0¾d
United States	100 cents(c or cts)	= 1 dollar ($)	50d
Mexico and South America*	100 centavos	= 1 peso, or Mexican dollar	48d
Japan	100 sen	= 1 yen	48d
Portugal	1000 reis	= 1 milreis (mlr)	53d
Brazil	1000 reis	= 1 milreis	27d
China	1000 cash	= 1 tael	6s 6d

Notes—France, Belgium, Switzerland, Italy, and Greece form what is known as the Latin Union, their coins are all alike in weight and fineness, differing only in name, and they circulate in any country of the Union

The *cent* of the United States is very different from the *centime* of the Latin Union, 1 *cent* = about 5 *centimes*

For most practical purposes, results *correct to the nearest* 5 *centimes* of the Latin Union are sufficiently accurate

Besides the English, the Indian is the only important *non-decimal* coinage

INDIAN MONEY TABLE

4 pice = 1 anna, 16 annas = 1 rupee.

Notes—The *nominal* value of a rupee is 1s 10½d, but its *exchange value* is now much less

The sum of 100000 rupees (Rs) is called a lac

In all British colonies English coins are current

Also in the Straits Settlements, Hong-kong, &c, a British *dollar*, equal in value to the Japanese *yen*, circulates.

All the above countries (except the United States) adopt in their currency the principle of monometallism, *i.e* *one* metal only (usually gold) has a fixed standard value

The United States still retains *bimetallism* (formerly in vogue in England and France), *i. e.* both silver and gold are *standards*

Note.—In a country (*e.g.* India) which has no gold coinage, the standard of value is *silver*.

* Except Brazil.

EXAMPLE i.—*Add* 89 *fr.* 78 *c*; 17 *fr.* 5 *c*; 113 *fr.* 50 *c.* and 8 *fr* 24 *c* *

$$\begin{array}{r} fr \\ 89\cdot78 \\ 17\cdot05 \\ 113\cdot50 \\ 8\cdot24 \\ \hline 228\cdot57 \end{array}$$

Ans. 228 *fr* 57 *c.*,
or approximately to the nearest 5 cents,
228 *fr* 55 *c.*

EXAMPLE ii.—*Multiply* 7 *M.* 20 *pf. by* 24.

$$\begin{array}{r} M. \\ 7\cdot20 \\ 24 \\ \hline 28\ 80 \\ 144\ 0 \\ \hline 172\cdot80 \end{array}$$

Ans. 172 *M.* 80 *pf.*

EXAMPLE iii.—*Divide* 637 *fr.* 56 *c by* 45.

$$\begin{array}{r} 5\,\underline{)\,637\cdot56} \\ 9\,\underline{)\,127\cdot512} \\ 14\cdot168 \end{array}$$

Ans 14 *fr* 16·8 *c.*
or, to the nearest centime, 14 *fr.* 17 *c.*
or, to the nearest 5 centimes, 14 *fr.* 15 *c.*

EXAMPLE iv.—*Taking a mark as equivalent to* 11¾*d., express* 427 *M.* 75 *pf. in* £, *s. d. to the nearest penny.*

This may conveniently be done by Practice.

From the value at 1*s* we obtain the value at 1*d*, and from that the value at ¼*d*. We then *cancel* the 1*d*. line and subtract.

$$\begin{array}{l|rl} & s & \\ 1d = \frac{1}{12} & 427\cdot75 & = \text{value at } 1s. \text{ per mark.} \\ \frac{1}{4}d = \frac{1}{4} & \cancel{35\cdot64} & = \ldots\ \cancel{1d.} \ldots \\ & 8\cdot91 & = \ldots\ \frac{1}{4}d \ldots \\ \hline & 418\cdot84 & = \ldots\ 11\frac{3}{4}d. \ldots \end{array}$$

Ans. £20, 18*s.* 10*d.*

EXAMPLE v.—*Express* £326, 13*s.* 7*d in dollars and cents, supposing a dollar to be equivalent to* 4*s.* 2*d.*

As 1 dollar = 50*d*, ∴ 1 cent = ½*d*

If, then, we reduce the English money to *half-pence* we obtain the equivalent number of *cents*

$$\begin{array}{r} \pounds\quad s.\quad d. \\ 326\ .\ 13\ .\ 7 \\ 20 \\ \hline 6533 \\ 12 \\ \hline 78403 \\ 2 \\ \hline 156806 = \$1568\cdot06 \end{array}$$

Ans $1568, 6 *cts*

* It is more usual abroad to *name the higher denomination only*, and instead of, *e.g.*, 8 *fr* 24 *c*, to write 8fr, 24, or *F* 8, 24, or 8 24 *fr*

LIX. THE METRIC SYSTEM.

The **Metric System** is the *decimal* system of Weights and Measures devised by the French at the time of the great Revolution. It is now in general use in all the chief countries of Europe except Russia* and England, and there it has been adopted for *scientific* purposes.

The *standard unit of length* in this system is called the **metre** (Greek, *measure*). The standard units of area, volume, capacity, and weight are all derived from the metre, which is thus the fundamental unit of the system to which it gives its name.† All other units of the system are either **decimal** multiples of, or **decimal** parts of, their respective standard units. Those which are decimal **multiples** are denoted by the *Greek* prefixes

deca- (10), **hecto-** (100), **kilo-** (1000)‡;

those which are decimal **parts**, by the *Latin* prefixes

deci- ($\frac{1}{10}$), **centi-** ($\frac{1}{100}$), **milli-** ($\frac{1}{1000}$).

For instance, a deca-metre is 10 metres, a deci-metre is $\frac{1}{10}$ of a metre, and a kilo-metre is 1000 metres, a milli-metre is $\frac{1}{1000}$ of a metre.

N.B.—The *Latin* prefixes denoting **parts** of a standard unit, *all end in the letter* i.

The metric, being a *decimal* system, possesses the following advantages over a non-decimal system which have already been noticed in the case of decimal money.

"Reduction" involves *no labour*, for it consists simply in either multiplying or dividing by a *power of ten*.

For instance, 5 *kilometres* is 5000 *metres*, 45 *metres* is ·045 of a *kilometre*, 6·5 *decametres* is 650 *decimetres*, 1·5 *decimetres* is ·15 of a *metre*.

Also, 8 *kilometres* 50 *metres* is either 8·050 *kilometres*, or 8050 *metres*.

Consequently, it is unnecessary to write the name of more than one denomination in the measure of any quantity. Thus the necessity for anything corresponding to the English "Compound" Rules is avoided, and all operations are performed by simple addition, subtraction, &c., in decimals.

Note.—The *decimal point* is not always used in foreign countries, a comma, or a gap, denoting its *position*, thus, $6^{m}05$, or 6,05m., stands for 6·05 metres, *i.e.* for 6 metres 5 centimetres.

---

* Russia and the United States of America, though possessing a *decimal coinage*, have not adopted a decimal system of Weights and Measures. In the United States the English Weights and Measures are still retained.

† The inventors of the system chose as the fundamental unit a ten-millionth part of a quarter-meridian of the globe, the length they thus obtained is preserved in a metal rod—the standard metre—kept at Paris.

‡ The Greek prefix myria- (10,000) is also sometimes used.

Moreover, when once the meanings of the *names* of the various units are known, the "tables" of the metric system (unlike our complex English tables) impose *no* tax on the memory

Note.—Those parts of the following tables which are printed in thick type are specially useful.

LENGTH.

Standard unit, the **Metre.**

Multiples

1 **kilo**-metre (*Km*) = **1000** metres
1 *hecto*-metre (*Hm*) = 100 metres
1 *deca*-metre (*Dm*) = 10 metres

Hence 10 Dm. = 1 Hm., 10 Hm = 1 Km
Also, **1 metre** = ·1 Dm. = ·01 Hm. = **·001 Km.**

Parts.

1 *deci*-metre (*dm.*) = $\frac{1}{10}$, or ·1, of a metre.
1 **centi**-metre (*cm.*) = $\frac{1}{100}$, or **·01**, of a metre.
1 *milli*-metre (*mm.*) = $\frac{1}{1000}$, or ·001, of a metre

Hence 10 mm. = 1 cm, 10 cm = 1 dm
Also, **1 metre** = 10 dm = **100 cm** = 1000 mm

Note—The kilometre, metre, and centimetre are the units of length commonly employed; the others are seldom used

For instance, the length **3675 metres** *might* be written and read as "3 *kilo*metres, 6 *hecto*metres, 7 *deca*metres, 5 metres"; but it is much more usual and convenient to write it thus, 3·675 *Km.*, and to read it thus, "3 *kilometres*, 675 *metres*".

EXAMPLE i—*Express* 47·6 *metres* (i) *in kilometres*; (ii) *in centimetres*

(i) [As 1000 metres = 1 kilometre, in order to express metres in kilometres, we have but to divide by 1000, i e *to move the decimal point* three *places to the* left. Hence—]

47·6 metres = ·0476 kilometres *Ans* (i)

(ii) [As 1 metre = 100 centimetres, in order to express metres in centimetres, we have but to multiply by 100, i e *to move the decimal point* two *places to the* right Hence—]

47·6 metres = 4760 centimetres *Ans* (ii)

EXAMPLE ii —*Find, in metres, the sum of* ·04 *Km*, 86·5 *Dm*, 3·4 *m*., 3·4765 *Km*, 213 *mm*

[We first express all the quantities in the same denomination, *metres*, and then add the resulting decimals.]

Ans 4385 113 *m*

$$\begin{array}{r} \text{m} \\ 40\cdot \\ 865\cdot \\ 3\cdot4 \\ 3476\cdot5 \\ 213 \\ \hline 4385\cdot113 \end{array}$$

EXAMPLE iii —*Multiply* 2 *Km* 85 *m by* 34

[We first express the given quantity in the single denomination *kilometres*, and then multiply the resulting decimal by 34]

Ans 70·89 *Km*, *or* 70 *Km* 890 *m*

$$\begin{array}{r} \text{Km} \\ 2\cdot085 \\ 34 \\ \hline 8\ 340 \\ 62\ 55 \\ \hline 70\cdot890 \end{array}$$

EXAMPLE iv.—*Divide* 217 *m* 50 *cm by* 23

[We first express the given quantity in the single denomination *metres*, and then divide the resulting decimal by 23, until the quotient obtained is sufficiently accurate for our purpose

(i) If, in this case, we carry the division to *two* places of decimals, the result is 9 *metres* 45 *centimetres*, with the remainder 15 *centimetres*

(ii) If the next figure in the quotient be obtained we have the more accurate result, 9 *metres* 45·6 *centimetres*, with the remainder 12 *millimetres*, *i e* 1·2 *centimetre*]

$$\begin{array}{rrr} & \text{m} & \text{m} \\ & 23)217\cdot50 & (9\ 45 \\ & 207 & \\ \hline & 10\ 5 & \text{m} \\ & 9\ 2 & 0(9\ 456 \\ \hline & 1\ 30 & \\ & 1\ 15 & \\ \hline \text{(i)} & ..\ 15 & 150 \\ & & 138 \\ \hline \text{(ii)}. & .. & 12 \end{array}$$

Correct to the nearest centimetre, the result is 9 *metres* 46 *centimetres*

EXAMPLE v —*Find the cost of* 16 *metres* 35 *centimetres of silk at* 5 *francs* 60 *centimes per metre*

[16 metres 25 centimetres = 16·25 *metres*, and 5 francs 60 centimes = 5·6 *francs* Hence]

$$\begin{aligned} \text{Required cost} &= \overset{\text{francs}}{5\cdot6} \times 16\cdot25 \\ &= 91\cdot56 \text{ francs} \\ &= 91 \text{ francs } 56 \text{ centimes } Ans \end{aligned}$$

$$\begin{array}{r} 16\cdot35 \\ 5\ 6 \\ \hline 98\ 10 \\ 817\ 5 \\ \hline 91\ 560 \end{array}$$

EXAMPLE vi —*If* 36 *metres* 75 *centimetres of cloth cost* 154 *francs* 35 *centimes, find, to the nearest* 5 *centimes, the value of* 21 *metres* 80 *centimetres of the cloth*

$$\begin{array}{llll} \text{metres.} & & \text{francs} & \\ 36\cdot75 & \text{cost} & 154\cdot35 & \\ \therefore 21\cdot8 & \ldots & 154\cdot35 \times \dfrac{21\cdot8^*}{36\ 75} = \overset{\text{fr}}{91\cdot56} & = 91\ fr\ 55\ c\ Ans. \end{array}$$

* The working, having already been exemplified under Decimals, is omitted here.

AREA

Standard unit, the **Square Metre**,

(*i.e.* an amount of surface equal to the area of a square each of whose sides is one metre long)

We have seen (page 30) that a table of *Square* measure is obtained by *squaring Long* measure.

Hence, as 1 metre = 100 centimetres,

∴ **1 square metre** (*sq m.*) = **10000 sq. centimetres** (*sq. cm.*)

And, as 1 decametre = 10 metres

∴ **1 square decametre** (*sq. Dm.*) = **100 square metres**

And so on.

In measuring land* the *square decametre* (*i e* 100 square metres) is taken as the principal unit, this is called an **Are**, and the Greek and Latin prefixes are applied to this unit

Thus 1 **hect**-are (*Ha.*) = 100 ares (*a.*).
1 dec-are (*Da.*) = 10 ares.
1 deci-are (*da.*) = $\frac{1}{10}$ of an are.
1 **centi**-are (*ca.*) = $\frac{1}{100}$ of an are (= 1 *sq metre*).

Note—The student must be careful here to avoid confusion:—
e g the decametre is a measure of *length*,
∴ a square *decametre* is not *ten* (but 100) square *metres*.
But the are is a measure of *surface*, ∴ a *decare* is ten *ares*.

VOLUME.

Cubic Measure

Standard unit, the **Cubic Metre**,

(*i.e.* the volume of a cube whose edge is one metre in length,

A table of *Cubic* measure is obtained by *cubing Long* measure.

Hence, as 1 metre = 100 centimetres,

∴ **1 cubic metre** (*c m.*) = **1000000 cubic centimetres** (*c.cm.*)

And, as 1 decimetre = $\frac{1}{10}$ of a metre,

∴ **1 cubic decimetre** (*c dm.*) = $\frac{1}{1000}$ **of a cubic metre.**

And so on

The cubic metre is sometimes called a **stere**, to which, of course, the Latin and Greek prefixes may be applied.

Thus 1 deca-stere = 10 steres; 1 deci-stere = $\frac{1}{10}$ of a stere

* In the case of a *very large* portion, the area would be expressed in square kilometres.

*Capacity.**

The principal unit of capacity is the *cubic decimetre*, which is called a **Litre**

1 **hectolitre** (*Hl*) = **100** litres
1 decalitre (*Dl*) = 10 litres
1 decilitre (*dl*) = $\frac{1}{10}$ of a litre.
1 centilitre (*cl*) = $\frac{1}{100}$ of a litre

N B—As a **litre** is a *cubic decimetre, i e* $\frac{1}{1000}$ of a cubic metre,

$\therefore$ **1000 litres = 1 cubic metre** (I)

WEIGHT

Standard unit, the **Gramme**

The gramme, like all the other standards of the Metric System, is derived from the fundamental unit of length, the *metre*

A **gramme** is the weight of a *cubic centimetre* of pure water when at its greatest density †

1 **kilogramme** (*Kg*) = **1000** grammes
1 hectogramme (*Hg*) = 100 grammes
&c

As the gramme is a very small unit, the *kilogramme* is the principal unit employed for general purposes (such as the English pound Av is used for) For large bulks the following units are employed — the **quintal** = 100 *kilogrammes*
and the **tonneau** (*T*) = 1000 *kilogrammes.*

N.B—As 1 decimetre = 10 centimetres,

$\therefore$ 1 *litre, i e* 1 *cubic* decimetre, = **1000** *cubic* centimetres

Hence, **a litre of water** weighs **1000 grammes** (II)

Note—It is unnecessary to give many easy examples in Area, Volume, and Weight, as they are worked in *exactly* the same way as those given under Length.

EXAMPLE vii.—*Find the cost of a plot of building land containing 2 hectares 17 ares, at 3 francs 50 centimes per square metre*

3 hectares 17 ares = 217 ares
= 21700 sq metres

$\therefore$ req^{d} cost = $\overset{fr}{3\cdot5} \times 21700$ = 75950 francs *Ans.*

---

* Used for *Liquids, &c.*, as stated on page 31

† *i e* weighed under certain conditions as regards both atmospheric pressure and temperature

EXAMPLE viii.—*Find the greatest* (i) *quantity*, (ii) *weight, of water which could be contained in a rectangular reservoir 23 metres 20 centimetres long, 16 metres 5 centimetres wide, and 2 metres 25 centimetres deep.*

(i) Volume of water $= 23 \cdot 2 \times 16 \cdot 05 \times 2 \cdot 25$, *cubic metres*
$= 23 \cdot 2 \times 16 \cdot 05 \times 2 \cdot 25 \times 1000$ *litres**
$= 232 \times 1605 \times 2 \cdot 25$ *litres*
$= 837810$ *litres*
$= \underline{8378 \cdot 1}$ *hectolitres Ans.* (i)

(ii) Hence weight of water $= 837810 \times 1000$ *grammes*†
$= 837810$ *kilogrammes*
$= \underline{837 \cdot 81}$ *tonneaux Ans* (ii)

CONVERSION OF METRIC INTO ENGLISH MEASURES, AND VICE VERSÂ.

Correct to five places of decimals

1 metre = 39·37079 inches.

And 1 English gallon contains 277·274 cubic inches.

By help of these two equivalents we can, approximately, by the methods of Chapter L, convert any given Metric, into the corresponding English, measure, or *vice versâ*

N B—Since 1 *metre* = 39·37079 *inches*,
$\therefore$ 1 square *metre* = $(39 \cdot 37079)^2$ *square inches*,
and 1 cubic *metre* = $(39 \cdot 37079)^3$ *cubic inches*

Note—The following *rough* approximations should be borne in mind. Though not accurate enough for use in actual calculations, they afford the means of making a rough rapid, or even mental, test of results calculated from more accurate data. In this way any great error in the result of a long calculation (such as would be caused by the misplacing of a decimal point) may at once be detected

A metre is about	39 inches,	a litre is about	$1\frac{3}{4}$ pints‡
a kilometre	5 furlongs,	a *hectolitre*	22 *gallons*
a centimetre .	$\frac{3}{8}$ of an inch	a kilogramme	$2\frac{1}{5}$ lbs Av,
a *hectare*	$2\frac{1}{2}$ *acres*,	a *gramme*	15 *grains*,
an *are*	4 *poles*	a *quintal*	2 *cwts*

* See (I), on page 232 † See (II), on page 232. ‡ i e a "reputed quart"

EXAMPLE IX.—*Express 5 metres 24 centimetres in feet and inches correct to the nearest eighth of an inch*

$$\begin{aligned} \text{5 metres 24 centimetres} &= 5{\cdot}24 \textit{ metres} \\ &= 5{\cdot}24 \times 39{\cdot}37079 \textit{ inches} \\ &= 206{\cdot}3 \textit{ inches} \\ &= \underline{17 \text{ ft } 2\tfrac{1}{4} \text{ in } \textit{Ans}} \end{aligned}$$

$$\begin{array}{r} 39{\cdot}37079 \\ \underline{4\;25} \\ 196\;86 \\ 7\;87 \\ \underline{1\;57} \\ 206{\cdot}30 \end{array}$$

EXAMPLE X.—*Express 17 miles 250 yards in kilometres correct to the nearest metre*

$$\begin{aligned} \text{17 mi. 250 yds} &= 30170 \textit{ yds} \\ &= 30170 \times 36 \textit{ in} \\ &= \frac{30170 \times 36}{39{\cdot}37079} \textit{ metres} \\ &= 27587 \textit{ metres} \\ &= \underline{27{\cdot}587 \text{ Km } \textit{Ans}} \end{aligned}$$

$$\begin{array}{r} 30170 \\ \underline{36} \\ 181020 \\ 90510 \end{array}$$

$$\begin{array}{rl} & \text{metres} \\ 39{\cdot}37079)\overline{1086120}(& 27586{\cdot}9 \\ \underline{787416} & \\ 298704 & \\ \underline{275595} & \\ 23109 & \\ \underline{19685} & \\ 3424 & \\ \underline{3149} & \\ 275 & \\ \underline{236} & \\ 39 & \end{array}$$

Note.—Remembering that a *Km* is about 5 fur., and applying this as a rough test, we see that our result is a reasonable one.

EXAMPLE XI.—*Express $2\frac{1}{2}$ hectolitres in gallons and pints correct to the nearest pint*

$$\begin{aligned} 2{\cdot}5 \; Hl = 250 \textit{ litres} &= \frac{250}{1000} \textit{ cubic metre} \quad \text{[See (I), page 232.]} \\ &= \tfrac{1}{4} \textit{ cubic metre} \\ &= \tfrac{1}{4} \times (39{\cdot}37079)^3 \textit{ cubic inches} \\ &= \tfrac{1}{4} \times \frac{(39{\cdot}37079)^3}{277{\cdot}274} \textit{ gallons}^* \\ &= \underline{55 \text{ gallons } \textit{Ans}} \end{aligned}$$

*We calculate the value of this expression to *one* place of decimals in the final quotient. See Chap L for the method.

EXAMPLE XII.—*Express 1 acre 3 ro. 15 po. in ares, correct to the nearest square metre.*

$$\begin{aligned} \text{1 ac. 3 ro. 15 po.} &= 8923{\cdot}75 \textit{ sq. yds} \\ &= 8923{\cdot}75 \times 9 \times 144 \textit{ sq. inches} \\ &= \frac{8923{\cdot}75 \times 9 \times 144}{(39{\cdot}37079)^2} \textit{ sq. metres}.\dagger \\ &= 7461 \textit{ sq. metres} \\ &= \underline{74{\cdot}61 \textit{ ares Ans}} \end{aligned}$$

† We calculate this, to the nearest integer, by the methods of Chap. L. See Ex. xxi, p. 184.

Note.—For further examples of this class see Chapter LXI.

LX. FOREIGN EXCHANGES.

A debt due to a person in a foreign country might be paid in any one of the following three ways—

(1) by transmitting gold coins (specie);
(2) by transmitting gold bars (bullion),
(3) by means of a Bill of Exchange (see Chap LVI).

If either the first or second method be chosen it is necessary (in order to determine the amount of gold which must be sent to discharge any particular debt) to know the comparative value of a gold* coin of each of the two countries, *i e* the ratio of the amounts of pure gold* in them. This is called the Par of *Exchange*

For instance, there is as much gold in 1000 *sovereigns* as there is in 2522 *ten-franc* pieces, and so the *par of exchange* between England and France is £1 = 25·22 *francs*

The trouble and expense, however, of transmitting specie or bullion is considerable, and so these methods are very seldom adopted; the third is the usual method.

Now the *cost*, in the coin of one country, of a Bill of Exchange, payable in the coin of another country, is *not* determined from the *Par* of Exchange. Foreign Bills, like other marketable things, fluctuate in price, their market rate rising or falling with any increase or decrease in the demand for them This market rate is called the Course of *Exchange*, or the Rate of *Exchange*, and the cost of a Foreign Bill depends upon the *course of exchange at the time* it is bought

EXAMPLE i.—*Exchange £472, 10s. into francs at 25·16 francs for £1.*

$$£472 \cdot 5 = \overset{\text{francs}}{25 \cdot 16} \times 472 \cdot 5 = 11888 \cdot 10 \text{ francs. } \textit{Ans.}$$

EXAMPLE ii.—*Exchange £320 into rupees when the rate of exchange is 1s. $2\frac{3}{8}$d. for 1 rupee.*

As $\overset{d}{14\frac{3}{8}}$ is equivalent to $\overset{\text{rupee}}{1}$

$\therefore 320 \times 240$ .. ... $\dfrac{320 \times 240}{14\frac{3}{8}}$ rupees

$= 5342\frac{14}{23}$ rupees

$=$ *Rs* 5342, 10 *annas*, nearly *Ans.*

---

* *Silver*, in the case of two countries having a *silver standard*

EXAMPLE III.—*Exchange 5380 marks into sterling at $20 \cdot 34\frac{1}{2}$m. for £1*

$$\begin{array}{lll} & \text{marks} & \text{£} \\ & 20 \cdot 345 \text{ are equivalent to} & 1 \\ \therefore 5380 & \ldots\ \ldots & 1 \times \dfrac{5380}{20 \cdot 345} = \text{£}264 \cdot 438\ldots \\ & & = \underline{\text{£}264,\ 8s\ 9d\ \ Ans} \end{array}$$

EXAMPLE IV —*Exchange 3560 tael into sterling at 2s 11d for 1 tael*

$$\begin{array}{lll} & \text{tael} & d \\ & 1 \text{ is equivalent to} & 35 \\ \therefore 3560 & \ldots\ \ldots & 3560 \times 35 = \underline{\text{£}519,\ 3s\ 4d\ \ Ans} \end{array}$$

Note—The student will notice a remarkable difference between the *exchange* value and the *nominal* value, as given on page 225, of the coin of a country having a *silver* standard This is mainly due to the depreciation of late years in the value of silver For instance, the *exchange* value of the rupee is, owing to this cause, far below its *nominal* value

Courses of exchange are published twice a week in the London daily newspapers, namely the rates then ruling "on 'Change" *in London* with respect to the money of various foreign countries, and also the rates *telegraphed from various foreign centres* as then ruling in them with respect to English money These "quotations" are usually given in one of two ways, in the case of some countries the value of £1 *in the foreign coinage* is given, in the case of others, the value of *the principal foreign coin* is given in pence, or in shillings and pence

For instance, "Paris, 25 $19\frac{1}{2}$" means 25 *francs* $19\frac{1}{2}$ *centimes* for £1.
"Vienna, 11·97" 11 *gulden* 97 *kreuzer* £1.
"Lisbon, $40\frac{1}{2}$" $40\frac{1}{2}$ *pence* 1 milreis.

Note—In the cases of St Petersburg, Amsterdam, and New York, two ways are used Quotations *telegraphed from* St Petersburg appear in the form "93·70", which means 93 *roubles* 70 *kopeks* for £10, but the *London* rate appears in the form "$25\frac{1}{4}$", which means $25\frac{1}{4}d$ for 1 rouble

The rate *telegraphed from* Amsterdam appears in the form "12 09", which means 12 *florins* 9 cents for £1, but the *London* rate appears in the form "12 $2\frac{3}{4}$", which means 12 *florins* $2\frac{3}{4}$ stivers* for £1

The rate *telegraphed from* New York appears in the form "4·86", which means 4 *dollars* 86 *cents* for £1, whereas the *London* rate takes the form "$48\frac{3}{8}$", i e $48\frac{3}{8}d$ for \$1

In such "quotations" the coin represented by unity is called the "fixed price", and its value in other coins the "variable price"

Note.—The extent to which the course of exchange between two countries fluctuates above or below their par of exchange is limited by the cost of transmitting *gold*, for no one would care to buy *bills* with which to defray a debt, if he could *at less cost transmit gold*, these limits are called specie points

* Here $2\frac{3}{4}$ is *not* a decimal of a florin, 1 *florin*=20 stivers.

EXAMPLE v.—*Exchange £218, 12s 6d. into marks at 20·38.*

$$\begin{array}{lll} & \text{£} & & \text{marks} \\ \text{As} & 1 & \text{is equivalent to} & 20\cdot38 \\ \therefore & 218\cdot625 & \ldots\ldots\ldots & 20\cdot38 \times 218\cdot625 \\ & & & = \underline{4455\cdot58\ M\ \ Ans} \end{array}$$

EXAMPLE vi.—*Exchange £87, 13s. 5d. into Dutch money at 12 1¾*

$$\begin{array}{ll} & \text{£} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{florins} \\ \text{As } 1 = 12\,\textit{florins}\ 1\tfrac{3}{4}\ \textit{stivers} = 12\tfrac{7}{80}\ \textit{florins} = & 12\cdot0875 \\ \therefore 87\cdot652 \quad \ldots\ldots \qquad \ldots\ldots \qquad \ldots = & 12\ 0875 \times 87\cdot652 \\ = & \underline{1059\cdot49 \text{ florins. } Ans} \end{array}$$

When the Course of Exchange quotations are for cheques, or bills payable *at sight* (*i.e.* for bills payable without delay) they are called **short** rates, when the quotations are for bills the payment of which is deferred, they are called **long** rates

For instance, "Paris, cheques, 25·16", "Vienna, sight, 12·97"; "Hong-Kong, T.T *, 2s. 1⅝d ", are specimen quotations of *short* rates.

"St Petersburg, 3 mo, 93·70"; "New York, 60 days, 48⅝", are specimen quotations of *long* rates.

The difference at any particular date between a long, and the corresponding short, rate is the *discount* (at the current rate per cent of discount) for the time named in the long rate.

N B.—In calculating this discount in order to find the short rate corresponding to a given long rate, or vice versâ, *results of sufficient accuracy are obtained by simply taking, to 3 places of decimals,* **the interest on the variable price** *mentioned, and either* **adding** *this interest to, or* **subtracting** *it from, the variable price, according as the case requires a greater or less variable price*

The fluctuations in the Course of Exchange between two countries are due to several causes, the chief of which is called the *Balance of Trade.*

Thus, if the exports from England to France were equal in value to those from France to England, accounts between the two countries could be settled *without transmitting any gold,* and the **short** Rate of Exchange would coincide with the Par of Exchange. But when the exports from France to England exceed in value those from England to France, and the *balance of trade* is in favour of France, the demand in London for bills, by which to pay for French goods, increases and their price rises.

Now if this were the sole cause the rate in London on Paris, and that in Paris on London would be the same on any particular date But this is seldom the case as there are persons who speculate in Bills of Exchange, thus increasing the demand for them sometimes in the one place more than the other Also news affecting the demand for Bills may reach the one place some hours before the other.

---

* Telegraphic transfers

EXAMPLE vii —*Find the cost in London of a bill for* 6550 *francs payable in Paris when the Course of Exchange, London on Paris, is* 25·21

Here the expression "London on Paris, 25 21" gives the rate which rules on 'Change in London, and means that £1 cash paid in London will buy a bill for 25·21 francs payable at Paris

Hence, as a bill for 25·21 francs payable in Paris costs £1 cash in London

∴ .... 6550 $\frac{6550}{25\ 21}$

$= £259·817$

$= £259,\ 16s\ 4d$ *Ans*

EXAMPLE viii —*Find the "sight" quotations corresponding to the following "long" rates,* (i) *London on Paris,* 3 *mo,* 25·31, (ii) *Berlin on London,* 8 *days,* 25·35, *the rate of discount being* 4%

(i) "London on Paris, 3 mo, 25·31", means that

£1 cash paid in London buys a bill for 25·31 *francs payable in* 3 *mo at Paris*

Now the interest on 25·31 francs for 3 mo at 4% is

$$25·31\text{ fr} \times \tfrac{1}{4} \times \tfrac{4}{100} = ·2531\text{ fr}$$

And as £1 will buy fewer francs *payable at sight than in* 3 *mo* we subtract ·2531 fr. from 25·31 fr, obtaining as the short rate required 25·06 fr nearly

Ans (i) 25 06

(ii) "Berlin on London, 8 days, 25·35", means that

25·35 *marks cash at Berlin buys a bill for* £1 *payable in* 8 *days at London.*

Now the interest on 25·35 marks for 8 days at 4% is ·02 marks nearly.

And as more marks must be paid for a bill for £1 payable *at sight* than for a bill for £1 payable *in* 8 *days*, we add ·02 marks to 25·35 marks, obtaining 25 37 marks as the required "short" rate

Ans (ii) 25 37

In actual transactions in bills the debtor has to pay *brokerage* and the *cost of the government stamp*

In England the former is usually $\frac{1}{10}$ % (*i e.* 1 per *mille**), and the latter 1s % (*i e.* $\frac{1}{2}$ per *mille*)

*1 per *mille*, or 1 *p m*, means 1 per *thousand*

EXAMPLE ix —*Find the cost (including the usual brokerage and stamp) in London of a bill for 845 roubles payable at sight at St. Petersburg, when the Course of Exchange is—London on St Petersburg, 3 mo, 93·70, and the Bank rate of discount is 4 per cent.*

[As £10 cash in London will buy a bill for *fewer* roubles payable *at sight* than in 3 *months*, we must *subtract* the discount]

Now the discount (*i e* the int on 93·70 *R* for 3 mo at 4%) = ·937 *R* nearly, and 93·70 — ·937 = 92·76 *R* nearly.

Hence

a *bill* for 92·76 roubles payable *at sight* at St. P costs £10 *cash* in London

∴ ... 845 ... ... $\dfrac{10 \times 845}{92{\cdot}76}$ ...

= 91·095.

But the brokerage at 1 *p m.* = £91·095 ÷ 1000 = ·091

And the stamp at $\frac{1}{2}$ *p m* = $\frac{1}{2}$ of £·091 = ·045.

∴, by addition, the total cost = 91·231..

= £91, 4*s.* 7*d* *Ans.*

There are various ways in which the payment of a debt by means of a foreign bill of exchange may be effected, and owing to the fact that the Course of Exchange between two commercial centres is not generally the same at both at the same date* (*e.g* London on Paris might be 25·21 when Paris on London is 25·22), the question arises how the debtor may most cheaply pay his foreign creditor; whether

(i) by *direct remittance*

For instance, a debtor A in London owing 1000 francs to a creditor B in Paris, may buy for cash *in London*, at the London rate, a bill for 1000 francs payable at Paris, and post it to B

Or (ii) by instructing his creditor to *draw* on him.

For instance, the debtor A in London may instruct his creditor B in Paris to *draw* a bill payable at London for A's acceptance. This bill B would "negotiate" (*i.e.* sell) in Paris.

Or (iii) by *indirect* exchange.

For instance, A in London, instead of treating directly with B in Paris, may discharge his debt through an agent C in some other place, say Hamburg, when the courses of exchange between London and Hamburg, and Hamburg and Paris, are such that the cost to A would be less this way than if he effected a direct exchange.

The rate of exchange which results in this third case (when one or more places intervene) between the two principal places is called an Arbitrated *Course of Exchange.*

* See page 237

EXAMPLE x —*A London merchant wishes to pay for 1000 florins' worth of goods, bought in Vienna, when the rates of Exchange are—London on Vienna, 3 mo, $12 \cdot 12\frac{1}{2}$, Vienna on London, sight, $11 \cdot 97\frac{1}{2}$, and the rate of discount in London is 3 per cent, will it be better for him to remit to Vienna, or for his creditor there to draw upon him in London?*

A bill for $12\ 12\frac{1}{2}$ *fl* payable *in 3 mo* at Vienna costs £1 cash in London

Now the int. on $12 \cdot 12\frac{1}{2}$ *fl* for 3 mo at 3% is $\cdot 09$ *fl* nearly,

and $12 \cdot 12\frac{1}{2} - \cdot 09 = 12\ 03\frac{1}{2}$,*

∴ a bill for $12 \cdot 03\frac{1}{2}$ payable *at sight* at Vienna costs £1 cash in London,

∴ .... 1000 .... $\dfrac{1000}{12 \cdot 03\frac{1}{2}}$ (i)

Again, $11 \cdot 97\frac{1}{2}$ *fl cash* is obtainable *in Vienna* in exchange for a *bill* for £1 payable *at sight in London*,

∴ 1000 *fl*. .. .. £$\dfrac{1000}{11\ 97\frac{1}{2}}$ (ii)

Now (i) is less than (ii), for its denominator is greater than that of (ii), while the numerators are equal

Hence, in this case, the merchant will pay his debt more cheaply by *remitting*.

EXAMPLE xi.—*Find the arbitrated rate of exchange at London on New York, viâ Paris and Berlin, when the direct rates are—*

London on Paris, $25 \cdot 26$ (francs for £1),
Paris on Berlin, $123 \cdot 54$ (francs for 100 marks),
Berlin on New York, $4 \cdot 22$ (marks for \$1).

[The course of exchange, London on New York, is expressed in *pence per dollar*, we therefore convert \$1 into *pence*, by the *Chain Rule* exemplified on p 146]

$$\begin{aligned} \text{Hence } \$1 &= 4 \cdot 22 \textit{ marks} \\ &= 4 \cdot 22 \times \frac{123\ 54}{100}, \textit{francs} \\ &= 4 \cdot 22 \times 1\ 2354, \textit{francs} \\ &= \frac{4 \cdot 22 \times 1 \cdot 2354}{25 \cdot 26}, \textit{pounds} \\ &= \frac{4 \cdot 22 \times 1 \cdot 2354}{25 \cdot 26} \times 240, \textit{pence} \\ &= \underline{49\tfrac{1}{2}d \text{ nearly } \textit{Ans}} \end{aligned}$$

---

* Subtracting, because £1 will buy fewer "*sight*" than "3 *mo*" florins.

In coins the precious metal is mixed with alloy (see page 25). The *degree of purity* of the mixture is called its **fineness**

For instance, British *standard gold* is "$\frac{11}{12}$ fine",* for it contains 22 parts by weight of *pure* gold mixed with 2 parts of *alloy*, and the ratio of 22 to 24 is expressed by $\frac{11}{12}$

EXAMPLE xii.—*Find the par of exchange between the United States and England, having given that the gold eagle (of* 10 *dollars),* $\frac{9}{10}$ *fine, weighs* 258 *grains, and that* 1869 *sovereigns are coined from* 480 *Troy ounces of standard gold* $\frac{11}{12}$ *fine*

[This is another example of the "Chain Rule"]

$$\begin{aligned}
£1 &= \frac{480}{1869} \text{ ozs. of } \textit{English standard} \text{ gold} \\
&= \frac{480}{1869} \times \frac{11}{12} \text{ ozs of } \textit{pure} \text{ gold} \\
&= \frac{480}{1869} \times \frac{11}{12} \times \frac{10}{9} \text{ ozs. of } \textit{U S standard} \text{ gold} \\
&= \frac{480}{1869} \times \frac{11}{12} \times \frac{10}{9} \times \frac{480}{258} \textit{ dollars} \\
&= \underline{\$4{\cdot}866 \text{ .. } \textit{Ans.}}
\end{aligned}$$

EXAMPLE xiii—*English standard (22 carat) gold is worth* £3, 17*s* 10½*d per Troy ounce,* 1395 *German reichsmarks,* 900 *(per mille) fine, are coined from* 1 *pfund (*500 *grammes) of pure gold,* 1 *Troy ounce is equivalent to* 31·1035 *grammes; hence obtain the mint par of exchange between England and Germany.*

£3, 17*s*. 10½*d* = £3·89375; and 22 *carat* gold is $\frac{11}{12}$ fine; hence, by the "Chain Rule",

$$\begin{aligned}
£1 &= \frac{1}{3{\cdot}89375} \text{ ozs. of } \textit{English standard} \text{ gold} \\
&= \frac{1}{3{\cdot}89375} \times \frac{11}{12}, \text{ ozs. of } \textit{pure} \text{ gold} \\
&= \frac{1}{3{\cdot}89375} \times \frac{11}{12} \times 31{\cdot}1035, \textit{ grammes} \text{ of } \textit{pure} \text{ gold} \\
&= \frac{1}{3{\cdot}89375} \times \frac{11}{12} \times 31{\cdot}1035 \times \frac{1395}{500}, \textit{ marks} \\
&= \underline{20{\cdot}43 \text{ .. } \textit{marks, Ans}}
\end{aligned}$$

---

* Or "22 *carats* fine". See note to page 25

LXI. THE METHOD OF NINE MULTIPLES.

The method consists in the formation of a table of the products of some constant number, or decimalized quantity, by each of the digits 1, 2, 9, and then using the table so as to obtain, *by addition alone,* the products of this constant by various other numbers

It is a labour-saving process capable of being applied to many practical purposes,—*e g* calculating the cost, or weight, &c, of various quantities at a fixed price, or weight, &c, per unit, changing metric into English weights and measures, and *vice versâ*, exchanges

For instance, suppose we frequently had occasion to obtain the product of the decimal ·24375 by various numbers Then we should make the following table consisting of 1, 2, 9 times this decimal, and preserve the table for future use

Table of multiples of ·24375

1	24375
2	·48750
3	·73125
4	·97500
5	1 21875
6	1 46250
7	1 70625
8	1 95000
9	2 19375

Now suppose we happen to require the product of this decimal by 385 7

From line 3 of the table (*by moving the decimal point* two *places to the* right) we obtain	300 times the decimal	= 73·125
From line 8 of the table (*by moving the decimal point* one *place to the* right) we obtain	80	= 19·500
From line 5 of the table we obtain	5	= 1·21875
From line 7 of the table (*by moving the decimal point* one *place to the* left) we obtain	·7	= ·170625
∴, by addition,	385 7	= 94 014375

Note —The table is easily formed, for line 4 is obtained by *doubling* line 2, line 7 by *adding* lines 3 and 4, &c,

EXAMPLE i.—*Make a table of multiples for use in calculating the cost of coal (tons and cwts.) at* 11*s.* 7*d. per ton, and use it in finding the cost of* (i) 453 *tons*, (ii) 72 *tons* 13 *cwts.*

11*s* 7*d* = £·579166 .

ton	£
1 .	·579166
2 .	1·158333
3	1·737500
4 .	2·316666
5 .	2·895833
6 . . .	3·475000
7	4·054166
8 .	4·633333
9 . ..	5·212500

[It is unnecessary to carry the table beyond 6 places of decs unless we require the cost of numbers of tons exceeding 1000.]

Now, from the table,

			£
(i)	400	tons cost	231·6666
	50	.. .	28·9583
	3	. .	1·7375
∴, by addition,	453	. .	262·362

= £262, 7*s* 3*d* *Ans.* (i)

[Since cwts are *twentieths* of a ton, cwts. are decimalized by multiplying their number by 5.]

(ii) 72 tons 13 cwts = 72·65 tons, and from the table—

			£
	70	tons cost	40·5417
	2	. . . ..	1·1583
	·6	. . .	·3475
	·05	. . . .	·0289
∴, by addition,	72·65	. .	42·076

= £42, 1*s.* 6*d* *Ans.* (ii).

EXAMPLE ii.—*Given that* 1 *metre* = 39·37079 *inches, make a table for use in converting feet into metres, and use it in the case of* (i) 1756 *ft*, (ii) 309 *ft* $7\frac{1}{2}$ *in*

ft	metres
1 =	·3047945
2 =	·6095890
3 =	·9143835
4 =	1·2191780
5 =	1·5239725
6 =	1·8287670
7 =	2 1335615
8 =	2·4383560
9 =	2·7431405

As 39·37079 in. = 1 metre,

$$\therefore 1 \text{ foot, or } 12 \text{ in.} = \frac{12}{39\cdot37079} \text{ metre}$$
$$= \mathbf{\cdot3047945} \text{ metre.}$$

Now, from the table,

$$\text{(i)} \quad 1756 \text{ ft} = \left\{ \begin{array}{r} \text{metres} \\ 304\cdot7945 \\ 213\cdot3562 \\ 15\cdot2397 \\ 1\cdot8288 \\ \hline 535\cdot219 \end{array} \right.$$

= 535 *m.* 22 *cm.* nearly *Ans.* (i).

$$\text{(ii)} \ 309 \text{ ft } 7\tfrac{1}{2} \text{ in} = 309\cdot625 \text{ ft} = \left\{ \begin{array}{r} \text{metres} \\ 91\cdot4383 \\ 2\ 7431 \\ \cdot1828 \\ \cdot0061 \\ \cdot0015 \\ \hline 94\cdot372 \end{array} \right.$$

= 94 *m.* 37 *cm.* nearly *Ans.* (ii).

LXII. SQUARE ROOT.

The **square root** of a given number is the number whose *square** is equal to the given number

For instance, 3 is the square *root* of 9, for 3^2, *i e* $3 \times 3, = 9$, and 7 is the square *root* of 49, for 7^2, *i e* $7 \times 7, = 49$

The square root of a number is indicated by the symbol $\sqrt{}$

For instance, $\sqrt{9}$ and $\sqrt{49}$ are read "*square root of* 9" and "*square root of* 49", or, shortly, "*root* 9", "*root* 49" †

The **squares** of the first twelve numbers are known from the multiplication table, namely,

$1^2 = 1,\ 2^2 = 4,\ 3^2 = 9,\ 4^2 = 16,\ 5^2 = 25,\ 6^2 = 36,\ 7^2 = 49,$
$8^2 = 64,\ 9^2 = 81,\ 10^2 = 100,\ 11^2 = 121,\ 12^2 = 144$

Hence, conversely, the following *square roots* are also known,

$\sqrt{1} = 1,\ \sqrt{4} = 2,\ \sqrt{9} = 3,\ \sqrt{16} = 4,\ \sqrt{25} = 5,\ \sqrt{36} = 6,\ \sqrt{49} = 7,$
$\sqrt{64} = 8,\ \sqrt{81} = 9,\ \sqrt{100} = 10,\ \sqrt{121} = 11,\ \sqrt{144} = 12$

Hence also $\sqrt{400} = 20,\ \sqrt{1600} = 40,\ \sqrt{2500} = 50$ &c,

and $\sqrt{10000} = 100,\ \sqrt{40000} = 200,\ \sqrt{160000} = 400,\ \sqrt{250000} = 500$, &c.

The following squares, with the corresponding square roots, should also be known — $13^2 = 169,\ 14^2 = 196,$

$15^2 = 225,\ 16^2 = 256,\ 17^2 = 289,\ 18^2 = 324,\ 19^2 = 361$

From these tables it is evident that comparatively few numbers have *integral* square roots.

For instance, as the square root of 16 is 4, and the square root of 25 is 5, the square root of any number, say 19, between 16 and 25 must be *greater than* 4, *and less than* 5, and so cannot be an *integer*

In fact, the square root of such a number (*e g* 19) cannot be *exactly* found, though, as we shall see later, we may find an approximation as close as we please

A number which has an *exact* square root is called a **perfect square.**

For instance, 81 is a *perfect square*, for its square root is 9, and $\frac{16}{25}$ is a *perfect square*, for, as $\frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$, its square root is $\frac{4}{5}$

When the factors of a **perfect square** are known, its square root is also known.

For instance, the square root of $3 \times 3 \times 7 \times 7 \times 2 \times 2$, *i e* of $(3 \times 7 \times 2)^2$, is evidently $3 \times 7 \times 2$, or 42.

---

* See p 12.

† The sign $\sqrt{}$ was originally the letter *r*, the initial of the word *radix*, Latin for "root"

EXAMPLE i—*Find, by factors, the square root of the perfect square* 156816.

We see that the given number is exactly divisible by 4, and that the quotient is also divisible by 4 Also that 9801 is divisible by 9, and the quotient by 9 (See Tests of Divisibility, p 52)

$$\begin{array}{r|l} 4 & 156816 \\ 4 & 39204 \\ 9 & 9801 \\ 9 & 1089 \\ 11 & 121 \\ & 11 \end{array}$$

Hence $156816 = 4 \times 4 \times 9 \times 9 \times 11 \times 11$.

$\therefore \sqrt{156816} = 4 \times 9 \times 11 = \underline{396}$ *Ans*

EXAMPLE ii.—*Find, by factors, the square root of* $\frac{5625}{5929}$.

$$\sqrt{\frac{5625}{5929}} = \sqrt{\frac{5 \times 5 \times 15 \times 15}{7 \times 7 \times 11 \times 11}} = \frac{5 \times 15}{7 \times 11} = \frac{75}{77} \text{ Ans}$$

Note—This method of finding square root is not of much practical use, for we cannot always say from inspection whether a number is a perfect square or not, or whether it has factors small enough to be easily found.

The following theorems are important.—

(I) *No perfect square can end in* 2, 3, 7, or 8

For the square root of a perfect square must end in one of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9.

Hence any *perfect square* must either end in 0, or in *the units' figure of the* square *of one of the other digits, i.e.* in 0, or in the units' figure of 1, 4, 9, 16, 25, 36, 49, 64, or 81.

Hence the right-hand figure of a perfect square *must* be either 0, 1, 4, 9, 6, or 5, and $\therefore$ *cannot* be either 2, 3, 7, or 8

Note—Hence, although we cannot, without trial, say whether such a number as 496325 is, or is not, a perfect square, we *can* at a glance tell that 47638 is *not* a perfect square.

(II) *The* square *of any given number has either double*, or *one less than double, the number of figures in the given number.*

For, since the square of 10 is 100,

$\therefore$ the square of any single digit has either *one* or *two* figures in it

And since the square of 100 is 10000,

$\therefore$ the square of any number between 10 and 100 has either *three* or *four* figures in it. And so on.

Hence the *number of figures* in the integral part of the square root of a number is found by counting the number of *couples of figures* in the number, beginning on the *right*, and including part of a couple, if such there be

For instance, the *number of figures* in the square root of 64'00'00 is *three*, and the number of figures in the square root of 9'00'00 is also *three*.

(III) *The square of the sum of two numbers is equal to the sum of their squares plus twice their product*

Take, for instance, the number 47 and regard it as the sum of 40 and 7.

$$\begin{aligned} \text{Then } (40+7)^2 &= 47 \times (40+7) \\ &= 47 \times 40 \qquad\qquad + 47 \times 7^{*} \\ &= (40+7) \times 40 \qquad + (40+7) \times 7 \\ &= 40 \times 40 + 7 \times 40 + 40 \times 7 + 7 \times 7^{*} \\ &= 40^2 \quad + \text{ twice } 40 \times 7 \quad + 7^2 \end{aligned}$$

N B.—Hence $47^2 - 40^2 = \textit{twice } 40 \times 7 + 7^2$
$= (\textit{twice } 40 + 7) \times 7$

Similarly, in the case of any other numbers—

e g $365^2 - 360^2 = (\textit{twice } 360 + 5) \times 5$.

$\therefore$ *if from a perfect square* (*e g* 47^2) *we subtract the square of part of its root* (*e.g* 40^2), *the remainder* (R)

= (*twice this part of the root + the remaining part*) $\times$ *the rem^g part*

Hence the following

General method of finding the Square Root of a number

For instance, *to find the square root of* 2209

Marking the number off into periods of two figures, beginning on the right-hand side, we see, by (II), that the required root is a number of *two* digits.

```
        22|09 ( 40 + 7
        1600
80 + 7 ) 609
          609
```

Also, we know (see page 244) that $\sqrt{1600} = 40$, and that $\sqrt{2500} = 50$,

$\therefore \sqrt{2209}$ must lie between 40 and 50,

$\therefore$ the *tens'* figure of the root is 4

Now, from 2209 we subtract 40^2, *i.e* 1600, and then we know [see (R) above] that the rem^r, 609, = (*twice* 40 + units' fig of root) $\times$ units' fig

We $\therefore$ double 40 and use this double, 80, as a *trial divisor* of 609

We thus obtain the units' figure of the root, 7, which we add to the 80, and then we multiply the completed divisor 87 by 7, obtaining the product 609.

Thus the square root of 2209 has been found to be 47

* See (II), page 10

Again, *to find the square root of* 133225

Marking off the given number into "periods", of which in this case there are *three*, we know, from (II), that the req^{d} root is a number of *three* digits

1st stage

As 133225 lies between 160000 and 90000,
i e between 400^2 and 300^2,
the hundreds', figure of the root is 3
Subtracting 300^2, i e 90000, from 133225,
we obtain the first remainder 43225

$$\begin{array}{r|r|r}
 & 13'22'25 & 300 \\
 & 90000 & 60 \\
\cline{2-2}
600 & 43225 & 5 \\
60 & & \\
\cline{1-1}
660 & 39600 & \\
\cline{2-2}
720 & 3625 & \\
5 & & \\
\cline{1-1}
725 & 3625 & \\
\cline{2-2}
\end{array}$$

2nd stage

We now *double* 300, and use the double, *i.e.* 600, as a *trial divisor* of 43225

Now, as 600 is contained more than 70 and less than 80 times in 43225, *we try* 7 as the tens' figure of the root But on multiplying 670 by 70 we obtain a product greater than 43225

Hence as 7 is too great for the second figure of the root, we are led to 6, the proper figure

So we place 60 in the quotient, also add 60 to 600; multiply the completed divisor 660 by 60, and then subtract the product from 43225, obtaining the second remainder 3625

3rd stage.

We now repeat the operations of the second stage

Doubling 360, we obtain 720, which we use as a trial divisor of 3625, whence we arrive at the units' figure of the root, 5, for 725 × 5 yields a product = 3625

Thus the square root of 133225 has been found to be 365

N B.—It follows, from (III) that the remainder at any stage, i e the difference between the given square and the square of the part of the root already obtained, is of the form (R), we therefore at each fresh stage form a new trial divisor by doubling *the part of the root already obtained;* and, having thence found the next figure of the root, *complete the divisor by the addition of this newly obtained part of the root.*

Note—In practice it is usual to omit the *place* ciphers (such as are omitted in ordinary Long division, see page 15), and to bring down the periods in succession, when the above examples appear thus —

$$\begin{array}{r|l}
 & 22'09\,(47 \\
 & 16 \\
\cline{2-2}
87) & 609 \\
 & 609 \\
\cline{2-2}
\end{array}
\qquad\qquad
\begin{array}{r|l}
 & 13'32'25\,(365 \\
 & 9 \\
\cline{2-2}
66) & 432 \\
 & 396 \\
\cline{2-2}
725) & 3625 \\
 & 3625 \\
\cline{2-2}
\end{array}$$

EXAMPLE III —*Find the square root of* 2220100

Mark off the number into periods, beginning on the right

1st stage

Place in the quotient the square root of the greatest perfect square which is less than the left-hand period, in this case 1

Subtract 1^2 from the left-hand period, and then bring down the next period, 22

2nd stage

Form a trial divisor by *doubling* the quotient, 1

```
       2'22'01'00'00 ( 1490
       1
  24 ) 1 22
         96
 289 ) 26 01
       26 01
       ------
             00

          Ans 1490
```

Now, remembering that the new figure of the root will be placed to the right of the 2 in the trial divisor, we say, or think, "Twenty-*something* into 122?" And if we try 6 or 5 as an answer to this question we find that 6 times 26, and 5 times 25, both yield too great a product, we therefore conclude that 4 is the second figure of the root

So we place 4 in both divisor and quotient,
multiply 24 by 4 and subtract the product 96 from 122
We then bring down the next period, 01

3rd stage

Form a new trial divisor by doubling the quotient, 14
Now, remembering that the new figure of the root will be placed to the right of 28 in the trial divisor, we say, or think, "Two hundred and eighty-*something* into 2601?" which leads us to 9 as the new figure of the root So we place 9 in both divisor and quotient and multiply 289 by 9, obtaining the product 2601

Thus 149 is the square root of 22201,
∴ 1490 ... 2220100

EXAMPLE IV —*Extract the square root of* 39790864

Here, when we reach the 3rd stage, we have as trial divisor one thousand two hundred and sixty-*something*, which is *not contained* in the remainder 1008, we therefore place 0 in *both divisor and quotient*, bring down the next period, and proceed as usual

```
          39'79'08'64 ( 6308
          36
   123 )  3 79
          3 69
 12608 )  10 08 64
          10 08 64
```

Hitherto we have applied the method to finding the square root of a *perfect square*, in which case the operation terminates, *i.e.* no final remainder occurs.

The process, however, can be applied to numbers *not* perfect squares, *i.e.* we can obtain either the integral part of the square root of *any* number, or its root correct to any number of places of decimals required.

EXAMPLE V.—*Find the perfect square nearest to* 65748.

Obtaining the integral part of the root of 65748, the remainder is 212

∴ the square of 256 *falls short* of 65748 by 212

Similarly, the square of 257 *exceeds* 65748 by 301.

∴ 65748 − 212, or

65536 is the reqᵈ number

$$\begin{array}{r|l} & 6'57'48\,(256 \\ & 4 \\ 45 & 2\,57 \\ & 2\,25 \\ 506 & 32\,48 \\ & 30\,36 \\ & 2\,12 \end{array} \qquad \begin{array}{r|l} & 6'57'48\,(257 \\ & 4 \\ 45 & 2\,57 \\ & 2\,25 \\ 507 & 32\,48 \\ & 35\,49 \end{array}$$

Square Root of a Decimal.

EXAMPLE VI.—*Find, correct to three places of decimals, the square root of* 401·7.

We mark off the *whole number* into periods as usual Then, having obtained the *integral* part of the reqᵈ root, 20, we insert the decimal point in the quotient, and proceed as before, appending as many periods of ciphers as we have occasion for

$$\begin{array}{r|l} & 4'01\cdot'70'00'00'00\,(20\cdot0424 \\ & 4 \\ 4004 & 1\,7000 \\ & 1\,6016 \\ 40082 & 98400 \\ & 80164 \\ 40084 & 1823600 \end{array}$$

Ans. 20·042.

Note —We continue the process until the figure in the *fourth* place in decimals is known, in order to see whether the required approximation is 20·042, or 20·043.

N.B.—In *marking the periods* in the case of a decimal we should always begin at the decimal point, proceeding to the *left* in the case of any *integral* part, and to the *right* for the *decimal*

For instance, in $\sqrt{\cdot15625}$ we *must* mark the periods thus:—·15'62'5

Square Root of a Vulgar Fraction

CASE I.—When the *denominator* is a *perfect square*, we find the square roots of numerator and denominator *separately*.

N.B—A *mixed number* must *first* be reduced to an *improper fraction*, in this case

EXAMPLE VII —*Find* (i) $\sqrt{\frac{1369}{5329}}$, *and* (ii) $\sqrt{1\frac{25}{144}}$.

(i)

$$\begin{array}{r|l} & 13'69\,(37 \\ & 9 \\ 67 & 4\,69 \\ & 4\,69 \end{array} \qquad \begin{array}{r|l} & 53'29\,(73 \\ & 49 \\ 143 & 4\,29 \\ & 4\,29 \end{array}$$

Ans (i) $\frac{37}{73}$.

(ii) $\sqrt{1\frac{25}{144}} = \sqrt{\frac{169}{144}}$

$= \frac{13}{12}$

$= 1\frac{1}{12}$ *Ans.* (ii).

EXAMPLE viii.—*Find* $\sqrt{\frac{726}{2166}}$.

[Here 2166 is *not* a perfect square, but as the fraction is not in its lowest terms, we try the experiment of reducing it, and thus obtain a denominator which *is* a perfect square]

$$\sqrt{\frac{726}{2166}} = \sqrt{\frac{121}{361}} = \underline{\frac{11}{19}\ \textit{Ans}}$$

CASE II.—When the *denominator* of the fraction is *not* a *perfect square*, we first reduce the fraction to a *decimal*, and then extract the square root of the result.

EXAMPLE ix —*Find* $\sqrt{\frac{4}{3}}$.

$$\sqrt{\tfrac{4}{3}} = \sqrt{1\dot{3}}$$

$$= \underline{1{\cdot}1547\quad \textit{Ans}}$$

$$\begin{array}{r l}
 & 1\ {}^{\prime}33^{\prime}33^{\prime}33^{\prime}33\ (1{\cdot}1547 \\
 & \underline{1} \\
21) & 33 \\
 & \underline{21} \\
225) & 1233 \\
 & \underline{1125} \\
2304) & 10833 \\
 & \underline{9216} \\
23087) & 161733
\end{array}$$

Note —Much labour would be wasted in such cases if we found the roots of num^r and den^r *separately*, as when this was done it would afterwards be necessary to divide the first result by the long decimal of the second result

Method of Contracting the work of finding an Approximate Square Root

(IV) *When any number of figures of a square root have been obtained by the ordinary process, one less than as many more figures may be obtained by* (contracted) division *

For instance, suppose we require the sq root of 2 to *six* places of decimals We obtain the first *four* figures of the root by the ordinary process *We then cancel the right-hand figure of the last trial divisor*, and obtain the next *three* figures of the root by *contracted division* (see Chap L), thus—

[Here the last figure is 3, or nearly 4

Thus the first seven figures of the root are 1·414213,

but the *nearest* approximation in six places of decimals is 1 414214]

$$\begin{array}{r l}
 & 2\ {}^{\prime}00^{\prime}00^{\prime}00\ (1{\cdot}414213 \\
 & \underline{1} \\
24) & 1\ 00 \\
 & \underline{96} \\
281) & 400 \\
 & \underline{281} \\
2\cancel{8}\cancel{2}\cancel{4}) & 11900 \\
 & \underline{11296} \\
 & 604 \\
 & \underline{565} \\
 & 39 \\
 & \underline{28} \\
 & 11
\end{array}$$

* For a proof of the truth of this theorem, see Appendix.

Note—The method of "Abridged Division" (see page 15), may be applied to square root

We give the above example worked in three other forms.—

(i) At full length, using ordinary division

(ii) the method of *abridged* division

(iii) Using the method of *abridged* division, and *contracting* the latter part.

(i)

```
          ( 1·414213
 2· 00 00 00 00 00 00
 1
24 ) 1 00
       96
281 ) 4 00
      2 81
2824 ) 1 19 00
       1 12 96
28282 ) 6 04 00
        5 65 64
282841 ) 38 36 00
         28 28 41
2828423 ) 10 07 59 00
           8 48 52 69
           1 59 06 31
```

(ii)

```
              (1·414213
  2·00 00 00 00 00 00
24 ) 1 00
281 ) 4 00
2824 ) 1 19 00
28282 ) 6 04 00
282841 ) 38 36 00
2828423 ) 10 07 59 00
            1 59 06 31
```

(iii)

```
           ( 1·414213
   2·00 00 00
24 ) 1 00
281 ) 4 00
28~~24~~ ) 1 19 00
             6 04
               39
               11
```

Note.—In the last example we obtained the square root of 2 to *six* places of decimals, and we see from (i) that the remainder is then only ·000001590631

Now we know from (III) that this rem^r $= 2 - (1·414213)^2$. Thus by proceeding far enough we can find a number whose *square* differs from 2 by as small an amount as we please. But *there must always be a rem^r*, for *there is no significant digit whose square ends in* 0

Such expressions as $\sqrt{2}$, $\sqrt{3·6}$, &c., where the roots cannot be *exactly* obtained, are called **surds**

In contradistinction to surds, numbers which are *not* surds are called *rational* numbers.

It follows from the definition of the sign $\sqrt{\ }$, that $\sqrt{2} \times \sqrt{2} = 2$.

Note.—$2\sqrt{3}$ means $2 \times \sqrt{3}$, or $\sqrt{3} \times 2$, and so on.

EXAMPLE x.—*Find the value of* $\frac{3}{2\sqrt{2}}$ *correct to* 3 *places of decimals.*

$$\frac{3}{2\sqrt{2}} = \frac{3 \times \sqrt{2}}{2\sqrt{2} \times \sqrt{2}}$$ [For the value of a fraction is not altered by multiplying both num^r and den^r by the same quantity See page 68]

$$= \frac{3\sqrt{2}}{2 \times 2} = \frac{3\sqrt{2}}{4} = \frac{3 \times 1·4142 ..}{4}$$ [From the example above]

$$= \frac{4·2426}{4} = 1·0606 = \underline{1·061\ Ans}$$

N B—Here the first step, which is called **rationalizing the denominator**, saves much labour, namely, the division of 3 by double the long decimal root of 2.

Such an expression as $(5 - \sqrt{3}) \times (5 + \sqrt{3})$ is *rational*,

for $(5 - \sqrt{3}) \times (5 + \sqrt{3}) = (5 - \sqrt{3}) \times 5 + (5 - \sqrt{3}) \times \sqrt{3}$*

$$= 25 - 5\sqrt{3} \quad + 5\sqrt{3} - 3$$
$$= 25 - 3 = 22$$

Hence, if such an expression as $5 - \sqrt{3}$† occurs in the denominator of a fraction whose value is required, we *rationalize the denominator* by *multiplying* both num^r and den^r by the *complementary factor* $5 + \sqrt{3}$†

EXAMPLE xi —*Find, the value of* $\frac{7}{3 + 2\sqrt{2}}$ *correct to three places of decimals.*

$$\frac{7}{3+2\sqrt{2}} = \frac{7 \times (3 - 2\sqrt{2})}{(3 + 2\sqrt{2}) \times (3 - 2\sqrt{2})} = \frac{7 \times (3 - 2\sqrt{2})}{9 - 8}$$
$$= \frac{7 \times (3 - 2 \times 1{\cdot}4142)}{1} = 7 \times \ 1715 = \underline{1{\cdot}201 \ Ans}$$

The *fourth* root of a given number is the number whose *fourth power* (see page 12) is equal to the given number

(The sign $\sqrt[4]{\ }$ is used to indicate the fourth root)

For instance, $\sqrt[4]{16} = 2$, for 2^4, *i e* $2 \times 2 \times 2 \times 2 = 4 \times 4 = 16$

Hence the fourth root of a number may be found by *extracting the square root of its square root*

The following are examples of the application of Square Root —

EXAMPLE xii —*A square field contains* 10 *acres, find the length of its side*

$$10 \text{ acres} = 10 \times 4840 \text{ square yards}$$

Hence, length of side $= \sqrt{10 \times 4840}$ yards $= \sqrt{100 \times 121 \times 4}$ yards
$$= 10 \times 11 \times 2 \text{ yards} = \underline{220 \text{ yards} \ \ Ans}$$

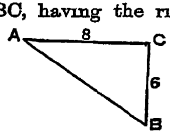
It is proved in Euclid, Book I, Proposition 47, that *in a right-angled triangle, the square on the side opposite to the right angle is equal to the sum of the squares on the other two sides*

Hence, if the lengths of any two sides of a right-angled triangle are known, the length of the remaining side can be calculated

For instance, if in the right-angled triangle ABC, having the right angle at C, AC is 8, and BC is 6, inches long, then the *square* of the length of AB $= 8^2 + 6^2$

$$= 64 + 36$$
$$= 100$$

$\therefore$ the length of AB $= \sqrt{100} = \underline{10 \text{ in}}$



---

* By (II), page 10, see also Appendix F (vii)

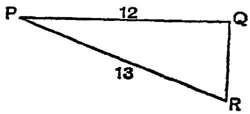
† Or *vice versâ*

Again, if in the right-angled triangle PQR having the right angle at Q, PR is 13, and PQ is 12 inches long, then the *square* of the length of QR $= 13^2 - 12^2$

$= 169 - 144$

$= 25$

$\therefore$ the length of QR $= \sqrt{25} = \underline{5 \text{ in}}$



EXAMPLE XIII —*A rectangular floor is* 11 *feet long and* 8 *feet* 3 *in. wide, find the distance between its opposite corners.*

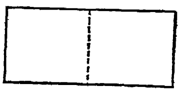
$$\text{Req}^{\text{d}} \text{ distance} = \sqrt{11^2 + (8\tfrac{1}{4})^2} \text{ ft} = \sqrt{\frac{121 \times 16 + 33 \times 33}{16}} \text{ ft.}$$

$$= \sqrt{\frac{121\,(16+9)}{16}} = \frac{55}{4} \text{ ft} = \underline{13 \text{ ft } 9 \text{ in } \textit{Ans}}$$

EXAMPLE XIV.—*The cost of turfing a rectangular cricket-ground twice as long as wide, at 3d. per square yard, is* £765, 12*s.* 6*d*, *find the length of fencing required to surround it.*

Number of sq. yds in area of ground

$$= \frac{£765,\ 12s\ 6d.}{3d.} = 61250.$$



As the ground is twice as long as wide it can be divided into two equal squares.

$\therefore$ area of one of these squares $= 30625$ sq yds

$\therefore$ breadth of ground $= \sqrt{30625} = 175$ yds.

Hence perimeter of ground $= 175 \times 6 = \underline{1050 \text{ yds } \textit{Ans.}}$

We will now consider the postponed case in Compound Interest (see page 206).

EXAMPLE XV.—*At what rate per cent compound interest will* £750 *amount to* £811, 4*s.* *in* 2 *years?*

$$£750 \times \left(\frac{100 + \text{req}^{\text{d}} \text{ rate}}{100}\right)^2 = \text{amount of } £750 \text{ in 2 yrs.*}$$

$$\therefore \left(\frac{100 + \text{req}^{\text{d}} \text{ rate}}{100}\right)^2 = \frac{£811\tfrac{1}{5}}{£750}$$

$$\therefore \frac{100 + \text{req}^{\text{d}} \text{ rate}}{100} = \sqrt{\frac{811\tfrac{1}{5}}{750}} = \frac{26}{25}$$

$$\textit{i.e. } 1 + \frac{\text{req}^{\text{d}} \text{ rate}}{100} = \frac{26}{25} \quad \therefore \frac{\text{req}^{\text{d}} \text{ rate}}{100} = \frac{1}{25}$$

$$\therefore \underline{\text{req}^{\text{d}} \text{ rate} = 4 \textit{ Ans.}}$$

[* By the formula on p 205]

LXIII. CUBE ROOT.

The **cube root** of a given number is the number whose *cube** is equal to the given number

For instance, 4 is the *cube root* of 64, for 4^3, i e $4 \times 4 \times 4 = 64$

The symbol $\sqrt[3]{}$ indicates the *cube* root

For instance, $\sqrt[3]{8}$ is read "the cube root of 8"

The following *cubes* should be known—

$1^3 = 1,\ 2^3 = 8,\ 3^3 = 27,\ 4^3 = 64,\ 5^3 = 125,\ 6^3 = 216,$
$7^3 = 343,\ 8^3 = 512,\ 9^3 = 729,\ 10^3 = 1000,\ 11^3 = 1331,\ 12^3 = 1728.$

Hence the following *cube roots* will be known—

$\sqrt[3]{1} = 1,\ \sqrt[3]{8} = 2,\ \sqrt[3]{27} = 3,\ \sqrt[3]{64} = 4,\ \sqrt[3]{125} = 5,\ \sqrt[3]{216} = 6$

$\sqrt[3]{343} = 7,\ \sqrt[3]{512} = 8,\ \sqrt[3]{729} = 9,\ \sqrt[3]{1000} = 10,\ \sqrt[3]{1331} = 11,\ \sqrt[3]{1728} = 12$

Hence also $\sqrt[3]{8000} = 20,\ \sqrt[3]{27000} = 30,\ \sqrt[3]{64000} = 40,$ &c
and $\sqrt[3]{1000000} = 100,\ \sqrt[3]{8000000} = 200,\ \sqrt[3]{27000000} = 300,$ &c

Note—From the above tables it is evident that comparatively few numbers have an *exact* cube root, for, since $\sqrt[3]{64} = 4$ and $\sqrt[3]{125} = 5$, the cube root of any number (e g 96) between 64 and 125 must be greater than 4 and less than 5, and is *inexact* (See Square Root, p 244)

A number which has an *exact* cube root is called a *perfect cube*

For instance, 343 is a *perfect cube*, for $\sqrt[3]{343} = 7$
and $\frac{27}{125}$ is a *perfect cube*, for $\sqrt[3]{\frac{27}{125}} = \frac{3}{5}$

When the factors of a perfect cube are known its cube root can at once be seen

For instance, $\sqrt[3]{11 \times 11 \times 11 \times 5 \times 5 \times 5} = 11 \times 5$, or 55

Hence in the case of *a perfect cube which can easily be factorized*, its cube root may be found thus—

Example i †—*Find, by factors, the cube root of the perfect cube* 583200

$$5832000 = 8 \times 729 \times 1000$$
$$\therefore \sqrt[3]{5832000} = 2 \times 9 \times 10 = \underline{180\ Ans}$$

Note—This method is of no great *practical* use, since we cannot always, without trial, tell whether the number whose cube root we require is a perfect cube, and whether it can easily be factorized, or not

By reasoning similar to that of Theorem (I) on Square Root it may easily be shown that *a perfect cube can end in any digit*

* See page 10. † See also Example K, page 63

And, as in Square Root, so in Cube Root, *the number of figures in the integral part of the cube root of any number can be foretold.*

For, as $\sqrt[3]{1000} = 10$, the *cube root* of any number *less* than 1000 consists of one figure only.

And, as $\sqrt[3]{1000000} = 100$, the *cube root* of any number *between* 1000 and 1000000 consists of two figures And so on

Hence, if we mark off the figures of the number whose cube root is required into *sets of three*, beginning on the right, the *number of sets* (including part of a set if such there be) will be the *number of figures* in its cube root

For instance, if we thus mark off the number 12 167, there are *two* such "periods", ∴ its cube root is a number of *two* digits

And as there are three such "periods" in 8 365'427, its cube root is a number of *three* digits

Note —If the table of cubes on page 254 is known, the cube root of any number *less than a million which is known to be a* perfect cube, can be obtained *by inspection.*

For instance, take the perfect cube 157'464.

We see that its cube root consists of *two* figures.

And since 157464 lies between 50^3 and 60^3

∴ the *tens'* figure of the cube root must be 5

Also since 157464 ends in 4, we know from the table of cubes that the *units'* figure of its cube root must be 4. Thus $\sqrt[3]{157464} = 54$.

The following important Theorem corresponds to (III) in Square Root.—

The cube of the sum of any two numbers is equal to the cube of the first, plus thrice the square of the first multiplied by the second, plus thrice the first multiplied by the square of the second, plus the cube of the second, number.

Take, for instance, the number 47 and regard it as the sum of the two numbers 40 and 7.

$$
\begin{aligned}
\text{Then } (40+7)^3 &= (40+7)^2 \times 47 \\
&= (40^2 + \textit{twice } 40 \times 7 + 7^2) \times 47^* \\
&= (40^2 + \textit{twice } 40 \times 7 + 7^2) \times (40+7) \\
&= (40^2 + \textit{twice } 40 \times 7 + 7^2) \times 40 \\
&\qquad + (40^2 + \textit{twice } 40 \times 7 + 7^2) \times 7^\dagger \\
&= 40^3 + \textit{twice } 40^2 \times 7 + \phantom{\textit{twice }} 40 \times 7^2 \\
&\qquad + 40^2 \times 7 + \textit{twice } 40 \times 7^2 + 7^3 \\
&= 40^3 + \text{thrice } 40^2 \times 7 + \text{thrice } 40 \times 7^2 + 7^3
\end{aligned}
$$

* By theorem (III) of Square Root. † By (II), page 10.

N B—Hence $47^3 - 40^3 = \textit{thrice } 40^2 \times 7 + \textit{thrice } 40 \times 7^2 + 7^3$
$= (\textit{thrice } 40^2 + \textit{thrice } 40 \times 7 + 7^2) \times 7$

Similarly in the case of any other numbers—

e g. $135^3 - 130^3 = (\textit{thrice } 130^2 + \textit{thrice } 130 \times 5 + 5^2) \times 5$

∴ *If from a perfect cube (e g 47^3) we subtract the cube of part of its root (e g 40^3), the remainder* (R)

= *(thrice the square of this part of the root + thrice this part × the rem^g part + the square of the rem^g part) × the rem^g part*

Hence the following

Method of finding the Cube Root of a number

For instance, *to find the cube root of* 103823

Marking the number off into "periods", beginning at the units' place, we see that the cube root is a number of *two* figures.

Now 103823 lies between 64000 and 125000

∴ $\sqrt[3]{103823}$ 40 and 50

the *tens'* figure of the root must be **4**.

$$
\begin{array}{r|l}
 & 103{}^{\prime}823\,(40+7 \\
 & \underline{64000} \\
4800 & 39823 \\
840 & \\
\underline{49} & \\
5689 & \underline{39823}
\end{array}
$$

Subtracting 40^3 from the number we obtain the remainder 39823.

We now [see (R)] take *thrice* 40^2, *i.e.* 4800, as a *trial divisor* of 39823

[Now 4800 is contained more than 8 times in 39823, but if we try 8 as the units' figure of the required root and thus *complete* the divisor according to the rule obtained from (R), the result when multiplied by 8 is *greater* than 39823]

So we try 7 for the units' figure of the root, and complete the divisor by adding to 4800, *thrice* 40×7, and 7^2, when the complete divisor 5689 results. This we now multiply by 7 obtaining 39823

Thus the cube root of 103823 is 47.

N B.—In the case of a *three*-figure root a second trial divisor would be formed, and afterwards completed, in exactly the same way for it follows from the above Theorem that the remainder at any stage (*i e* the difference between the given number and the cube of the part of its root already obtained) is of the form (R)

We therefore at each stage take as a **trial divisor**

thrice the square of the part of the root already obtained,

and having thence found the next figure of the root, *we complete the divisor* by adding on

thrice the part of the root previously obtained × the new figure;

and *the square of the new figure.*

Note.—In practice the *place* ciphers may be omitted (as in Square Root), and the periods brought down in succession.

We will illustrate the process in this form, in finding the cube root of 16974593.

We first mark off the number into periods thus,— 16'974'593.

1st stage.

We then place in the "quotient" 2, *i.e.* the greatest number whose cube is less than the left-hand period, 16.

We subtract 2^3, *i.e.* 8, from 16, and bring down the next period, 974.

```
16'974'593(2
 8
 8974
```

Now as we bring down one period only at a time, we do not, in forming the first *trial divisor*, give 2 its *real* local value (*i.e.* 200), but merely its local value *relative* to the next figure of the root (*i.e.* 20).

2nd stage.

So we take as first trial divisor *thrice* 20^2, *i.e.* 1200.

Now 1200 is contained 7 times in 8974, but if we *complete* the divisor by the help of 7, the result when multiplied by 7 is greater than 8974. The same also, we find on trial, is the case with 6. Thus we arrive at 5 as the new figure of the root.

```
      16'974'593(2
       8
1200)8974
```

We place 5 in the quotient, and now complete the divisor with

thrice 20×5, *i.e.* 300,
and 5^2, *i.e.* 25.

Adding 1200, 300, and 25 we obtain 1525 as the complete divisor.

We now multiply 1525 by 5, subtract the result from 8974, and then bring down the last period, 593.

```
      16'974'593(25
       8
1200 | 8974
 300 |
  25 |
1525 | 7625
       1349539
```

3rd stage.

We now take *thrice* 250^2, *i.e.* 187500, as a new trial divisor.

We thence obtain 7 as the third figure of the root.

To complete the divisor we add to 187500

thrice 250×7, *i.e.* 5250
and 7^2, *i.e.* 49

obtaining 192799 as the complete divisor.

We now multiply 192799 by 7 obtaining 1349593 as the product.

```
         16'974'593(257
          8
  1200 | 8974
   300 |
    25 |
  1525 | 7625
187500 | 1349593
  5250 |
    49 |
192799 | 1349593
```

Thus the required cube root is found to be 257.

All that was said in the chapter on Square Root as regards the treatment of Fractions and Decimals applies also to Cube Root, and need not therefore be repeated here.

For instance, $\sqrt[3]{5\frac{13}{343}} = \sqrt[3]{\frac{1728}{343}} = \frac{12}{7} = 1\frac{5}{7}$.

EXAMPLE ii —*Find the cube root of* 8869743000

Here our first trial divisor, 1200, is *not* contained in the first remainder 869, so we place a 0 in the quotient, bring down the next period and form a *new trial divisor*, i e thrice 200^2.

Hence $\sqrt[3]{8869743} = 207$
$\therefore \sqrt[3]{8869743000} = 2070$

	8'869'743'000 (2070
	8
120000	869743
4200	
49	
124249	869743
	000

Ans 2070

EXAMPLE iii.—*Find three significant figures of* $\sqrt[3]{\cdot 00064}$

We mark off periods, beginning at the decimal point

As the left-hand period consists of three ciphers, the first figure of the decimal cube root (a *non-significant* one) is 0
Now 640 lies between 512 and 729
i e. 8^3 and 9^2
so the first *significant* figure of the root is 8
We subtract 8^3 from 640 and proceed with the work just as if the case were that of an *integer*, until two more figures of the root are obtained

	·000'640'000'000 (·0861
	512
19200	128000
1440	
36	
20676	124056
2218800	3944000

Ans ·0861.

The work of finding an *approximate* cube root may be somewhat shortened, for *when* 3, 4 *figures of a cube root have been obtained by the ordinary process,* 1, 2 . *more figures, respectively, may be obtained by* (contracted) division

Suppose, for instance, we require the cube root of $4\frac{1}{6}$ correct to 5 places of decimals.
We obtain the cube root to *three* places of decimals by the ordinary process. We have then *four* figures of the root and our last divisor is 772381, and the remainder 1157137 We now cancel the right-hand figure of the divisor, and obtain the next *two* figures of the root by Contracted Division, thus—

	4·'166'666'666 (1·60915
	1
300	3 166
180	
36	
516	3 096
7680000	70666666
43200	
81	
772328$\cancel{1}$	69509529
	1157137
	772328
	384809

Note.—There are other ways of approximating to the cube root of a number. The cube (or *any other*) root may be found approximately by the help of *Logarithms*, or by "*Horner's method*". Horner's method is exemplified in the Appendix.

The *sixth* root of a number may be found by taking *the cube root of its square root.*

For instance, $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3 = (2^3)^2$

Thus the *square root* of 2^6 is 2^3, and the *cube root of this* is 2, which is evidently the *sixth* root of 2^6.

Similarly the *ninth* root of a number may be found by taking *the cube root of its cube root*

EXAMPLE IV.—*Find the least number which must be added to* 148776, *that the sum may be a perfect cube.**

We begin the operation of finding the cube root, and thence discover that the given number falls short of the cube of 53 by 101

```
               148'776(53
               125
         7500 | 23776
          450 |
            9 |
         7959)  23877
```

Ans 101.

EXAMPLE V.—*Find the area of the entire surface of a cubical block the volume of which is* 17 *cub ft.* 415 *cub. in.*

$$
\begin{aligned}
17 \text{ cub. ft. } 415 \text{ cub in} &= 29791 \text{ cub. in.} \\
\therefore \text{ length of side of cube} &= \sqrt[3]{29791} \text{ in.} \\
&= 31 \text{ in.} \\
\therefore \text{ area of one face} &= 31 \times 31 \text{ sq. in.} \\
&= 961 \text{ sq. in.} \\
\therefore \text{ area of entire surface} &= 961 \times 6 \text{ sq. in} \\
&= 5766 \text{ sq in.} \\
&= 40 \text{ sq ft } 6 \text{ sq. in. } \textit{Ans.}
\end{aligned}
$$

EXAMPLE VI.—*Three numbers are as* 3 : 4 : 5; *the sum of their cubes is* 74088; *find the numbers.*

Since the numbers are as 3 : 4 : 5

$\therefore$ their *cubes* are as $3^3 : 4^3 : 5^3$

i e as 27 : 64 : 125.

We therefore divide 74088 into parts proportional to 27, 64, and 125. These parts will be found to be 343×27, 343×64, 343×125.

Hence the req[d] numbers are $\sqrt[3]{343 \times 27}$, $\sqrt[3]{343 \times 64}$, $\sqrt[3]{343 \times 125}$

i e 7×3, 7×4, 7×5

or 21, 28, 35 *Ans.*

---

* Compare with Ex. K, page 63

LXIV. SCALES OF NOTATION.

A *Scale of Notation* is a systematic method of representing numbers by means of *figures*

We know (see pages 2, 3) that the **radix**, or base, in the Common, or Decimal, Scale of Notation is the number *ten*, and that, consequently, *ten* different figures, and ten only, are used in this scale

Also, that any other number may be made the *radix* of a Scale of Notation similar to the Common Scale

For instance, just as, in the *common* scale, 473 means

4 *hundreds* + 7 *tens* + 3 *units*,

$$i.e.\quad 4 \times 10^2 + 7 \times 10 + 3,$$

so, in the scale of radix *eight*, 473 means

$$4 \times 8^2 + 7 \times 8 + 3$$

i.e. 4 *sixty-fours* + 7 *eights* + 3 units

And, that the number of different figures required for any scale is the same as the number chosen as radix

Thus in Scale Eleven, *ten* must be represented by a single figure, so the letter t is used.

And in Scale Twelve *ten* and *eleven* must be represented by single figures, so t and e are used.

For instance, in the scale of radix *twelve*, 3*t*7*e* stands for *three* times (*twelve*)^3 + *ten* times (*twelve*)^2 + *seven* times *twelve* + *eleven*

EXAMPLE i.—*Add* 8454, 7608, 726, *and* 1853 *in the Scale of Nine.*

3 + 6 + 8 + 4 = twenty-one* = 2 nines + 3 *units*; set down 3, and carry 2,		(*Nine*)
2 + 5 + 2 + 0 + 5 = fourteen	= 1 nine + 5 units, set down 5, and carry 1,	8454
1 + 8 + 7 + 6 + 4 = twenty-six	= 2 nines + 8 *units*, set down 8, and carry 2,	7608
2 + 1 + 7 + 8 = eighteen	= 2 nines + 0 units.	726
		1853
		20853 *Ans.*

EXAMPLE ii.—*Subtract* 7*t*58 *from* *e*90*t* *in Scale* 12

8 from t leaves 2, set down 2,	(*Twelve*)
5 from *twelve* leaves 7, set down 7, and carry 1,	*e*90*t*
1 + t from 9 + *twelve* leaves t, set down t, and carry 1,	7*t*58
1 + 7 from e leaves 3.	3*t*72 *Ans*

* The *mental* work, it will be noticed, is really performed in the *common* scale; though it *might*, of course be performed in Scale Nine by constructing a system of *Numeration* (see Numeration on page 2), and learning the corresponding Addition Table for the scale —*e.g.* "3 and 6 make *nine*, *nine* and 8 make '*eighteen*', eigh*teen* and 4 make two-*ny*-three, set down 3 and carry 2."

EXAMPLE iii.—*Multiply* 4256 *by* 54 *in Scale Seven.*

$6 \times 4 =$ twenty-four $= 3$ sevens $+ 3$ *units*,
set down 3, and carry 3,
$5 \times 4 + 3 =$ twenty-three $= 3$ sevens $+ 2$ *units*;
set down 2, and carry 3,
$2 \times 4 + 3 =$ eleven $= 1$ seven $+ 4$ *units*,
set down 4, and carry 1.
$4 \times 4 + 1 =$ seventeen $= 2$ sevens $+ 3$ units, &c

(*Seven*)

```
  4256
    54
 23423
31012
333543  Ans
```

EXAMPLE iv.—*Divide* 344*t*7 *by* 8 *in Scale Eleven.*

8 into (3 elevens + 4), *i.e.* 8 into *thirty-seven*, 4, and 5 over,
8 into (5 elevens + 4), *i.e.* 8 into *fifty-nine*, 7, and 3 over,
8 into (3 elevens + t), *i.e.* 8 into *forty-three*, 5, and 3 over,
8 into (3 elevens + 7), *i.e.* 8 into *forty*, 5

(*Eleven.*)

```
8)344t7
  4755 Ans
```

To transfer a given number from one scale to another.

If we divide any number in the common scale, say 366, again and again by any one number, say 5, we know (see page 16) that

the first remainder is a number of *units*
the second . . . . . *fives*
the third . . . . . . *twenty-fives*
And so on

```
5)366
5)73 + 1
5)14 + 3
   2 + 4
```

i.e. 366 = 2 *hundred-and-twenty-fives* + 4 *twenty-fives* + 3 *fives* + 1 *unit*
$= 2 \times 5^3 + 4 \times 5^2 + 3 \times 5 + 1$
$\therefore$ 366 in scale 10 = 2431 in scale 5.

And if we divide a number, say 1435, in any scale, say Scale 8, again and again by any one number, say 6, *performing the division in Scale* 8, we obtain a similar result.

Thus 1435 in scale 8
$= 3 \times 6^3 + 4 \times 6^2 + 0 \times 6 + 5$
$= 3405$ in scale 6

(Radix Eight.)

```
6)1435
6)204 + 5 units
6)26 + 0 sixes
   3 + 4 thirty-sixes
```

Hence the following Rule—

(I) *Divide the given number successively* (until the final quotient is less than the divisor) *by* the radix of the new scale *in which it is to be expressed*, performing the division in the scale in which the number is given, then the *remainders*, in order, are the figures of the number in its new scale, the *first* remainder being the *units'* figure.

EXAMPLE v.—*Change* 2738 *from the Common Scale to Scale* 4

[By the above Rule we have but to divide 2738 successively by 4, performing the division in the ordinary way]

$$\begin{array}{r} 4\,\underline{)2738} \\ 4\,\underline{)684} + 2 \\ 4\,\underline{)171} + 0 \\ 4\,\underline{)42} + 3 \\ 4\,\underline{)10} + 2 \\ 2 + 2 \end{array}$$

Hence 2738 (scale 10) = $\underline{222302 \text{ (scale 4) } Ans}$

EXAMPLE vi —*Change* 2653 *from Scale* 7 *to the Common Scale*

[By the above Rule we divide 2653 successively by ten, *performing the division in Scale Seven*]

(Radix Seven)

$$\begin{array}{r} \text{ten}\,\underline{)2653} \\ \text{ten}\,\underline{)203} + 8 \\ \text{ten}\,\underline{)13} + 1 \\ 1 + 0 \end{array}$$

Hence 2653 (scale 7) = $\underline{1018 \text{ (scale 10) } Ans}$

Note—In changing to the *common* scale, instead of the above process we may proceed thus—

$$\begin{aligned} 2653 \text{ in scale } 7 &= 2 \times 7^3 + 6 \times 7^2 + 5 \times 7 + 3 \\ &= 2 \times 243 + 6 \times 49 + 5 \times 7 + 3, \text{ in scale } 10 \\ &= \underline{1018 \text{ in scale } 10 \quad Ans} \end{aligned}$$

The various scales are often described by words derived from the names of the Latin numerals Thus Scales 2, 3, 4, 5, 6, 7, 8, 9, 11 are the *Binary*, *Ternary*, *Quaternary*, *Quinary*, *Senary*, *Septenary*, *Octenary*, *Nonary*, *Undenary* Scales respectively, Scale 10 (the Common Scale) is the *Denary*, or the *Decimal*, and Scale 12, the *Duodenary*, or the *Duodecimal*, Scale

EXAMPLE vii.—*Transfer* 201221 *from the Ternary to the Undenary Scale*

[By the above Rule we divide 201221 successively by *eleven*, *performing the division in Scale* 3]

(Radix 3)

$$\begin{array}{r} \text{eleven}\,\underline{)201221} \\ \text{eleven}\,\underline{)1210} + t \\ 11 + 4 \end{array}$$

Hence 201221 in scale 3
= $\underline{44\,t \text{ in scale } 11 \quad Ans}$

Note—In such cases as this we may also, if preferred, proceed otherwise, first changing the given number into the *common* scale, and thence into the required scale Thus—

$$\begin{aligned} 201221 \text{ in scale } 3 &= 2 \times 3^5 + 0 \times 3^4 + 1 \times 3^3 + 2 \times 3^2 + 2 \times 3 + 1 \\ &= 2 \times 243 + 0 + 27 + 18 + 6 + 1, \text{ in scale } 10 \\ &= 538 \text{ in scale } 10 \end{aligned}$$

We now change 538 (scale 10) to scale 11, thus—

$$\begin{array}{r} 11\,\underline{)538} \\ 11\,\underline{)48} + t \\ 4 + 4 \end{array}$$

Hence 201221 (scale 3) = 538 (scale 10)
= $\underline{44t \text{ (scale 11) } Ans}$

In order to transfer a *Vulgar* Fraction from one scale to another, we change both numerator and denominator *separately* into the new scale

RADIX FRACTIONS.

A Radix Fraction in any scale corresponds to a *Decimal* fraction in the Common Scale

For instance, just as $\cdot 413$ in scale 10 means $\frac{4}{10} + \frac{1}{10^2} + \frac{3}{10^3}$,

so $\cdot 413$ in scale 7 means $\frac{4}{7} + \frac{1}{7^2} + \frac{3}{7^3}$.

A *vulgar* fraction is changed into a *radix* fraction *in the same scale* by dividing the numerator by the denominator (as in Decimals) performing the division in the given scale

To transfer a Radix fraction from one scale to another.

If we multiply a *decimal*, say $\cdot 875$, by any number less than *ten*, say 4; then $\cdot 875 \times 4 = 3\cdot 5$

$$\therefore \cdot 875 = \frac{3\cdot 5}{4} = \frac{3}{4} + \frac{\cdot 5}{4}$$

But $\cdot 5 \times 4 = 2$

$$\therefore \cdot 5 = \frac{2}{4}, \quad \therefore \quad \frac{\cdot 5}{4} = \frac{2}{4^2}$$

$$\text{Hence } \cdot 875 = \frac{3}{4} + \frac{2}{4^2}$$

i e $\cdot 875$ *in scale* 10 $=$ $\cdot 32$ *in scale* 4.

Again, if we multiply a *radix fraction* of any scale, say $\cdot 43$ in scale 9, by any number less than *nine*, say 6, performing the work in scale 9;

then, in scale 9, $\cdot 43 \times 6 = 2\cdot 8$

$$\therefore \cdot 43 = \frac{2\cdot 8}{6} = \frac{2}{6} + \frac{\cdot 8}{6}$$

But, in scale 9, $\cdot 8 \times 6 = 5\cdot 3$

$$\therefore \cdot 8 = \frac{5\cdot 3}{6}, \qquad \therefore \frac{\cdot 8}{6} = \frac{5\cdot 3}{6^2} = \frac{5}{6^2} + \frac{\cdot 3}{6^2}$$

and, in scale 9, $\cdot 3 \times 6 = 2$, $\qquad \therefore \cdot 3 = \frac{2}{6}$, and $\therefore \frac{\cdot 3}{6^2} = \frac{2}{6^3}$

$$\text{Hence } \cdot 43 = \frac{2}{6} + \frac{5}{6^2} + \frac{2}{6^3}$$

i e $\cdot 43$ *in scale* 9 $=$ $\cdot 252$ *in scale* 6.

Hence the following Rule—

(II) **Multiply the fractional** *part of the given radix fraction by the* **radix of the new scale,** and the fractional part of the result by the same radix, and so on, then the *integral* parts of the successive products are the figures in order of the transformed radix fraction.

EXAMPLE VIII.—*Express the decimal* 286·78125 *as a radix fraction in Scale* 8.

N.B.—The integral and fractional parts must be dealt with separately

Integral part.		Fractional part	
By Rule (I)	$8\,)\,286$ $8\,)\,35 + 6$ $4 + 3$	By Rule (II)	$\cdot78125$ 8 $6\cdot25000$ 8 $2\cdot00000$
286 = 436 in scale 8		·78125 = ·62 in scale 8	

Ans. 436·62 in scale 8

EXAMPLE IX.—*Express the Senary radix fraction* ·45 *as a decimal.*

[By Rule (II) we multiply successively by *ten*, *performing the work in scale* 6.]

Hence ·45 in scale 6 = $\cdot80\dot{5}$ in scale 10 *Ans.*

$$\begin{array}{r} (Six)\\ \cdot45\\ \text{ten}\\ \hline 8\cdot02\\ \text{ten}\\ \hline 0\cdot32\\ \text{ten}\\ \hline 5\cdot32\\ \text{ten}\\ \hline 5\cdot32 \end{array}$$

EXAMPLE X.—*Change* $e0t\cdot t4$ *from the scale of* 12 *to the scale of* 8.

[By Rule (I) we divide $e0t$ by 8, *performing the work in scale* 12.]

$$\begin{array}{l} (Twelve)\\ 8\,)\,e0t\\ 8\,)\,147 + 2\\ 8\,)\,20 + 7\\ \quad 3 + 0 \end{array}$$

[By Rule (II) we multiply the *fractional* part successively by 8, *performing the work in scale* 12.]

$$\begin{array}{r} (Twelve)\\ \cdot t4\\ 8\\ \hline 6\cdot t8\\ 8\\ \hline 7\cdot14\\ 8\\ \hline 0\cdot t8 \end{array}$$

Hence $e0t\cdot t4$ in scale twelve = $3072\cdot\dot{6}7\dot{0}$ in scale 8. *Ans.*

EXAMPLE XI.—*Express the decimal* $\cdot08\dot{3}$ *as a radix fraction in Scale* 6.

Multiply the fractional part successively by 6, and, performing the work in the common scale, we find that

$\cdot08\dot{3}$ in scale 10 = $\cdot02\dot{5}$ in scale 6

Now just as $\cdot\dot{9}$ in scale 10 = 1 (see page 121), so $\cdot\dot{5}$ in scale 6 = 1

$$\begin{array}{r} \cdot08333\ldots\\ 6\\ \hline 0\cdot4999\\ 6\\ \hline 2\cdot9999\ldots\\ 6\\ \hline 5\cdot9999\ldots \end{array}$$

Hence $\cdot08\dot{3}$ in scale 10 = ·03 in scale 6. *Ans.*

To express a recurring radix fraction as a vulgar fraction in the same scale

The method follows from that given on page 121

For instance, consider the mixed recurring radix fraction $\cdot 5\dot{4}\dot{3}$ in scale 8

$$\text{Since } \cdot 5\dot{4}\dot{3} = 5\ 434343$$

$$\therefore (\text{eight})^3 \text{ times } \cdot 5\dot{4}\dot{3} = 543 \cdot 4343$$

$$\text{and eight times } \cdot 5\dot{4}\dot{3} = 5 \cdot 434343$$

$$\text{Hence by subtraction, } [(\text{eight})^3 - \text{eight}] \text{ times } \cdot 5\dot{4}\dot{3} = 543 - 5$$

$$\therefore \cdot 5\dot{4}\dot{3} = \frac{543 - 5}{(\text{eight})^3 - \text{eight}}$$

$$= \frac{543 - 5}{770} \text{ in scale 8.}$$

Note —This result corresponds exactly with that on page 121

i e as in scale 10 there are in the den^r as many *nines* as there are recurring figs
so in scale 8 *sevens*

And so on

Hence we can write down at once the vulgar fraction equivalent to a given recurring radix fraction in any scale

e g. $\cdot\dot{5}7\dot{3}$ in scale 9 $= \dfrac{573}{888}$ in scale 9, and $2 \cdot 45\dot{3}$ in scale 6 $= 2\dfrac{453 - 45}{500}$ in scale 6.

EXAMPLE xii —*In what scale of notation is the common scale number* 4783 *expressed by the figures* 11257?

Here the *units'* figure of the number, when expressed in the scale whose radix is required, is 7.

$\therefore$ 4783 — 7, *i e* 4776 must be exactly divisible by the req^d radix.

Also the req^d radix must be a number greater than 7, since this figure is used in the scale.

But the *only small* factors of 4776 are 2, 3, 4, 6, and 8

$\therefore$ the scale is that of 8. *Ans.*

EXAMPLE xiii —*How may weights of* 1, 3, 3^2, 3^3, 3^4, *&c, lbs be used in a balance so as to weigh* 257 *lbs?*

We express 257 in scale 3.

Thus $257 = 100112$ in scale 3
$= 3^5 + 3^2 + 3 + 2$
$= 3^5 + 3^2 + 3 + (3 - 1)$

$$\begin{array}{r|l} 3 & 257 \\ 3 & 85 + 2 \\ 3 & 28 + 1 \\ 3 & 9 + 1 \\ 3 & 3 + 0 \\ & 1 + 0 \end{array}$$

Thus we see that one weight of 3^5 lbs, one of 3^2 lbs, two of 3 lbs in one scale-pan, and 1 lb *in the other* scale-pan, will weigh the given amount

LXV. MISCELLANEOUS EXAMPLES.

MIXTURES *

A (i)—*If 9 lbs of tea costing* 1s 8d *per lb, 6 lbs costing* 1s $10\frac{1}{2}$d *per lb, and 5 lbs costing* 2s 4d. *per lb be mixed, what is the cost per lb of the mixture?*

The total cost of the mixture $= 20^d \times 9 + 22\frac{1}{2}^d \times 6 + 28^d \times 5 = 455d$

The total weight of the mixture $=$ $9 + 6 + 5$, lbs $= 20$ *lbs*

$\therefore$ the cost of 1 lb of the mixture $= \frac{455}{20}d = 22\frac{3}{4}d =$ 1s $10\frac{3}{4}$d *Ans*

Note—Hence, when the quantity and cost of each ingredient is given, the cost of a unit of the mixture is found by dividing the *total cost* by the *total* number of units in its *quantity*, *i e* we find the *average* cost per unit. (See Ex. D, page 21)

A (ii)—*In what ratio must teas which cost* 1s 4d *and* 1s 9d *per lb be mixed so that the mixture may cost* 1s $7\frac{1}{2}$d *per lb?*

The cheaper tea costs 7 *halfpence* per lb *less* than the mixture,
the dearer tea costs 3 per lb *more* than the mixture

[We must therefore so arrange that the gain on the amount of cheaper tea in the mixture *balances* the loss on the amount of the dearer tea]

$\therefore$ with each 3 lbs of cheaper we must take 7 lbs of dearer tea *Ans*

[For the cost of 3 lbs of the cheaper is 7×3 halfpence *less*, and the cost of 7 lbs of the dearer is 3×7 halfpence *more*, than the corresponding cost of mixture, and these balance each other]

A (iii).—*How may teas at* 1s 3d, 1s 4d, *and* 1s 9d *per lb be mixed so as to form a mixture worth* 1s 7d *per lb?*

The first kind costs 4 *pence* per lb *less* than the mixture,
second . 3 . *less*
. . third 2 *more*

[We must so arrange the quantities that the *loss* on the third kind, *balances* the *gain* in the *other two kinds*, of tea]

Now 2 lbs of the third will thus balance 1 lb of the first kind,
and 3 lbs. 2 lbs second

Hence, if with each 5 lbs of the third we take 2 lbs of the second and 1 lb of the first, the mixture will be of the req[d] value

Note—Questions of this kind (in which *three*, or more, ingredients occur) are indeterminate, that is, they admit of a variety of solutions *e g* instead of the above numbers the following will be found to satisfy the question:—7, 2, 2, or 8, 4, 1, &c

* Sometimes called *Alligation* For other examples see Ex. iii, p 47, and Exs ix, x, p 191

A (iv) —*How may spirits costing* 14*s*, 16*s*., 18*s*., *and* 18*s* 6*d*. *per gallon be mixed so that a profit of* 20 *per cent may be made by selling the mixture at* 21*s*. *per gallon?*

The *cost* of a gallon of mixture $= 21s. \times \frac{100}{120}$* $= 17s\ 6d$.

	sixpences			
Now the first kind costs	7	per gallon	*less*	than the mixture,
second	3	"	*less*	"
third	1	"	*more*	"
fourth	2	"	*more*	"

Hence 1 gal of the *first* balances 7 gals of the *third*,
and 2 gals " *second* " 3 gals " *fourth*

Ans 1, 2, 7, 3 gals respectively

Note —There are many other possible solutions, *e g* 1, 1, 4, 3, &c

---

A (v) —*A mixture of* 35 *gallons of spirit and water contains* 80 *per cent of spirit, how much spirit must be added to raise the percentage of spirit to* 85?

As the mixture contains 80% of spirit, it contains 20% of water

∴ the number of gallons of water in it $= \frac{20}{100}$ of 35 gallons
$= 7$ gallons.

But there is to be 85 per cent of spirit, and ∴ 15 per cent of water, in the new mixture.

Hence 15 per cent of the increased quantity of mixture = 7 gals.

i e $\frac{15}{100}$ of the number of gallons in the new mixture = 7

∴ the number of gallons in the new mixture $= 7 \times \frac{100}{15} = 46\frac{2}{3}$ gals.

∴ the number of gallons added $= 46\frac{2}{3} - 35 = 11\frac{2}{3}$ gals *Ans.*

A (vi) —*A* 36-*gallon cask is full of spirit,* 3 *gallons are drawn out, and the cask filled up with water, this process is repeated a second, and a third, time; how many gallons of spirit are then left in the cask?*

After the 1st operation, the cask contains 33 gallons of spirit,

and ∴ $\frac{33}{36}$, or $\frac{11}{12}$, of each gallon of mixture is spirit

After the 2nd operation, the cask contains $33 - \frac{11}{12}$ of 3
$= 30\frac{1}{4}$, gallons of spirit

and ∴ $\frac{30\frac{1}{4}}{36}$, or $\frac{121}{144}$, of each gal. of the mixture is spirit.

∴, after the 3rd operation, the cask contains $30\frac{1}{4} - \frac{121}{144}$ of 3
$= 27\frac{35}{48}$, gallons of spirit *Ans.*

---

* See (II), p 187.

UNIFORM MOTION* CIRCULAR TRACKS

B (i) —*If I leave home at a certain time and walk to the railway-station at the rate of* 4 *miles an hour, I shall be* 2 *minutes too late for my train, if at the rate of* $4\frac{1}{2}$ *miles an hour, I shall have* 8 *minutes to spare, how far have I to walk?*

At 4 miles per hour I walk 1 mile in $\frac{1}{4}$ hour

. $4\frac{1}{2}$ 1 $\frac{2}{9}$

∴ at the faster rate I save $\frac{1}{4} - \frac{2}{9}$, or $\frac{1}{36}$, hour in each mile

But $2 + 8$, min. or $\frac{1}{6}$ hour, in the req^d distance.

$$\therefore \text{ the req}^d \text{ distance} = \overset{\text{mile}}{1} \times \frac{\frac{1}{6}}{\frac{1}{36}} = \underline{6 \text{ miles } \textit{Ans}}$$

B (ii) —*The road from A to B rises. A man walked from A to B at the rate of* 3 *miles per hour, rested half an hour at B, and returned at the rate of* 5 *miles an hour, reaching A* 3 *hours* 26 *min after he started Find the distance from A to B*

The man takes $\frac{1}{3}$ hour to walk 1 mile in going

and $\frac{1}{5}$ .. 1 returning

∴ his total time per mile is $\frac{1}{3} + \frac{1}{5}$, or $\frac{8}{15}$, hour

But his total time for the req^d dist is $3\frac{13}{30} - \frac{1}{2}$, or $\frac{88}{30}$ hrs

$$\therefore \text{ the req}^d \text{ dist} = \overset{\text{mi}}{1} \times \frac{\frac{88}{30}}{\frac{8}{15}} = \underline{5\tfrac{1}{2} \text{ mi } \textit{Ans}}$$

B (iii) —*A and B travel in the same direction round a circular course* 220 *yards round, starting together from the same point, their rates being* 12 *and* 16 *miles an hour respectively, how soon will they again be together at the starting-point?*

As A goes 12 miles in 60 min

∴ A makes a complete circuit, $\frac{1}{8}$ mi, in $\dfrac{60}{8 \times 12} = \frac{5}{8}$ min

Similarly B $\dfrac{60}{8 \times 16} = \frac{15}{32}$ min.

[Hence the req^d interval is the smallest number of minutes which *exactly* contains $\frac{5}{8}$ and $\frac{15}{32}$, i e the L C M of $\frac{5}{8}$ and $\frac{15}{32}$, or of $\frac{20}{32}$ and $\frac{15}{32}$, min.†]

Now the L C M of 20 and 15 is 60,

∴ the L.C M of $\frac{20}{32}$ and $\frac{15}{32}$ is $\frac{60}{32} = 1\frac{7}{8}$ *Ans* $1\frac{7}{8}$ min

---

*For other Examples see pages 143, 144 †Compare Ex D, p 95, for the method

B (iv)—*A and B start together from the same point and travel in the same direction round a circular track 220 yards in circumference, A at the rate of 14, and B at the rate of 17, miles an hour, when and where will they be together again for the first time?*

Also, if they travelled in opposite directions, what intervals would elapse between their successive meetings?

(i) B gains 3 miles an hour on A;

∴ B gains a complete round, $\frac{1}{8}$ mi, on A in $\frac{1}{24}$ hour $= 2\frac{1}{2}$ min.

Now in $\frac{1}{24}$ hour A goes $\frac{1}{24}$ of 14 mi. $= 4\frac{2}{3}$ furlongs, or "laps" of 220 yds

∴ when B first overtakes A, A has completed 4 *laps* and $\frac{2}{3}$ of a fifth, and is ∴ $\frac{1}{3}$ of 220 yds $= 73\frac{1}{3}$ yards from the starting-point.

Ans (i) in $2\frac{1}{2}$ min., $73\frac{1}{3}$ yds from starting-point

(ii) When they move in opposite directions they separate, or approach each other, at the rate of $14 + 17 = 31$ miles per hour

∴ they *jointly* complete a circuit, $\frac{1}{8}$ mi., in $\frac{1}{8 \times 31}$ hour $= 14\frac{16}{31}$ secs

Hence they meet at intervals of $14\frac{16}{31}$ secs. *Ans* (ii)

B (v)—*A, B, and C start together from the same point and travel in the same direction round a circular course of 100 yards, at rates of 21, 19, and $17\frac{2}{3}$ feet per second respectively, when and where will all three next be together again?*

A gains 2 feet per sec on B,

∴ A gains a complete circuit on B, i e overtakes him, in $\frac{300}{2}$ secs $= 150$

A gains $3\frac{1}{3}$ feet per sec on C,

∴ A gains a complete circuit on C, in $\frac{300}{3\frac{1}{3}} = 90$, secs.

Hence all three will next be together in the L C M of 150 and 90, secs, i.e. in 450 secs $= 7\frac{1}{2}$ min.

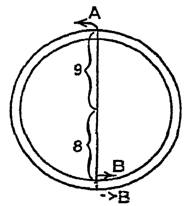
And in 450 secs. A goes 450×7 yds $= 3150$ yds $= 31\frac{1}{2}$ circuits

Hence all three are next together in

$7\frac{1}{2}$ min, and at 50 yards from the starting-point

N B—It would not do in this case to find as in B (iii) the L C M of the times they severally take to complete a circuit, for that would give the interval after which all would be together at the *starting-point* whereas they are all together, at another point, *sooner* To find when all are together for the *first* time after the start, we must always, as above find the *L C M of the times the fastest traveller takes to gain a complete circuit on each one of the others.*

B (vi).—*A and B ride at the same rate, 18 miles per hour, in the same direction, round concentric circular tracks, the circumference of A's track being 440 yards, and the diameter of B's being to that of A's in the ratio of* 8 : 9. *If they start from opposite points, what interval will elapse before B hides A from the view of a spectator at the centre?*



B's *angular* motion (*i e* his apparent motion as viewed by the spectator at the centre O) will be unchanged, if we suppose him *transferred to A's track* and moving on it at a rate *increased in the ratio of the diameters*

The problem, then, is to find how long B will take to catch A, who has a start of 220 yards, A moving at the rate of 18 mi per hour, and B supposed to move on the same track at the rate of $18 \times \frac{9}{8} = 20\frac{1}{4}$, mi. per hour

	mi.		min
Now B will gain	$2\frac{1}{4}$	in	60
∴ B	$\frac{1}{8}$		$60 \times \dfrac{\frac{1}{8}}{2\frac{1}{4}} = \underline{3\frac{1}{3} \text{ min } \textit{Ans}}$

WORK UNIFORM GROWTH, &c

C (i) —*If* 8 *men with* 6 *boys can dig a piece of ground in* 2 *days, and* 6 *men with* 2 *boys can dig it in* 3 *days, how long would* 5 *men with* 3 *boys take to dig it?*

	men		boys				days	
	8	+	6	do	the whole work	in	2	
∴	8	+	6	do	$\frac{1}{2}$ of the work		1	(i)
Again	6	+	2	do	the whole work	in	3	
∴	6	+	2	do	$\frac{1}{3}$ of the work		1	(ii)

[We now take such multiples of the statements (i) and (ii) as to obtain *the same number* of men (or boys) in both In this case, then, we multiply (i) by 3 [*i e* three times as many men + three times as many boys would do three times as much work in a day as is stated in (i)], and (ii) by 4]

	men		boys			day
Hence, from (i),	24	+	18	do	$\frac{3}{2}$ of the work in	1
and from (ii),	24	+	8	do	$\frac{4}{3}$.	1

[Now in these two new statements there is the same number of men, so the extra amount of work in the former must be due to the extra number of boys.]

∴, by subtraction, 10 boys do $\frac{3}{2} - \frac{4}{3}$, or $\frac{1}{6}$, of the work in 1 day

∴ 1 boy does $\frac{1}{60}$ . . . . . . 1 . .

But from (ii) 6 men + 2 boys do $\frac{1}{3}$ of the work 1 day.

∴ 6 men alone do $\frac{1}{3} - \frac{2}{60}$, or $\frac{3}{10}$, . .. 1 . .

∴ 1 man does $\frac{1}{20}$ . ... 1 .

Hence 5 men + 3 boys do $\frac{5}{20} + \frac{3}{60}$, or $\frac{3}{10}$, 1 .

∴ 5 men + 3 boys the whole $1 \times \frac{10}{3}$ days

$= 3\frac{1}{3}$ days *Ans.*

C (ii) —*If* 10 *oxen were put into a field, which has remained empty for a time, the grass would last them for* 20 *days, but it would last* 8 *oxen for* 35 *days, how long would it last* 12 *oxen, the grass being supposed to grow at a uniform rate?*

[Here we must distinguish between the daily growth of grass while the oxen are in the field, and the previously accumulated growth during the time the field was empty, this latter we shall call the *original* grass]

		oxen		
20 days' growth + the orig grass feeds		10	for 20 days	
∴ 20 + .		10 × 20, or 200,	1 day	(i).
Again,				
35 days' growth + the orig grass feeds		8	for 35 days.	
∴ 35 . +		8 × 35, or 280,	1 day	(ii).

[Now in the statements (i) and (ii) the amount of *original* grass is the same in both, so the extra oxen in (ii) are provided for by the extra *daily* growth]

Hence from (i) and (ii) by subtraction,

15 days' growth feeds 80 oxen for 1 day

∴ 1 day's growth $\frac{80}{15}$, or $\frac{16}{3}$, 1 (iii).

Hence in the req^{d} case

$12 - \frac{16}{3}$, or $\frac{20}{3}$, oxen *must be provided for* by the *original* grass.

But, by (i), 20 days' growth + the original grass feeds 200 oxen. for 1 day

And, from (iii), 20 days' growth alone feeds $\frac{16}{3} \times 20$, or $\frac{320}{3}$, . 1

∴ the original grass feeds $200 - \frac{320}{3}$, or $\frac{280}{3}$, 1

∴ $\frac{20}{3}$ $1 \times \frac{280}{20}$ dys.

= 14 days *Ans.*

Note —The principle of the above question may, of course, be clothed in words in various ways, for instance, the *figures* involved in the solution of the following question are the same as those given above, the *words* alone needing to be changed :—

A cistern partly full, into which a steady stream of water is flowing, has

a number of equal holes in the bottom, which can be opened or closed at will If 10 were opened the cistern would be emptied in 20 minutes, if 8 were opened it would be emptied in 35 minutes Twelve are opened how soon will it be emptied?

EQUATION OF PAYMENTS

The *Equated time of Payment* of several sums of money due at different dates is the interval at the end of which all might fairly be paid together

For instance, if £100 is due in 3 months and £200 is due in 8 months, the equated time of payment is the number of months (between 3 and 8) at the end of which a payment of £300 would balance the two debts

Now as £300 at the end of this interval is equivalent to £100 in 3 mo + £200 in 8 mo, the *Present Value** of the £300 must be equal to the *sum of the present values* of the £100 and the £200

If, then, we suppose 5 per cent to be the rate at which discount is calculated, we have

$$£300 - £300 \times \frac{\text{req}^{\text{d}} \text{ no of months}}{12} \times \frac{5}{100}$$

$$= £100 - £100 \times \frac{3}{12} \times \frac{5}{100} + £200 - £200 \times \frac{8}{12} \times \frac{5}{100},$$

which reduces to

$$£300 \times \text{req}^{\text{d}} \text{ no of months} = £100 \times 3 + £200 \times 8$$

$$\therefore \text{req}^{\text{d}} \text{ no of months} = \frac{£100 \times 3 + £200 \times 8}{£300}$$

N B.—The *rate per cent* does not enter into the result

Hence the following practical rule for finding the Equated time of Payment (or *Average* term of Credit as it is sometimes called)

Multiply each debt by the number of months (or days) which elapse before it is due, and divide the sum of these products by the sum of the debts

D.—*The equated time of payment of £200 cash, a bill of exchange for £350 due in 60 days, and another bill due in 90 days, is 45 days, find the face-value of the second bill*

By the above rule,

$$\text{the equated time} = \frac{£200 \times 0\dagger + £350 \times 60 + req^d\ sum \times 90}{£200 + £350 + req^d\ sum}.$$

Hence from the question

$$\frac{£350 \times 60 + req^d\ sum \times 90}{£550 + req^d\ sum} = 45,$$

$$\text{or } £2100 + 90 \text{ times } req^d\ sum = (£550 + req^d\ sum) \times 45$$

$$= £24750 + 45 \text{ times } req^d\ sum$$

$$\therefore 45 \text{ times } req^d\ sum = £24750 - £2100 = £3750$$

$$\therefore req^d\ sum = \frac{£3750}{45} = \underline{£833,\ 6s\ 8d\ \ Ans}$$

* Bankers' discount. † As £200 is due in *ready money* its multiplier is 0

E —*The sum of* 103 *consecutive numbers is* 105472, *find the least of them*

[The *middle* number of any* odd number of consecutive numbers is the *average** of them, *e g*

$\frac{3+4+5+6+7+8+9}{7} = \frac{42}{7} = 6$, the middle number. Hence—]

$$\text{The req}^{\text{d}} \textit{ middle } \text{number} = \frac{105472}{103} = 1024,$$

and there are 51 of the *consecutive* numbers below, and 51 above, this.

$\therefore$ the least is $1024 - 51 = \underline{973}$ *Ans*

Note —The same principle applies in the case of a series of numbers formed by the successive addition of any other constant number instead of unity, *e g* such a series as 17, 20, 23, 26, &c Series so formed are said to be in Arithmetic Progression. Moreover, as the sums of the first and last terms; 2nd and last but one, and so on, are all equal, *the sum of any number of terms* of such a series is found by *multiplying the sum of the 1st and last terms by half the* number *of terms*

e g To find the sum of 86 terms of the series, 1, 3, 5, 7, &c

Here the **2**nd term = the 1st + 2, the **3**rd term = the 1st + *twice* 2, the **4**th term = the 1st + *thrice* 2, and so on

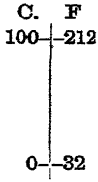
Hence the **86**th term = the 1st + **85** times 2,

and the sum of 86 terms $= (1 + 1 + 85 \times 2) \times 43 = 7396$.

F.—*In the Centigrade thermometer freezing-point is marked zero and boiling-point* 100, *in the Fahrenheit, freezing-point is marked* 32 *and boiling-point* 212, *find* (i) *the reading F. corresponding to* 45° *C.,* (ii) *the reading C. corresponding to* 59° *F.*

If a thermometer is graduated according to the Centigrade scale on the left, and according to the Fahrenheit scale on the right, the space between freezing and boiling points is divided into **100** equal parts on the left, and into **180** equal parts on the right,

i.e 100 C steps = 180 F. steps.



(i) $\therefore$ 45 steps C. $= \frac{180}{100} \times 45 = 99$ steps F.

And the req^d. reading F is found by counting on 99 steps above 32, *i e* 45° C. = (32 + 99) F = $\underline{131° \text{ F}}$ *Ans.*

(ii) As 59 = 32 + 27, the given reading is 27 F steps above freezing-point, so we have to find how many steps C correspond to 27 steps F But 180 steps F = 100 steps C,

$\therefore$ 27 steps F $= \frac{100}{180} \times 27 = 15$ steps C, *i e* 59° F = $\underline{15° \text{ C}}$ *Ans.*

Note —If a given reading F is *less* than 32°, it corresponds to some number of degrees *below zero* C *E g* to find the reading C for 14° F

As 14 = 32 − 18, we must find how many C steps are equal to 18 F. steps, namely, 10, $\therefore$ 14° F corresponds to 10° *below zero* C.

---

*See p 21.

APPENDIX.

A **Complementary Methods**.—The Arithmetical *Complement* of a given number is the number which, when added to it, makes up the unit of next higher order, *e g* 3 is the *complement* of 7, since $7 + 3 = 10$, 14 is the *complement* of 986, since $986 + 14 = 1000$

In Subtraction the complementary method, exemplified below, might well replace any other form of "wording" in the teaching of beginners

EXAMPLE.—*Subtract* 2653 *from* 8129

"*Wording*"	3 and **6** (set down 6) make 9,	8129
	5 and **7** (set down 7) make 12, carry 1,	2653
(1 and 6, 7),	7 and **4** (set down 4) make 11, carry 1,	5476 *Ans*
(1 and 2, 3),	3 and **5** (set down 5) make 8	

Note.—This method of subtracting is convenient in such cases as those of the Ex on p 8, Ex iii on p 39, and in "Abridged" Division

The use of "complementary" methods often effects considerable saving of labour *e g* See Ex. A (v), p 19, A (viii), p 20 (called *Synthetic* Division), Ex x, p 42, Ex. ix, p 77 Exs vii, viii, p 98, Ex ii, p 101, Note (iii), p 104

B. **General Definition of Multiplication**, applicable to a *fractional* as well as to an integral multiplier

To multiply one number by a second is to do to the first *that which is done to* unity *to obtain the second*

Integers *e g*, 8×6,

To obtain 6 we repeat unity *six* times, *i e* $1 + 1 + 1 + 1 + 1 + 1, = 6$

$\therefore$, by the above definition, $8 \times 6 = 8 + 8 + 8 + 8 + 8 + 8 = 48$

Fractions *e g*, $\frac{3}{4} \times \frac{5}{7}$,

To obtain $\frac{5}{7}$, unity is divided into *seven* equal parts, of which *five* are taken,

$$i\,e\ \ \frac{5}{7} = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7},$$

$\therefore$, by the above definition, $\frac{3}{4} \times \frac{5}{7} = \dfrac{\frac{3}{4}}{7} + \dfrac{\frac{3}{4}}{7} + \dfrac{\frac{3}{4}}{7} + \dfrac{\frac{3}{4}}{7} + \dfrac{\frac{3}{4}}{7}$

But $\frac{3}{4} \div 7$ is $\frac{3}{28}$,* $\therefore \frac{3}{4} \times \frac{5}{7} = \frac{3}{28} + \frac{3}{28} + \frac{3}{28} + \frac{3}{28} + \frac{3}{28} = \frac{15}{28}$, or $\frac{3 \times 5}{4 \times 7}$

C **Prime Numbers**.—All the primes below any given number may be found by the following method, known as the "*Sieve of Eratosthenes*" †

Write down the numbers in natural order as far as required, thus —

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, &c

Mark every *second* number after 2; these are the multiples of 2,

" " *third* " " 3, " " " 3,

" " *fifth* " " 5, " " " 5,

" " *seventh* " " 7, " " " 7,

" " *eleventh* " " 11, " " " 11,

and so on, taking each of the unmarked numbers in turn

Then the numbers which finally remain *unmarked* are *primes*, since no number left unmarked will be divisible by any number, except unity, less than itself

---

* See p 78. † A Greek of Alexandria, who lived about 200 B C

D Proofs of the Tests of Divisibility. (See page 52)

(I) Any number may be regarded as a multiple of *ten* + some *units*,

e g 4756 is 475 *tens* + 6 *units*,

and, as *ten* is divisible by 2, ∴ any no of *tens* is also divisible by 2;

∴ any number is divisible by 2, if its units' digit is divisible by 2, *i e* if *its units' digit is even*

(II) Proof exactly similar to that of (I)

(III) Any number may be regarded as a multiple of a *hundred* + the no. of *units* represented by the *two right-hand* digits,

e g 83475 is 834 *hundreds* + 75 *units*,

and, as 100 is divisible by 4, or by 25, ∴ any number of *hundreds* is divisible by 4, or by 25,

∴ any number is divisible by 4, or by 25, *if the number represented by the two right-hand digits is divisible by 4, or by 25*

(IV) Consider any number, *e g.* 23587

$$23587 = 20000 + 3000 + 500 + 80 + 7.$$

Now, as $10000 = 9999 + 1$, ∴ $20000 = 9999 \times 2 + 2 =$ a multiple of $9 + 2$,
as $1000 = 999 + 1$, ∴ $3000 = 999 \times 3 + 3 =$ a multiple of $9 + 3$,
as $100 = 99 + 1$, ∴ $500 = 99 \times 5 + 5 =$ a multiple of $9 + 5$,
and as $10 = 9 + 1$, ∴ $80 = 9 \times 8 + 8 =$ a multiple of $9 + 8$,
also $7 = 7$

Hence, by addition, $23587 =$ some mult of $9, + 7 + 8 + 5 + 3 + 2$.

∴ 23587 is divisible by 9 *if the sum of its digits is divisible by 9.*

And any other number might have been dealt with in the same way.

Note —The proof of the test of divisibility by 3 is identical with this

(V) Consider any no, *e g.* $564328 = 500000 + 60000 + 4000 + 300 + 20 + 8$.

Now, as any number represented by an *even* no. of *nines* is divisible by 11,
and as $100000 = 99990 + 11 - 1$, ∴ $500000 =$ a multiple of $11 - 5$,
as $10000 = 9999 + 1$, ∴ $60000 =$ a multiple of $11 + 6$,
as $1000 = 990 + 11 - 1$, ∴ $4000 =$ a multiple of $11 - 4$;
as $100 = 99 + 1$, ∴ $300 =$ a multiple of $11 + 3$,
and as $10 = 11 - 1$, ∴ $20 =$ a multiple of $11 - 2$,
also $8 = 8$

Hence, by addition, $564328 =$ some multiple of $11 + 8 + 3 + 6 - (2 + 4 + 5)$,

∴ 564328 is divisible by 11, *if the difference between the sum of the units', hundreds', &c, digits, and the sum of the tens', thousands', &c, digits is divisible by* 11.

And any other number might have been dealt with in the same way.

The Tests of Divisibility may be summarized thus —

1 *If the divisor is a factor of*

(i) the *radix* (*e g.* 2, 5, factors of 10), the criterion is *the* units' *figure*,

(ii) the *square of the radix* (*e g.* 4, 25, factors of 100), the criterion is *the number represented by the* two *right-hand figures*,

(iii) the *cube of the radix* (*e g.* 8, 125, factors of 1000) the criterion is *the number represented by the* three *right-hand figures* And so on.

2 *If the divisor is* one less *than the radix*, the criterion is *the sum of the digits*

3 *If the divisor is* one more than the radix, the criterion is *the difference between the sums of the alternate digits*

Hence Tests corresponding to those of the Decimal Scale of Notation may be applied in any scale, *e g* —

A no *in Scale* 6 is divisible by 5, if the *sum of its digits* is divisible by 5:
. . 12 3, *units' figure* is divisible by 3; &c.

E. Casting out Nines —The reason of this process, applied as a test of multiplication, may be shown thus —

Any number may be regarded as a multiple of 9 + a remainder consisting of one of the digits (including 0) less than 9 Hence

The multiplicand = a multiple of 9 + some rem^r, say 7 (i),
and the multiplier = a multiple of 9 + some rem^r, say 4 (ii)
∴ the product = (i) × (ii)
= a mult of 81 + 7 times a mult of 9
+ 4 times a mult of 9 + 4 × 7*
= some mult of 9, + 4 × 7

Hence the rem^r when the product is divided by 9 = the rem^r obtained by dividing 4 × 7 by 9

And these rem^{rs} we know, by D (IV), are obtained by dividing the *sum of the digits* of each number by 9

Note 1 —Casting out Nines may be applied as a test of Addition and Subtraction, *e g* in Addition Cast out 9 s from each of the numbers added, add the resulting rem^{rs}, and cast out nines from their sum, and the final rem^r should be the same as the rem^r obtained by casting out 9's from the sum *of the given numbers*

Note 2 —It is *possible* that a result may satisfy the test of casting out Nines and yet not be correct, since *an error in it amounting to 9 or a multiple of* 9 would not affect the *remainder* obtained by dividing it by nine

Note 3 —Results may be tested in a similar way by *Casting out Elevens* The way in which this would be done is indicated in D (V)

F. Some General Properties of Numbers —Besides those already considered the following are noteworthy —

(i) The sum (or diff) of *two even*, or of *two odd*, numbers must be *even*.
(ii) The sum (or diff) of *an even, and an odd*, number must be *odd*
(iii) The sum of any *even* number of numbers must be *even*
(iv) The sum of any *odd* number of *odd* numbers must be *odd*
(v) The product of *even* nos , or of *even and odd* nos , must be *even*
(vi) The product of *two*, or more, *odd* nos must be *odd* And conversely, in *exact* division, if the *dividend* is *odd* the *quotient* is *odd* also.

Note —The above are evident from the results of the Addn and Multn Tables, since they depend only on the *units* figures of the numbers

(vii) *The product of the sum and difference of two numbers is equal to the difference of their squares*

Take, *e g*, any two numbers, say 23 and 7
Then, the product of their sum and diff
= (23 − 7) × (23 + 7)
= (23 − 7) × 23 + (23 − 7) × 7*
= 23 × 23 − 7 × 23 + 23 × 7 − 7 × 7*
= $23^2 - 7^2$
= the diff of their squares

Similarly we might deal with *any other* two numbers

EXAMPLE 1 —*Simplify* $\dfrac{37{\cdot}8 \times 37\ 8 - 62{\cdot}2 \times 62\ 2}{37{\cdot}8 - 62{\cdot}2}$ *with as little labour as possible*

By converse of (vii), $\dfrac{(37{\cdot}8)^2 - (62{\cdot}2)^2}{37{\cdot}8 - 62{\cdot}2} = 37{\cdot}8 + 62\ 2 = \underline{100\ \textit{Ans.}}$

---

* By (II), p. 10

EXAMPLE 2 —*Show that if 2^{20} be decreased by unity the result is a multiple of* 11

By (vii), $2^{20} - 1 = (2^{10} + 1) \times (2^{10} - 1)$
$= (2^{10} + 1) \times (2^5 + 1) \times (2^5 - 1)$

Now $2^5 = 32$, $\therefore 2^5 + 1 = 33$, a multiple of 11

EXAMPLE 3 —*Prove that the difference of the squares of any two odd numbers is a multiple of* 4

By (viii), the diff of the squares of the two odd nos
= (the sum of the nos) × (their diff)
= (an *even* no) × (an *even* no)   By (i)
= (an odd no × 2) × (an odd no. × 2)
= a multiple of 4.

EXAMPLE 4 —*If any prime number greater than 3 be increased, and also decreased, by unity, one or other of the results is a multiple of* 6.

For every prime (except 2) is *odd*, $\therefore$ both results are *even* and divisible by 2 But the prime − 1, the prime, and the prime + 1 are three consecutive nos, and when the numbers are arranged in natural order every 3rd no is divisible by 3 $\therefore$ one of *any three consecutive* nos. must be divisible by 3 But this is not the case with the prime.

Hence one of the two "results" is divisible by 3, and is also even
$\therefore$ it is a multiple of 6

EXAMPLE 5 —*If the sum of an odd number of fractions, whose numerators and denominators are all odd, is an integer, that integer must be odd*

Consider, e g, the *three* fractions $\frac{5}{7}$, $\frac{3}{11}$, $\frac{9}{13}$,

$$\text{their sum} = \frac{5 \times 11 \times 13 + 3 \times 7 \times 13 + 9 \times 7 \times 11}{7 \times 11 \times 13} \qquad (Q)$$

Now, by (vi), each of the products $5 \times 11 \times 13$, $3 \times 7 \times 13$, $9 \times 7 \times 11$ is *odd*; and $\therefore$, by (iv) their *sum* is *odd*.

Also $7 \times 11 \times 13$ is *odd* $\therefore$, by (vi), the quotient (Q) is *odd*.

Similarly the theorem may be proved in the case of any other set of fractions fulfilling the given conditions

---

EXAMPLE 6 —*Show that every perfect cube is either divisible by 7, or when divided by 7 gives remainder* 1 *or* 6

Every no = a mult. of 7 +, or −,   0, 1, 2 or 3.
$\therefore$ Every *cube* = a mult. of 7 +, or −,   0, 1, 8 or 27 *
= some mult. of 7 +, or −, 0, 1, 1 or 6.

EXAMPLE 7 —*If a perfect square end in an odd digit, the last figure but one is even*

For any odd no may be regarded as a mult of *ten* + an odd digit, say 3

Then its square = (a mult of $10 + 3)^2$
= a mult of 100 + *twice* a mult of $10 \times 3 + 3^2$†
= some *even* mult of 10 + 9

Hence the last figure but one is even

---

* This is evident if we expand, say (mult of $7 + 3)^3$, compare p 255.
† By Theorem (III), p 246

EXAMPLE 8 —*Show that* 1331 *is a perfect cube in any scale of notation whose radix is greater than* 3

We have seen, on p 255, that the cube of any number in the Decimal scale, *e g.*, 47^3, is of the form $4 \times 10^3 + 3 \times 10^2 \times 7 + 3 \times 10 \times 7^2 + 7^3$

$$\text{Hence, } 1 \times 10^3 + 3 \times 10^2 \times 1 + 3 \times 10 \times 1^2 + 1^3 = 1331, \text{ in scale } 10,$$
$$1 \times 6^3 + 3 \times 6^2 \times 1 + 3 \times 6 \times 1^2 + 1^3 = 1331, \text{ in scale } 6,$$
$$1 \times 5^3 + 3 \times 5^2 \times 1 + 3 \times 5 \times 1^2 + 1^3 = 1331, \text{ in sc } 5, \text{ \&c.}$$

Thus 1331 is a perfect cube in any scale whose radix is greater than 3

G Recurring Decimals (See p 122)

Note 1 —When the recurring period of one of a set of fractions, which yield pure recurring decimals with the maximum no of figures in their periods, is known, the periods of all others of the set can be written down at once

e g $\frac{1}{17}$ yields a decimal with the maximum number of figures, 16, in its period, hence, if $\frac{1}{17}$ be found as a decimal, and the figures of its period arranged in a ring (as is done for *sevenths* on p 122), then $\frac{2}{17}$, $\frac{3}{17}$, &c , can at once be read off

Note 2 —As the figures of the quotient *must* recur when all possible remainders have appeared in the division, but *may* recur sooner, it follows that, in the latter case, the number of figures in the actual period is a *factor* of the number in the maximum,

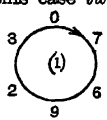
e g $\frac{1}{13}$ = 076923, a *six*-figure period, where 6 is a factor of 12, the *maximum* in the case of *thirteenths*

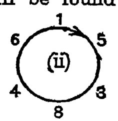
In such cases, as many rings would be needed as the shorter period is contained in the maximum, in this case *two* It will be found that Ring (i) gives

$\frac{1}{13}, \frac{3}{13}, \frac{4}{13}, \frac{9}{13}, \frac{10}{13}, \frac{12}{13}$;

and Ring (ii),

$\frac{2}{13}, \frac{5}{13}, \frac{6}{13}, \frac{7}{13}, \frac{8}{13}, \frac{11}{13}$





H Continued Fractions

—On p 89 is given a method of reducing a Continued Fraction to a Simple Fraction We here consider the converse process, which will be evident from the following example —

Reduce $\frac{274}{79}$ *to a Continued Fraction*

$$\frac{274}{79} = 3 + \frac{37}{79} = 3 + \frac{1}{\frac{79}{37}} = 3 + \frac{1}{2 + \frac{5}{37}} = 3 + \cfrac{1}{2 + \cfrac{1}{\frac{37}{5}}} = 3 + \cfrac{1}{2 + \cfrac{1}{7 + \frac{2}{5}}} \text{ \&c.}$$

Now in this operation it is evident that the successive remainders resulting from the divisions, viz , 37, 5, 2, &c , become less and less, so that we must at last arrive at the remainder 1, when the operation ends

Hence every *rational* fraction (vulgar or decimal) can be expressed as a *terminating* continued fraction

Note —The above process exactly corresponds to the ordinary method of finding the G C F of 274 and 79, the numbers 3, 2 7, &c , being the successive *quotients*

The fraction which results from stopping at any stage, in the above process, and discarding the remainder at that stage, is called a *Convergent* to the continued fraction

Thus, in the above example, the *first* convergent is $3 = \frac{3}{1}$,

second $3 + \frac{1}{2} = \frac{7}{2}$,

third $3 + \frac{1}{2 + \frac{1}{7}} = \frac{52}{15}$.

The following properties of convergents are noteworthy.

(1.) *The convergents are alternately less and greater than the complete fraction.*

e g, in the above ex., the 1*st* convergent is *too small*, since the part $\frac{1}{4}$, &c., is omitted,
the 2*nd* „ *too great*, since the *den*[r] 2 is *less* than $2+\frac{1}{3}$,
the 3*rd* „ *too small*, since the *den*[r] $2+\frac{1}{3}$ is too great, &c

(2.) *The difference between any two successive convergents is a fraction whose numerator is unity, and denominator the product of their den*[rs].

$$e\,g,\ \frac{7}{2}-\frac{52}{15}=\frac{105-104}{2\times 15}=\frac{1}{2\times 15},\ \text{and}\ \frac{111}{32}-\frac{52}{15}=\frac{1665-1664}{32\times 15}=\frac{1}{32\times 15}$$

From (1) and (2) it follows that

(3.) *Any convergent is nearer to the complete fraction than the preceding convergent, that it differs from the complete fraction by less than half of* $\frac{1}{\text{prod. of den}^{\text{rs}}\text{ of convergents}}$, *and that it is in lowest terms.*

Hence the successive convergents are successive approximations to the value of the continued fraction

To convert a Surd *into a Continued Fraction*, *e g*, $\sqrt{10}$.

Observing that the greatest perfect square below 10 is 9, we proceed thus—

$$\sqrt{10}=3+\sqrt{10}-3=3+\frac{\sqrt{10}-3}{1}=3+\frac{(\sqrt{10}-3)\times(\sqrt{10}+3)}{1\times(\sqrt{10}+3)}\quad\Big\}\ \text{See p. 252}$$

$$=3+\frac{1}{\sqrt{10}+3}=3+\frac{1}{6+\sqrt{10}-3}=3+\cfrac{1}{6+\cfrac{1}{\sqrt{10}+3}};\ \text{and so on}$$

$$\therefore \sqrt{10}=3+\cfrac{1}{6+\cfrac{1}{6+\cfrac{1}{6+\&c}}}$$

Thus a surd produces a *recurring* continued fraction.

Note 1.—By the definition of a surd it is evident that it *cannot* produce a *terminating* continued fraction, for *that* we have seen can be reduced to a *rational* simple fraction, whereas a *surd* is *incommensurable*, *i e* cannot be *exactly expressed in figures*

Note 2.—A Continued Fraction is often written in the form $-3+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\cdots$

K. Proof of Theorem (IV), Square Root. (See p 250.)

By Theorem (III), page 246,

$$365^2=360^2+2\times 360\times 5+5^2,\quad \therefore\ \frac{365^2-360^2}{2\times 360}=5+\frac{5^2}{2\times 360}.$$

Hence, in the case of any *three*-figure square root, if when *two* figures of the root have been obtained, we divide the *remainder* (*i e* the diff. between the given number and the square of the part of the root already obtained) by the last complete divisor (*i e* by double the part of the root already obtained), the

$$\text{quotient}=\text{the remaining fig}+\frac{(\text{rem}^{\text{g}}\text{ fig})^2}{\text{double the part of the root already obtained}},$$

and this latter portion of the quotient must be a *proper* fraction, since the num[r] can never exceed 9^2, and the den[r] can never be less than 200

$\therefore$ the *integral* part of the above quotient = the last fig of the root.

Thus the theorem holds good in the case of a *three*-figure root.

Similarly it may be shown to hold good in any other case.

L. Cube Root by Horner's Method—The *reason* of the method belongs to the Theory of Equations, the method is, however, easy to *apply*

The work is arranged in four columns, in the first we place 1, in the second and third, 0, and in the fourth, the number (whose cube root is required) marked off into "periods"

EXAMPLE 1.—*Find the cube root of* 24389

The integral part of the cube root of the first period is 2 Set 2 on the right

1st stage

Multiply 1 by the root-fig 2, and add to 2nd column

Multiply the result by the root-fig 2, and add to 3rd column

Multiply this result by the root-fig 2, and subtract from 1st period

1	0	0	24'389 (29 *Ans*
	2	4	8
	2	4	16 389
	2	8	16 389
	4	**1200**	
	2	621	
	60	1821	
	9		
	69		

Trial Divisor stage

Multiply 1 by the root-fig 2, and add to 2nd column

Multiply the result by the root-fig 2, and add to 3rd column, afterwards appending *two* ciphers *

Again multiply 1 by the root-fig 2, and add to 2nd column, afterwards appending *one* cipher *

Bring down the next period, and obtain the next fig of root, 9, by help of the *trial divisor* 1200

2nd stage (We now repeat with the new root-fig 9 the process of stage 1)

Multiply 1 by 9 and add to 2nd column

Multiply the result by 9 and add to 3rd column

Multiply this result by 9 and subtract from 4th column

Note—In forming the 2nd and 3rd columns the multiplications and *additions*, and, in the 4th column, the multiplications and *subtractions*, may easily be combined (see Ex., p 12)

EXAMPLE 2.—*Find the cube root of* 3 *to three places of decimals*

Written work in full as in Ex 1

1	0	0	3·'000'000'000 (1 442
	1	1	1
	1	1	2 000
	1	2	1 714
	2	**300**	256 000
	1	136	241 984
	30	436	14 016 000
	4	152	
	34	**58800**	
	4	1696	
	38	60496	*Ans* 1·442
	4	1712	
	420	**6220800**	
	4		
	424		
	4		
	428		

Condensed, as suggested in the Note to Ex 1

1	0	0	3' (1 442
	1	1	2 000
	2	**300**	256 000
	30	436	14 016 000
	34	**58800**	
	38	60496	
	420	**6220800**	
	424		
	428		

Ans 1·442

Note—If this cube root had been reqd to *five* places of decs the two extra figs could be obtained by *Contracted* division (See p 258.)

* The reason for these ciphers will be seen by comparing with the example on p 257

EXERCISES.—PART I.

I. NUMERATION, NOTATION.

Read, or write in *words*,

1. 30800.
2. 51020.
3. 18200.
4. 70020.
5. 384610.
6. 423654.
7. 2000000
8. 3003000
9. 700100.
10. 160011.
11. 503010.
12. 675082.
13. 3503071.
14. 4004004.
15. 32005701
16. 60600752.
17. 10101010
18. 122012.
19. 33333333.
20. 50000000.
21. 18800008.
22. 19099090.
23. 567802005.
24. 10007063.
25. 3756421871.
26. 8300235007.
27. 308056300072.
28. 1000000000000
29. 236750846374.
30. 3008700050000

Write in *figures*,

31. Seventeen thousand and twenty.
32. One hundred and two thousand, seven hundred.
33. Six hundred and fifty thousand
34. Five hundred and forty-three thousand and eleven.
35. Three million seven hundred thousand and seventy.
36. Thirteen million, one thousand three hundred.
37. Five million, six hundred and ninety-four thousand, three hundred and eighty-seven.
38. Twenty-three million, one hundred and ten.
39. Four hundred and one million, forty thousand, four hundred and forty.
40. Seven hundred and sixty-eight million six hundred and seventy-five thousand five hundred and eighty-six
41. Eighty million, eighteen thousand
42. Seven thousand and eleven million sixty thousand, three hundred.
43. One thousand and three million, one thousand and three
44. Forty-nine thousand two hundred and eighty-one million, five hundred and fifty thousand, six hundred and sixty.

Write in *figures*,

45. One hundred thousand and seventy million, fourteen thousand and eight.

46. Three hundred and sixty thousand million, three hundred and six thousand, and thirty-six

47. Two hundred thousand and twenty million, twenty thousand, two hundred.

48. One billion, one million, one thousand and one

49. Seventeen billion eight hundred and ninety thousand and forty-five million, seven hundred and ninety-six thousand and sixteen

50. Nineteen billion, one thousand and ninety million, one hundred thousand, nine hundred.

Read, or write in Arabic figures,

51	XIX	61.	LXXXIX.	71.	DCCXC
52.	XXIV	62	XCV	72.	DCLXXIX.
53	LVIII	63	CXII	73.	MDCCLXIV.
54	XLV.	64	CCIV	74	MDCXLVII
55	LXXVI.	65	CLXVI	75	MDCCCLXXXV.
56	LXVIII	66	CXLIV.	76	MDLXVI
57.	XLIX	67.	CCLXXXIV	77	MDLXXXVIII
58	LXXXIII.	68.	DXL	78	MDCLXVI
59	CIV.	69.	CCCXC.	79	MDCCLXXXIX
60.	CLX.	70.	DLV.	80.	MDCCCXCIX

Express in Roman numerals,

81	34.	86.	203	91.	1241	96	1100
82.	46	87.	555	92.	1336	97.	1413
83	73.	88	423	93.	1759	98.	1689
84.	87.	89.	324	94	1867	99	1888
85.	99.	90.	560.	95.	1891.	100.	5000.

II. THE SIMPLE RULES.

Addition.

1.	2.	3.	4.	5.
973	59432	237	240738	348170
49	917	46	59618	91754
61358	6480	32752	485	2760
876	753	47	79	96287
56	96075	187246	4328	462
208	99	7609	19576	5720
1095	2160	9642	275843	18234
283	84317	35132	3629	29216
94	908	23109	73	4056
9048	7563	36	69384	80099

6.	7.	8.	9.	10.
13455	6704	50462	246	59386
6124	78534	4000	10988	370
448	3460	31078	8537	12
90653	79473	2053	54561	2694
7127	36324	275	4123	49057
48	5968	5268	40345	371804
67422	5374	63516	67087	96287
5100	63239	44363	564	1590
589	41258	5086	62742	329105
3694	48485	65045	56653	40462
87318	47256	54678	62583	708
3256	23245	56743	51646	402917
939	9172	6726	96	348170
16285	365	93623	88219	13465
6903	1335	7036	8136	150918

Find the *sum* of—

11. 56841, 7345, 860, 7843217, 14, 500109, 6234, 503, 8040, and 172.

12. 13721, 179, 1270, 430920, 1275, 7079, 32, 7700, 14030211, and 71.

13. 3204, 8571, 396, 14, 9763, 2842, 971, 8309, 2135, 614, 73, 428, 1867, 3905, and 428

14. 910, 4016, 18791, 20645, 17453, 46, 2712, 406, 57916, 71865 4262, 90014, 370, 12, and 2694

15. 24910, 4016, 18791, 20645, 17453, 46, 2712, 40689, 57916, 71865, 4262, 90014, 370, 12, 2694, and 49057

16. 3204, 8571, 396, 14, 9763, 2842, 971, 8309, 2135, 614, 73, 428, 1867, 3905, 428, 6054, 378, 4915, 3284, and 418

Find the *sum* of—

17. One million, fifteen thousand and eighty; four hundred and nine thousand, seven hundred and ninety, two hundred and forty-two thousand, six hundred and thirty-nine, seventeen thousand, one hundred; and seven thousand, one hundred and seven

18. Sixty thousand and seventy-six, one hundred and twelve thousand, nine hundred and four, eighty-seven thousand, two hundred and ninety, seven hundred thousand, six hundred and twenty-seven; and eight thousand, eight hundred and eighty-eight.

19. Three hundred and forty thousand and fifty, five millions, nine hundred and twenty-two thousand and nine, seven hundred and four thousand, three hundred and four, twenty thousand and five, sixty-five thousand, six hundred

20. Seventy million, seventy thousand, eight hundred and nine, eight million and eight, nine hundred and eighty-five thousand, nine hundred and eighty, sixty thousand, eight hundred and five, and nine million, eight thousand, four hundred and forty-four.

Add *across**—

21. 21+43+109+8.
22. 32+5+91+210
23. 180+15+2000+11
24. 47+8+967+150
25. 17+93+8+154+62
26. 9+142+16+11+140
27. 42+31+102+9+67.
28. 750+110+38+70+3
29. 92+201+859+4121+7.
30. 45+309+70+257+8008
31. 37+201+1073+16+4.
32. 1035+67+138+200+19.
33. 9+33+801+7605+80+7
34. 84+307+666+3041+28+1.
35. 235+25+76+972+270+5
36. 9378, 927, 70, 4837, 67, 234
37. 23, 3916, 204, 1795, 3070, 69
38. 590617, 475, 180, 106, 25, 2
39. 201, 384, 17, 6001, 889, 7801.
40. 678, 294, 5, 4291, 328, 55, 1.
41. 56741, 37, 414, 2190, 125, 2013, 4191
42. 31129, 1341, 1014, 758, 9429, 4711, 10.
43. 2120, 10672, 2923, 89, 19098, 3058, 6120.
44. 31280, 645, 39811, 3064, 11421, 3125, 849
45. 2152, 3205, 220, 930, 4073, 11909, 386, 2072.
46. 487, 6590, 7803, 2499, 16075, 925, 769, 6291
47. 876, 1096, 5018, 830, 6537, 1070, 673, 856, 210, 845.
48. 13721, 2256, 2076, 4323, 13276, 74, 7079, 320, 486, 1405
49. 14245, 819, 9195, 2573, 2406, 1275, 7495, 197, 770, 892
50. 451, 6003, 876429, 790, 8035, 42, 8, 70312, 170, 8764

* These Exercises should be worked as they stand, the *answer only* being *written*

SUBTRACTION.

51. Subtract 4378 from 10649.
52. Subtract 6847 from 9203.
53. From 12007 subtract 11438.
54. From 30201 subtract 7745.
55. Take 14730 from 25000.
56. Take 50687 from 67127.
57. From 123456 take 65432.
58. From 987654 take 123456.
59. Find 750420 — 93874.
60. Find 165207 — 158709.
61. Find 857142 — 142857.
62. Find 987654 — 321098.
63. Find 3080715 — 2991032.
64. Find 4743021 — 4446666.

65. Find the difference between 8975 and 9857.
66. Find the difference between 30405 and 15069
67. How much greater is 10000 than 9875?
68. How much greater is 40871 than 3900?
69. How much less is 3085 than 12713?
70. How much less is 5893 than 17000?
71. What number must be added to 13708 to make 38190?
72. What number must be added to 120781 to make 138005?
73 What number must be taken from 4708 to leave 1809?
74. What number must be subtracted from 84701 to leave 896?
75. By what number does 12341 exceed 9087?
76. Find the excess of 44444 over 8797.
77. Find the remainder when 4731 is decreased by 4097.
78. Find the remainder when 803 is deducted from 10071.
79. The sum of two numbers is 55555; the greater is 34567: find the less.
80. The sum of two numbers is 10,000; the greater is 8203: find the less.
81. The sum of two numbers is 76804; the less is 1992: find the greater.
82. The sum of two numbers is 14004; the less is 399· find the greater.
83. The difference between two numbers is 589; the greater is 42876 find the less.
84. The difference between two numbers is 2003, the greater is 40010: find the less.
85. By how much does the sum of 36052 and 27581 exceed their difference?
86. By how much does the sum of 60572 and 534891 exceed their difference?

87 From three hundred and two thousand and eleven take eighty-seven thousand seven hundred and eight

88. Take thirty-seven thousand from one million five thousand.

89. What number must be added to seven hundred million, four hundred and six thousand, five hundred and four to produce eight hundred million?

90. Find the difference between three thousand million, seven hundred and six thousand, and one million, eight hundred and six thousand and seven.

Find the value of—

91. 281—179+325
92. 78+432—493
93. 46—8—17+3—22.
94. 23—16+41—19—27
95 72—4+13—23—7—49.
96 8+171—29—102+16
97. 63—21+17—24+8—19
98. 379+801—978—175+2
99. 8+19—23+74—89+17
100. 176—23—84—15+4—56
101. 14—27+122—86.
102 37—73+85—41
103. 2—71—801+1230—97
104 14—23+2—13—9+38
105. 3—11—17—86+45—1+80
106 301—482+65—139+297.
107. 223—4851+1764+3020
108 102—7689+57+8008
109. 67—18973—351+24531—3
110 681—3451+8795—870+76

111. 6—7+8—9+1—2+3—4+5+1—12+8—17+20.
112. 67—76—89+98—54+43+12—41—53+99.

Find, in *one* operation,

113. 4369—875—654—1207
114. 5001—967—856—2330
115. 738962—7482—371806—359
116 57431—3209—7698—24807.

117. Take the sum of 367, 8298, 774207, and 999 from a million.

118. What number must be added to the sum of 82760, 509, 7360 and 93, to make up a total of one hundred thousand?

119. Take the sum of thirty-five millions six hundred and twenty-nine thousand and seventy-three, eight millions two thousand and six; and five millions seven hundred and four, from forty-nine millions fifty thousand six hundred and eleven

120. The populations of the different parishes of a large town are 16640, 321, 3750, 3906, 5144, 2684, 13360, 391, and 5797 respectively, by how many does the population of the town fall short of sixty thousand?

MULTIPLICATION.

Read off (or write), *at sight,* the product of—

121. 87×10	127. 150×100	133 $1074 \times 10.$
122. $56 \times 100.$	128. 308×10	134. $7030 \times 100.$
123. 19×1000	129. $430 \times 100.$	135. $500 \times 100.$
124. $175 \times 10.$	130. 1002×10	136. $710 \times 1000.$
125. 30×100	131. $17 \times 10000.$	137. $270 \times 10000.$
126. 700×100	132. 403×100	138. $800 \times 10000.$

Multiply—

139. 123456789 by 9.
140 987654321 by 9.
141. 857142 by 14.
142. 142857 by 49.
143 123456789 by 63.
144. 987654321 by 36.
145. 97302 by 7016
146. 85762 by 3109.
147. 2895 by 35700
148. 82653 by 19800.
149. 20356740 by 3960
150. 4205970 by 6390.

Find, using *not more than three* lines in multiplying,

151. 179430×507900
152. 80410×479030
153. $30700 \times 54603.$
154. $60080 \times 124071.$
155. $96800 \times 203750.$
156. 380700×765043

157. Find the product of 867490 and 80900.

158. Find the product of 806010 and 100102

159. Multiply thirty million and thirty thousand by four hundred thousand and eighty.

160. Multiply one million nine thousand and eighty-seven by six hundred thousand five hundred and forty-three.

161. Multiply 2468 by 3057.
Hence, without any further work, write down the product of 2468 and 500.

162. Multiply 129847 by 468.
Hence, without further *multiplication*, obtain the product of 129847 and 4008

163. Multiply the sum of fourteen thousand two hundred and thirty-one, and thirteen thousand two hundred and twelve by their difference

164. Find the product of the sum and difference of two hundred and thirty-eight thousand four hundred and twenty-eight, and one hundred and five thousand and sixty-nine.

Read off (or write), *at sight*, the continued product of—

165 17 × 10 × 10
166. 24 × 10 × 100
167. 8 × 1000 × 10.
168. 75 × 10 × 10 × 10.
169 300 × 10 × 10
170. 810 × 10 × 100
171. 47 × 2 × 5
172. 89 × 5 × 2.
173. 79 × 2 × 50
174 2 × 2 × 5 × 5
175 7 × 5 × 20
176 23 × 25 × 4.

Find the continued product of—

177. 17, 26, and 54.
178. 19, 35, and 87
179. 95, 462, and 932.
180. 6363, 5252, and 4141
181. 7070, 6006, and 5342
182. 19, 28, 307, 460, and 5000

Find the *square* of—

183. 19.
184. 55
185 103
186. 210
187. 161
188 235.
189 3108
190. 4690.
191. 5432
192. 7777

Find the *cube* of—

193. 17.
194. 21.
195. 64
196 98.
197 215
198 444
199 6280
200. 1111.

Division

Read off (or write), *at sight*, the quotient and remainder of—

201 423 ÷ 10.
202. 7681 ÷ 100
203. 4865 ÷ 1000.
204. 2030 ÷ 10
205. 5140 ÷ 100
206. 4700 ÷ 10
207. 35000 ÷ 100
208 7006 ÷ 10.
209 8046 ÷ 1000
210. 76008 ÷ 100
211. 36001 ÷ 100
212. 23400 ÷ 10
213. 30101 ÷ 100
214. 27070 ÷ 100
215 580000 ÷ 10
216 34050 ÷ 1000.
217. 46321 ÷ 10000
218. 120008 ÷ 1000

Find, by *one short* division, the quotient and remainder of—

219. 436251 ÷ 7.
220. 780575 ÷ 9.
221. 2360513 ÷ 8.
222. 680507 ÷ 11.
223. 8465307 ÷ 12.
224. 668800795 ÷ 12.
225 86431 ÷ 20
226. 57942 ÷ 30
227. 85767 ÷ 60.
228 670432 ÷ 110
229. 8763251 — 900
230. 3805407 ÷ 500
231. 37091 ÷ 20
232 761053 ÷ 40.
233 558710 ÷ 70
234 146357 ÷ 120.
235. 4605643 ÷ 7000.
236. 77635081 ÷ 8000.

Divide—

237. 87401 by 23.
238. 226314 by 31.
239 9516156 by 52
240. 5942718 by 74.
241. 1719035 by 87
242. 5973467 by 243.
243 3597840 by 789.
244. 16122344 by 536.
245 7855568 by 4173.
246. 56831974 by 3278.
247 4280960342 by 15003.
248. 237000675 by 63041.
249. 25600160001 by 159601.
250. 888888888888 by 63492.

Divide, using *no unnecessary ciphers*—

251. 386451 by 230.
252. 77513 by 4300.
253 32541700 by 79000.
254. 687524 by 53000.
255. 393051 by 13100.
256. 34287000 by 17000.
257 7654302 by 310000.
258. 8463200 by 6700.
259. 7674528 by 43100
260. 77777777 by 98700.

Find, by using two *short* divisions, the quotient and the complete remainder of—

261. $9315261 \div 16$.
262. $2910141 \div 18$.
263. $917551 \div 24$.
264. $61385 \div 28$
265. $921124 \div 35$.
266. $571882 \div 36$.
267. $9178854 \div 49$.
268. $106513 \div 48$
269. $2494614 \div 63$
270. $1911200 \div 66$
271. $72295 \div 42$.
272. $478752 \div 81$
273. $607191 \div 72$.
274. $7997152 \div 64$.
275. $18051231 \div 96$.
276. $22519 \div 84$.
277. $111725 \div 99$.
278. $232181 \div 132$.
279. $4234172762 \div 144$.
280. $814359509 \div 121$.

Divide, using *short* divisions to obtain the quotient and complete remainder,

281. 21011 by $3 \times 5 \times 7$
282. 43387 by $4 \times 6 \times 7$.
283 199916 by $7 \times 8 \times 9$
284 392252 by $8 \times 9 \times 5$
285. 92828 by $9 \times 9 \times 9$
286 62432 by $11 \times 11 \times 11$.
287. 710818 by $3 \times 5 \times 7 \times 9$
288 51327 by $2 \times 7 \times 8 \times 11$.
289 401807 by $12 \times 12 \times 12$
290. 603992 by $7 \times 7 \times 7 \times 7$.
291. 30721 by 490.
292 86203 by 560
293. 14730687 by 8100.
294. 15630293 by 10800.

295 How many *twenty-threes* make 1771?

296 How many *thirty-sevens* make 11248?

297. Find the remainder when 784135 is divided by 47.

298 Find the remainder when 4096371 is divided by 83

299. How often can 61 be subtracted from 19032?

300. How many times can 430 be taken from 146200?

301. The dividend is 3452164, the quotient 1858, find the divisor

302. The dividend is 833382, the quotient 4026, find the divisor

303 How often is 6704 contained in 302129168?

304. How many times is 9897 contained in 823479885?

305. What remains when 97 has been taken as often as possible from 49381?

306. What is the final remainder after 371 has been subtracted as many times as possible from 93075?

307. By what number must 3094 be multiplied to produce 48288058?

308. What number, multiplied by 32643, produces 218773386?

309. The product of two numbers is 501000500, one of them is 3850, find the other

310. The product of two numbers is 57380625, one of them is 7575; find the other

311. Divide two million four hundred and ninety thousand two hundred and one by four thousand and eighty-nine

312 Divide three thousand five hundred and ninety-five millions five hundred and twenty-one thousand six hundred and forty-five by fifty thousand seven hundred and nine

313. The dividend is 582167, the quotient is 762, and the remainder is 761, find the divisor

314 The dividend is 3954007, the quotient is 7060, and the remainder is 407; find the divisor

315 By what number must 2765 be divided in order that the quotient may be 32 and the remainder 13?

316 The quotient resulting from the division of 2338425 by a certain number is 346, and the remainder is 157 Find the divisor

317. The quotient is 231, the divisor 321, and the remainder 123, what is the dividend?

318. The divisor is 243, the quotient is 1306, and the remainder is 92; find the dividend.

319. The divisor is 1890, the quotient 365, and the remainder 52; find the dividend.

320. The divisor is 90213, the quotient 738, and the remainder 1857. Find the dividend.

III. MISCELLANEOUS EXERCISES.

1. Write 7707070 in words
2. Write Thirteen million fifty thousand and eleven in figures
3. Add together 23, 4217, 806, 1785, 149, 30781, and 6034
4. Subtract 170492 from 203510.
5. Multiply 5009 by 4100
6. Divide 1004561024 by 711.
7. Divide 706234 by 63 using short divisions.
8. Simplify $43 - 17 - 21 - 25 + 29 + 3$.
9. In the three volumes of a book there are 1256 pages; the first volume contains 432 pages, the second 396 pages, how many pages are there in the third volume?
10. If, from a sack containing 8900 nuts, 36 nuts were given to each of 245 children, how many nuts would be left in the sack?

---

11. Write in words 41305087
12. Write in figures Seventeen million four hundred and fifty thousand three hundred and two.
13. Add 142857, 285714, 428571, 571428, 714285, and 857142
14. From 312750 take 17096
15. Multiply 94100608 by 37009
16. Divide 5640832 by 4064
17. Find the complete remainder when 55555 is divided by the factors of 88.
18. Simplify $4751 - 235 - 178 - 45 - 86 - 1937$
19. William I. died in 1087 Queen Victoria began to reign in 1837 at the age of 18 How many years after the death of William I was Queen Victoria born?
20. In a page of a newspaper there are 8 columns; in each column 190 lines; and in each line 42 letters How many letters are there in the page?

---

21. Write in words MDCCLXXVII
22. Write 1851 in Roman numerals
23. Find the sum of seventy-nine, ninety, nineteen, eleven hundred, seven thousand and ten, ninety-seven thousand, eighteen hundred and sixty, and twelve.
24. Find the difference between two thousand and two hundred thousand
25. Find the product of 5807 and 2479.

26 How many times is 101 contained in 707707?

27 Cube 61

28 What is that number which when multiplied by 13 gives the product 59514?

29 A man aged 42 is three times as old as his son, how old was he when his son was born?

30 A train consisted of 17 carriages, each capable of seating 22 persons There were 293 passengers in the train How many seats were vacant?

---

31. Write in figures MCCCCXIII.

32. Express 1603 in Roman numerals

33. Add together eight hundred and seventy, twenty-one, one thousand six hundred and twelve, thirteen thousand nine hundred and eighty, fifty-three, and ten thousand and nine

34. From five hundred and eighty millions five hundred and three thousand and ninety-nine, subtract thirty millions sixty-three thousand and sixty-six

35. Find the continued product of 47, 407, and 4007

36. How many times can 111 be subtracted from 333333?

37. Square 7063

38. The multiplier is 17 and the product is 511377 Find the multiplicand

39. Seventeen years ago a man was four times as old as his son, whose present age is 29 What is the father's present age?

40. In a certain town there are 117 more females than males The number of males is 4232 Find the population of the town.

---

41. Write down separately in words the value of each 5 in the number 523517.

42. Find $148500 \div 12375$

43. Find $1230401 - 487920$

44. Find 70209×302050

45. Find $87365 + 968476$

46. Write down *at sight* the value of $74 \times 10 \times 2 \times 10 \times 5$.

47. Simplify $42 - 13 + 24 - 35 + 26 - 47 + 6$

48. Find, without waste of labour, the quotient and remainder of $47506803 \div 73000$

49. Find, shortly, $357 + 357 + 357 + 357 + 357 + 357 + 357$

50. The sum of two numbers is 1872, the less is 746; find their difference

---

51. Write down separately in words the value of each 3 in the number 230731.

52. Find $2345538 \div 90213$.

53. Find $60728 + 13409$.

54. Find 70083×43572.

55. Find $176305 - 84250$.

56. Write down *at sight* the value of $460000 \div 2000$.

57. Simplify $41 - 35 + 8 - 56 + 23 - 17 + 41$.

58. Find, without waste of labour, the value of $2396 \times 2 \times 7 \times 50$.

59. Find, shortly, $384 - 47 - 47 - 47 - 47 - 47 - 47 - 47 - 47$.

60. The difference between two numbers is 953; the greater is 4687; find their sum.

---

61. Express 2300025015 in words.

62. Take the sum of 55, 666, and 7777 from 8888

63 Multiply 4283 by 987
Hence, without further work, write down the product of 4283 and 90, also the product of 4283 and 8000.

64. Divide 7398758 by 121, using short divisions.

65. Find, by inspection, the continued product $79030 \times 5 \times 2 \times 2 \times 5$.

66. Find the value of $856 - 394 + 289 + 436 - 752 - 123$.

67. How many times must 312 be added to 4321 that the sum may be 49561?

68 Show that the square of 40 together with the square of 9 is equal to the square of 41.

69. How many figures are made in writing down all the numbers from 1 to 99?

70. A has 63 marbles and B has 29 How many must A give B that they may each have the same number?

---

71. Express in figures, Thirteen million four hundred thousand and fifty-seven

72. Simplify $3835 - 943 - 807 - 1279 - 56 - 644$.

73 Find the product of the sum and difference of 6713 and 6371.

74 By how much does the square of 31 exceed the square of 29?

75 Find, by inspection, the quotient and remainder of $37053 \div 100$

76. Divide 2369360 by 7 and the result by 8.
Hence, without further division, write down the quotient of $2369360 \div 560$, also the quotient of $2369360 \div 70$.

77. How many times must 79 be taken from 3716 in order to leave 319?

78 Take any three consecutive numbers and show that the sum of the greatest and least is equal to twice the middle number

79 Write down the greatest possible number of four figures

80. Find a number which is as much less than 375 as it is greater than 213

---

81 Express 874 in Roman numerals

82 Write three and a half millions in figures

83. What number must be added to seven millions four hundred and six thousand five hundred and four that the sum may be eight millions?

84 Divide the product of 8576453 and 6839 by 10000

85 Divide 678413 by 17
Hence, without further work, write down the quotient and remainder of $678413 \div 1700$

86 Simplify $13 - 27 + 8 - 41 + 23 - 14 + 53$

87. What number, divided by 71, gives quotient 701 and remainder 17?

88 By how many does the square of 31 exceed the product of 29 and 33?

89. Seven years ago a man was six times as old as his son who will be 21 eight years hence What is the father's present age?

90 How far would a boy have to walk to bring 10 oranges, placed in a row and 12 yards apart, one by one to a basket close to the first orange?

---

91 Express in figures Twenty-three million thirty thousand and five

92. Find the difference between three score and ten, and two and a half dozen.

93 Take the sum of 1307, 587, 9630, and 23 from 22222

94. Find the value of $730 \times 62 \times 51 - 4$

95. Divide 570021 by 770, using short division

96. By what number must 594 be multiplied that the result may be equal to the product of 702 and 429?

97. Divide 359907 by 789, and prove the result by casting out nines

98. Find the value of $5^2 + 6^2 + 7^2 + 8^2$

99. A chapter of a book began at the top of page 979 and ended at the bottom of page 1010 how many pages were in the chapter?

100. A farmer has 13 horses, twice as many pigs, five times as many oxen, and sixteen times as many sheep; how many animals does he possess?

101. Express 7306002015 in words.

102. Divide 675432 by 43000.

103. By how much does the product of 567 and 809 exceed their sum?

104. If the divisor, quotient, and dividend be respectively 123 312, and 38389, what is the remainder?

105. If a man were 7 years older than he is, he would be twice as old as his son, who was 38 last year. How old is the father?

106. Prove that the cube of 41 is equal to the sum of the squares of 236 and 115.

107. Find, shortly, $4631 + 247 + 247 + 247 + 247 + 247 + 247$.

108. Simplify $8 \times 7 + 5 \times 9 + 3 \times 2$.

109. Multiply 8763 by 99 with as little labour as possible.

110. Find, shortly, the product of 37173 and 25.

---

111. Write in words the number $\overline{\text{X}}$DCCCLXI.

112. How many figures are there in all the numbers from 1 to 200?

113. In a division sum the divisor is eight times, and the quotient seven times, the remainder. The remainder is 452. Find the dividend.

114. How many boxes, each capable of holding 173, would be required for 100000 oranges, supposing that a few were allowed to remain unpacked?

115. A book of 356 pages has on the average 40 lines on a page, and 52 letters in a line. How many letters are in the book?

116. Edward has 41 marbles, Frank has 29, how many must Edward give Frank that they may each have the same number?

117. Six years hence a man will be three times as old as his son whose present age is 13. How old was the father when the son was born?

118. Simplify $3 \times 15 - 8 \times 13 + 5 \times 12$.

119. Multiply 27864 by 11 with as little labour as possible.

120. Find, shortly, the product of 8643 and 125.

---

121. What do you understand by 34?

122. Write in words 70007007077.

123. Add together the sum, difference, product, and quotient of 537152 and 763.

124. What number subtracted 73 times in succession from 1497 will leave 37 remaining?

125. How old is a man who, 28 years ago, was 29 times as old as his son whose present age is 30?

126 Divide 240 into two parts, one of which shall be seven times as great as the other

127. Find, by inspection, the value of, $2 \times 2 \times 3 \times 7 \times 5 \times 5$

128. Find a number which exceeds the square of half 42 by unity.

129. Multiply, as shortly as you can, 7864 by 999

130. Multiply 3612 by 497 by a short method

---

131. In the number 658457, by how many does the value of one of the 5's exceed that of the other?

132 Divide 143 into two parts, the difference of which shall be 7

133 What number multiplied by 119 gives the same product as 187 multiplied by 133?

134. The total number of guests at a ball was 163 there were 17 more ladies than gentlemen, how many ladies were there?

135. The continued product of 35, 36, and a third number is 165060, find the third number

136 Take any seven consecutive numbers, and show that their sum is seven times the middle number

137 Divide 1554584788 by 2197

138. Find the value of $7 + 3 \times (12 + 5)$

139. Divide 97 into two parts whose difference shall be 13

140. Multiply 2781 by 357 in two lines

---

141. Write $\overline{\text{X}}$DCCXLIX in words

142. Find the sum of the greatest and least numbers each of five digits

143. By how much does the cube of 73 exceed the square of 623?

144. The dividend is 18732510, and the quotient is 2643, find the divisor and remainder

145. The product of 79 and 86 is 6794 Find, without multiplication, the product of 78 and 86.

146. Find, shortly, $3876 - 237 - 237 - 237 - 237 - 237 - 237$.

147. James had 14 more marbles than William; he gave William 6 how many more than William had he then?

148. Simplify $73 \times 13 - 12 \times 72 + 1$

149. Multiply 387654 by 98 as shortly as possible

150. Find $47091000 \div 125$ without actually dividing.

---

151. Subtract seventeen times three thousand and nine from half a million.

152. Find the value of $78351 - 89463 - 16231 + 30005$.

153. The sum of two numbers is 1767, the less is 572; find their difference.

154. How many bags, each holding 7316 pebbles, must be thrown together to make a heap of more than a million?

154. The sum of two numbers, one of which is treble of the other, is 7228, find them

155. What number is contained as many times in 15929 as 881 is contained in 14977?

156. Write down *at sight* the value of $93 \times 8 \times 125$.

157. What number is that to the double of which if 9 be added the result is 55?

158. Divide 2716 into two parts, one of which shall be three times as great as the other

159. Simplify $(7 + 3) \times 12 + 5$.

160. Find, shortly, 932571×125.

---

161. Find the difference between the greatest and least numbers which can be expressed by the digits 1, 5, 7, 3, 9.

162. Divide 823479885 by 9897.

163. Find, by inspection, $5 \times 10 \times 2 \times 25 \times 2 \times 5$.

164. The total number of boys and masters in a school is 140. There are 7 masters What is the average number of boys for each master?

165. The total number of votes polled for two candidates at an election was 7397, the successful candidate's majority was 551, how many votes did each receive?

166. Ten years hence a man will be twice as old as his son who was 13 twenty years ago. Find their present ages.

167. Divide 6507 into two parts, the difference of which shall be 723.

168. Find the value of $19 - (12 + 3)$.

169. Multiply 36165 by 287 in two lines.

170. Find. shortly, $47091375 \div 125$.

---

171. Find the sum of the four largest numbers that can be expressed by the four figures 3, 5, 2, 7.

172. Find the product of the sum and difference of 735 and 357.

173. By what number must 27658 be divided that the quotient may be 285 and the remainder 13?

174. In a force of 1000 volunteers 40 are officers· what is the average number of men to each officer?

175. The difference between two numbers is 36, their sum is 110; find them.

176 If from the square of a certain number we subtract 5 the result when divided by 4 yields quotient 11, find the number

177. The sum of the ages of two boys is 23 years, seven years ago one of them was 6; how old are they?

178. Simplify $8 \times 5 - 7 \times (6 - 3)$

179. Find 9368×125 without multiplying.

180. Multiply 2357 by 568 in two lines

---

181. Write in figures Two billion twenty million two hundred

182. What divisor of 145291 yields quotient 481 and remainder 29?

183. Find the complete remainder when 84359752 is divided by $7 \times 5 \times 11 \times 3$ by successive short divisions

184. Find, shortly, the product of 99999 and 23758

185. Find the sum of the seven consecutive numbers the least of which is 351

186 In a match at chess the winner won three games out of every four which he played, he won 18 games altogether, how many did he lose?

187 A cricketer made the following scores during a cricket week — 17, 9, 0, 41, 3, 26 What was his average for the week?

188. If from a certain number we subtract 3, divide the remainder by 3, add 3 to the quotient, and then multiply the resulting sum by 3, the product is 342 Find the number

189. Simplify $8 \times (5 + 7) - 49 \div 7$

190. Multiply 47653 by 512648 in three lines

---

191 Express 6666 in Roman characters

192. Multiply 810901 by 809101, and prove the result by casting out nines.

193 Divide the difference between 80000 and 57735 by 365

194. A certain number when divided by 5, 7, and 11 in succession yielded remainders 2, 4, and 9 respectively, what was the complete remainder?

195. I distributed 79 oranges among 21 children, giving each boy 3 and each girl 5; how many girls were there?

196. The sum of two numbers is 5115, their difference 953, find them

197 A, B, and C have 89 marbles between them, A and B have 43, B and C have 53; how many has each?

198 Find, by inspection, $2 \times 2 \times 5 \times 2 \times 5 \times 7$.

199. Find, shortly, the product of 2367 and 998

200. Find the value of $(14 - 5)(13 - 6)$

---

201. What number must be added to 4037 that the result may be equal to the sum of 19, 505, 650, and 9003?

202. Find, by inspection, $7 \times 11 \times 25 \times 2 \times 2$.

203. Divide 19094867 by 4009. Hence write down the quotient of $3058867 \div 4009$, also the quotient of $252567 \div 4009$

204. The product of 79 and 86 is 6794 hence find, without multiplication, the product of 79 and 85.

205. A farmer one season lost one out of every six lambs born; 235 lived; how many died?

206. A class of 17 boys in an hour worked correctly 12, 16, 14, 9, 6, 3, 11, 8, 13, 13, 5, 0, 12, 8, 17, 6, 0, sums respectively; what was their average?

207. At an election there were two candidates and 4501 votes were recorded. The successful candidate's majority was 397. How many persons voted for him?

208. Find, shortly, the product of 3467 and 9990.

209. Simplify $79 - 2 \times (19 + 17 - 18)$

210. Find, shortly, $566327 \div 25$

---

211. Find the difference between the sum and the continued product of 22, 444, and 5555.

212. Divide 7891242387 by 3493, and prove the result by casting out nines.

213. The quotient is seven times the divisor, the divisor is seven times the remainder, and the sum of all three is 741; find the dividend.

214. Find the sum of 20 consecutive numbers beginning with 32514.

215. The difference between two numbers is 4321, their sum is 8765, find them.

216. Ten years ago the sum of the ages of a man and a boy was 31 years. The man is 15 years older than the boy. Find the present age of each.

217. Simplify $5 + (16 - 3) \times 4 - 1$.

218. Find, shortly, 44678×98.

219. Find, shortly, $938357 \div 2500$.

220. Multiply 380652 by 378546 in three lines.

---

221. If the base of our system of notation were 6 instead of 10, how many units would be represented by 423?

222. Find the product of the sum and difference of the greatest and least numbers of four digits.

223. Divide the sum of the numbers 34651, 96327, 65412, and 61810 by the sum of their digits.

224. A sum in division was correctly worked on a slate, and then partly rubbed out, when all that remained was what is here given Restore the twelve missing figures, whose places are indicated by the asterisks

$$\begin{array}{r}
4\,*\,)\,*\,*\,8\,*\,(\,*\,7 \\
\,8\, \\
\hline
\,6\, \\
\,\,9 \\
\hline
*\,0 \\
\hline
\end{array}$$

225. The sum of two numbers is 999999, their difference is 714285. find them

226. A man aged 50 will, seven years hence, be twice as old as his son will then be, how old is his son now?

227. At an election the total number of votes polled for three candidates was 9452, the successful candidate received 561 more votes than one, and 1702 more votes than the other unsuccessful candidate. What was the exact state of the poll?

228. Find, shortly, the product of 36493 and 9999.

229. Divide, shortly, 463285 by 99

230. Multiply 234678 by 378426 in three steps

---

231. What would 1234 represent if 5 were the base of our system of notation?

232. Find the product of the greatest and least numbers which can be represented by the four digits 1, 3, 5, 7

233. Divide 3422 by 29, and explain the process.

234. The quotient is five times the divisor, and the divisor is three times the remainder. The difference between the quotient and remainder is 868 Find the dividend.

235. A multiplication sum, having been correctly worked on a slate, was partly rubbed out, and all that remained was what is shown here Restore the twelve missing figures whose places are shown by the asterisks

$$\begin{array}{r}
4\,*\,* \\
3\,* \\
\hline
3\,6\,*\,* \\
\,\,7\,* \\
\hline
\,\,3\,*\,* \\
\hline
\end{array}$$

236. A bag contains 450 nuts. The nuts are distributed among 69 children, each boy receiving 5 and each girl 7, and there is then one nut left in the bag. How many boys and how many girls are there?

237. Seven planks are laid side by side, three are placed across them, and so on, in successive layers of 7 and 3, the last being the 23rd layer. How many planks are there in the stack?

238 If the sum of 83, 176, 871, and 120 be added to a certain number, if the result be divided by 43, and the difference between the quotient and 308 be multiplied by 5, the final result is 1335 Find the number

239. Multiply 182357 by 192648 as shortly as possible

240. Find, as shortly as possible, the difference between 495×753 and 495×387.

IIIᴬ. ELEMENTARY DECIMALS.

Give, in words, the meaning of—

1. ·3; ·7, ·9; ·01; ·04, ·06; ·002; ·005; ·008.
2. 2·1, 3·07, 5·009; 12·6; 20·4; 2·04; 103·5.

Write in figures—

3. Four *tenths*; Seven *tenths*; Nine *tenths*
4. Three *hundredths*; Eight *hundredths*, Five *hundredths*.
5. One *thousandth*; Six *thousandths*; Two *thousandths*
6. Four, and one *tenth*, Seven, and five *tenths*
7. Three, and seven *hundredths*, Six, and four *thousandths*
8. Twenty, and five *tenths*; Thirteen, and nine *hundredths*.
9. Give, in words, the number of *tenths* in One unit; Three units; 2; 1·3; 4·6; 3·7.
10. Give, in words, the number of *hundredths* in One tenth; Seven tenths; ·3; ·8; ·15; ·74.
11. Give, in words, the number of *thousandths* in One hundredth; Eight hundredths; ·02; ·06; ·09; ·014; ·028; ·065, ·123; ·508; ·43; ·59.

Write in figures—

12. Twenty-three *hundredths*; Fifty-two *hundredths*
13. One hundred and forty-seven *thousandths*.
14. Two hundred and eight *thousandths*
15. Thirty-four *thousandths*; Nineteen *thousandths*.
16. Two, and seventeen *hundredths*; Five, and forty-one *thousandths*.

Multiply, at sight, the following decimals.—

17. ·34, ·67, 2·15, 4·83, 2·7, 3·1, ·06, ·09, ·4, ·8, ·003, ·008, each by 10.
18. 3 456, ·731, 1·205, ·035, 4·61, ·07, ·2, 5·3, ·009, 1·06, each by 100

Divide, at sight, the following decimals.—

19. 1·4, 3·5, 4·36, 37·2, ·9, ·3, 17·03, 10·7, ·01, ·75, each by 10.
20. 246·7, 301·8, 42·5, 21·3, 8·4, 3·5, ·2, ·7, ·15, ·06, each by 100.

Add together—

21 3·14, 4·7, 1·06, ·53.
22. 1 203, ·87, ·035, 4·9
23. 18 4, 1·84, ·184, 0184
24. 103·7, 10·37, 1·037, ·1037.
25 45·6, 3·08, 937, 14
26. 8, ·16, ·035, 16·6
27. 2·525, ·08, 17·3, ·015.
28 24·5, 8·75, ·95, 10
29. ·21, 6·44, 18·4, ·95.
30. 6·736, 14·02, ·5, ·044.

Subtract—

31. ·96 from 8·75.
32. 1·85 from 3·246.
33. ·084 from ·43
34. 1·725 from 10·5.
35 4·63 from 4·805
36. ·765 from 7·65
37 ·725 from 1·175.
38 1 84 from 5
39. 3·7 from 11·245
40. 10·75 from 17·05.

Find the value of—

41. 2·46 + 13·8 − 7·235.
42. 1·405 + 23·7 − ·865.
43. 2 + 5·7 − 1·25 − 4·6
44. 8·5 + ·635 − 3 − 2·875.

Multiply—

45. 2·47 by 6.
46 1·375 by 9.
47. 13·65 by 12.
48. 1·036 by 23
49. ·285 by 47.
50 20·46 by 65
51. ·154 by 820.
52. 4 35 by 126
53 1·024 by 205.
54. 2·08 by 7300

Divide, as far as three places of decimals, if the division does not end sooner—

55. 43·235 by 5.
56. 2·528 by 8
57. ·53 by 6.
58. ·174 by 20.
59. 23·52 by 16
60. ·765 by 45.
61. 10·5 by 28
62. ·8 by 63
63 57·97 by 17
64. 9·269 by 23.
65. 50·5 by 41.
66 36·25 by 58
67 205 8 by 74
68 2·35 by 87
69 256·4 by 124.
70. 13·1 by 157.

Note.—For further Exercises in Decimals the student is referred to pages 96–99.

III^B. THE METRIC SYSTEM

Read off the number of *metres* in—

1. (i) 3 kilometres, (ii) 7 decametres; (iii) 2 hectometres.

2. (i) 5 *Km.* 750 *m*, (ii) 4 *Km.* 5 *Hm.*; (iii) 6 *Dm* 3 *m.*

3. (i) 1·685 *Km.*, (ii) ·95 *Km.*, (iii) ·5 *Km*; (iv) ·06 *Km.*

Read off as a decimal of a *kilometre*—

4. (i) 358 metres; (ii) 250 metres, (iii) 28 metres, (iv) 5 metres.

5. (i) 3 *Dm* 4 *m*, (ii) 17 *Dm.*, (iii) 2 *Hm* 5 *m*, (iv) 4·5 *m*

Read off the number of *centimetres* in—

6 (i) 17 metres, (ii) 2·45 metres, (iii) ·2 metre; (iv) ·08 metre.

7. (i) 1 *m.* 3 *dm*, (ii) 4 *Dm* 8 *m.*; (iii) ·025 *Km*, (iv) 250 *mm*

Read off as a decimal of a *metre*—

8 (i) 12 centimetres; (ii) 8 centimetres; (iii) 70 centimetres.

9. (i) 4 *cm.* 5 *mm*, (ii) 6 *dm.* 2 *cm*; (iii) 436 *mm*; (iv) 20 *mm.*

10. Express in *metres*. (i) 9 *Dm* 8 *m.* 7 *dm.*; (ii) 6 *m.* 5 *cm.*

11. Express in *millimetres*. (i) 3 *dm.*; (ii) 4 *m.* 20 *cm*; (iii) 1 *m.* 5 *cm.*

12. Express in *grammes*: (i) 13 *Kg.*; (ii) 4 *Kg.* 75 *g*, (iii) 1·65 *Kg.*

13. Express in *kilogrammes*. (i) 1850 *g.*; (ii) 436 *Dg*; (iii) 3 *Kg.* 25 *g.*

14 Express in *litres*. (i) 3 *Hl.*; (ii) 2 *Hl.* 15 *l.*, (iii) 32·5 *Hl.*

15 Express in *hectolitres*: (i) 435 *l*; (ii) 1580 *l.*, (iii) 7000 *l.*

16. Find (in kilometres and metres) the sum of:
4 *Km.* 250 *m*; 1786 *m*; 3·25 *Km.*, 3 *Km* 25 *m.*; and 10·3 *Km.*

17. Find (in metres and centimetres) the sum of
3 *m.* 40 *cm.*; 6·7 *m*, 1 *m.* 3 *cm*; 14·05 *m*; and 24 *cm.* 5 *mm.*

18. Find (in kilogrammes and grammes) the sum of.
3 *Kg.* 7 *Dg*, 14 *Kg.* 70 *g*, 2·6 *Kg*; and 1875 *g*

19. Find (in metres and centimetres) the difference between:
(i) 4·8 *m.* and 195 *cm.*; (ii) 3 *Dm.* and 7 *dm.*

20. Find (in hectolitres and litres) the difference between:
(i) 23 *Hl* 50 *l* and 1708 *l*; (ii) 1·7 *Dl.* and 17 *dl.*

Multiply—

21. 13 *m* 20 *cm* by 8.

22. 4 *m* 5 *cm.* by 34.

23. 2 *Km* 80 *m* by 17.

24. 3 *Km* 475 *m.* by 24

25 4 *Kg.* 250 *g* by 74.

26. 2 *Hl.* 5 *l* by 48.

Divide—

27. 207 *m.* 41 *cm* by 7.

28. 34 *Km.* 200 *m.* by 12.

29. 26 *Dm.* 5 *m* 2 *dm.* by 65

30. 1 *Km.* 3 *Hm.* 1 *Dm.* 4 *dm* by 325.

31. 418 *Kg* 500 *g.* by 18.

32. 6 *Hl.* 29 *l* by 85.

33. Find, to the nearest metre, the quotient of 14 *Km* 35 *m* by 31

34 Find, to the nearest millimetre, the quotient of 13 *m* 40 *cm* by 217.

35. How many times is 3 *m* 4 *dm* contained in 2 *Km* 8 *Hm* 5 *m*?

36. How many times is 3 *cm* 8 *mm* contained in 10 *m* 7 *cm*?

37 How many bits each 14 centimetres long can be cut from a cord 125 metres long, and what is the length of the remnant?

38 How many times can a jug which holds 65 centilitres be filled from a cask containing a hectolitre of wine, and how much wine is then left in the cask?

39 A metre is about 39 4 inches how many complete yards are there in 120 metres?

40 A kilogramme is nearly 2·2 lbs. how many complete pound packets can be made from 217 kilogrammes of coffee?

Find the cost of—

41 37 things at 1 franc 5 centimes each

42. 19 things at 3 marks 80 pfennige each

43. 15 metres of silk at 4 francs 15 centimes per metre

44 24 metres of cloth at 3 francs 95 centimes per metre

45. 27 kilogrammes of coffee at 3 *fr* 35 *c* per kilogramme

46. 17·5 kilogrammes of sugar at 65 centimes per kilogramme.

47. 3 litres 25 centilitres of milk at 30 centimes per litre

48. 6 metres 75 centimetres of cloth at 4 *fr* 50 *c* per metre

49. 74 hectolitres 50 litres of wine at 1 *fr* 75 *c* per litre

50. 21 *Kg.* 500 *g* of butter at 1 *M* 85 *pf* per kilogramme

51. 3725 articles at 80 centimes per hundred.

52. 4250 articles at 3 marks 40 pfennige per hundred

53. If 7 metres of silk cost 39 *fr* 20 *c*, find the cost per metre

54. If 36 *Kg* of coffee cost 81 francs, find the cost per kilogramme

55 If the cost of 17 *Km* of road be 73,100 *fr*, find the cost per metre

56. If 28 *Kg.* of lead are worth 34 *M* 44 *pf*, find the value of 1000 *Kg*

57 How much butter, at 2 francs 5 centimes per kilogramme, can be bought for 116 francs 85 centimes?

58. How many litres of wine, worth 320 francs per hectolitre, can be bought for 5040 francs?

Find, to the nearest centime, or pfennig, the value of—

59 7 *m.* 4 *cm.* of a material worth 8 *fr* 55 *c* per metre

60 3 *Kg.* 75 *g.* of a substance worth 15 *fr.* 48 *c* per kilogramme.

61 1 metre of a material of which 13 *m.* 75 *cm.* cost 85 *fr.* 50 *c.*

62 1 decagramme of a commodity when 23·8 *Kg* cost 800 *M.*

IV. REDUCTION.

MONEY.

Reduce—

1. £113, 17s $10\frac{3}{4}d$ to farthings.
2. £819, 14s $3\frac{1}{2}d$ to farthings.
3. £1111, 11s $11\frac{1}{4}d$ to farthings
4. £999, 19s $9\frac{3}{4}d$ to farthings.
5. £36, 3s $10\frac{1}{2}d$. to halfpence
6. £47, 10s $8\frac{1}{2}d$. to halfpence.
7. £201, 15s. $4\frac{1}{2}d$ to halfpence
8. £222, 2s $2\frac{1}{2}d$ to halfpence.
9. £29, 19s. 9d to threepences
10. £43, 13s. 6d. to threepences.
11. £108, 17s. 3d to threepences.
12. £441, 6s. 9d to threepences
13. £17, 17s 8d to fourpences.
14. £88, 8s 8d to fourpences
15. £145, 19s. 4d to fourpences
16. £302, 2s 4d to fourpences.
17. £51, 15s 6d to sixpences
18. £85, 18s 6d. to sixpences.
19. £257, 11s 6d to sixpences.
20. £666, 6s 6d. to sixpences
21. £572, 18s. to florins.
22. £631, 14s. to florins.
23. £3051, 16s to florins.
24. £4235, 8s to florins.
25. £25, 15s. to crowns.
26. £68, 5s. to crowns.
27. £472, 10s. to crowns.
28. £833, 10s. to crowns.
29. £235, 7s 6d. to half-crowns.
30. £643, 2s. 6d. to half-crowns.
31. £457, 12s. 6d to half-crowns.
32. £326, 5s. to half-crowns.
33. £721, 17s. 6d to half-crowns
34. £830, 10s to half-crowns.
35. £21, 2s $4\frac{1}{2}d$. to three-halfpences
36. £42, 14s 9d. to threepences.
37. £77431, 10s to half-sovereigns.
38. £963, 16s. to double-florins.

Reduce—

39. 42035 pence to £, s d
40. 36941 pence to £, s d
41. 100000 farthings to £, s d
42. 444444 farthings to £, s d.
43. 5000 halfpence to £, s d.
44. 7673 halfpence to £, s. d.
45. 60301 halfpence to £, s. d
46. 57003 halfpence to £, s d.
47. 6845 twopences to £, s. d.
48. 9719 fourpences to £, s d.
49. 22222 threepences to £, s. d.
50. 77777 threepences to £, s. d.
51. 37545 sixpences to £, s. d
52. 8603 sixpences to £, s. d.
53. 12371 florins to £, s. d.
54. 17659 florins to £, s. d.
55. 487 crowns to £, s. d
56. 555 crowns to £, s d
57. 1234 half-crowns to £, s. d
58. 4321 half-crowns to £, s. d.
59. 7638 half-crowns to £, s d
60. 9708 half-crowns to £, s. d.
61. 10207 half-crowns to £, s d
62. 20005 half-crowns to £, s. d.
63. 4741 double florins to £, s d
64. 30821 half-sovereigns to £, s. d.
65. 2771 three-halfpences to £, s. d
66. 56307 halfpence to £, s. d.

Reduce—

67. 1781 guineas to shillings
68 4631 guineas to shillings.
69. 473 guineas to pence.
70 175 guineas to farthings
71. 981 florins to twopences
72. 1276 florins to sixpences
73. 487 half-crowns to pence.
74 867 crowns to halfpence
75 7413 crowns to threepences
76. 3295 hf -crowns to sixpences.
77. 4224 sixpences to half-crowns
78 311 half-crowns to farthings
79 7563 halfpence to florins
80 11234 pence to half-crowns.
81. 4872 sixpences to guineas
82. 3875 pence to hf -sovereigns.
83. 77240 threepences to florins
84 2761 fourpences to crowns
85. 100000 farthings to guineas
86. 8888 halfpence to guineas

Reduce—

87. 4440 guineas to pounds
88 8340 guineas to pounds
89. 2873 guineas to pounds
90. 12407 guineas to pounds
91. £13986 to guineas
92. £16317 to guineas
93 £40631 to guineas
94 £36003 to guineas
95. 8762 sixpences to fourpences
96. 5083 fourpences to sixpences.
97. 7031 half-crowns to shillings.
98 8888 florins to half-crowns
99. 998 shillings to half-guineas
100 1032 hf -crns to fourpences
101 674512 half-crowns to florins
102 63021 guineas to half-crowns
103. 15351 crowns to half-guineas
104 14196 florins to half-guineas
105. 189550 hf -guineas to hf -crns
106 463252 half-crowns to florins

107. What would penny stamps for 500 letters cost?

108. What would halfpenny stamps for 3000 circulars cost?

109. How many penny stamps can be bought for £5?

110. How many halfpenny stamps can be bought for 3 half-crowns?

111 What would be the cost of a dinner for 412 persons at two shillings a head?

112. How many people could receive half a crown a-piece from a fund of £142, 12*s* 6*d* ?

113 How many five-pound notes should be given in exchange for 1560 half-crowns?

114 How many pencils at 1½*d*. each could be bought for 9*s* 6*d* ?

115. What income-tax, at 6*d* in the pound, is paid by a man who has £550 a year?

116. How much income-tax, at 4*d* in the pound, would be paid by a man whose income was £953?

TIME.

Reduce—

117. 19 hrs. 33 min 42 sec to seconds.

118. 23 hrs. 41 min 5 sec to seconds

119. 17 days 13 hrs 47 min. to minutes

120. 23 days 11 hrs. 39 min. to minutes.

121. 6 wks 2 days 18 hrs to hours.

122. 5 wks. 3 days 15 hrs to hours.

123. 3 com. yrs. 217 days 17 hrs. to hours

124. 2 com. yrs 189 days 13 hrs to hours

125. 2 com yrs 37 days 1 hr 47 min to minutes.

126. 3 wks 6 days 10 hrs. 13 secs to seconds

Reduce—

127. 31423 secs to hours &c.

128. 57029 secs. to hours &c.

129. 17962 mins to days &c.

130. 20765 mins. to days &c.

131. 320841 secs. to days &c.

132. 479073 secs. to days &c.

133. 475905 mins. to weeks &c.

134. 27630820 secs. to weeks &c.

135. 1372573 mins. to yrs., dys. &c

136. 2007603 mins. to yrs., dys &c.

137. How many hours are there altogether in the months of April and May?

138. How many minutes are there in the three months September, October, and November?

139. How many hours are there between 11 A.M. on Monday and 7 P.M. on the following Wednesday?

140. How many hours are there between 9 A.M. on Tuesday and 9 P.M. on the following Saturday?

141. How many minutes are there between 7.30 A.M. and 5.15 P.M. on the same day?

142. How many seconds are there between 5 minutes past 2 o'clock P.M. and a quarter to 5 P.M. on the same day?

143. How many days were there from the beginning of the year till the end of August 1890?

144. How many days were there in the first 9 months of the year 1884?

145. How many days were there altogether in the five years 1887, 1888, 1889, 1890, and 1891?

146. How many hours were there altogether in the years 1891 and 1892?

Avoirdupois Weight

Reduce—

147. 17 cwt. 1 qr 13 lbs to lbs	148. 9 cwt 3 qrs 24 lbs to lbs.
149. 1 qr. 11 lbs 7 ozs to ozs	150. 2 qrs 7 lbs 13 ozs to ozs
151. 13 cwts 13 lbs 13 ozs to ozs	152 17 cwt 8 lbs 12 ozs to ozs
153. 17 tons 3 cwts 20 lbs to lbs	154. 23 tons 17 cwt 15 lbs to lbs
155. 3 qrs. 10 lbs 7 ozs to ozs	156 19 lbs 11 ozs 7 drs to drams

157. 2 tons 13 cwt 1 qr 15 lbs 7 ozs to ounces
158. 7 tons 11 cwt 3 qrs 25 lbs 11 ozs. to ounces
159 13 tons 6 cwt 3 qrs 24 lbs 10 ozs 7 drs to drams
160 52 tons 6 cwt 2 qrs 17 lbs 15 drs to drams

161 3 cwt. 3 qrs. 1 st. to stones	162 7 tons 15 cwts 1 qr to stones.
163. 1 ton 11 cwt. 111 lbs to lbs	164 4 tons 3 cwt 73 lbs to lbs
165 5 cwt. 93 lbs 6 ozs. to ounces	166 18 cwt 89 lbs 11 ozs to ozs
167. 2 cwt. 1 qr 5 lbs. to grains	168 5 cwt 2 qr. 17 lbs to grains

Reduce—

169. 1111 ozs to quarters &c	170. 1503 ozs to quarters &c
171 4371 drams to pounds &c	172. 5029 drams to pounds &c
173. 10729 lbs to tons, cwts &c	174. 47568 lbs to tons, cwts &c
175. 16417 ozs. to cwts. qrs &c	176 19382 ozs to cwts qrs &c
177. 568430 drs. to cwts. qrs &c	178. 780451 drs to cwts qrs &c
179 23075 ozs to cwts qrs. &c.	180. 33333 ozs to cwts qrs &c
181. 43970 ozs. to tons, cwts &c	182 80307 ozs to tons, cwts &c
183. 648327 ozs to tons &c	184 4765183 ozs to tons &c.
185. 7657938 ozs. to tons &c	186 1147274 drams to tons &c.
187. 5555555 drams to tons &c	188. 9179065 drams to tons &c

Troy Weight.

Reduce—

189. 3 lbs. 7 ozs. 14 dwt. 5 grs to grains.
190. 7 lbs. 7 ozs. 7 dwt 7 grs to grains

191. 5 lbs. 11 ozs. 18 dwt. to grs	192. 29 ozs 235 grs to grains.
193. 37 ozs 340 grs. to grains.	194. 45 ozs 417 grs to grains

Reduce—

195 7525 grs. to lbs. ozs dwt &c.	196 25525 grs to lbs ozs dwt &c.
197. 87863 grs. to lbs ozs. dwt. &c.	198 87776 grs to ozs Troy
199. 63745 grs. to ozs Troy.	200 90671 grs to ozs. Troy

Long Measure.

Reduce—

201. 17 yds. 2 ft. 11 in. to inches.
202. 29 yds. 1 ft. 7 in. to inches.
203. 56 yds. 1 ft. 9 in. to inches.
204. 88 yds. 2 ft. 10 in. to inches.
205. 4 mi 380 yds to yards.
206. 13 mi 1056 yds. to yards.
207. 3 mi. 931 yds 2 ft. to feet.
208. 5 mi. 1125 yds. 1 ft. to feet.
209. 7 mi 1601 yds. 1 ft. to feet.
210. 10 mi. 1066 yds 2 ft. to feet.

Reduce—

211. 441 in to yds. &c.
212. 693 in. to yds. &c.
213. 1305 in to yds &c.
214. 2071 in. to yds. &c.
215. 76541 yds to mi and yds
216. 84563 yds to mi. and yds.
217. 36581 ft to mi, yds &c.
218. 47891 ft to mi, yds. &c.
219. 358323 in. to mi., yds. &c.
220. 765045 in. to mi., yds. &c.

Reduce—

221. 24 po to yards.
222. 38 po. to yards.
223. 27 po. 2½ yds. to yards.
224. 39 po. 4½ yds. to yards.
225. 18 po. 3 yds. 1 ft. to ft
226. 22 po. 5 yds. 2 ft. to ft.
227. 31 po. to half-yards.
228. 37 po to half-yards.
229. 19 po. 4 yds. to half-yds.
230. 28 po 3 yds. to half-yds.
231. 36 po. to inches.
232. 38 po. 3 yds. to inches.
233. 23 po 2 yds. 9 in. to in
234. 37 po 1 yd. 11 in. to in.
235. 17 po 4 yds. 1 ft 7 in. to in
236. 29 po. 5 yds 2 ft. 8 in. to in.
237. 7 po. 3 yds. 1 ft. to feet
238. 9 po. 4 yds. 2 ft. to feet.
239. 13 po. 4 yds. 1 ft 6 in to ft.
240. 17 po 3 yds. 2 ft 6 in to ft.
241. 8 mi 5 fur. 103 yds. to yards
242. 11 mi. 1 fur 1 ft. to feet.
243. 17 mi. 7 fur. 1 ft. to feet.
244. 19 mi. 1 fur. 119 yds. to yds.
245. 5 mi. 2 ft. 3 in. to in.
246. 7 mi. 4 fur. 78 yds. to yds.
247. 7 mi. 5 fur. 32 po. 4 yds. to yards.
248. 6 mi. 7 fur. 24 po. 5 yds. to feet.
249. 10 mi. 1 fur 10 po. 1 yd 1 ft. to inches.
250. 22 mi 2 fur 22 po. 2 yds 2 ft. to feet.
251. 3 mi 5 fur. 17 po 1 yd 1 ft. to inches.
252. 1 mi 3 fur. 37 po. 3 yds 2 ft. to inches.
253. 3 mi 1 fur. 39 po 3 yds. 2 ft 8 in to inches.
254. 17 mi 3 fur 19 po. 4 yds. to inches.
255. 25 mi. 6 fur. 17 po 4 yds. 3 in. to inches.
256. 43 mi. 5 fur. 23 po. 11 in. to inches.

Reduce—

257 1033 half-yards to poles

258 986 half-yards to poles.

259 3587 half-yards to poles

260 2634 half-yards to poles.

261. 242 yards to half-yds

262 642 yards to half-yds

263. 660 yards to half-yds

264 1634 yards to half-yds

265. 1023 yards to poles

266. 693 yards to poles

267. 735 yards to poles

268 1617 yards to poles

269. 856 yards to poles &c

270. 1234 yards to poles &c

271. 1763 inches to poles &c

272 2904 inches to poles &c

273 3759 inches to poles &c

274. 6888 inches to poles &c

275 273 feet to poles &c

276 381 feet to poles &c

277. 496 feet to poles &c

278 503 feet to poles &c

279. 571 feet to poles &c

280 687 feet to poles &c

Reduce to miles, furlongs, poles &c

281 154737 inches

282. 241985 inches

283 268543 inches

284 527895 inches

285 674381 inches.

286. 3126749 inches

287. 9367875 inches

288 3744576 inches

289. 10000 yards.

290 44444 yards

291. 57383 yards

292 1234567 yards

293. 7777777 yards.

294 112566 feet

295. 1000000 feet.

296. 103962 feet

297. 1847638 feet.

298. 6754321 feet

299. 101010101 inches.

300 100000000 inches.

Square Measure

Reduce—

301. 3 sq yds 7 ft 19 in to sq in

302 17 sq yds 8 ft. 79 in to sq in

303. 2 sq yds. 1 ft 128 in to sq in.

304. 15 sq yds 3 ft 141 in to sq in

305. 72 ac. 3 ro. 19 po to sq po

306. 131 ac 1 ro 37 po to sq po

307. 143 ac. 1 ro 25 po to sq po

308 89 ac 3 ro 21 po to sq po

Reduce—

309. 6453 sq in. to sq yds. &c

310. 8507 sq in to sq yds. &c

311. 70538 sq in. to sq yds. &c.

312 123756 sq in to sq yds &c.

313 3651 sq po to acres &c

314 7408 sq po to acres &c.

315. 73217 sq po. to acres &c.

316. 90573 sq. po. to acres &c.

Reduce—

317. 24 sq. po. to sq yds.
318. 36 sq. po to sq. yds.
319. 20 sq po. 18 sq yds to sq yds
320. 32 sq po. 27 sq. yds. to sq. yds.
321. 18 sq po to sq yds.
322. 22 sq. po. to sq. yds.
323. 17 sq po to sq yds.
324. 29 sq po to sq yds.
325. 14 sq. po 17½ sq yds. to sq yds
326. 21 sq po 13 sq yds to sq. yds.
327. 4 sq po. 23 sq yds to sq. ft
328. 8 sq po 30 sq. yds. to sq. ft
329. 6 sq po 4½ sq ft to sq. ft
330. 10 sq. po 27 sq. yds. to sq ft
331. 17 sq. po 5¾ sq yds to sq. ft
332. 19 sq. po. 12¼ sq yds. to sq ft
333 30 sq po. to sq in.
334. 21 sq. po. to sq in.
335 27 sq po to sq in.
336. 39 sq po to sq. in.
337. 5 sq po. 17 sq yds to sq in.
338. 7 sq po 28 sq yds to sq. in.
339. 7 ac 2 ro 32 po to sq. yds.
340 4 ac 3 ro. 36 po. to sq yds.
341. 2 ac. 19 po to sq. yds.
342. 4 ac. 37 po to sq yds.
343. 5 ac 870 sq. yds. to sq. yds
344. 9 ac 2003 sq. yds. to sq. yds.
345. 17 ac 138 sq. yds. to sq yds.
346. 27 ac. 450 sq. yds. to sq. yds

347. 3 ac. 2 ro. 27 po. 27 yds. 7 ft. 25 in. to square inches.
348. 8 ac. 2 ro. 34 po. 3 ft. 87 in. to square inches.
349. 53 ac. 21 po. 8 ft 125 in. to square inches.
350. 3 ac. 3 ro. 33 po. 3 yds. 3 ft. 33 in to square inches.

Reduce—

351. 616 sq yds. to sq po
352. 851 sq yds to sq po
353. 267 sq yds. to sq po &c.
354. 353 sq yds. to sq po &c
355. 917 sq. yds to sq po &c
356. 1061 sq yds. to sq po. &c.
357. 882 sq ft to sq. po &c
358. 1234 sq. ft. to sq po &c.
359. 180123 sq in to sq. po. &c.
360. 246843 sq in to sq po. &c

Reduce to acres, roods, poles &c.

361. 11495 sq. yds.
362. 83061 sq. yds.
363. 37495 sq yds.
364. 657345 sq. ft.
365 562936 sq. ft
366. 1000000 sq. ft.
367. 7865432 sq in
368. 16019400 sq in.
369. 25607809 sq in.
370. 54650895 sq in.
371. 895487 sq yds
372. 3333333 sq ft.
373. 333274481 sq. in.
374. 99597888 sq. in.

Cubic Measure

Reduce to cubic inches—

375. 17 cub ft 1335 in
376. 18 cub ft 1073 in
377. 1 cub yd 24 ft 760 in
378. 1 cub yd 18 ft 631 in
379. 4 cub. yds 13 ft 5 in
380. 9 cub yds 21 ft 875 in.

Reduce to cubic yards &c.

381. 143562 cub in
382. 764591 cub in
383. 847325 cub in.
384. 987407 cub in
385. 4831850 cub. in
386. 8001304 cub in

Capacity

387. 13 gal. 3 qt. 1 pt to pints
388. 19 gal 1 qt 1 pt to pints.
389. 18 gal. 1 qt to pints
390. 23 gal 1 pt to pints
391. 875 pints to gallons &c
392. 678 pints to gallons &c
393. 1232 pints to gallons &c
394. 1805 pints to gallons &c
395. 7 qrs 3 bush 2 pks. to pecks.
396. 9 qrs. 6 bush 2 pks to pecks
397. 121 qrs. 1 bush 1 pk 1 qt to quarts
398. 89 qrs 3 bush 3 pks 1 gal 1 pt to pints
399. 2559 pints to qrs, bush. &c
400. 33333 qts to qrs. &c.

Miscellaneous

401. Express 225 ozs Troy, in lbs. Avoirdupois
402. Reduce 2 miles 71 chains 36 links to links
403. Reduce 3 yds 3 qrs 3 nails to inches
404. Reduce 7 square miles to square yards
405. In 3 barrels of beer how many pints?
406. Reduce 17 reams 13 quires 11 sheets to sheets.
407. Reduce 31° 47′ 29″ to seconds
408. How many knots are equivalent to 38 miles?
409. Reduce 14520 square chains to acres
410. How many Troy ozs. are equal to 6 cwts.?

V. THE COMPOUND RULES.

Compound Addition

	£	s.	d.		£	s.	d.		£	s.	d.
1.	71	3	2	2.	408	13	10½	3.	245	6	2¼
	32	17	1½		176	13	7		8139	7	4¼
	361	8	9¼		109	19	10½		94	3	4½
	917	10	8		36	11	4		9065	13	8¾
	2752	15	3¾		79	8	6¾		1067	13	8
	526	1	1½		8	13	10¼		2935	8	1¾
	45	2	11¾		5	8	2		239	2	5
	52	1	3		21	18	9½		163	9	11¾
	3041	2	7¼		40	15	6¾		752	15	3

	£	s.	d.		£	s.	d.		£	s.	d.
4.	416	9	8	5.	287	14	7	6.	207	4	6
	63	9	11		19	3	8		94	3	4
	385	13	6		21	18	9		211	6	8
	65	13	8		139	7	4		9035	2	10
	7	13	8		5	8	1		409	12	8
	612	3	2		39	2	5		61	10	8
	46	3	6		6904	7	3		492	13	7
	1130	1	4		752	15	3		56	4	10
	79	12	9		7245	6	2		287	9	11
	287	6	8		85	2	2		9603	2	5
	2307	4	2		109	19	10		408	13	10
	784	9	11		2015	1	2		8043	7	9

	£	s.	d.		£	s.	d.		£	s.	d.
7.	9416	9	8	8.	8204	15	2	9.	4321	18	9
	215	9	5		476	3	9		8139	7	4
	691	13	9		1067	13	8		2935	8	1
	239	2	5		734	5	1		239	2	5
	134	6	6		4614	3	3		6904	7	3
	9374	5	7		3198	10	8		752	15	3
	752	15	3		57	2	4		7245	6	2
	2935	8	1		1316	9	8		485	2	2
	4321	18	9		4917	10	8		109	19	10
	196	3	2		6917	11	9		2015	1	2
	651	4	3		976	2	5		210	14	8
	360	5	5		8416	9	10		6218	12	8
	4201	6	8		5618	7	6		783	8	7
	476	13	4		9314	2	9		1397	7	1
	2376	0	9		1618	2	5		691	13	9

10.

£	s	d
643	17	6
485	2	2
8139	7	4
6409	12	8
239	2	5
4561	10	8
612	3	2
2935	8	1
6218	12	8
109	19	10
46	3	6
4201	6	8
94	3	4
2015	1	2

11.

£	s	d
817	12	6
6127	9	10
408	13	10
1067	13	8
8385	13	6
631	7	7
8279	12	9
360	5	5
9532	8	7
1130	1	4
3	2	5
6904	7	3
563	4	1
163	9	11

12.

£	s	d
4391	16	9
4123	17	6
211	6	8
9035	2	10
7245	6	2
207	4	6
9065	13	8
56	4	10
3276	5	9
691	13	9
2307	4	2
362	3	4
1287	14	7
5976	2	5

13.

£	s	d
827516	3	9
734321	18	9
16391	1	4
76899	13	7
897	15	3
11835	0	3
403750	5	8
274032	12	1
528636	4	11
675940	18	2
94636	1	1
16391	1	4
691040	3	7
153748	2	9
95483	17	10
765491	9	5
673159	0	1
31140	15	6
310397	8	7

14.

£	s	d
29215	9	5
326408	13	10
639875	12	3
810732	17	1
106747	18	3
74321	18	9
42361	8	9
23109	19	10
6679	14	5
810327	4	4
97243	17	11
239763	2	1
112935	8	1
93198	10	8
310397	8	7
228416	9	10
20242	0	7
299653	1	8
7187	7	7

15

£	s	d
16391	1	4
6679	14	5
23109	19	10
376514	10	5
528636	4	11
47983	19	3
810732	17	1
274032	12	1
94636	1	1
291414	12	4
17218	9	6
714961	13	11
93198	10	8
153748	2	9
34728	12	6
765491	9	5
673159	0	1
31140	15	6
299653	1	8

16. Add together £8385, 13*s* 6*d*, £8019, 3*s* 8½*d*, £9416, 2*s* 8*d*, £207, 4*s* 6¼*d*, £612, 3*s* 2¾*d*, £46, 3*s* 6*d*, £1130, 1*s* 4*d*, £8279, 12*s* 9½*d*, and £4321, 18*s* 9½*d*.

17 Find the sum of £3241, 11*s* 6*d*, £163, 9*s* 11*d*, £5, 13*s* 6*d*, £9065, 13*s* 8*d*, £1067, 13*s*. 8*d*, £612, 3*s* 2*d*, £46, 3*s* 6*d*, £30, 1*s*. 4*d*, £8279, 12*s* 9*d*, £87, 6*s* 8*d*, £307, 4*s* 2*d*, £2784, 9*s* 11*d*, £5, 16*s* 4*d*, £27, 9*s* 10*d*., and £201, 6*s* 8*d*

18. Find the sum of £106747, 18*s* 3*d*, £29215, 9*s* 5*d*, £76899, 13*s* 7*d*, £321, 18*s*. 9*d*, £750, 5*s* 8*d*, £6, 3*s* 9*d*, £61, 8*s*. 9*d*, £97, 11*s* 7*d*, £243, 17*s* 11*d*, £408, 13*s* 10*d*, £321, 18*s* 9*d*, £374, 5*s* 7*d*, £9763, 2*s* 1*d*., £9642, 5*s* 2*d*, £83, 17*s*. 10*d*, £263, 9*s*. 5*d*., £397, 8*s*. 7*d*., £97, 8*s*. 7*d*., and £649603, 2*s*. 5*d*

19.

days	hrs.	min.	secs
17	21	36	51
11	20	52	57
1	14	39	25
22	19	49	46
38	7	17	38

20.

days.	hrs.	min.	secs.
4	17	25	39
3	21	55	17
1	19	48	56
2	16	51	3
	3	7	42

21.

yrs.	days.	hrs
4	223	17
5	317	12
1	47	23
19	199	7
2	13	15

22.

yrs.	days.	hrs.
17	201	13
9	333	21
7	149	11
27	73	9
14	351	19

23

cwts.	qrs.	lbs.	ozs.
1	2	13	5
7	1	15	11
	3	14	2
11	0	17	9
1	1	5	14

24.

qrs.	lbs.	ozs.	drs
1	17	13	2
	27	11	15
3	9	7	3
2	26	14	13
	19	12	12

25.

tons.	cwts.	qrs.	lbs.
5	13	2	19
1	16	1	23
18	9	3	10
4	16	0	27
12	7	1	9

26.

tons.	cwts.	qrs.	lbs	ozs
3	17	2	25	13
1	1	3	11	7
14	10	0	20	2
7	15	1	8	11
	5	2	19	15

27.

cwts.	lbs.	ozs.
1	102	7
2	89	10
5	41	15
1	98	5
	81	1

28.

tons.	cwts.	lbs.	ozs.
3	17	89	8
1	12	29	11
5	19	98	6
	2	23	15
12	14	88	3

29.

lbs Tr	ozs	dwts.	grs
2	1	7	15
7	11	13	1
19	2	7	19
3	5	2	5
4	0	17	23

30.

ozs Tr	grs
27	412
30	389
1	89
2	461
15	123

31.

yds	ft	in
3	1	7
1	2	11
7	0	9
	2	10
2	1	8

32.

yds	ft	in
4	2	6
13	1	10
1	1	11
8	2	5
3	2	11

33.

mi	yds
4	225
1	1026
2	1320
3	486
5	1237

34.

mi	yds	ft	in
7	85	1	7
1	180	2	3
2	1100	1	8
11	1625	0	2
2	977	1	11

35.

po	yds.	ft.	in
31	5	2	11
2	3	1	7
17	4	1	8
8	3	2	10
29	1	1	9

36

mi	fur	po	yds	ft.
7	7	27	5	2
3	6	15	2	1
9	1	7	1	0
11	4	29	2	2
	5	38	4	2

37.

fur.	po	yds	ft	in.
1	31	3	1	7
2	33	4	2	10
3	17	3	2	8
7	12	2	2	9
4	5	1	0	11

38.

mi	chains	links
3	39	55
1	68	82
2	27	41
10	74	93
5	59	70

39.

sq yds	sq ft	sq in
7	8	79
1	3	112
11	1	45
5	0	131
4	7	68

40.

sq yds	sq ft	sq in
1	5	88
2	8	112
1	6	109
3	2	93
10	4	125

41.

ac.	ro.	sq. po
79	2	31
2	1	28
1	3	5
221	0	37
2	2	21

42.

ac	ro	sq po
13	1	27
5	3	29
41	2	16
9	0	38
52	1	8

43.

ac	ro	sq po	sq yds
1	2	27	20
2	1	18	25
	3	31	16
5	0	7	12
	1	16	8

44.

ac	ro	sq po	sq yds.
7	1	25	20
1	3	28	29
2	3	35	17
3	1	32	28
	2	21	13

45.

ac	ro.	sq po	sq yds.	sq ft
4	3	29	29	8
7	1	34	17	5
11	2	17	18	1
	1	24	23	6
	3	12	14	1

46.

sq po	sq yds.	sq ft	sq in
31	28	7	112
7	16	5	84
28	25	3	56
19	27	8	138
23	30	1	79

47.

cub yds.	cub ft.	cub in.
14	5	107
2	17	1234
7	24	389
21	12	43
1	18	497

48.

cub yds.	cub ft.	cub in.
22	22	222
173	15	1191
25	19	985
17	5	637
204	26	1680

49.

gals.	qts.	pts.
3	1	1
17	3	0
5	2	1
13	1	1
7	3	0

50.

qrs.	bush.	pecks.	gals.
13	5	3	1
5	7	2	1
18	2	2	0
29	6	1	1
16	3	0	1

COMPOUND SUBTRACTION.

51. Subtract £1, 11*s.* 11$\frac{3}{4}$*d.* from £10, 1*s.* 2$\frac{1}{2}$*d.*

52. Subtract £21, 11*s* 11$\frac{3}{4}$*d.* from £25, 16*s.* 4*d.*

53. From £4586, 7*s.* 9$\frac{1}{4}$*d.* take £1397, 18*s.* 10$\frac{1}{2}$*d.*

54. From £8133, 11*s.* 6$\frac{1}{2}$*d* take £533, 13*s* 8$\frac{3}{4}$*d.*

55. Find the difference between £3, 17*s.* 10$\frac{1}{2}$*d.* and £5.

56. Find the difference between £113, 9*s.* 2*d.* and £97, 17*s* 8$\frac{3}{4}$*d.*

57. What must be added to £29, 13*s* 5$\frac{1}{2}$*d* to make £31, 12*s* 2*d.*?

58. What must be taken from £87, 6*s* 2$\frac{1}{4}$*d.* to leave £79, 9*s* 4$\frac{1}{2}$*d.*?

59. By how much does £1001, 1*s.* 1*d.* exceed £909, 19*s* 9*d*?

60. By how much does £978, 19*s* 2*d* fall short of £1000?

Subtract—

61 2 hrs 40 min. 25 secs from 11 hrs. 8 min. 15 secs.
62. 12 hrs 37 min 51 secs from 19 hrs 5 min 35 secs
63 8 days 17 hrs 45 min from 12 days 3 hrs 21 min
64 2 days 13 hrs 35 min 47 secs from 5 days 7 hrs

65 8 lbs 13 ozs 7 drams from 13 lbs 5 ozs 1 dram
66 1 qr 23 lbs 11 ozs from 3 qrs 11 lbs 3 ozs
67. 17 tons 14 cwts. 2 qrs from 25 tons 6 cwts 1 qr.
68. 23 tons 17 cwts 3 qrs 19 lbs from 35 tons 13 cwts 1 qr
69 5 tons 17 cwts 2 qrs. 15 lbs from 7 tons 14 cwts 2 qrs. 10 lbs
70 3 cwts. 1 qr 27 lbs 15 ozs from 1 ton 1 qr 11 lbs. 11 ozs
71 17 tons 15 cwts 108 lbs 10 ozs from 23 tons 56 lbs
72. 13 tons 17 cwts 1 qr. 19 lbs. 9 ozs 13 drs from 20 tons.

73. 3 lbs Tr 5 ozs 17 dwts 19 grs from 11 lbs Tr 1 oz 14 dwts.
74. 13 ozs Tr 256 grs. from 27 ozs Tr. 120 grs

75. 3 yds 2 ft 9 in from 17 yds 1 ft 4 in
76. 14 yds 1 ft 10 in from 23 yds 0 ft 7 in
77. 5 mi. 967 yds. 2 ft from 13 mi 241 yds 1 ft.
78. 10 mi 1380 yds 7 in from 32 mi 47 yds 2 ft
79. 156 mi. 4 fur 216 yds from 191 mi 2 fur 2 yds
80. 3 fur. 33 po 3 yds from 1 mi 17 po 5 yds
81. 7 mi. 7 fur. 37 po 3 yds 1 ft 7 in from 8 mi 26 po
82. 2 mi. 22 po 2 ft from 5 mi 5 fur. 5 yds 5 in

83. 3 sq yds. 7 sq ft 117 sq in from 8 sq yds 2 sq ft
84. 13 sq. yds. 8 sq. ft 132 sq in from 21 sq yds 89 sq in
85. 23 ac. 3 ro 29 po from 71 ac 2 ro 13 po.
86. 198 ac 2 ro. 37 po from 305 ac 2 ro 14 po
87. 2 ro 27 po 28 sq yds from 3 ro 25 po 18 sq. yds
88. 3 ro 19 po 30 sq yds from 3 ac 2 ro 1 po
89. 17 ac. 3130 sq yds. from 23 ac 1760 sq yds.
90. 99 ac. 1873 sq. yds from 1 sq mi. 230 ac
91. 3 ro 29 po. 27 sq. yds 8 sq ft. from 2 ac 2 ro
92. 39 po. 17 sq. yds. 98 sq in from 1 ac 1 ro 1 sq ft
93. 2 ro. 32 po 12 sq yds 6 sq ft 10 sq in from 2 acres
94. 3 ac. 3 ro. 33 po 3 sq yds. 3 sq. ft. 33 sq. in from 5 acres.

95. 12 cub. yds. 22 ft. 1222 in. from 21 cub. yds. 2 ft. 221 in.
96. 276 cub. yds. 987 in. from 456 cub. yds. 17 ft.

97. 19 gal. 3 qts. 1 pt. from 39 gal. 1 qt.
98. 3 bush. 2 pks. 1 gal from 7 bush. 1 pk.
99. 357 qrs. 7 bush. from 402 qrs. 3 bush.
100. 89 qrs. 3 bush. 1 pk from 157 qrs. 1 bush.

Find the value of—

101. 2*s* 6*d.* + 8*s.* 7½*d* − 4*s.* 3*d.* + 9*s.* 11¼*d.* − 11*s.* 7¾*d.*
102. 14*s.* 2*d.* − 19*s.* 8½*d.* − 17*s.* 11¾*d* − 10*s.* 10½*d.* + 18*s.* 1¼*d.* + 18*s*
103. £1, 12*s.* − £1, 4*s* 7*d* + £2, 15*s.* 4*d.* − £1, 3*s* 11*d* + £3, 19*s* 5*d.*
104. £2, 3*s.* 4*d.* − £5, 17*s.* + £3, 1*s.* 6*d* − £4, 15*s* 10*d* + £6. 6*s.*
105. £24781, 13*s.* 9¼*d* − £31265, 8*s.* 3½*d* + £8503, 11*s* 3*d*
106. £85608, 17*s.* 2½*d.* + £459, 12*s* 11¾*d.* − £77953, 18*s.* 6*d*
107. 4*s* 5*d.* − 9*s.* 8½*d.* + 11*s.* 2¼*d* − 13*s.* 11¾*d.* + 8*s* 6*d* − 2*s* 10*d.* + 15*s* 1¾*d.* − 7*s.* 7*d.* + 5*s.* 1½*d.*
108. 13*s.* 9*d.* − 8*s* 11*d.* − 14*s.* 1*d.* + 2*s.* 6½*d.* + 17*s.* 8*d.* − 4*s.* 9¼*d.* − 18*s* 5½*d.* + 16*s.* 11*d* − 1*s.* 10¼*d* + 19*s.* 2¾*d.*

Find, in one operation, the value of—

109. £20 − £1, 17*s.* 6*d.* − £2, 15*s.* 11*d.* − £7, 13*s.* 8*d.* − £5, 11*s.* 3*d.*
110. £7, 7*s* 7*d.* − £1, 13*s.* 4½*d* − £2, 11*s.* 10¾*d.* − £1, 8*s* 7½*d.* − 14*s.* 9*d*
111. £31, 5*s* 3*d.* − £11, 7*s* 9½*d* − £4, 13*s* 3*d* − £1, 15*s* 1¼*d.* − £9, 12*s*
112. £100 − £21, 10*s* − £5, 3*s* 6*d.* − £17, 19*s* 8*d* − £49, 19*s* 2*d*
113 £5 − (£1, 2*s* 6*d* + 14*s* 5*d* + £1, 17*s* 11½*d* + 13*s* 9¼*d*)
114. £13, 3*s.* 3*d* − (£2, 14*s* 7*d.* + £1, 12*s.* 9*d.* + 17*s.* 5¾*d* + 16*s.* 8½*d*)

115 Take the sum of £2, 11*s* 8*d.*, £11, 19*s.* 2*d*, £4, 13*s.* 9*d*, and £3, 8*s.* 11*d.* from £25, 2*s.* 6*d*

116. From £37, 15*s.* 3*d.* take the sum of £1, 17*s* 5½*d*, 13*s.* 8¼*d.* £13, 16*s.* 9*d.*, and £5, 19*s.* 7½*d*

117. By how much does the sum of £1, 13*s.* 6½*d.*, £2, 4*s.* 10*d*, 18*s* 8½*d*, £2, 12*s.* 6*d.*, 15*s.* 9*d.*, and £10, 18*s* fall short of £20?

118. Subtract the sum of £2, 13*s* 3½*d*, £5, 7*s* 11½*d*, £18, 19*s* 3¾*d.*, £2, 12*s.* 6*d*, and £31, 19*s* 10½*d.* from £86, 4*s* 2*d*

119. How much is left out of £10 after paying bills of 13*s.* 6*d.*, £2, 19*s.* 7*d*, £1, 14*s.* 8*d.*, £3, 12*s.* 11*d.* and 4*s.* 10*d.*?

120. What is left out of £50 after paying bills of £5, 3*s* 8*d.*, £14, 16*s.* 7*d.*, £19, 7*s.* 10*d*, £4, 18*s* 9*d.*, £1, 11*s.*, and £2, 14*s.* 1*d.*?

Compound Multiplication.

Multiply—

121. £17, 13s $5\frac{1}{2}d$ by 9	122. £18, 15s $10\frac{1}{4}d$ by 11.
123. £154, 16s $8\frac{3}{4}d$ by 7	124 £62, 17s $9\frac{1}{2}d$ by 12.
125. £93, 9s $7\frac{1}{2}d$. by 15	126 £149, 14s $6\frac{3}{4}d$ by 18
127 £43, 12s $4\frac{3}{4}d$. by 24	128. £71, 18s. $7\frac{3}{4}d$ by 30
129 £2, 16s. $8\frac{1}{2}d$ by 63	130. £5, 11s $6\frac{1}{2}d$ by 96
131. £250, 13s 4d by 66	132 £247, 13s $5\frac{1}{4}d$ by 64
133. £22, 15s $5\frac{1}{2}d$ by 100	134. £16, 14s 11d by 144
135. £12, 9s 11d by 200	136 £8, 10s 9d by 700
137. £7, 13s. 9d by 1000	138. £2, 11s 3d by 3000.
139. £53, 7s 11d. by 17.	140. £36, 12s. $1\frac{1}{2}d$ by 23.
141. £4, 13s $8\frac{1}{2}d$ by 43	142 £2, 16s $3\frac{1}{4}d$ by 51.
143 £72, 14s. 5d. by 76	144. £60, 18s 10d by 86
145. £14, 6s. $7\frac{1}{4}d$. by 68	146 £11, 8s $2\frac{1}{2}d$ by 94
147 £231, 10s $4\frac{1}{2}d$ by 85	148 £748, 11s $3\frac{1}{4}d$ by 92
149. £19, 15s 1d. by 130	150. £25, 8s $8\frac{1}{2}d$ by 170
151. £3, 13s 10d by 340	152. £24, 3s 7d by 360
153. £5, 12s 4d, by 1007	154. £8, 14s. 5d by 2050
155 £124, 7s. $11\frac{1}{2}d$ by 19	156 £3, 16s $2\frac{1}{4}d$ by 69.
157. £388, 12s $9\frac{1}{2}d$ by 29	158. £2017, 17s $5\frac{1}{2}d$ by 39.
159. £76, 3s. 11d by 59.	160. £18, 9s. 8d by 89.
161. £237, 4s 9d by 119	162. £74, 17s $8\frac{1}{2}d$ by 297.
163. £1, 15s. 4d by 253	164. £1, 17s $3\frac{1}{2}d$ by 322.
165. £2, 13s 5d by 365.	166 £3, 8s 9d by 365
167. £1084, 7s 6d by 672	168 £67, 4s 2d by 1335.
169. £1, 10s 8d by 2037.	170 £3, 9s 5d by 4708.

Find, shortly,

171. 191 times £49, 19s 6d.	172 201 times £59, 19s 9d.
173. 279 times 4s $11\frac{1}{2}d$	174. 187 times 9s $10\frac{3}{4}d$
175 311 times 19s $11\frac{3}{4}d$.	176. 253 times £1, 19s $10\frac{1}{2}d$.
177. 1023 times £7, 18s 8d	178. 3008 times £21, 18s 9d.
179. 2457 times £1, 17s. 6d.	180 1897 times £2, 17s.

Find, without separating the multiplier into parts,

181. 873 times £2, 0*s.* 2*d.*
182. 769 times £7, 0*s.* 4*d.*
183. 2813 times 16*s.* 2*d.*
184. 3597 times 12*s.* 1*d.*
185. 716 times £5, 0*s.* 7½*d.*
186. 987 times £13, 0*s.* 7*d.*
187. 777 times £7, 7*s.* 7*d.*
188. 1111 times £11, 11*s.* 11*d.*
189. 2573 times £17, 1*s.* 8*d.*
190. 3011 times £12, 3*s.* 3½*d.*

Multiply—

191. 2 hrs. 33 min. 49 secs. by 51.
192. 1 hr. 17 min. 15 secs. by 107.

193. 12 tons 17 cwts. 1 qr. by 12.
194. 1 qr. 12 lbs. 14 ozs. by 10.
195. 2 lbs. 7 ozs. 11 drs. by 42.
196. 9 cwts. 0 qr. 16 lbs. by 24.

197. 5 tons 12 cwts. 2 qrs. 13 lbs by 13.
198. 1 cwt. 27 lbs. 15 ozs. by 17.
199. 18 tons 3 cwts. 2 qrs. 9 ozs. by 23.
200. 11 cwts. 3 qrs. 15 lbs. 4 ozs. by 46.
201. 4 tons 15 cwts. 2 qrs. 27 lbs. by 195.
202. 6 cwts. 1 qr. 21 lbs. 5 ozs. by 321.

203. 13 yds. 1 ft. 7 in. by 9.
204. 5 mi. 3 fur. 27 po. by 12.
205. 1 mi. 7 fur. 39 po. by 41.
206. 7 yds. 2 ft. 10 in. by 53.

207. 7 mi. 6 fur. 22 po. 4 yds. by 27.
208. 2 po. 3 yds. 2 ft. 3 in. by 54.
209. 53 mi. 5 fur. 23 po. 4 yds. by 19.
210. 5 mi. 3 fur. 17 po. 4 yds. 2 ft. by 29.
211. 3 mi. 7 fur. 31 po. 3 yds. by 103.
212. 17 po. 4 yds. 2 ft. 9 in. by 114.

213. 7 A. 3 R. 20 P. by 12.
214. 12 A. 1 R. 32 P. by 9.
215. 8 sq. ft. 117 sq. in. by 14.
216. 6 sq. ft. 94 sq. in. by 18.

217. 1 ac. 2 ro. 3 sq. po. 4 sq. yds. 5 sq. ft. by 11.
218. 2 ac. 3 ro. 31 sq. po. 23 sq. yds. 7 sq. ft. by 8.
219. 1 ac. 3 ro. 7 sq. po. 4 sq. yds. by 17.
220. 209 ac. 3 ro. 25 sq. po. 3 sq. yds. by 13.
221. 4 ac. 1 ro. 27 sq. po. 19 sq. yds. 4 sq. ft. 72 sq. in. by 27.
222. 19 sq. po. 11 sq. yds. 5 sq. ft. 128 sq. in. by 84.

223. 3 cub. yds. 25 cub. ft. 1118 cub. in. by 30.
224. 13 qrs. 5 bush. by 1144.

COMPOUND DIVISION

Divide, using short division—

225. £6, 9*s* $5\frac{3}{4}d$ by 11
226 £214, 11*s* 3*d* by 12
227. £271, 15*s* $9\frac{1}{4}d$ by 8
228 £2031, 17*s* $11\frac{1}{2}d$ by 9
229. £3475, 2*s* 3*d* by 6
230. £56831, 0*s* 8*d* by 7
231. £23471 by 9.
232. £695031 by 8

233. £75, 19*s* 6*d*. by 24
234. £118, 1*s* 11*d* by 28
235. £786, 4*s* 7*d*. by 60
236. £308, 15*s* 10*d* by 72
237. £2105, 18*s* $3\frac{1}{2}d$ by 99
238. £4021, 10*s* $5\frac{3}{4}d$ by 144
239. £3050, 9*s* $10\frac{1}{2}d$ by 81
240. £6595, 4*s* 8*d* by 44
241. £9652, 6*s* 0*d* by 96
242. £15942, 16*s* 6*d* by 108
243 £145845, 17*s* $9\frac{3}{4}d$ by 15
244. £560127, 17*s* $9\frac{3}{4}d$ by 45

Divide—

245. £39, 11*s* $3\frac{1}{4}d$ by 19
246. £379, 19*s* $7\frac{1}{4}d$ by 19
247. £208, 2*s* $6\frac{1}{4}d$ by 23
248. £2065, 19*s* 6*d* by 23
249. £153, 1*s* $3\frac{3}{4}d$. by 29.
250. £183, 12*s* $1\frac{1}{2}d$ by 58
251. £146, 14*s* $10\frac{1}{2}d$ by 53
252. £22912, 9*s* $0\frac{1}{2}d$ by 53
253. £29, 11*s*. $4\frac{1}{2}d$ by 83
254. £68, 9*s* $11\frac{1}{4}d$ by 69
255. £14736, 19*s* 7*d* by 73
256. £78791, 11*s* 9*d* by 73
257 £21544, 14*s* $2\frac{1}{2}d$ by 97
258 £102354, 14*s* $8\frac{1}{4}d$ by 93
259. £9418, 12*s* 3*d* by 118
260 £85071, 19*s* 4*d* by 136.
261 £203745, 12*s* $1\frac{1}{2}d$ by 147
262 £317235, 1*s* $6\frac{3}{4}d$ by 145
263. £118332, 19*s* $5\frac{1}{4}d$ by 321.
264. £98703, 19*s* 11*d* by 359
265. £1412, 1*s* $10\frac{1}{2}d$ by 365
266 £10839, 7*s* $2\frac{1}{4}d$ by 365
267. £266780 by 365
268 £1850321 by 730
269 £34161, 17*s* 11*d* by 2705
270. £84304, 5*s* $3\frac{1}{2}d$ by 2801
271. £1459, 13*s* by 4723
272 £68493, 2*s* by 7891
273. £1624270, 6*s* 3*d*. by 935
274 £226125587, 3*s* $5\frac{1}{2}d$ by 602
275. £12801 by 7300.
276 £86107, 10*s* 8*d* by 7300

Divide—

277. £1998, 17*s* 1*d*. by 100
278 £6521, 9*s* 2*d* by 100
279. £11705, 12*s* 6*d*. by 100
280 £32065, 14*s* $2\frac{1}{2}d$ by 100
281. £2371, 17*s* $9\frac{1}{2}d$ by 100
282 £40312, 13*s* $4\frac{1}{4}d$ by 100
283 £113, 17*s* 11*d*. by 100
284 £97, 16*s* $0\frac{1}{2}d$ by 1000.
285. £876, 11*s* $10\frac{1}{2}d$ by 1000
286. £27906, 13*s* 4*d* by 100.

Divide—

287. 365 days 5 hrs. 48 min. 48 secs. by 12.
288. 47 days 15 hrs. 23 min. 20 secs. by 20
289. 1146 days 14 hrs. 38 min 20 secs. by 100
290. 146097 days 0 hrs. 5 min. 30 secs. by 365.

291. 13 tons 14 cwts. 1 qr. 12 lbs. 4 ozs. by 6.
292. 7 tons 6 cwts. 3 qrs. 17 lbs. 2 ozs. by 9.
293. 201 tons 3 cwts 1 qr. by 56.
294. 201 tons 13 cwts. 1 qr. 4 lbs. by 64.
295. 12 tons 3 cwts 1 qr 25 lbs. by 37.
296. 309 tons 12 cwts 2 qrs. 19 lbs. by 47
297. 10 tons 8 cwts. 3 qrs. 11 lbs. 12 ozs. 15 drs. by 69.
298. 30 tons 6 cwts. 1 qr. 24 lbs. 8 ozs. by 168.
299. 56 tons 6 cwts. 1 qr. 13 lbs. by 321
300. 756 tons 13 cwts. 3 qrs. 17 lbs. 11 ozs. 8 drs. by 251.

301. 247 yds. 1 ft. 6 in. by 33.
302. 3 mi. 102 yds. 2 ft. by 24.
303. 8 mi. 3 fur. 25 po. 3 yds. 1 ft. 6 in. by 6.
304. 17 mi. 5 fur 21 po. 4 yds. 1 ft. 3 in. by 11.
305. 1 mi. 2 fur 35 po. 4 yds. 2 ft. 11 in. by 17.
306. 18 mi 3 fur 23 po. 1 yd. 2 ft. 6 in. by 35.
307. 75 mi. 1 fur 39 po. 3 yds. by 75.
308. 756 mi. 634 yds. 0 ft. 8 in by 208
309. 1693 mi. 1660 yds. by 737.
310. 3 mi. 6 fur. 35 po. 3 yds. 6 in. by 551.

311. 29 sq. yds. 7 sq ft. 72 sq in. by 18.
312. 23 sq yds. 6 sq. ft. 48 sq. in. by 96.
313. 24 ac 1 ro 33 sq. po 18 sq yds. 2 sq. ft. 42 sq in by 10.
314. 12 ac 3 ro 27 sq po. 21 sq yds. 6 sq ft. 1 sq in. by 7.
315. 4851 ac 3 ro. 36 sq po. 28 sq. yds. by 94.
316. 198 ac 3 ro 16 sq. po 3 yds. 0 ft. 72 in. by 187.
317. 1 ac. 2 ro 8 sq. po. 11 yds. 2 ft. 108 in. by 367.
318. 40301 ac. 0 ro. 38 sq. po 9 sq. yds. by 251.

319. 944 cub. yds. 1 cub ft. 1104 cub. in. by 59
320. 53 qrs. 6 bush 2 pks. 3 qts. 1 pt. by 217.

Find—

321. £48, 6*s* ÷ 11*s* 6*d*

322 £44, 19*s* ÷ 9*s* 8*d*

323. £3294, 4*s* ÷ £18, 2*s*

324. £289, 17*s* ÷ 4*s* 3*d*

325. £437, 18*s* 9*d* ÷ 8*s* 9*d*

326 £8485 ÷ £565, 13*s* 4*d*

327. £152, 7*s* 7½*d*. ÷ 3*s* 4½*d*

328 £38, 0*s* 5*d* ÷ 1*s* 7¼*d*

329. £43, 6*s* 8*d* ÷ £3, 6*s* 8*d*.

330. £28, 2*s* 6*d* ÷ £1, 17*s* 6*d*

331. £103, 14*s* 9*d* ÷ £17, 5*s* 9½*d*

332. £203, 10*s* ÷ £12, 14*s* 4½*d*

333. 7 hrs 20 min ÷ 7 min 20 secs

334 17 hrs 30 min ÷ 1 hr 15 min

335 8 yds. 2 ft ÷ 1 ft 1 in

336 15 yds. 1 ft 8 in ÷ 2 ft 4 in.

337. 39 ac. 8 po ÷ 2 ro 8 po

338 30 bush 3 gal ÷ 3 gal. 3 pts.

Divide—

339. £904, 11*s*. 4*d* by £56, 10*s* 8½*d*

340. £500, 1*s*. 7½*d* by £27, 15*s* 7¾*d*

341. £5647, 18*s* 11¼*d* by £3, 4*s* 5¼*d*

342. £3226, 0*s* 10*d* by £3, 4*s* 6¼*d*

343. 10 tons 7 cwts. by 12 cwts 3 qrs 21 lbs

344. 6 tons 8 cwts. 8 lbs. by 2 cwts 2 ozs

345. 57 tons 5 cwts. 1 qr. 7 lbs by 1 ton 12 cwts 2 qrs 25 lbs

346 23 ozs. 15 dwts. 7 grs by 7 dwts 19 grs

How many times is—

347. £4, 7*s* 6*d* contained in £161, 17*s* 6*d* ?

348. £3, 17*s*. 6*d* contained in £399, 2*s* 6*d*.?

349 195 yds. 1 ft. 8 in. contained in 1 mile?

350. 8 po. 1 yd contained in 1 mile?

351. 2 yds. 2 ft. contained in 27 mi 551 yds 2 ft ?

352. 1 rood 24 poles contained in 320 acres?

353. 1 sq. ft. 127 in. contained in 7 sq yds 8 ft 77 in ?

354. 5 cub. ft. 1037 in. contained in 1 cub yd 1 ft 1 in ?

355 How many payments of 14*s* 11¾*d* amount to £89, 2*s* 6¼*d* ?

356. How many lbs of tea at 1*s* 10½*d* a lb can be bought for £45, 5*s*. 7½*d* ?

357. How many tons of coal at 13*s* 9*d* per ton could be bought for £413, 3*s* 9*d* ?

358. How many persons could each be supplied with a pint and a half of milk from a vessel containing 7 gallons 2 quarts?

359 How many bits each 4½ inches long could be cut from 12 yards of tape?

360 How many plots of 2 ro 16 po 8 yds are there in 10 ac. 36 po 15 yds.?

VI. MISCELLANEOUS EXERCISES.

1. Reduce £237, 4*s* 9$\frac{1}{4}$*d.* to farthings, and prove the result.
2. How many more seconds are there in July than in June?
3. Add £119, 6*s* 8$\frac{1}{2}$*d*, £34, 17*s* 2*d*, £33, 11*s* 0$\frac{3}{4}$*d*, £21, 0*s* 10$\frac{1}{2}$*d.*, £17, 16*s* 4$\frac{1}{4}$*d*, £9, 18*s.* 7$\frac{3}{4}$*d*, and £2, 16*s* 2$\frac{1}{2}$*d*
4. Take the sum of £13, 4*s* 7$\frac{1}{2}$*d* and £25, 19*s* 10$\frac{3}{4}$*d* from £42, 4*s.* 6*d.*
5. Multiply £4628, 9*s.* 8$\frac{1}{4}$*d* by 9.
6. Divide £85307, 18*s* 0*d.* by 48.
7. What was the income of a man who spent on the average £2, 3*s* 6*d.* a day, and saved £50 in the year?
8. Divide the difference between £24299, 14*s.* 5*d.* and £14487, 16*s.* 11*d* by 100.
9. How many twopence-halfpenny stamps can be bought for ten shillings?
10. How many times is 3 cwts. 3 qrs. 15 lbs. contained in 1 ton 7 cwts. 21 lbs.?

---

11. Reduce 12 tons 3 cwts. 2 qrs. 11 lbs. to pounds, and prove the result.
12. How many minutes are there in the last six months of the year?
13. Find the sum of £201, 3*s* 5$\frac{1}{2}$*d.*, £18, 0*s.* 9$\frac{3}{4}$*d.*, £178, 6*s.* 11$\frac{1}{4}$*d.*, £125, 17*s.* 1$\frac{1}{2}$*d*, £574, 4*s* 0*d*, and £13, 19*s.* 7$\frac{1}{4}$*d.*
14. After paying bills of £20, 13*s* 4$\frac{1}{2}$*d.*, £30, 15*s.* 6$\frac{1}{2}$*d.* £46, 13*s* 11$\frac{1}{2}$*d.* what is left out of £100?
15. Multiply £608, 13*s* 7$\frac{1}{4}$*d* by 72.
16. Divide £74, 15*s* 7$\frac{1}{2}$*d.* by 37.
17. What sum of money when divided by 17 gives quotient 3*s* 4$\frac{1}{2}$*d.* and remainder 2*s.* 5$\frac{3}{4}$*d.*?
18. Find half the difference between 111111 farthings and £111, 11*s.* 11$\frac{1}{4}$*d*
19. If eggs are sold at 7 for sixpence, how many will three half-crowns purchase?
20. How many times is 2 ft 3 in. contained in 19 yds. 2 ft. 5 in.?

---

21. Show that there are as many farthings in £59, 12*s* 1$\frac{3}{4}$*d.* as there are pounds in 25 tons 10 cwts. 3 qrs. 19 lbs.
22. How many pence are there in £19 + 19 half-sovereigns + 19 half-crowns + 19 shillings + 19 sixpences?
23. From 300 half-guineas take 300 half-crowns.
24. Multiply 2 tons 9 cwts 15 lbs. 9 ozs. by 27.
25. Divide £7777, 17*s.* 1*d* by 17, and prove the result.

26. Find, shortly, the value of 648 things at 19*s* 2*d* each

27. If 2 cwts 1 lb cost £116, 19*s* $0\frac{1}{4}d$, what is the cost of 1 lb?

28. If a sovereign were distributed equally among 320 children what would each receive?

29 How many cricket-balls at 5*s* 6*d* each could be bought for £166, 7*s* 6*d*?

30. How many times does a wheel, which is 7 ft 4 in. in circumference, turn round in going a distance of 3 miles?

---

31. How many weeks, days, &c, are there in as many seconds as there are ounces in fifty tons?

32. Express in £, *s* *d* the sum of 1000 sovereigns, 1000 shillings, 1000 pence, and 1000 farthings

33 Reduce 5346 half-guineas to farthings

34 Multiply 8 tons 14 cwts 2 qrs 14 lbs 12 ozs by 38.

35 If £99, 2*s* $9\frac{3}{4}d$ is divided equally among 47 people, what does each receive?

36 Find, as shortly as you can, the value of 731 articles at 9*s* $11\frac{1}{2}d$ each.

37 A leg of mutton, at 9*d* per lb, cost 5*s* $7\frac{1}{2}d$, what did it weigh?

38 What would be the cost of giving a penny bun and a halfpenny orange to each of a thousand school-children?

39 Divide 16*s*. $9\frac{1}{2}d$ by 1*s* $3\frac{1}{2}d$, and prove the result

40. How many bottles, each holding 3 pints, could be filled from a cask containing 36 gallons?

---

41. How many half-crowns would be a fair exchange for 1560 florins?

42. What is the value of the contents of a purse which contains a five-pound note, a cheque for £3, 8*s* 7*d*, 3 sovereigns, 3 half-sovereigns, a half-crown, a florin, 3 shillings, 3 sixpences, a threepenny bit, 7 penny stamps, and 7 halfpenny stamps?

43. Find the cost of 1183 slates at 4*d* each?

44. If a man's income is £685, and he saves £274, 7*s* 6*d*, what is his average daily expenditure?

45. Find the sum of money nearest to £7 which could be equally divided among 93 persons

46 Take the sum of £25, 16*s* $1\frac{3}{4}d$, £2, 17*s* $11\frac{1}{2}d$, £9, 5*s* $3\frac{1}{2}d$, and 7*s*. $7\frac{1}{2}d$ from £52

47. What sum of money would be spent in paying £149, 17*s* 10*d* to each of 23 persons?

48. A man received £4, 0*s*. 6*d* as wages for 28 days how much did he earn per day?

49. Postcards are sold at 6 for $3\frac{1}{2}d.$: how many can be bought for 10*s.* 6*d* ?

50. A draper bought 240 yards of cloth at 14*s* 9*d.* a yard, and sold it at 15*s.* 7*d.* a yard. how much did he gain?

---

51. Which is greater 17 half-crowns or 21 florins?

52. A man left home having in his purse two ten-pound notes 3 sovereigns, 2 half-sovereigns, 7 shillings, and 3 pence. He paid bills of £4, 3*s.* 5*d*, £10, 7*s.* 6*d*, £2, 17*s* 4*d*, and £1, 4*s*. and he bought a dozen halfpenny stamps. How much money did he bring back?

53. Find the cost of 2465 articles at 6*d.* each

54. A man's income is £550 a year: he spends on the average £1, 7*s* 3*d.* daily. What does he save annually?

55. Divide one thousand five hundred and forty-nine pounds nineteen shillings and fourpence farthing into thirty-one equal parts.

56. From £1000, 3*s* $4\frac{3}{4}d.$ take the sum of £17, 8*s* $6\frac{1}{4}d.$, £25, 14*s.* $6\frac{3}{4}d$, and £12, 19*s.* $11\frac{3}{4}d.$

57. Find, as shortly as you can, 311 times £39, 19*s* $11\frac{1}{2}d$

58. If £465 12*s* 6*d.* be divided equally among 100 persons, what does each receive?

59. How many persons could each be paid 7*s.* 7*d.* out of £294, 12*s.* 3*d* ?

60. Find the value of 70 articles all alike, if 5 of them are worth £1, 19*s.* 2*d*

---

61 How many hours are there in a leap-year?

62 How many half-ounces are there in 6 tons 12 cwts. 1 qr. 17 lbs. 5 ozs 8 drams?

63. Reduce 4972165 seconds to days, hrs. &c

64. Add 176 cwts. 3 qrs 15 lbs, 48 cwts. 2 qrs. 11 lbs.; 35 cwts. 3 qrs. 7 lbs.; 7 cwts. 2 qrs. 21 lbs., and 4 cwts. 1 qr 23 lbs.

65. Subtract 235 yds. 2 ft. 7 in. from 1 mile.

66. Multiply 2 dwts. 2 grs. by 101.

67. Divide 526 tons 11 cwts. 1 qr 25 lbs. 3 ozs. by 197

68. If 3 pennies weigh an ounce Avoirdupois, how many will balance a four-stone weight?

69. The ages of the 9 boys in a class are 11 yrs. 3 mo, 12 yrs. 3 mo., 9 yrs. 10 mo, 10 yrs, 12 yrs. 1 mo, 11 yrs. 7 mo, 9 yrs. 7 mo, 13 yrs., and 12 yrs. 5 mo Find the average age.

70. If the length of a man's step be $27\frac{1}{2}$ inches, how many steps will he take in walking 4 miles?

---

71 How many cwts qrs &c, are there in 10192000 grains?

72. How many half-minutes are there in 1 day 5 hrs 36 min. 30 secs?

73. Express 364713 feet in miles and yards

74. Add 47 A 3 R 17 P, 19 A 1 R 37 P, 5 A 2 R 19 P, 13 A. 1 R. 25 P, and 43 A. 3 R 22 P

75. Subtract 2 gross 8 dozen and 10 from 4 gross 5 doz and 7

76. Multiply 1 yd 1 ft 1 in by 101

77. If 20 railway wheels weigh 2 tons 15 cwts 1 qr 2 lbs, what is the weight of one wheel?

78. If 3 pennies weigh an ounce Avoirdupois, what would 240000 pence weigh?

79. The heights of 7 boys are 5 ft 3 in, 5 ft 2 in, 4 ft 10 in, 4 ft. $9\frac{1}{2}$ in, 5 ft. $1\frac{1}{2}$ in, 5 ft 5 in, 5 ft, what is the average height?

80. How many medals, each weighing 1 oz. 8 dwts could be made from 100 ozs 16 dwts of metal?

---

81. How many guineas are there in 9730 crowns?

82. Add together 17 halfpence, 17 pence, 17 threepences, 17 sixpences, 17 shillings, 17 florins, 17 half-crowns, 17 double-florins, 17 crowns, 17 half-sovereigns, and 17 sovereigns

83. Multiply £427, 17*s* $5\frac{1}{2}d$ by 91

84. Multiply £29, 19*s* $10\frac{3}{4}d$ by 192, as shortly as you can

85. Multiply $3\frac{3}{4}d$ by fifty millions five thousand and six

86. Find the profit on 841 sheep bought at £1, 19*s* 9*d* a head, and sold at £2, 0*s*. 3*d* a head

87. Divide £387, 13*s* $9\frac{1}{2}d$ by 60, using short divisions

88. Divide £376540583, 3*s* 6*d* by 972

89. How many persons could each be paid £17, 17*s* 9*d*. out of a fund which amounted to £554, 10*s* 3*d*?

90. If two oranges cost three-halfpence, how many can be bought for 5*s* 6*d*?

---

91. Which is the greater, the number of farthings in £3,549,596, 14*s* $6\frac{3}{4}d$., or the number of two-shilling pieces in three hundred and forty millions, seven hundred and sixty-one thousand, two hundred and eighty-six pounds?

92. What is left out of a ten-pound note after paying bills of £2, 11*s*. $7\frac{1}{2}d$, £1, 3*s* 11*d*, £1, 13*s* $3\frac{1}{2}d$, 17*s* $5\frac{1}{2}d$, and £2, 7*s* 9*d*?

93 Find the value of 10000 herrings when two are worth $1\frac{1}{2}d$

94 Multiply £3, 5*s*. $11\frac{3}{4}d$ by 752.

95. Find, as shortly as you can, 999 times £5, 5s. 5d.

96. Find the profit on 1479 lbs. of tea bought at 1s $10\frac{1}{2}d$ per lb., and sold at 2s 2d per lb.

97. Divide £49396, 16s $6\frac{1}{4}d$ by 531.

98 Find the sum of money which, when multiplied by 1000, amounts to £10542, 14s 2d.

99. Among how many persons must £641, 14s $11\frac{1}{4}d.$ be divided that the share of each may be £2, 15s. $6\frac{3}{4}d.$?

100. A sum of £12, 2s 7d was divided equally among a certain number of persons, and the shares of three of them amounted to 17s. 9d How many persons were there?

---

101. Reduce 2 mi. 6 fur. 4 yds. to feet.

102. How far do I travel third class at a penny a mile if I pay £2, 3s. 7d for my ticket?

103. How long would it take to count a billion at the rate of 100 per minute?

104. How much coal would supply 12 fires for 27 weeks, each fire consuming 1 cwt. 1 qr. 14 lbs. weekly?

105. A person's annual income is £555, 15s. 5d.; he saves £171 a year: what is his average daily expenditure?

106. Find the cost of 3 tons 2 cwts. 6 lbs. of sugar at $2\frac{1}{2}d.$ per lb.

107. Divide 42 mi. 7 fur. 7 po. 2 yds. 8 in. by 19.

108. Divide 5 ac. 3 ro. 24 po by 1 ro 19 po

109. Divide £2 between George and John, so that George may have seven times as much as John

110. If there are 34 lines on each side of a sheet of ruled foolscap paper, how many lines are there in a ream of such paper?

---

111. Reduce 15 mi. 3 fur 29 po $4\frac{1}{2}$ yds. to inches.

112. Multiply 1 mi. 1 fur. 1 po. 1 yd. 1 ft. 1 in. by 19.

113. How much would 3 tons 18 cwts. 1 qr of butter cost at 14d. per lb?

114. If it took a man, working 9 hours a day and counting 120 a minute, 17 days to count a heap of pins, how many were there in the heap?

115. Divide £337426, 5s 4d by 832.

116. If a man spends on the average 9s 7d daily and saves £25 yearly, what is his income?

117. Find the cost of 3 gross of note-books at 8d per dozen.

118. A field of 18 ac. 36 po is divided into allotments, each containing 1 rood 14 poles how many allotments are there?

119. Divide £34, 16*s* 9*d* between two persons so that one may have eight times as much as the other

120. If a silver tankard weighs 175 ozs Troy, what is its weight Avoirdupois?

---

121 In 1956742 ozs, how many tons, cwts, &c ?

122. Which is greater 250000 farthings, or 2083 half-crowns?

123. What is the size of an estate which consists of 142 ac 2 ro of arable land, 113 ac 1 ro of pasture, 71 ac of wood, and 89 ac. 1 ro of moor?

124. If a man with a salary of 25 guineas per quarter spends at the rate of 5*s* 3*d* per day, how much does he save in a year?

125. Divide 69 mi 7 fur. 39 po 2 ft by 492

126. A flock of 170 sheep were bought at £2, 18*s* 6*d* each, half of them were sold at £2, 19*s* 9*d* each, and the rest at £3, 2*s* 0*d*. each, what profit was made?

127. How many times is 1 cub ft 117 cub in contained in 3 cub yds. 25 cub ft 1332 cub in ?

128. Divide £4, 4*s* 4*d* between A and B, so that A may have £1, 19*s* 10*d* more than B

129. A sum of money was divided between three persons, A, B, and C, in such a manner that the shares of A and B together amounted to 13*s*, those of A and C to 14*s*, and those of B and C to 15*s* How much had each?

130. Three acres and a cow cost £170 The land costs seven times as much as the animal. Find the cost of the land per acre

---

131. Show that there are as many halfpence in £3, 11*s* 7*d* as there are half-inches in 23 yds 2 ft 7 in

132. Reduce 3241985 inches to miles, furlongs, poles, &c

133. Divide the sum of £17, 3*s* 2*d*, £12, 19*s* 5½*d*, £21, 17*s* 11*d*, and £24, 12*s* 10*d* by 47

134. If a man's income be £37, 15*s* per calendar month, and he spend at the rate of £1, 4*s* 7½*d* per day, how much will he save in a year?

135. Divide 13 tons 17 cwts. 3 qrs. 20 lbs into two parts, one of which shall be double of the other

136 A green-grocer bought 420 oranges for £1, 1*s*, he sold half of them at 9*d*. a dozen, and the rest at 7 for sixpence, what did he gain?

137. How many allotments of 2 ro 4 po each can be made from a field of 7 ac. 3 ro 24 po.?

138. Divide £18, 0*s* 2*d*. between 11 men, giving one of them 10*s*. more than each of the others.

139. The total cost of a coat, hat, and umbrella was £6 The coat and hat alone would have cost £5, 1s; the hat and umbrella alone, £1, 16s Find the cost of each article.

140. Divide £24, 6s 6d. into two parts, one of which shall contain as many shillings as there are half-crowns in the other.

---

141. Find in pounds the difference between one million eight hundred and three thousand eight hundred and forty farthings, and one hundred and fifty thousand four hundred threepenny pieces.

142. Reduce 11 ro 11 po. 11 sq. yds. to square inches, and *prove* the result.

143 Find, as shortly as you can, the value of 8632 articles at £2, 18s 11½d each.

144. How much a day is £4651, 9s 4½d a year?

145. Divide 15 cwts. 1 qr 2 lbs. into two parts, one of which shall be nine times as large as the other

146. Two friends set out on a tour with £12, 16s 9d between them, one had £1, 7s 5d more than the other; what had each?

147. A grocer buys 15 cwts. of rice for £24, 10s At what price per lb. must he sell it to gain £7?

148 The circuit of a racing path is 137½ yards how many times round it will make 5 miles?

149. A postman whose pay is 15s a week is fined 1s 6d. whenever he is late, and at the end of 13 weeks he has received £8, 15s 6d. How often had he been late?

150. Divide £10 among 16 men, 16 women, and 16 boys, giving each woman 2s less, and each boy 5s less than each man.

---

151. If a sheet of notepaper contains 54 square inches, how many sheets would cover an acre of ground? Also, what would they weigh if 11 sheets weigh 1 ounce?

152. If the Mint buys sufficient silver to coin seventy-four million eight hundred and sixty-seven thousand five hundred florins, but when half of the florins are finished, coins the rest of the silver into half-crowns, how many of these will there be?

153. Find, with as little labour as possible, the smallest yearly income which permits of an average daily expenditure of £1, 19s. 8¼d

154. What sum of money must be subscribed by each of 100 persons to raise a fund of £251, 2s. 11d.?

155. Divide £82, 4s. 6d. between A and B, so that A may have 77 times as much as B

156. Twelve chairs and a table cost £12, 7s. 6d, the chairs cost 12s. 6d each; find the cost of the table.

157 The circumferences of the wheels of a bicycle are 6 ft 5 in and 7 ft 4 in; how many more turns does the small wheel make than the large one in running 7 miles?

158 A grocer buys a cwt of sugar for £1, 13*s* 4*d* and sells it for £2, 2*s*. 8*d*, what does he gain per lb?

159. A bill of £9, 8*s* $10\frac{1}{4}d$ was paid in equal shares by a number of persons, of whom 14 together paid £3, 11*s* $5\frac{1}{2}d$ how many persons were there altogether?

160. Divide £9, 1*s* 6*d* among 11 men, 22 women, and 33 boys, giving each man three times as much as each woman, and each woman three times as much as each boy

---

161. How many days are there in seven consecutive years, beginning with 1892?

162. Reduce 1234 lbs Avoirdupois to ounces and grains Troy

163. Find, by inspection, the difference between £43, 10*s* and 48,000 farthings

164. What is the weight of a mile of wire if 3 inches of it weigh an ounce Av?

165. February 23rd, 1892, was Tuesday, what day of the week was August 19th, 1891?

166 Divide £16, 16*s* between 35 persons, giving to one of them twice as much as to each of the others

167. If with every 6 lbs of black tea, which I buy at 1*s* 8*d* per lb, I mix half a pound of green tea at 2*s* 9*d* per lb, what does the mixture cost me per lb?

168 If thirty-three telegraph posts, placed at equal distances, extend a mile, how far apart are they?

169. Divide £33, 5*s* among 30 men, 30 women, and 30 boys, giving every two men as much as 3 women, and every two women as much as 3 boys.

170. A sum of £28, 5*s* 0*d* was divided among 71 persons, some of whom received 8*s* 4*d* each, and the rest 7*s* 6*d* each How many received 8*s* 4*d*?

---

171 How many days are there from January 13th, 1880 to July 7th, 1893?

172. How many times does the hammer of a clock, which strikes the hours only, hit the bell in a week?

173. Write down, at sight, 241 times £3, 0*s* 2*d*

174 February 19th, 1892 fell on a Friday; what day of the month was the second Wednesday in May that year?

175. Divide £45 between two persons, so that for every sovereign one has the other may have half-a-crown

176. An innkeeper buys 10 gallons of spirit at 12*s* a gallon, 15 at 4*s*. 6*d*, and 18 at 5*s* 9*d*. How does he sell the mixture per gallon, if he gains £2, 2*s* 3*d* altogether?

177. If $1\frac{1}{2}$ inches of wire make a pin, how many yards of wire would be used in making 80 gross of pins?

178. If £5 be divided among 6 men, 12 women, and 17 boys, so that two men receive as much as five boys, and two women as much as three boys; how much will each receive?

179. A sum of £24, 7*s* 11*d*. was divided among a number of men and boys, the boys each receiving 3*s* 1*d* and the men 10*s*; how many men were there, the total number of persons being 73?

180. The *Saturday Review* of Feb. 6, 1892, was numbered 1893; what was the number on a copy bearing the date Dec. 27, 1890?

---

181. What is the cost of the *Times* daily newspaper for the year 1893 at 3*d*. per copy? The year began on a Sunday

182. Two pieces of cloth of the same length cost £11, 3*s*. 6*d*. and £14, 8*s*. 0*d*. respectively, the price of the first was 6*s*. $2\frac{1}{2}$*d*. per yard; what was the price of the second per yard?

183. Find the cost of a steel hammer weighing 225 tons, at the rate of 7 lbs. for 1*s*

184. Divide £92, 16*s* among 19 men and 17 women, giving each woman £1 less than each man.

185 How many shillings, each weighing 3 dwts 12 grs., could be made from 37 lbs of silver mixed with 3 lbs of alloy?

186. A takes 1980 steps in walking a mile, and B takes 1920 steps. How much longer is B's stride than A's?

187. If a dealer bought eggs, to the value of £3, at 11 for a shilling and sold them at 13*d*. a dozen, how much would he gain or lose?

188. A person bought equal quantities of sugar at $3\frac{1}{2}$*d*. per lb., tea at 3*s*. 9*d* per lb, butter at 1*s*. $11\frac{1}{2}$*d* per lb., and coffee at 1*s* 4*d* per lb. The whole cost £1, 16*s* 8*d* How much of each did he buy?

189. A wine merchant bottles off a 36-gallon cask of claret into an equal number of reputed quart bottles and half-bottles; how many dozen of each has he?

190. January 9, 1892, fell on a Saturday; what day of the week was June 20, 1837?

---

191. What was paid for the *Standard* penny daily newspaper by a person who began to buy it on January 1st, and ceased to do so after March 26th in the year 1892?

192. If 6 dozen table-spoons weigh 11 lbs 6 ozs 12 dwts and the same number of tea-spoons weigh 3 lbs 7 ozs 10 dwts, by how much does the weight of a table-spoon exceed that of a tea-spoon?

193 If 11 thalers, 6 florins, 3 guineas, and 5 dollars amount to £6, 0*s* 7*d*, and a dollar is worth 4*s* 2*d*, what is the value of a thaler?

194. A wine-merchant bought a 36-gallon cask of brandy at 27*s* 6*d* per gallon, diluted it with water and then sold it at 30*s* per gallon, making a profit of £10, 10*s* How much water did he add?

195. Two boys start for a quarter-mile race, one takes four strides of 40 inches each while the other takes five of 33 inches, which wins and by how much?

196. The live stock on a farm consists of a certain number of horses worth 60 guineas each, an equal number of pigs worth £2, 10*s* each, three times as many cows worth £18, 10*s* each, and fifteen times as many sheep worth £1, 15*s* each The whole value of the live stock is £1030, 15*s* How many are there of each kind?

197. The *Journal of Education* is published monthly, the issue of February, 1892, was numbered 271, when was No 186 published?

198. A wine-merchant bottles off 42 gallons of beer into an equal number of imperial quart, pint, and half-pint bottles, how many dozen of each has he?

199. A man bought 374 eggs at 2 a penny, and some others at 3 for twopence. He paid altogether £1, 9*s* 11*d* for them How many eggs did he buy?

200. On April 14, 1892, a man had lived 14610 days, find the date of his birth

VII. FACTORS, MULTIPLES, PRIMES.

Without actually dividing, find by which of the numbers 2, 5, 10: 4, 25, 3, 9, each of the following numbers is exactly divisible.—

1. 2016.	6. 31452	11. 66312.	16. 17265
2. 6750.	7. 17348.	12. 24680	17. 321000.
3. 5638.	8. 20520.	13. 13579.	18. 444444.
4. 5625.	9. 73818.	14. 71799.	19. 853050.
5. 7109.	10. 19875.	15. 138600.	20. 714051.

Which of the following numbers is a multiple of 11?—

21. 38964035 and 76057183. 22. 413780495 and 709082649

Find, without actually dividing, the remainder of—

23. $73578 \div 4$, $597361 \div 25$, $84127 \div 3$ and $267488 \div 9$

24. $30857 \div 4$, $8978377 \div 25$, $76426 \div 3$ and $789635 \div 9$.

Find, by inspection, the prime factors of—

25. 12.	33. 51.	41. 28.	49. 99.
26. 18.	34. 57.	42. 36.	50. 110.
27. 20.	35. 63.	43. 40.	51. 121.
28. 24.	36. 65.	44. 45.	52. 125.
29. 27.	37. 70.	45. 52.	53 130.
30. 34	38. 88	46. 69.	54. 104.
31. 39.	39 90.	47 81.	55. 108.
32. 42.	40. 100	48 91.	56. 144.

Resolve into prime factors—

57. 112.	67. 369.	77. 1728.	87. 56430
58. 120.	68 448.	78 2240.	88. 21970
59. 111.	69. 441.	79. 4840.	89. 13013.
60. 128.	70. 567.	80. 1892.	90. 14641
61. 136	71. 693.	81. 1893.	91. 111111.
62. 148.	72. 882.	82. 5760	92. 21978.
63. 161.	73 999.	83. 3024	93. 14847
64. 171.	74. 1000.	84. 6435	94. 873873.
65. 252.	75. 1331.	85. 2871	95. 123456.
66. 255	76. 1760.	86. 7777.	96. 999999.

97. How many primes are there between 1 and 100?

98. How many primes are there between 100 and 200?

99. Is 1009 a prime number? 100. Is 1513 a prime number?

VIII. GREATEST COMMON FACTOR.*

Find, *by inspection*, the G C F of—

1. 35 and 42
2. 70 and 90
3. 25 and 65
4. 50 and 75
5. 12 and 16.
6. 16 and 24
7. 24 and 32
8. 36 and 48
9. 34 and 51
10. 39 and 65.
11. 80 and 100.
12. 45 and 30.
13. 41 and 82
14. 23 and 69.
15. 38 and 57
16. 28 and 42
17. 99 and 66
18. 66 and 88
19. 56 and 96
20. 600 and 800
21. 34 and 43
22. 51 and 15
23. 69 and 96
24. 62 and 93
25. 101 and 202
26. 102 and 201
27. 777 and 999
28. 606 and 808
29. 1313 and 1919
30. 2346 and 3162

Find the G C F. of—

31. 315 and 385
32. 255 and 935
33. 428 and 824
34. 432 and 324.
35. 432 and 3888
36. 1056 and 576
37. 837 and 999
38. 849 and 1132.
39. 847 and 1001
40. 677 and 8801.
41. 949 and 1387
42. 1081 and 1311
43. 1008 and 1036
44. 2240 and 5760
45. 1760 and 4840
46. 1547 and 5712
47. 3465 and 4655
48. 1869 and 3471
49. 2187 and 8019
50. 7623 and 8316
51. 5763 and 5198
52. 6741 and 2289
53. 4611 and 5307
54. 4833 and 7533
55. 6468 and 2009
56. 7701 and 9231
57. 8401 and 8374
58. 9805 and 8374
59. 15933 and 10011
60. 28101 and 92017
61. 185625 and 38115
62. 929181 and 1012891
63. 87318 and 206910
64. 437248 and 489159
65. 6545253 and 12151659
66. 214258289 and 262833917.
67. 86, 215 and 387
68. 126, 198 and 270
69. 196, 308 and 420
70. 85, 153 and 595
71. 252, 264 and 444.
72. 1221, 2109 and 5402
73. 6003, 3519 and 145107
74. 8487, 39123 and 46782
75. 581, 83083 and 146993
76. 84411, 98649 and 102717
77. 108, 288, 360 and 924
78. 102, 187, 357 and 680
79. 384, 1280, 752 and 352
80. 713, 1219, 1817 and 2001

---

* Often called Greatest Common *Measure* (G C *M*).

IX. LEAST COMMON MULTIPLE.

Find, *by inspection*, the L.C.M. of—

1. 4 and 6.
2. 8 and 12.
3. 10 and 15
4. 9 and 18
5. 8 and 16.
6. 12 and 36
7. 20 and 30
8. 15 and 20
9. 20 and 25
10. 22 and 33
11. 2, 4 and 6
12. 2, 4 and 8.
13. 3, 4 and 5.
14. 4, 5 and 6
15. 4, 5 and 8
16. 3, 6 and 9
17. 7, 21 and 42
18. 4, 6 and 8.

Find the L.C.M. of—

19. 35 and 77
20. 55 and 88
21. 15 and 35
22. 63 and 99
23. 84 and 96
24. 49 and 84
25. 26 and 65
26. 34 and 51.
27. 38 and 57
28. 74 and 111
29. 85 and 119
30. 23 and 29
31. 17 and 71.
32. 13 and 91
33. 112 and 120.
34. 112 and 128.
35. 18, 45 and 63.
36. 22, 33 and 55
37. 32, 6 and 27
38. 42, 63 and 105
39. 15, 27 and 39
40. 15, 21, and 35
41. 14, 16 and 112
42. 16, 17 and 18.
43. 6, 8, 16 and 21.
44. 10, 15, 20 and 25.
45. 4, 10, 18 and 24.
46. 15, 20, 33 and 110
47. 6, 8, 9, 10 and 12.
48. 3, 6, 9, 12 and 15.
49. 63, 12, 84 and 14.
50. 18, 28, 30 and 42.
51. 12, 16, 18, 24 and 36
52. 10, 12, 14, 36 and 42.
53. 14, 28, 40 and 48
54. 12, 16, 21 and 70.
55. 7, 11, 21, 33 and 42
56. 36, 60, 96 and 108
57. 42, 64, 70 and 112
58. 18, 24, 30 and 56.
59. 65, 91 and 104
60. 121, 143 and 169
61. 6, 8, 10, 12, 14 and 16.
62. 6, 8, 10, 12, 16 and 20.
63. 7, 14, 28, 49, 140 and 210.
64. 300, 400, 500, 600 and 800.
65. 6, 10, 12, 15, 20, 24, 25.
66. 12, 16, 20, 24, 28, 32, 36.
67. 30, 32, 36, 42, 44 and 50.
68. 63, 84, 105, 126 and 252.
69. 121, 132, 143, 154, 165.
70. 34, 51, 68, 85, 102, 119.
71. 885 and 5900
72. 1763 and 2021.
73. 1728 and 1760.
74. 973 and 12649
75. 29181 and 16851
76. 187, 781 and 497.
77. 323, 493 and 551.
78. 1078, 1617 and 1386
79. 690, 713, 736 and 759.
80. 2733, 4555, 6377 and 10021.

X. MISCELLANEOUS EXERCISES.

1. Express 31416 as the product of its prime factors
2. Show that 811 is a prime number
3. Find, *by inspection*, the G C M of 59 and 5959
4. Find the least number of which both 168 and 560 are factors.
5. What number is the same multiple of 13 that 6579 is of 17?

---

6. Separate 132288 into its elementary factors
7. Prove that 449 is a prime number
8. Find, *by inspection*, the G C M of 707 and 1212
9. Find the greatest number which divides 33495 and 106260
10. Prove 16327 to be the same multiple of 29 that 113163 is of 201

---

11. Find the sum of all the prime numbers between 75 and 115
12. Find the G C M of 104, 286 and 663
13. Find the L C M of 77, 707 and 7007
14. Show that there are six numbers, besides unity and the number itself, by which 455 is exactly divisible
15. What is the smallest sum of money which must be subtracted from £102, 10*s.* 9*d* in order that the remainder may exactly contain £1, 13*s* 7*d*?

---

16. Find the sum of all the prime numbers between 300 and 400
17. Find the G C M of 798, 5016 and 8664
18. Find the L C M of 200, 500, 800, 1100 and 1700
19. Find a common multiple of 15, 25 and 35, which is less than 2000 and greater than 1500
20. Find the least sum of money which added to £243, 5*s* 7½*d* makes the result exactly divisible by 431.

---

21. Express 693693 as the continued product of its prime factors
22. Prove that 115991 and 118009 are prime to each other.
23. Find the largest divisor of 201, 501, 801, 1101 and 1701
24. Find the least number which exactly contains both 451 and 533
25. Find, without division, the remainder of $432561 \div 9$

---

26. Resolve 114660 into its prime factors.
27. Write down at sight the G C F of $11 \times 17 \times 23$, $13 \times 19 \times 17$ and $17 \times 11 \times 19$
28. Find the smallest number which is exactly divisible by all the numbers between 1 and 13.

29. Find the largest number which is exactly contained in each of the numbers 798, 5016 and 8645

30. Find, without dividing, the remainder of $73894251 \div 4$

---

31. Find the complete remainder when 32547631 is divided by the prime factors of 1155

32. Is 70608050304 a multiple of 11?

33 How many times does the L.C.M. of 42, 56, 84 and 98 contain their G.C.M.?

34 Find the L.C.M. of 1s 2d, 1s 4d, 1s 6d and 1s 8d

35 Find the number nearest to 76478 which is divisible by 103.

---

36 Prove that 425 and 1062 are prime to each other

37. Is 9 a factor of 476583207?

38 Find the difference between the G.C.M. and L.C.M. of 111, 185 and 259

39. Find the L.C.M. of 2 tons 1 cwt. 2 qrs. and 7 tons 5 cwts 1 qr.

40. What is the least number which must be added to a million that the result may be exactly divisible by 365?

---

41. Find the multiple of 37 which is nearest to 1500.

42. Find the G.C.M. of 2 days 23 hours 40 mins, and 3 days 10 hours 25 mins

43 Find the smallest number which when divided by 2, 3, 4, 5, 6, 8, 9, 10, 11, or 12, gives remainder 1 in each case

44. Find, shortly, the sum of 567×36 and 567×64.

45. The G.C.M. of 493 and another number is 17; their L.C.M. is 18241, find the other number

---

46. Find the least number of three digits which exactly contains 41.

47. Find the G.C.M. of 5 cwts. 3 qrs. 24 lbs. 7 ozs. and 4 cwts. 1 qr. 18 lbs. 1 oz

48. Find the least number which can be divided by 9, 12, 15, or 20, with remainder 7 in each case.

49. Find, shortly, the difference between $90424 \div 73$ and $83051 \div 73$.

50. The G.C.F. of two numbers, each of four digits, is 431, their L.C.M. is 15085. Find the numbers.

---

51 Find the number nearest to 1000 which exactly contains 37.

52. Find the sum of all multiples of 8 greater than 300 and less than 400.

53. Find the greatest number which divides 3243 with remainder 3, and 4255 with remainder 7.

54. Three bells toll at intervals of 7, 12 and 15 seconds respectively If they all begin to toll together, after how many minutes will they again toll together?

55 Show that 108900 is a perfect square

---

56 Find the greatest number of three figures which is divisible by 41 without remainder

57. Find the sum of all the multiples of 9 which are greater than 150 and less than 200

58. What number divides 10224 with remainder 621, and 11886 with remainder 537?

59. If a franc is worth $9\frac{1}{2}d$ and a rupee $1s\ 2\frac{1}{4}d$, what is the smallest number of francs which contains an exact number of rupees?

60 Find the greatest number of grains which is exactly contained in both a pound Troy and a pound Avoirdupois

---

61. Find the smallest number of five digits which is a multiple of 473

62. Find the greatest length which exactly measures either 106 yds. 1 ft., or 135 yds 2 ft.

63 Find the least number which when divided by 77 gives remainder 76, when divided by 88 gives remainder 87, and when divided by 99 gives remainder 98

64. Find two numbers which have G C M 91, and L C M 1911

65. The product of two numbers, one of which is double the other, is 3528, find them

---

66. Find the greatest and least numbers each of six digits which are exactly divisible by 789

67. Find the shortest space of time which exactly contains either 1 day 12 hrs. 17 min, or 1 day 17 hrs 28 min

68 What is the least number which when divided by 26, 65, 91, or 143 leaves remainder 1 in each case?

69 The G C M. of two numbers is 137, the sum of the two numbers is 959, find them. How many correct answers are there?

70. By what must 40768 be multiplied to form a perfect square?

---

71. The numbers 390507 and 391337 are not prime to each other, find, without division, their G C F

72. Find *all* the common factors of 37037 and 23023

73. Two heaps of the same kind of shot weigh 180 tons 4 cwts. 2 qrs, and 124 tons 8 cwts. 2 qrs, respectively what is the greatest possible weight of each shot?

74 The product of two numbers is 76614; their G C F is 113, find their L.C M.

75. The product of three consecutive numbers is 10626, find them.

XI. FRACTIONS.

Notation.

Read, or write in words—

1. $\frac{1}{7}$; $\frac{2}{5}$; $\frac{3}{8}$, $\frac{5}{9}$; $\frac{1}{11}$; $\frac{4}{13}$.
2. $\frac{1}{20}$; $\frac{3}{7}$, $\frac{9}{10}$; $\frac{4}{19}$; $\frac{3}{40}$; $\frac{17}{30}$.
3. $\frac{2}{15}$, $\frac{10}{11}$; $\frac{19}{20}$; $\frac{3}{1000}$; $\frac{19}{800}$.
4. $\frac{12}{13}$; $\frac{1}{100}$, $\frac{69}{70}$; $\frac{7}{19}$; $\frac{311}{350}$.

Write in figures—

5 One-tenth, One-twelfth; Seven-twentieths; Thirteen-sixteenths.
6 Five-ninths; Twelve-nineteenths, Twenty-three hundredths.
7. Two-fifths; Seventy-three seven-hundredths, Three thousandths.
8. Three-sevenths; Nineteen ninety-ninths; Eight eighty-oneths.

Read off (or write down) at sight the number of—

9 shillings in $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{10}$; $\frac{1}{20}$, of a sovereign.
10. pence in $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{12}$; of a shilling.
11. ounces in $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$; of a pound Avoirdupois.
12. lbs. in $\frac{1}{2}$; $\frac{1}{7}$; $\frac{1}{14}$; $\frac{1}{28}$; of a quarter.
13. shillings in $\frac{3}{20}$; $\frac{7}{20}$; $\frac{11}{20}$; $\frac{13}{20}$; $\frac{19}{20}$; of £1.
14. pence in $\frac{5}{12}$; $\frac{7}{12}$; $\frac{3}{12}$, $\frac{11}{12}$; $\frac{8}{12}$; of a shilling
15. ounces in $\frac{3}{16}$; $\frac{5}{16}$; $\frac{7}{16}$; $\frac{13}{16}$; $\frac{11}{16}$; of a lb Av.
16. lbs. in $\frac{3}{28}$; $\frac{11}{28}$; $\frac{19}{28}$; $\frac{5}{28}$; $\frac{23}{28}$; of a quarter.
17 hours in $\frac{1}{2}$; $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{12}$; $\frac{1}{24}$; of a day.
18. minutes in $\frac{1}{2}$; $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$; $\frac{1}{10}$, $\frac{1}{12}$; $\frac{1}{15}$, $\frac{1}{20}$; $\frac{1}{30}$; of an hour
19. hours in $\frac{7}{24}$; $\frac{11}{24}$, $\frac{9}{24}$; $\frac{13}{24}$, $\frac{23}{24}$, of a day
20. minutes in $\frac{17}{60}$, $\frac{19}{60}$, $\frac{13}{60}$; $\frac{29}{60}$, $\frac{51}{60}$, of an hour.

Find the value of—

21. Half of 68; One-third of 93; Two-fifths of 365.
22. One-fifth of 80; One-seventh of 343; Five-elevenths of 187.
23. Three-fourths of a ton; Two-thirds of £24; One-tenth of 2s. 6d.
24 Three-quarters of an hour; Nine-tenths of £30; One-fifth of 7s 6d.
25 £$\frac{3}{10}$, £$\frac{7}{10}$; £$\frac{9}{10}$; £$\frac{2}{5}$, £$\frac{4}{5}$; £$\frac{3}{5}$, £$\frac{1}{240}$; £$\frac{7}{240}$; £$\frac{23}{240}$
26. £$\frac{1}{4}$; £$\frac{1}{10}$; £$\frac{11}{10}$, £$\frac{9}{20}$; £$\frac{13}{20}$; £$\frac{1}{40}$; £$\frac{11}{240}$; £$\frac{19}{240}$; £$\frac{29}{240}$.
27. $\frac{3}{4}$s.; $\frac{2}{3}$s.; $\frac{5}{6}$s; $\frac{1}{24}$s; $\frac{7}{24}$s.; $\frac{11}{24}$s.; $\frac{7}{48}$s; $\frac{11}{48}$s.; $\frac{1}{8}$s.; $\frac{5}{8}$s.
28 $\frac{3}{4}$ min., $\frac{5}{12}$ day, $\frac{3}{10}$ ton; $\frac{3}{10}$ hour; $\frac{7}{36}$ yard; $\frac{5}{18}$ sq ft.
29. £$\frac{13}{60}$; $\frac{5}{16}$s.; $\frac{17}{30}$ hour, $\frac{13}{40}$ ton, $\frac{5}{6}$ foot; $\frac{3}{10}$ of an acre.
30. £$\frac{19}{30}$; $\frac{13}{16}$s; $\frac{2}{3}$ of a year; $\frac{5}{14}$ cwt.; $\frac{5}{11}$ mile; $\frac{3}{16}$ of a gallon.

XII. FRACTIONS.

Improper Fractions and Mixed Numbers

Read, or write in words—

1. $5\frac{3}{8}$, $7\frac{4}{5}$, $9\frac{1}{11}$, $12\frac{3}{13}$, $15\frac{7}{20}$ 2 $1\frac{2}{33}$, $12\frac{17}{50}$, $10\frac{5}{9}$, $28\frac{7}{25}$.

Write in figures—

3 Two and three-fifths, Seven and one-third
4. Eight and seven-tenths, One and one-twelfth
5 Eleven and three-fourths, Seventeen and a quarter
6. Twenty and three-twentieths, Nine and eight-ninths

Turn, mentally, into integers—

7. $\frac{77}{7}$, $\frac{24}{8}$, $\frac{132}{11}$, $\frac{51}{3}$; $\frac{38}{19}$ 8. $\frac{100}{5}$, $\frac{100}{25}$, $\frac{808}{101}$, $\frac{81}{3}$, $\frac{56}{14}$

Turn, mentally, into mixed numbers—

9 $\frac{5}{3}$; $\frac{8}{5}$; $\frac{11}{2}$, $\frac{7}{6}$, $\frac{10}{7}$, $\frac{17}{4}$ 10. $\frac{7}{2}$, $\frac{9}{5}$, $\frac{14}{11}$, $\frac{21}{8}$, $\frac{16}{7}$; $\frac{13}{3}$
11. $\frac{22}{7}$; $\frac{18}{5}$; $\frac{30}{11}$, $\frac{41}{12}$, $\frac{80}{9}$.   12. $\frac{37}{6}$, $\frac{48}{11}$, $\frac{91}{12}$; $\frac{100}{9}$, $\frac{123}{10}$

Turn, mentally, into improper fractions—

13 $1\frac{1}{5}$, $2\frac{1}{3}$, $3\frac{3}{4}$, $5\frac{4}{7}$, $6\frac{5}{8}$ 14 $1\frac{1}{9}$, $3\frac{2}{5}$, $4\frac{5}{6}$, $5\frac{1}{2}$, $2\frac{5}{12}$
15 $4\frac{3}{4}$, $5\frac{5}{6}$, $3\frac{2}{7}$, $11\frac{1}{2}$, $10\frac{5}{8}$ 16 $7\frac{1}{7}$, $4\frac{2}{9}$, $1\frac{7}{20}$, $21\frac{1}{2}$, $9\frac{7}{9}$

Express as improper fractions—

17. $3\frac{13}{25}$	25 $9\frac{7}{15}$	33 $9\frac{1}{33}$	41 $100\frac{3}{8}$
18 $2\frac{5}{38}$	26. $15\frac{7}{9}$	34 $21\frac{7}{13}$	42 $5\frac{103}{108}$
19 $2\frac{17}{24}$	27. $12\frac{11}{12}$	35 $2\frac{51}{53}$	43 $97\frac{1}{51}$
20 $3\frac{19}{20}$	28 $4\frac{7}{19}$	36 $27\frac{7}{9}$	44 $27\frac{7}{19}$
21. $4\frac{20}{27}$	29 $5\frac{16}{17}$	37 $11\frac{23}{41}$	45. $53\frac{179}{200}$
22 $4\frac{9}{47}$	30 $11\frac{10}{11}$	38 $35\frac{5}{12}$	46. $50\frac{721}{831}$
23. $12\frac{11}{13}$	31 $13\frac{12}{13}$	39 $12\frac{29}{50}$	47. $5\frac{301}{1197}$
24 $2\frac{2}{71}$	32 $7\frac{5}{29}$	40. $30\frac{17}{40}$	48 $5\frac{2194}{2359}$

Express as mixed numbers—

49. $\frac{1000}{7}$	57. $\frac{4231}{100}$	65. $\frac{673}{35}$	73. $\frac{3797}{50}$.
50 $\frac{1001}{9}$	58. $\frac{53267}{1000}$	66. $\frac{827}{45}$	74 $\frac{5811}{40}$
51 $\frac{1010}{3}$	59 $\frac{603}{13}$	67. $\frac{7423}{300}$	75 $\frac{11325}{41}$
52 $\frac{1000}{11}$	60. $\frac{215}{14}$	68 $\frac{1973}{700}$	76 $\frac{42008}{31}$
53 $\frac{351}{12}$	61 $\frac{701}{15}$	69. $\frac{873}{110}$	77 $\frac{3622}{147}$
54. $\frac{777}{8}$	62 $\frac{843}{17}$	70 $\frac{8573}{5000}$	78 $\frac{7960}{207}$
55 $\frac{2361}{10}$	63 $\frac{789}{23}$.	71 $\frac{817}{552}$	79 $\frac{42653}{558}$
56. $\frac{579}{20}$	64 $\frac{1335}{29}$.	72. $\frac{605}{324}$	80. $\frac{18579}{844}$.

XIII. FRACTIONS.

REDUCTION.

Reduce, mentally—

1. Two *thirds* to *ninths.*
2. One *fourth* to *twelfths*
3. Seven *tenths* to *fiftieths.*
4. Three *sevenths* to *sixty-thirds.*
5. Seven *eighths* to *fortieths*
6. 4 *elevenths* to *fifty-fifths.*
7. 3 *fifths* to *fifty-fifths.*
8. 5 *sixths* to *thirtieths.*
9. 13 *fourteenths* to *seventieths*
10. 23 *twentieths* to *sixtieths.*
11. Six *eighths* to *fourths.*
12. Twelve *twentieths* to *fifths.*
13. Four *twenty-fourths* to *sixths*
14. Six *thirtieths* to *tenths.*
15. Twelve *sixtieths* to *fifteenths.*
16. 9 *twenty-oneths* to *sevenths.*
17. 15 *twenty-fifths* to *fifths.*
18. 8 *twentieths* to *fifths.*
19. 18 *thirtieths* to *tenths*
20. 25 *fortieths* to *eighths.*

Supply the proper figures in place of the asterisks in—

21. $\frac{2}{3} = \frac{*}{6}$.
22. $\frac{3}{4} = \frac{*}{12}$.
23. $\frac{2}{5} = \frac{*}{25}$
24. $\frac{5}{6} = \frac{*}{24}$
25. $\frac{9}{10} = \frac{*}{50}$.
26. $\frac{7}{11} = \frac{*}{44}$
27. $\frac{1}{12} = \frac{*}{84}$
28. $\frac{5}{9} = \frac{*}{63}$
29. $\frac{11}{13} = \frac{*}{39}$
30. $\frac{2}{5} = \frac{*}{100}$
31. $\frac{1}{4} = \frac{21}{*}$.
32. $\frac{3}{14} = \frac{15}{*}$.
33. $\frac{3}{17} = \frac{6}{*}$.
34. $\frac{5}{11} = \frac{20}{*}$
35. $\frac{8}{9} = \frac{64}{*}$.
36. $\frac{5}{19} = \frac{15}{*}$
37. $\frac{19}{25} = \frac{76}{*}$.
38. $\frac{20}{31} = \frac{60}{*}$
39. $\frac{9}{17} = \frac{45}{*}$.
40. $\frac{4}{19} = \frac{12}{*}$.
41. $\frac{10}{12} = \frac{5}{*}$.
42. $\frac{24}{28} = \frac{6}{*}$
43. $\frac{25}{45} = \frac{5}{*}$.
44. $\frac{40}{35} = \frac{8}{*}$.
45. $\frac{12}{20} = \frac{3}{*}$
46. $\frac{16}{24} = \frac{2}{*}$.
47. $\frac{40}{60} = \frac{2}{*}$
48. $\frac{75}{100} = \frac{3}{*}$
49. $\frac{28}{32} = \frac{7}{*}$
50. $\frac{21}{49} = \frac{3}{*}$.
51. $\frac{120}{132} = \frac{*}{11}$.
52. $\frac{63}{72} = \frac{*}{8}$.
53. $\frac{15}{35} = \frac{*}{7}$.
54. $\frac{24}{64} = \frac{*}{16}$.
55. $\frac{33}{39} = \frac{*}{13}$.
56. $\frac{48}{84} = \frac{*}{14}$.
57. $\frac{16}{56} = \frac{*}{14}$.
58. $\frac{57}{75} = \frac{*}{25}$.
59. $\frac{80}{144} = \frac{*}{9}$
60. $\frac{25}{200} = \frac{*}{40}$.

Express—

61. $\frac{7}{11}$ with denominator 55.
62. $\frac{9}{13}$ „ „ 65
63. $\frac{17}{21}$ „ „ 84
64. $\frac{5}{7}$ „ „ 119
65. $\frac{7}{12}$ „ „ 156
66. $\frac{41}{53}$ „ „ 159
67. $\frac{5}{7}$ with numerator 40
68. $\frac{9}{11}$ „ „ 108
69. $\frac{2}{17}$ „ „ 100
70. $\frac{13}{14}$ „ „ 169.
71. $\frac{12}{16}$ with denominator 8.
72. $\frac{24}{32}$ „ „ 4.
73. $\frac{40}{60}$ „ „ 3.
74. $\frac{56}{84}$ „ „ 12.
75. $\frac{75}{125}$ „ „ 25.
76. $\frac{110}{120}$ „ „ 24.
77. $\frac{18}{20}$ with numerator 9.
78. $\frac{42}{63}$ „ „ 6.
79. $\frac{91}{169}$ „ „ 7.
80. $\frac{220}{330}$ „ „ 2.

XIV. FRACTIONS.

Reduction to Lowest Terms.

Reduce to lowest terms—

1. $\frac{6}{12}$.	2. $\frac{8}{12}$.	3. $\frac{9}{12}$	4. $\frac{10}{12}$.
5 $\frac{15}{20}$	6. $\frac{18}{20}$	7. $\frac{12}{20}$	8. $\frac{16}{20}$
9. $\frac{8}{24}$	10. $\frac{9}{24}$	11. $\frac{15}{24}$	12. $\frac{16}{24}$
13 $\frac{6}{21}$	14 $\frac{14}{21}$	15. $\frac{12}{30}$	16 $\frac{18}{30}$
17 $\frac{28}{35}$.	18. $\frac{27}{36}$	19 $\frac{25}{35}$	20 $\frac{22}{33}$
21. $\frac{24}{36}$	22 $\frac{18}{36}$	23 $\frac{30}{36}$	24 $\frac{27}{36}$
25. $\frac{12}{33}$	26. $\frac{26}{39}$.	27 $\frac{34}{51}$	28. $\frac{14}{42}$
29 $\frac{16}{48}$.	30. $\frac{32}{48}$	31 $\frac{21}{48}$	32 $\frac{36}{48}$
33. $\frac{75}{100}$.	34. $\frac{80}{100}$.	35. $\frac{125}{500}$	36 $\frac{625}{1000}$
37 $\frac{49}{63}$.	38 $\frac{88}{121}$.	39 $\frac{81}{99}$	40. $\frac{54}{66}$
41. $\frac{27}{72}$	42 $\frac{60}{108}$	43 $\frac{64}{96}$	44. $\frac{45}{120}$
45. $\frac{39}{65}$	46. $\frac{50}{75}$	47 $\frac{51}{85}$	48 $\frac{57}{95}$
49. $\frac{900}{1200}$.	50 $\frac{1500}{3500}$	51 $\frac{260}{390}$	52 $\frac{3800}{5700}$
53 $\frac{101}{707}$	54 $\frac{111}{444}$	55 $\frac{1001}{7007}$	56. $\frac{2002}{3003}$
57 $\frac{303}{808}$	58. $\frac{777}{999}$	59 $\frac{1313}{1717}$	60. $\frac{3333}{5555}$
61 $\frac{119}{153}$	62. $\frac{117}{169}$	63 $\frac{133}{205}$	64 $\frac{141}{329}$
65. $\frac{273}{364}$	66. $\frac{292}{365}$	67. $\frac{305}{366}$	68 $\frac{384}{672}$
69. $\frac{392}{441}$.	70. $\frac{299}{391}$	71. $\frac{377}{403}$	72. $\frac{299}{377}$
73. $\frac{215}{440}$	74 $\frac{414}{441}$	75 $\frac{713}{943}$	76 $\frac{501}{2171}$
77. $\frac{595}{1071}$	78. $\frac{501}{1837}$	79 $\frac{594}{2079}$	80. $\frac{949}{1387}$
81 $\frac{261}{3103}$	82. $\frac{111}{1101}$	83 $\frac{3330}{5328}$	84. $\frac{4844}{5536}$
85. $\frac{1160}{1924}$	86 $\frac{7623}{8316}$	87 $\frac{1133}{1339}$	88. $\frac{1331}{1430}$
89 $\frac{1653}{1863}$	90. $\frac{2592}{3456}$	91. $\frac{1892}{1936}$	92. $\frac{2109}{3813}$
93. $\frac{2471}{9922}$	94. $\frac{1919}{2223}$	95. $\frac{2007}{12265}$	96 $\frac{24389}{26071}$.
97 $\frac{21189}{36414}$	98. $\frac{64961}{68419}$	99. $\frac{237655}{387285}$	100. $\frac{11413}{22066}$

Express as mixed numbers in simplest form—

101. $\frac{803}{14}$	102. $\frac{402}{9}$	103 $\frac{725}{15}$.	104. $\frac{747}{18}$
105. $\frac{441}{72}$.	106 $\frac{495}{65}$.	107. $\frac{924}{172}$	108 $\frac{987}{105}$
109. $\frac{1382}{33}$	110 $\frac{1967}{28}$.	111. $\frac{1440}{84}$	112 $\frac{1313}{91}$
113. $\frac{417}{411}$	114. $\frac{1407}{1400}$.	115. $\frac{4191}{771}$	116. $\frac{3553}{779}$
117. $\frac{8894}{385}$	118. $\frac{6123}{598}$.	119 $\frac{27474}{20485}$.	120. $\frac{100110}{31866}$.

XV. FRACTIONS.

REDUCTION TO LEAST COMMON DENOMINATOR.

Reduce to their least common denominator—

1. $\frac{5}{6}$ and $\frac{7}{8}$
2. $\frac{7}{12}$ and $\frac{11}{18}$.
3. $\frac{3}{20}$ and $\frac{7}{30}$.
4. $\frac{11}{15}$ and $\frac{13}{20}$.
5. $\frac{5}{16}$ and $\frac{11}{24}$.
6. $\frac{9}{50}$ and $\frac{13}{75}$
7. $\frac{17}{48}$ and $\frac{21}{64}$
8. $\frac{16}{63}$ and $\frac{19}{84}$
9. $\frac{13}{24}$ and $\frac{17}{36}$.
10. $\frac{7}{39}$ and $\frac{4}{51}$
11. $\frac{11}{36}$ and $\frac{17}{54}$.
12. $\frac{22}{57}$ and $\frac{41}{95}$.
13. $\frac{1}{6}$, $\frac{3}{8}$ and $\frac{5}{12}$
14. $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{8}$.
15. $\frac{3}{8}$, $\frac{7}{12}$ and $\frac{9}{16}$
16. $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$.
17. $\frac{5}{14}$, $\frac{8}{21}$, $\frac{11}{28}$.
18. $\frac{11}{24}$, $\frac{15}{36}$, $\frac{19}{48}$.
19. $\frac{2}{15}$, $\frac{3}{20}$, $\frac{4}{25}$.
20. $\frac{7}{30}$, $\frac{9}{40}$, $\frac{11}{50}$
21. $\frac{41}{48}$, $\frac{51}{56}$, $\frac{61}{72}$
22. $\frac{11}{42}$, $\frac{11}{63}$, $\frac{11}{84}$
23. $\frac{1}{68}$, $\frac{1}{85}$, $\frac{1}{102}$
24. $\frac{4}{39}$, $\frac{4}{65}$, $\frac{4}{91}$

Reduce to their least common denominator—

25. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$.
26. $\frac{7}{8}$, $\frac{5}{16}$, $\frac{3}{32}$ and $\frac{1}{64}$.
27. $\frac{1}{10}$, $\frac{2}{15}$, $\frac{3}{20}$ and $\frac{4}{25}$.
28. $\frac{21}{22}$, $\frac{29}{33}$, $\frac{19}{44}$ and $\frac{18}{55}$.
29. $\frac{5}{13}$, $\frac{8}{21}$, $\frac{11}{28}$ and $\frac{17}{42}$.
30. $\frac{5}{14}$, $\frac{4}{15}$, $\frac{14}{45}$ and $\frac{15}{56}$
31. $\frac{71}{202}$, $\frac{141}{404}$ and $\frac{281}{808}$
32. $\frac{83}{333}$, $\frac{167}{666}$ and $\frac{251}{999}$.
33. $\frac{3}{35}$, $\frac{3}{40}$, $\frac{3}{56}$ and $\frac{3}{70}$.
34. $\frac{3}{55}$, $\frac{4}{65}$, $\frac{7}{121}$ and $\frac{8}{143}$.
35. $\frac{307}{1760}$ and $\frac{301}{1728}$.
36. $\frac{13}{1212}$, $\frac{16}{1515}$ and $\frac{19}{1818}$.

Which is greater—

37. $\frac{5}{7}$ or $\frac{7}{9}$?
38. $\frac{9}{14}$ or $\frac{13}{20}$?
39. $\frac{7}{8}$ or $\frac{93}{104}$?
40. $\frac{17}{24}$ or $\frac{83}{108}$?
41. $\frac{31}{36}$ or $\frac{79}{90}$?
42. $\frac{23}{84}$ or $\frac{37}{144}$?
43. $\frac{1}{9}$ or $\frac{88}{801}$?
44. $\frac{5}{7}$ or $\frac{51}{71}$?
45. $\frac{10}{143}$ or $\frac{11}{156}$?
46 $\frac{16}{19}$ or $\frac{14}{17}$?
47. $\frac{25}{37}$ or $\frac{52}{73}$?
48. $\frac{17}{117}$ or $\frac{23}{153}$?

Arrange in *ascending* order of magnitude—

49. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{8}{11}$ and $\frac{17}{22}$.
50. $\frac{3}{7}$, $\frac{5}{12}$, $\frac{10}{21}$ and $\frac{19}{42}$.
51. $\frac{13}{28}$, $\frac{16}{35}$ and $\frac{19}{42}$
52. $\frac{7}{10}$, $\frac{8}{11}$, $\frac{15}{22}$ and $\frac{29}{55}$.
53. $\frac{3}{16}$, $\frac{5}{27}$, $\frac{7}{36}$ and $\frac{25}{144}$
54. $\frac{3}{68}$, $\frac{4}{85}$ and $\frac{5}{119}$.

Arrange in *descending* order of magnitude—

55. $\frac{26}{33}$, $\frac{53}{66}$ and $\frac{80}{99}$.
56. $\frac{7}{12}$, $\frac{9}{16}$, $\frac{11}{18}$ and $\frac{13}{24}$
57 $\frac{4}{5}$, $\frac{11}{15}$, $\frac{19}{25}$ and $\frac{27}{35}$.
58. $\frac{17}{26}$, $\frac{33}{52}$, $\frac{42}{65}$ and $\frac{87}{130}$.

59 Compare $\frac{19}{50}$, $\frac{28}{75}$, $\frac{31}{90}$

60. Compare $\frac{1}{11}$, $\frac{11}{111}$, $\frac{111}{1111}$

XVI. ADDITION OF FRACTIONS.

Read off, or write down, the sum of—

1 3 *elevenths* and 5 *elevenths*

2. 5 *twelfths* and 6 *twelfths*

3. 7 *thirtieths* and 16 *thirtieths*

4 19 *ninetieths* and 13 *ninetieths*.

5 $\frac{5}{24}$, $\frac{7}{24}$ and $\frac{1}{24}$.

6 $\frac{3}{17}$, $\frac{5}{17}$ and $\frac{4}{17}$

7. $\frac{3}{29}$, $\frac{5}{29}$ and $\frac{11}{29}$.

8. $\frac{7}{43}$, $\frac{9}{43}$ and $\frac{10}{43}$

9 $\frac{9}{100}$, $\frac{11}{100}$ and $\frac{13}{100}$

10 $\frac{5}{27}$, $\frac{7}{27}$, $\frac{8}{27}$ and $\frac{2}{27}$.

Find, mentally, the sum of—

11 $\frac{1}{2}$ and $\frac{1}{3}$.

12. $\frac{1}{3}$ and $\frac{1}{4}$

13 $\frac{2}{3}$ and $\frac{1}{4}$

14 $\frac{1}{4}$ and $\frac{2}{5}$

15 $\frac{1}{2}$ and $\frac{5}{12}$.

16 $\frac{2}{3}$ and $\frac{1}{6}$

17. $\frac{1}{3}$ and $\frac{7}{12}$

18 $\frac{3}{4}$ and $\frac{1}{6}$.

19. $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{5}{12}$

20. $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{11}{20}$

21. $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{12}$.

22. $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$

Add together—

23. $\frac{3}{8}$ and $\frac{7}{12}$

24 $\frac{4}{21}$ and $\frac{5}{28}$

25. $\frac{5}{9}$ and $\frac{7}{11}$.

26. $\frac{5}{8}$ and $\frac{5}{7}$

27. $\frac{11}{18}$ and $\frac{17}{27}$

28. $\frac{10}{21}$ and $\frac{12}{35}$

29. $\frac{1}{4}$ and $\frac{5}{28}$

30 $\frac{3}{8}$ and $\frac{1}{24}$

31 $\frac{3}{5}$ and $\frac{4}{35}$

32 $\frac{5}{12}$ and $\frac{7}{60}$

33 $\frac{6}{7}$ and $\frac{10}{21}$

34 $\frac{5}{9}$ and $\frac{25}{36}$

35. $\frac{1}{6}$, $\frac{3}{7}$ and $\frac{5}{21}$

36. $\frac{2}{5}$, $\frac{1}{6}$ and $\frac{1}{60}$

37. $\frac{1}{8}$, $\frac{1}{9}$ and $\frac{5}{144}$

38 $\frac{1}{16}$, $\frac{1}{24}$ and $\frac{41}{48}$

39 $\frac{5}{8}$, $\frac{1}{40}$ and $\frac{23}{80}$

40 $\frac{8}{15}$, $\frac{11}{40}$ and $\frac{41}{120}$

Find, in simplest form, the value of—

41. $\frac{2}{3}+\frac{5}{12}+\frac{1}{4}$

42 $\frac{3}{4}+\frac{4}{5}+\frac{5}{6}$

43 $\frac{3}{10}+\frac{7}{20}+\frac{11}{30}$

44. $\frac{8}{9}+\frac{1}{27}+\frac{5}{18}$

45 $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$

46. $\frac{1}{2}+\frac{3}{8}+\frac{4}{15}+\frac{7}{24}$

47. $\frac{1}{10}+\frac{11}{100}+\frac{17}{1000}$

48. $\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}+\frac{3}{10000}$

49 $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$

50. $\frac{1}{2}+\frac{1}{3}+\frac{1}{8}+\frac{1}{24}$

51. $\frac{3}{4}+\frac{13}{16}+\frac{13}{32}+\frac{7}{480}$

52 $\frac{1}{3}+\frac{3}{8}+\frac{2}{5}+\frac{5}{12}+\frac{3}{7}$

53 $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+\frac{9}{10}+\frac{11}{12}$

54 $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$

55. $\frac{7}{15}+\frac{13}{25}+\frac{19}{24}$

56 $\frac{13}{42}+\frac{5}{72}+\frac{17}{63}+\frac{7}{24}+\frac{9}{56}$.

57. $\frac{12}{17}+\frac{8}{51}+\frac{2}{3}$

58 $\frac{5}{17}+\frac{2}{51}+\frac{3}{34}$

59 $\frac{7}{13}+\frac{5}{26}+\frac{11}{39}$.

60 $\frac{7}{65}+\frac{17}{39}+\frac{4}{15}$

61. $2\frac{1}{3}+3\frac{1}{4}+4\frac{1}{5}$.

62 $1\frac{2}{3}+1\frac{3}{4}+1\frac{4}{5}$

63. $17\frac{1}{2}+12\frac{5}{6}+9\frac{1}{3}$

64 $2\frac{2}{3}+13\frac{3}{7}+4\frac{4}{21}$

65. $\frac{7}{16}+\frac{2}{3}+\frac{5}{48}+3\frac{19}{24}$.

66 $2\frac{3}{7}+5\frac{1}{4}+\frac{8}{21}+1\frac{5}{12}$

67. $120\frac{1}{2}+791\frac{2}{3}+12\frac{7}{12}$.

68 $785\frac{1}{8}+81\frac{5}{6}+133\frac{1}{3}$

69. $90\frac{7}{30}+90\frac{7}{40}+90\frac{7}{50}$

70. $17\frac{1}{3}+12\frac{2}{5}+\frac{11}{100}+99\frac{1}{2}$.

71. $5\frac{4}{9} + \frac{3}{8} + \frac{7}{12} + 2\frac{5}{24}$.
72. $3\frac{4}{15} + 17\frac{1}{12} + 9\frac{1}{2} + 15\frac{2}{3}$
73. $1\frac{1}{3} + 2\frac{2}{9} + 3\frac{4}{27} + 4\frac{5}{81}$
74. $6\frac{1}{2} + 5\frac{3}{4} + 4\frac{5}{8} + 3\frac{7}{16} + 2\frac{9}{32}$.
75. $13\frac{7}{38} + 14\frac{8}{57}$
76. $1\frac{3}{28} + 2\frac{5}{42} + 3\frac{13}{154}$.
77. $3\frac{1}{3} + 4\frac{3}{8} + 2\frac{1}{9} + \frac{19}{72}$.
78. $3\frac{1}{7} + 3\frac{3}{10} + 2\frac{3}{11} + 2\frac{5}{22} + \frac{2}{35}$
79. $1\frac{1}{4} + 2\frac{5}{12} + \frac{4}{15} + \frac{7}{25} + \frac{11}{30}$
80. $7\frac{9}{20} + 12\frac{4}{15} + 4\frac{7}{12} + \frac{9}{40} + 3\frac{7}{10}$
81. $200\frac{1}{400} + 300\frac{1}{600} + 400\frac{1}{800}$.
82. $404\frac{2}{15} + 2\frac{1}{25} + 1002\frac{7}{45}$
83. $46231\frac{7}{110} + 2063\frac{11}{70}$
84. $35\frac{3}{4} + 762\frac{7}{8} + 8456\frac{2}{3}$.
85 $5\frac{1}{3} + \frac{15}{7} + \frac{5}{6} + 3\frac{1}{14}$.
86. $\frac{2}{3} + 6\frac{5}{7} + \frac{13}{9} + \frac{8}{21}$.
87. $\frac{3}{7} + \frac{5}{8} + \frac{23}{14} + 4\frac{1}{4}$.
88. $\frac{4}{15} + \frac{5}{8} + \frac{17}{10} + 4\frac{5}{6}$.
39. $\frac{14}{3} + \frac{26}{4} + 5\frac{1}{5} + 2061\frac{11}{15} + 7\frac{2}{25}$.
90. $\frac{145}{225} + \frac{39}{90} + 7 + 5\frac{4}{45} + 3$[illegible]
91. $\frac{5}{144} + \frac{5}{156} + 3\frac{5}{168}$
92. $1\frac{3}{25} + 11\frac{33}{125} + 111$[illegible]
93. $1\frac{7}{1225} + 2\frac{5}{49} + 3\frac{3}{245}$
94. $1\frac{19}{219} + 1\frac{18}{365} + 1\frac{19}{292} + 1\frac{17}{146}$.
95. $17\frac{17}{18} + 18\frac{18}{17}$.
96. $10\frac{5}{1768} + 20\frac{5}{2173}$.
97. $3\frac{234}{432} + 7\frac{252}{288}$.
98. $11\frac{2163}{5047} + 17\frac{868}{1953}$.
99. $425913\frac{175}{1484} + 796753\frac{28}{2151}$.
100. $7\frac{1}{3542} + 8\frac{1}{3813} + 9\frac{1}{5084}$.

XVII. SUBTRACTION OF FRACTIONS.

Find, mentally, the difference between—

1 $\frac{8}{11}$ and $\frac{3}{11}$
2 $\frac{9}{17}$ and $\frac{5}{17}$
3. $\frac{17}{25}$ and $\frac{4}{25}$
4. $\frac{30}{49}$ and $\frac{25}{49}$.
5. $\frac{7}{8}$ and $\frac{3}{4}$.
6. $\frac{5}{12}$ and $\frac{5}{24}$
7. $\frac{3}{8}$ and $\frac{5}{16}$
8. $\frac{7}{15}$ and $\frac{7}{30}$
9. $\frac{7}{8}$ and $\frac{1}{32}$.
10. $\frac{5}{7}$ and $\frac{9}{56}$.
11. $\frac{17}{28}$ and $\frac{2}{7}$.
12. $\frac{19}{64}$ and $\frac{3}{16}$.
13. $\frac{17}{18}$ and $\frac{4}{9}$.
14. $\frac{23}{48}$ and $\frac{3}{8}$.
15. $\frac{101}{112}$ and $\frac{1}{56}$.
16. $\frac{79}{80}$ and $\frac{3}{40}$
17. $13\frac{1}{3}$ and 7.
18. $12\frac{3}{8}$ and 9
19. $7\frac{5}{6}$ and $2\frac{5}{6}$
20. $11\frac{5}{21}$ and $3\frac{5}{21}$.
21. $2\frac{5}{6}$ and $\frac{2}{3}$.

Find, mentally, the value of—

22. $3 - \frac{7}{12}$.
23. $5 - \frac{4}{7}$.
24. $7 - \frac{3}{8}$
25. $11 - \frac{3}{16}$.
26. $15 - \frac{9}{20}$
27 $12 - \frac{11}{14}$.
28. $17 - \frac{19}{35}$
29. $8 - 3\frac{2}{5}$.
30 $7 - 1\frac{5}{6}$
31. $12 - 6\frac{1}{7}$
32. $14 - 12\frac{2}{9}$
33. $17 - 7\frac{7}{17}$.
34. $19 - 1\frac{1}{19}$.
35. $23 - 7\frac{15}{28}$.
36. $41 - 32\frac{9}{40}$.
37. $12\frac{1}{2} - 7\frac{1}{4}$.
38. $8\frac{3}{4} - 1\frac{5}{8}$.
39. $7\frac{3}{5} - 2$.
40 $12\frac{17}{28} - 7$.
41. $5\frac{3}{7} - 2\frac{2}{3}$.
42. $7\frac{2}{5} - 3\frac{1}{4}$.

Find the difference between—

43. $\frac{7}{30}$ and $\frac{17}{36}$
44. $\frac{23}{63}$ and $\frac{17}{42}$
45. $8\frac{5}{24}$ and $12\frac{7}{32}$
46. $12\frac{13}{42}$ and $17\frac{9}{28}$
47. $\frac{7}{12}$ and $\frac{19}{32}$
48. $\frac{13}{40}$ and $\frac{19}{56}$
49. $\frac{5}{39}$ and $\frac{7}{52}$
50. $\frac{7}{65}$ and $\frac{11}{91}$
51. $21\frac{7}{9}$ and $21\frac{8}{11}$.
52. $67\frac{4}{21}$ and $67\frac{5}{28}$
53. $79\frac{5}{8}$ and $78\frac{5}{9}$.
54. $110\frac{7}{9}$ and $109\frac{7}{10}$

Find, in simplest form, the value of—

55. $3\frac{1}{7} - 1\frac{1}{5}$
56. $7\frac{3}{8} - 2\frac{3}{5}$
57. $12\frac{2}{5} - 8\frac{3}{4}$
58. $14\frac{2}{7} - 5\frac{2}{3}$
59. $20\frac{1}{4} - 6\frac{5}{8}$
60. $23\frac{1}{2} - 10\frac{7}{8}$
61 $8\frac{4}{5} - 4\frac{9}{11}$
62 $31\frac{5}{8} - 22\frac{5}{7}$
63. $75\frac{5}{21} - \frac{9}{28}$
64. $33\frac{11}{36} - 29\frac{29}{45}$
65. $11\frac{11}{78} - 7\frac{10}{13}$
66. $16\frac{2}{15} - 11\frac{7}{18}$.
67. $100\frac{1}{100} - 4\frac{57}{140}$
68. $472\frac{7}{130} - \frac{19}{39}$
69 $34\frac{3}{77} - 33\frac{3}{14}$
70 $13\frac{9}{280} - 12\frac{6}{35}$
71 $19\frac{19}{84} - 14\frac{15}{28}$
72 $120\frac{7}{60} - 9\frac{43}{220}$
73 $63\frac{5}{76} - 34\frac{46}{57}$
74 $17\frac{11}{153} - 15\frac{19}{34}$
75 $111\frac{12}{13} - 7\frac{30}{31}$
76 $19\frac{5}{432} - \frac{11}{54}$
77 $15\frac{1}{17} - \frac{32}{33}$
78 $17\frac{7}{707} - 7\frac{7}{777}$.
79 $5\frac{17}{30} - \frac{443}{400}$
80 $578\frac{3}{14} - \frac{4931}{70}$.
81 $\frac{23}{286} - \frac{51}{637}$.
82. $\frac{170}{171} - \frac{207}{209}$
83 $112\frac{20}{1210} - \frac{130}{1100}$
84. $8569\frac{25}{750} - \frac{875}{1000}$
85 $9876 - \frac{9874}{9875}$.
86. $12345 - \frac{12344}{12345}$.
87. $\frac{8}{2717} - \frac{7}{2431}$.
88. $\frac{231}{264} - \frac{594}{693}$
89 $\frac{1001}{1386} - \frac{935}{2295}$
90. $\frac{1065}{38766} - \frac{165}{45045}$

Simplify—

91. $\frac{13}{60} - \frac{13}{42} + \frac{13}{90}$
92. $\frac{19}{35} - \frac{38}{45} + \frac{19}{63}$.
93. $\frac{5}{14} - \frac{5}{16} - \frac{1}{112}$
94. $2\frac{9}{10} - 1\frac{7}{16} + 1\frac{17}{40}$
95. $1\frac{7}{102} + \frac{5}{34} - 1\frac{18}{85}$.
96. $\frac{4}{9} - \frac{5}{18} + \frac{7}{36} - \frac{11}{72}$
97. $\frac{11}{180} - \frac{7}{12} + \frac{5}{9} - \frac{1}{30}$
98. $\frac{1}{3} - \frac{4}{21} + \frac{1}{2} - \frac{7}{11}$
99. $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{47}{60}$.
100. $\frac{3}{8} - \frac{5}{12} + \frac{8}{15} - \frac{3}{20} - \frac{1}{120}$
101. $3\frac{1}{2} - 1\frac{7}{8} + 4\frac{5}{12} - 2\frac{11}{30}$
102 $3\frac{10}{11} + 5\frac{7}{15} - 2\frac{9}{22} - 4\frac{9}{10}$
103. $2\frac{3}{4} + 1\frac{5}{8} + 2\frac{3}{10} - 3\frac{5}{24} + 1\frac{8}{15}$.
104 $\frac{1}{6} + 2\frac{1}{7} + 13\frac{3}{10} + \frac{4}{21} - 4\frac{33}{35}$.
105. $1\frac{3}{8} + 3\frac{1}{3} - 5\frac{5}{6} + 2\frac{1}{4} - 1\frac{1}{12}$.
106. $2\frac{1}{5} + 1\frac{4}{7} - 4\frac{1}{2} - \frac{3}{5} + 2\frac{8}{35}$
107. $3\frac{1}{2} - 4\frac{1}{3} + 5\frac{1}{4} - 6\frac{1}{5} + 7\frac{1}{6}$
108 $12\frac{1}{12} - 13\frac{1}{13} + 26\frac{1}{26} - 24\frac{1}{24}$
109. $3\frac{1}{4} - 4\frac{3}{8} + 5\frac{5}{12} - 6\frac{7}{16} + 3\frac{1}{6}$
110 $7\frac{5}{7} - 8\frac{4}{5} - 9\frac{11}{21} + 10\frac{7}{15} + 1\frac{3}{5}$
111. $20 - \frac{2}{3} - \frac{3}{4} - \frac{4}{5} - \frac{5}{6}$.
112 $10 - \frac{3}{5} - \frac{5}{7} - \frac{7}{10} - \frac{9}{14}$.
113. $\frac{1}{8} - \frac{1}{64} - \frac{1}{512} - \frac{1}{4096}$
114 $\frac{1}{3} - \frac{1}{9} - \frac{1}{27} - \frac{1}{81} - \frac{1}{243}$
115 $1\frac{1}{4} - 2\frac{7}{8} - 3\frac{7}{9} - 4\frac{7}{12} + 11\frac{1}{6}$.
116 $2\frac{2}{5} - 3\frac{3}{10} - 4\frac{4}{15} - 7\frac{7}{25} + 15\frac{1}{30}$
117. $34\frac{1}{4} + 22\frac{1}{6} - 16\frac{7}{8} - 14\frac{7}{9} - 12\frac{11}{12}$.
118 $4\frac{1}{4} - 5\frac{3}{16} - 6\frac{5}{64} - 7\frac{7}{256} + 18\frac{1}{512}$
119. $\frac{311}{7} - \frac{211}{5} + \frac{111}{3} - \frac{11}{2} + 1$.
120. $\frac{2400}{7} - \frac{342}{49} + \frac{48}{343} - \frac{16}{2401}$.

XVIII. FRACTIONS.

Multiplication and Division by an Integer.

Read off, or write down, in lowest terms—

1. $\frac{3}{7} \times 2$
2. $\frac{5}{11} \times 2.$
3. $\frac{7}{19} \times 2.$
4. $\frac{8}{17} \times 2.$
5. $\frac{2}{15} \times 4.$
6. $\frac{3}{16} \times 3$
7. $\frac{1}{11} \times 7.$
8. $\frac{5}{23} \times 4.$
9. $\frac{13}{27} \times 8.$
10. $\frac{13}{100} \times 7.$
11. $\frac{3}{8} \times 2.$
12. $\frac{5}{12} \times 2.$
13. $\frac{1}{10} \times 2$
14. $\frac{3}{14} \times 2.$
15. $\frac{5}{21} \times 3.$
16. $\frac{2}{15} \times 3.$
17. $\frac{7}{100} \times 4$
18. $\frac{11}{100} \times 5$
19. $\frac{2}{35} \times 7$
20. $\frac{3}{32} \times 8.$
21. $\frac{6}{11} \div 2.$
22. $\frac{8}{15} \div 2$
23. $\frac{10}{17} \div 2$
24. $\frac{14}{19} \div 2$
25. $\frac{12}{13} \div 3$
26. $\frac{15}{16} \div 3$
27. $\frac{16}{21} \div 4$
28. $\frac{28}{45} \div 4$
29. $\frac{25}{26} \div 5$
30. $\frac{14}{41} \div 7.$
31. $\frac{7}{15} \div 2$
32. $\frac{3}{10} \div 2.$
33. $\frac{5}{26} \div 2$
34. $\frac{11}{34} \div 2.$
35. $\frac{7}{15} \div 4.$
36. $\frac{23}{25} \div 3.$
37. $\frac{5}{12} \div 8$
38. $\frac{19}{24} \div 4.$
39. $\frac{11}{20} \div 6.$
40. $\frac{10}{11} \div 7.$

Multiply—

41. $\frac{5}{21}$ by 14.
42. $\frac{4}{35}$ by 26.
43. $\frac{7}{60}$ by 48.
44. $\frac{11}{48}$ by 64
45. $\frac{22}{57}$ by 38.
46. $\frac{23}{84}$ by 36
47. $\frac{7}{19}$ by 21
48. $\frac{13}{17}$ by 40
49. $\frac{15}{41}$ by 34
50. $\frac{27}{46}$ by 69.
51. $6\frac{3}{4}$ by 2.
52. $7\frac{4}{9}$ by 3.
53. $4\frac{5}{12}$ by 9
54. $3\frac{7}{15}$ by 10
55. $8\frac{2}{3}$ by 11
56. $7\frac{7}{9}$ by 15
57. $8\frac{11}{30}$ by 45.
58. $10\frac{13}{21}$ by 28.
59. $12\frac{17}{18}$ by 12.
60. $14\frac{5}{14}$ by 49.
61. $\frac{127}{146}$ by 365.
62. $\frac{258}{365}$ by 219.
63. $58\frac{17}{26}$ by 390
64. $79\frac{83}{144}$ by 168

Divide—

65. $\frac{16}{17}$ by 12.
66. $\frac{6}{7}$ by 27.
67. $\frac{14}{25}$ by 35.
68. $\frac{28}{33}$ by 42.
69. $\frac{20}{21}$ by 3.
70. $\frac{17}{20}$ by 5.
71. $\frac{22}{31}$ by 77.
72. $\frac{84}{95}$ by 63.
73. $\frac{17}{50}$ by 13.
74. $\frac{23}{25}$ by 25.
75. $1\frac{1}{3}$ by 9
76. $3\frac{3}{7}$ by 12.
77. $5\frac{5}{8}$ by 18.
78. $7\frac{1}{3}$ by 33.
79. $6\frac{3}{8}$ by 34.
80. $12\frac{2}{3}$ by 8.
81. $10\frac{5}{7}$ by 25.
82. $7\frac{5}{13}$ by 24.
83. $6\frac{5}{12}$ by 42.
84. $3\frac{7}{8}$ by 51.
85. $240\frac{2}{3}$ by 209.
86. $91\frac{1}{4}$ by 292.
87. $461\frac{1}{3}$ by 332
88. $488\frac{1}{7}$ by 323.

89. Simplify $1347\frac{3}{4} \times 12$

90. Simplify $1347\frac{3}{4} \div 12.$

XIX. MULTIPLICATION OF FRACTIONS.

Find the value of—

1. $\frac{3}{8}$ of $\frac{7}{9}$	6. $\frac{16}{21}$ of $\frac{8}{9}$	11. $\frac{8}{9}$ of $3\frac{3}{4}$.	16 $\frac{21}{28}$ of $1\frac{1}{8}$
2. $\frac{4}{7}$ of $\frac{3}{10}$	7. $\frac{8}{9}$ of $\frac{9}{8}$	12. $\frac{21}{40}$ of $1\frac{1}{9}$	17 $\frac{26}{91}$ of $6\frac{1}{2}$.
3. $\frac{8}{15}$ of $\frac{9}{28}$	8. $\frac{8}{11}$ of $\frac{11}{16}$	13 $\frac{4}{5}$ of $1\frac{1}{39}$	18 $\frac{7}{8}$ of $60\frac{4}{7}$
4. $\frac{14}{25}$ of $\frac{45}{56}$	9 $\frac{13}{39}$ of 3	14. $\frac{11}{38}$ of $8\frac{1}{7}$	19 $\frac{112}{365}$ of $2\frac{3}{35}$
5. $\frac{9}{11}$ of $\frac{13}{17}$	10. $\frac{1}{12}$ of $\frac{17}{34}$	15. $\frac{35}{36}$ of $10\frac{2}{7}$	20 $\frac{95}{219}$ of $19\frac{4}{15}$.

Multiply—

21 $\frac{11}{42}$ by $\frac{28}{55}$	26 $\frac{84}{144}$ by $\frac{64}{112}$.	31. $1\frac{7}{8}$ by $1\frac{1}{3}$	36 $15\frac{4}{5}$ by $3\frac{8}{9}$
22 $\frac{33}{56}$ by $\frac{35}{88}$.	27. $\frac{51}{85}$ by $\frac{25}{27}$	32. $2\frac{2}{5}$ by $2\frac{1}{7}$	37 $3\frac{4}{5}$ by $3\frac{4}{5}$
23 $\frac{21}{48}$ by $\frac{32}{63}$.	28 $\frac{65}{100}$ by $\frac{52}{169}$	33. $10\frac{2}{7}$ by $2\frac{11}{12}$	38. $7\frac{1}{7}$ by $7\frac{1}{7}$
24. $\frac{44}{81}$ by $\frac{36}{77}$.	29 $\frac{45}{70}$ by $\frac{28}{18}$	34 $12\frac{1}{12}$ by $1\frac{1}{15}$	39 $6\frac{16}{23}$ by $6\frac{8}{11}$
25 $\frac{96}{108}$ by $\frac{60}{80}$	30 $\frac{35}{84}$ by $\frac{36}{15}$	35. $4\frac{3}{13}$ by $3\frac{6}{11}$	40. $6\frac{28}{45}$ by $1\frac{58}{77}$.

Simplify—

41 $255\frac{1}{2} \times 1\frac{1}{7} \times 10$	42. $132\frac{4}{5} \times 3\frac{1}{8} \times \frac{1}{10}$
43 $12\frac{1}{12} \times 7\frac{1}{5} \times \frac{1}{2}$.	44 $7\frac{7}{44} \times 2\frac{17}{63} \times 2$
45 $\frac{36}{65} \times \frac{52}{99} \times \frac{11}{80}$	46 $\frac{39}{46} \times \frac{85}{91} \times \frac{69}{102}$.
47. $\frac{50}{161} \times \frac{11}{400} \times \frac{7}{24}$	48. $\frac{84}{85} \times \frac{51}{132} \times \frac{91}{117}$
49. $\frac{1}{12}$ of $11\frac{1}{3}$ of $\frac{54}{85}$	50. $\frac{5}{11}$ of $25\frac{1}{3}$ of $\frac{77}{95}$
51 $\frac{2}{5}$ of $7\frac{1}{2} \times \frac{1}{8}$ of $11\frac{1}{5}$	52 $\frac{3}{4}$ of $1\frac{5}{11} \times \frac{2}{7}$ of $8\frac{7}{9}$
53. $3\frac{2}{3} \times 3\frac{3}{5} \times \frac{5}{11} \times \frac{7}{18}$	54. $2\frac{1}{7} \times 1\frac{1}{3} \times 2\frac{1}{4} \times 14$
55. $5\frac{1}{6} \times 9\frac{3}{62} \times \frac{5}{9} \times 5\frac{2}{5}$	56 $9\frac{4}{5} \times 2\frac{6}{7} \times 5\frac{1}{3} \times 2\frac{5}{8}$
57 $2\frac{14}{27}$ of $1\frac{11}{15} \times 2\frac{7}{34} \times 1\frac{5}{13}$	58 $3\frac{1}{7}$ of $3\frac{3}{10} \times 2\frac{3}{11} \times 2\frac{5}{22} \times \frac{2}{35}$
59 $\frac{15}{86}$ of $2\frac{11}{16}$ of $5\frac{1}{24} \times 11\frac{7}{11}$	60 $\frac{34}{57}$ of $\frac{33}{85}$ of $\frac{76}{117} \times 1\frac{31}{104}$
61 $\frac{32}{99} \times 1\frac{11}{31} \times 1\frac{23}{32} \times 1\frac{23}{70}$	62 $\frac{35}{132} \times 7\frac{5}{13} \times 2\frac{10}{21} \times \frac{11}{30}$
63. $2\frac{31}{37} \times 5\frac{16}{19} \times 2\frac{11}{20} \times 1\frac{19}{85}$	64 $5\frac{4}{23} \times 4\frac{16}{25} \times 2\frac{13}{51} \times 1\frac{17}{58}$
65 $10\frac{9}{16} \times \frac{9}{143} \times 1\frac{7}{13} \times \frac{44}{45}$	66 $\frac{63}{88} \times 36\frac{41}{56} \times \frac{30}{221} \times 2\frac{14}{45}$
67. $\frac{51}{91}$ of $1\frac{27}{65} \times \frac{77}{171}$ of $2\frac{46}{53}$	68 $\frac{19}{255}$ of $1\frac{6}{29}$ of $1\frac{13}{38}$ of $2\frac{18}{47}$
69. $\frac{117}{441}$ of $\frac{111}{1001}$ of $\frac{343}{407}$.	70. $\frac{209}{338}$ of $\frac{91}{187}$ of $\frac{130}{133}$ of $\frac{17}{60}$

Find the continued product of—

71. $\frac{93}{128}$, $1\frac{163}{187}$, $\frac{22}{35}$ and $2\frac{6}{31}$	72 $4\frac{3}{7}$, $\frac{16}{35}$, $1\frac{289}{341}$ and $1\frac{5}{72}$
73. $\frac{65}{123}$, $1\frac{5}{6}$, $\frac{75}{121}$ and $1\frac{43}{325}$	74 $5\frac{14}{23}$, $1\frac{13}{56}$, $3\frac{15}{43}$ and $\frac{49}{243}$
75 $\frac{18}{19}$, $182\frac{1}{2}$, $\frac{11}{219}$ and $4\frac{29}{45}$.	76 $1\frac{7}{26}$, $8\frac{13}{14}$, $\frac{7}{198}$ and $1\frac{14}{23}$
77 $1\frac{67}{186}$, $\frac{144}{161}$, $2\frac{55}{112}$ and $1\frac{97}{99}$	78 $\frac{5}{21}$, $\frac{13}{15}$, $\frac{48}{109}$, $\frac{105}{208}$ and $21\frac{4}{5}$
79 $\frac{91}{1440}$, $\frac{15}{17}$, $\frac{51}{77}$, $\frac{99}{100}$ and $\frac{256}{585}$	80. $\frac{440}{441}$, $\frac{407}{507}$, $\frac{338}{341}$ and $\frac{147}{148}$

XX. DIVISION OF FRACTIONS.

Divide—

1. 9 by $\frac{3}{4}$. | 6 128 by $\frac{4}{5}$ | 11 $\frac{2}{3}$ by $\frac{3}{4}$. | 16. $\frac{11}{12}$ by $\frac{1}{24}$.
2. 6 by $\frac{3}{5}$ | 7. 972 by $\frac{9}{10}$ | 12 $\frac{2}{5}$ by $\frac{5}{8}$. | 17. $9\frac{1}{11}$ by $9\frac{1}{11}$
3. 12 by $\frac{4}{5}$. | 8 324 by $\frac{6}{7}$ | 13. $\frac{7}{9}$ by $\frac{2}{3}$ | 18. $10\frac{2}{7}$ by $10\frac{2}{7}$.
4. 36 by $\frac{12}{13}$ | 9 560 by $\frac{14}{15}$ | 14 $\frac{11}{12}$ by $\frac{3}{8}$ | 19. $\frac{87}{112}$ by $\frac{29}{42}$.
5. 77 by $\frac{11}{12}$ | 10. 910 by $\frac{26}{29}$ | 15. $\frac{6}{7}$ by $\frac{9}{14}$. | 20. $1\frac{29}{55}$ by $\frac{108}{121}$

Find the value of—

21. $7 \div \frac{3}{4}$ | 26 $\frac{1}{64} \div \frac{5}{8}$ | 31 $11\frac{3}{5} \div 4\frac{1}{7}$ | 36. $11\frac{2}{3} \div 1\frac{3}{4}$.
22. $10 \div \frac{3}{5}$ | 27. $\frac{13}{96} \div \frac{65}{84}$ | 32. $2\frac{4}{7} \div 1\frac{13}{14}$. | 37 $20\frac{1}{4} \div 4\frac{1}{2}$
23 $14 \div \frac{4}{5}$ | 28. $\frac{17}{84} \div \frac{85}{96}$ | 33 $14\frac{4}{9} \div 11\frac{2}{3}$. | 38 $20\frac{1}{4} \div 2\frac{2}{9}$
24. $16 \div \frac{12}{13}$ | 29. $133 \div 1\frac{2}{5}$ | 34. $27\frac{4}{5} \div 3\frac{4}{5}$. | 39. $\frac{33}{52} \div 5\frac{29}{39}$
25. $38 \div \frac{19}{20}$. | 30. $155 \div 8\frac{6}{7}$. | 35. $5\frac{3}{4} \div 15\frac{1}{3}$. | 40. $\frac{29}{36} \div 3\frac{65}{84}$.

Simplify—

41. $\dfrac{\frac{3}{4}}{\frac{3}{7}}$. 42. $\dfrac{\frac{7}{8}}{\frac{4}{5}}$. 43. $\dfrac{\frac{7}{12}}{\frac{8}{9}}$. 44. $\dfrac{\frac{5}{11}}{\frac{15}{22}}$. 45 $\dfrac{\frac{13}{14}}{\frac{8}{21}}$. 46. $\dfrac{\frac{3}{17}}{\frac{2}{51}}$.

47. $\dfrac{\frac{24}{49}}{\frac{9}{14}}$. 48. $\dfrac{\frac{15}{16}}{\frac{25}{36}}$. 49. $\dfrac{3\frac{3}{8}}{\frac{15}{16}}$. 50. $\dfrac{4\frac{1}{6}}{\frac{20}{27}}$. 51. $\dfrac{\frac{7}{9}}{2\frac{4}{5}}$. 52. $\dfrac{\frac{3}{8}}{7\frac{1}{5}}$.

53. $\dfrac{\frac{3}{7}}{5}$. 54 $\dfrac{\frac{2}{7}}{9}$ 55. $\dfrac{\frac{11}{12}}{22}$ 56 $\dfrac{\frac{8}{11}}{24}$. 57. $\dfrac{9}{\frac{4}{5}}$ 58. $\dfrac{7}{\frac{3}{11}}$.

59. $\dfrac{12}{\frac{9}{10}}$. 60. $\dfrac{18}{\frac{4}{11}}$. 61 $\dfrac{7\frac{1}{2}}{5}$. 62. $\dfrac{2\frac{5}{8}}{3}$. 63 $\dfrac{10\frac{1}{9}}{14}$. 64 $\dfrac{10\frac{5}{8}}{34}$.

65. $\dfrac{15}{2\frac{3}{4}}$. 66 $\dfrac{27}{2\frac{1}{4}}$ 67. $\dfrac{21}{5\frac{5}{8}}$. 68. $\dfrac{18}{6\frac{6}{7}}$. 69. $\dfrac{2\frac{4}{9}}{2\frac{2}{3}}$. 70. $\dfrac{7\frac{1}{3}}{2\frac{2}{7}}$.

71. $\dfrac{9\frac{3}{8}}{2\frac{4}{5}}$ 72. $\dfrac{11\frac{1}{5}}{2\frac{6}{11}}$ 73. $\dfrac{17\frac{2}{3}}{22\frac{5}{7}}$ 74. $\dfrac{\frac{23}{24}}{12\frac{15}{16}}$. 75. $\dfrac{31\frac{13}{27}}{\frac{34}{81}}$ 76. $\dfrac{8\frac{44}{55}}{4\frac{8}{11}}$

77. $5\frac{44}{47} \div \frac{107}{141}$ | 78. $5\frac{11}{17} \div 1\frac{59}{85}$. | 79 $1\frac{7}{110} \div 4\frac{14}{55}$
80. $1\frac{25}{171} \div 5\frac{16}{57}$. | 81. $2\frac{17}{44} \div 2\frac{73}{121}$ | 82. $3\frac{17}{23} \div 1\frac{11}{161}$.
83. $3\frac{9}{14} \div \frac{221}{861}$. | 84. $33\frac{3}{25} \div 2\frac{91}{265}$ | 85. $706\frac{3}{4} \div 110\frac{1}{7}$.
86. $17\frac{48}{115} \div 26\frac{29}{230}$. | 87. $15\frac{163}{171} \div 33\frac{44}{133}$ | 88. $106\frac{79}{215} \div 9\frac{261}{301}$.

89. $\frac{38}{39}$ of $12\frac{1}{7}$ of $2\frac{1}{19} \div \frac{5}{14}$ 90 $\frac{1}{9}$ of $3\frac{1}{125}$ of $3\frac{4}{7} \div \frac{47}{345}$.

XXI. FRACTIONS.

Simplification of Fractional Expressions

Simplify—

1 $42 \div \frac{4}{7}$ of $9\frac{4}{5}$

2 $64 \div \frac{2}{9}$ of $2\frac{2}{5}$

3 $12\frac{9}{10} \div \frac{1}{3}$ of $6\frac{3}{4}$

4 $\frac{2}{3}$ of $4\frac{1}{2} \div \frac{4}{13}$ of $3\frac{1}{4}$

5. $\frac{7}{55}$ of $\frac{13}{20} \div \frac{52}{33}$ of $1\frac{1}{20}$

6 $5\frac{3}{5}$ of $4\frac{8}{9} \div \frac{1}{6}$ of $3\frac{2}{3}$

7. $6\frac{3}{7}$ of $4\frac{1}{5} \div 1\frac{1}{8}$ of $3\frac{1}{6}$

8 $9\frac{4}{5}$ of $3\frac{1}{8} \div 1\frac{7}{8}$ of $8\frac{5}{6}$

9. $1\frac{3}{5} \div \frac{3}{4}$ of $2\frac{2}{3}$ of $2\frac{1}{7}$

10 $7\frac{7}{12} - \frac{1}{2}$ of $4\frac{1}{3}$ of $3\frac{3}{4}$ of $2\frac{4}{5}$

11 $1\frac{1}{4} + 3\frac{1}{5}$ of $2\frac{5}{8}$

12 $3\frac{3}{8}$ of $2\frac{2}{9} + 4\frac{1}{2}$

13. $\frac{3}{7}$ of $\frac{5}{12} + \frac{1}{2}$ of $1\frac{9}{14}$

14 $\frac{9}{10}$ of $\frac{21}{50} - \frac{9}{10}$ of $\frac{1}{50}$

15. $1\frac{1}{4}$ of $4\frac{1}{6} - 2\frac{1}{12}$ of $2\frac{1}{10}$

16 $2\frac{1}{3}$ of $5\frac{1}{6} - 1\frac{7}{8}$ of $6\frac{1}{15}$

17. $\frac{6}{7} + 6\frac{1}{7} + \frac{6}{7}$ of $6\frac{1}{7}$

18. $\frac{8}{9} + 7\frac{1}{9} + \frac{8}{9}$ of $8\frac{1}{9}$

19. $7\frac{7}{8} + 6\frac{3}{5}$ of $5\frac{2}{3} - 4\frac{1}{2}$

20 $\frac{2}{3}$ of $\frac{1}{7}$ of $8\frac{3}{4} + \frac{1}{4}$ of $\frac{2}{5}$

21. $20 - \frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$

22 $50 - 5\frac{1}{2}$ of $5\frac{1}{7}$ of $1\frac{3}{11}$ of $1\frac{1}{12}$

23. $17\frac{4}{9} + \frac{5}{27} + 4\frac{2}{3} + 3\frac{1}{3}$ of $\frac{1}{9}$

24 $3\frac{5}{14} + 5\frac{4}{21} + \frac{11}{24} + 1\frac{1}{3}$ of $\frac{5}{56}$

25. $6\frac{1}{2}$ of $5\frac{3}{4} + 4\frac{5}{8}$ of $3\frac{7}{16} + 2\frac{9}{32}$

26 $\frac{3}{4}$ of $9\frac{1}{3} + \frac{7}{8} + 8\frac{4}{5}$ of $\frac{25}{26}$ of $8\frac{10}{11}$.

27. $8\frac{1}{2} - 4\frac{2}{3} + \frac{3}{5} - 1\frac{1}{6}$ of $\frac{2}{5}$

28. $12\frac{1}{2} - 14\frac{3}{4} + 5\frac{7}{8} - \frac{5}{6}$ of $2\frac{2}{5}$

29 $\frac{3}{4}$ of $\frac{5}{6} + \frac{5}{6}$ of $\frac{7}{8} + \frac{7}{8}$ of $\frac{9}{10}$

30 $\frac{3}{8}$ of $\frac{6}{11} - \frac{9}{14}$ of $2\frac{6}{11} + \frac{1}{4}$ of $6\frac{1}{11}$.

---

31 $2\frac{2}{3} + 7\frac{1}{2} \times \frac{3}{5}$

32 $\frac{5}{6} \times 1\frac{1}{5} + 5\frac{3}{4}$

33. $1\frac{1}{16} \times 1\frac{1}{17} + 1\frac{1}{15} \times 1\frac{1}{14}$

34 $5\frac{1}{3} \times 6\frac{1}{4} \times 7\frac{1}{5} - 8\frac{1}{6}$

35 $8\frac{1}{2} + 2\frac{3}{4} \div \frac{5}{11}$

36 $\frac{8}{15} \div 5\frac{3}{5} + 2\frac{1}{21}$

37. $17\frac{1}{9} \div 3\frac{3}{10} - 1\frac{1}{6} \times \frac{6}{7}$

38 $\frac{7}{12} + 2\frac{1}{6} + 5\frac{1}{2} \div 4\frac{2}{5}$

39. $\frac{11}{17} \times 21\frac{6}{7} - \frac{11}{19} \times 10\frac{5}{9} + \frac{11}{21} \div \frac{33}{61}$

40 $\frac{3}{7} - \frac{2}{49} \times \frac{1}{3} - \frac{1}{9} \div \frac{2}{3} - \frac{2}{9} \times \frac{1}{7} - \frac{1}{49}$.

41. $\dfrac{3\frac{1}{3}}{3\frac{3}{4}} + \frac{7}{8} \times \frac{4}{21} + 17.$

42 $1\frac{1}{3} + \frac{8}{3} \times \frac{41}{34} + \dfrac{4}{5\frac{1}{10}}$

43. $\dfrac{1\frac{3}{4}}{3\frac{4}{5}} - \dfrac{2\frac{2}{3}}{4\frac{3}{4}} + \dfrac{2\frac{2}{9}}{6\frac{1}{3}}$

44 $\dfrac{1\frac{3}{4}}{3\frac{4}{5}} + \dfrac{2\frac{2}{3}}{4\frac{3}{4}} - \dfrac{2\frac{2}{9}}{6\frac{1}{3}}$

45. $\dfrac{2\frac{1}{2}}{7} + \dfrac{2}{7\frac{1}{2}} + \dfrac{\frac{1}{9}}{3\frac{1}{3}} + \dfrac{1\frac{1}{2}}{4\frac{3}{8}}$

46 $\dfrac{1}{11\frac{2}{3}} + \dfrac{1}{13\frac{1}{2}} + \dfrac{1}{15\frac{3}{4}} + \dfrac{1}{17\frac{1}{2}}$.

47. $3\frac{1}{2} \times \dfrac{4\frac{1}{3}}{5\frac{1}{4}} + 8\frac{1}{9}$

48 $7\frac{3}{4} + \frac{5}{7} \times \dfrac{4\frac{2}{3}}{2\frac{1}{3}} - 1\frac{2}{5}$

49. $\frac{3}{8} \div 6 + \frac{63}{128} \times \frac{1}{3} \times 2\frac{2}{3} + \dfrac{1}{6\frac{6}{7}} \div \frac{7}{8} + \frac{27}{81}$

50 $\dfrac{2\frac{5}{7}}{5} - 10\frac{5}{6} \div 1\frac{12}{13} + \frac{3}{7} \times 3\frac{3}{4} \times 3\frac{4}{5} - 1\frac{1}{60}$

BRACKETS.

51. $\frac{3}{13}$ of $(2\frac{2}{3} + 2\frac{3}{4})$.

52. $(4\frac{2}{3} + 3\frac{1}{2}) \times 1\frac{2}{7}$

53. $(3\frac{3}{8} - 2\frac{2}{5}) \div 1\frac{1}{25}$

54. $(\frac{5}{6} + \frac{2}{9} - \frac{11}{15}) \div 2\frac{5}{12}$.

55. $(3\frac{1}{3} + 7\frac{1}{7}) \times (\frac{2}{5} + 9\frac{3}{4})$.

56. $(2\frac{1}{2} + \frac{1}{6}) \div (3\frac{2}{3} - \frac{1}{3})$.

57. $5\frac{2}{3}$ of $\frac{1}{15} \div (6\frac{1}{5} - \frac{2}{25})$

58. $\frac{1}{8}$ of $1\frac{1}{3} \div (1\frac{1}{6} + 6\frac{2}{3})$

59. $(4\frac{5}{8} + 3\frac{3}{8})(4\frac{5}{8} - 3\frac{3}{8})$.

60. $(8\frac{3}{14} - 7\frac{1}{7})(8\frac{7}{15} - 7\frac{7}{10})$.

61. $4\frac{5}{8} - (3\frac{5}{6} - 1\frac{3}{4})$.

62. $11 - (3\frac{3}{8} + 5\frac{5}{6}) - (5\frac{3}{8} - 3\frac{5}{8})$.

63. $(\frac{2}{5} - \frac{3}{10}) \times (\frac{3}{10} - \frac{4}{15}) \times (\frac{4}{15} - \frac{5}{20})$

64. $(\frac{1}{3} + \frac{1}{4}) \times (\frac{1}{4} + \frac{1}{5}) \times (\frac{1}{5} + \frac{1}{7})$.

65. $\frac{1}{5}$ of $(\frac{3}{5} + \frac{6}{7}) + \frac{1}{7}$ of $(\frac{2}{5} + \frac{4}{7})$

66. $2\frac{11}{12} \times (1 - \frac{25}{49}) + 1\frac{3}{11} \times (1 - \frac{16}{49})$.

67. $(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5}) \div (\frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{5})$

68. $(\frac{8}{15} + \frac{7}{18} - \frac{17}{105} + \frac{1}{3}) \div (\frac{11}{30} - \frac{8}{35} + \frac{11}{15})$

69. $(\frac{3}{7} - \frac{2}{49})$ of $(\frac{1}{3} - \frac{1}{9}) \div (\frac{2}{3} - \frac{2}{9})$ of $(\frac{1}{7} - \frac{1}{49})$.

70. $(\frac{3}{13} - \frac{5}{26})$ of $(\frac{7}{26} - \frac{9}{39}) \div (\frac{11}{52} - \frac{12}{65})$ of $(\frac{9}{39} - \frac{11}{52})$.

---

71. $\dfrac{3\frac{3}{5} \text{ of } 4\frac{4}{5}}{2\frac{1}{4} \text{ of } 3\frac{5}{7}}$.

72. $\dfrac{2\frac{2}{3} \text{ of } 3\frac{3}{4} \text{ of } 4\frac{4}{5}}{5\frac{5}{6} \text{ of } 6\frac{6}{7}}$.

73. $\dfrac{5\frac{1}{3} \text{ of } 6\frac{1}{2} \text{ of } 7\frac{1}{5}}{2\frac{1}{15} \text{ of } 3\frac{1}{5} \text{ of } 4\frac{1}{3}}$.

74. $\dfrac{1\frac{7}{9} \times 3\frac{5}{7} \times 5\frac{3}{5}}{3\frac{1}{5} \times 2\frac{1}{6} \times 4\frac{3}{8}}$.

75. $\dfrac{3\frac{1}{2} - 1\frac{7}{8}}{3\frac{1}{4} \times 2\frac{1}{2}}$.

76. $\dfrac{3\frac{1}{7} - \frac{2}{3}}{\frac{4}{49} \times 7\frac{7}{12}}$.

77. $\dfrac{9\frac{1}{4} - 2\frac{7}{10}}{2\frac{3}{4} + 3\frac{4}{5}}$.

78. $\dfrac{4\frac{1}{7} - 2\frac{1}{4}}{6\frac{1}{2} - 2\frac{1}{7}}$.

79. $\dfrac{3\frac{1}{5} - 2\frac{1}{3}}{5\frac{2}{7} - 1\frac{1}{9}}$

80. $\dfrac{2\frac{1}{2} + 3\frac{1}{3} - 4\frac{1}{4}}{5\frac{1}{5} + 7\frac{1}{12}}$.

81 $\dfrac{3\frac{7}{8} - 4\frac{1}{3} + 2\frac{1}{12}}{9\frac{1}{36} - 7\frac{19}{27}}$.

82. $\dfrac{2\frac{1}{9} - 1\frac{1}{8}}{7\frac{1}{2} - 6\frac{2}{3} + \frac{11}{72}}$.

83. $\dfrac{\frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28}}{\frac{1}{2} + \frac{3}{4} + \frac{6}{7} + \frac{13}{14} + \frac{27}{28}}$.

84. $\dfrac{\frac{1}{7} + \frac{1}{14} + \frac{1}{28} + \frac{1}{56}}{\frac{1}{7} - \frac{1}{14} + \frac{1}{28} - \frac{1}{56}}$.

85 $\dfrac{2\frac{2}{3} - 1\frac{1}{2}}{2\frac{2}{3} + 1\frac{1}{2}} + \dfrac{2\frac{3}{5} \text{ of } 9}{2\frac{4}{11} \text{ of } 2\frac{3}{4} \text{ of } 5}$.

86. $\dfrac{\frac{2}{3} \text{ of } 1\frac{2}{3} \text{ of } 2\frac{2}{5}}{\frac{7}{9} + \frac{5}{12} - \frac{3}{4}} + \dfrac{5\frac{2}{5} \text{ of } 7\frac{2}{9}}{8\frac{7}{24} - 3\frac{5}{12}}$

87. $\dfrac{1\frac{2}{3} + \frac{3}{5}}{\frac{2}{3} \text{ of } 2\frac{1}{8}} \times \dfrac{10 - 7\frac{1}{2}}{3\frac{1}{3} - 1\frac{1}{5}} \times 1\frac{1}{15}$.

88. $\dfrac{\frac{1}{9} \text{ of } 1\frac{1}{4} \text{ of } 4\frac{1}{2}}{\frac{5}{56} \text{ of } 1\frac{1}{3} \text{ of } 3\frac{1}{2}} - \dfrac{3\frac{1}{4} + 4\frac{1}{3}}{6\frac{1}{2} + 1\frac{1}{12}}$

89. $\dfrac{3\frac{1}{4} - 2\frac{1}{3}}{3\frac{1}{6} + 1\frac{1}{2}} \div 1\frac{4}{7}$

90. $\dfrac{\frac{1}{2} + \frac{1}{3} - \frac{1}{4}}{\frac{1}{3} + \frac{1}{4} - \frac{1}{5}} \div 1\frac{12}{23}$.

91 $4\frac{2}{9} \times 6\frac{3}{7} \div \dfrac{2\frac{1}{2}}{7}$

92. $3\frac{3}{4} \times 4\frac{4}{5} \times 5\frac{5}{6} \div \dfrac{9\frac{3}{5}}{6\frac{6}{7}}$

93 $\dfrac{3\frac{3}{4}}{60} \div \dfrac{\frac{3}{4}}{\frac{1}{2} \text{ of } \frac{1}{6}}$.

94. $3\frac{1}{3} \div \dfrac{1 - \frac{15}{16}}{\frac{1}{5} - \frac{1}{8}}$.

95. $\dfrac{4\frac{5}{6}}{\frac{5}{6} + \frac{3}{8}} \div \dfrac{4\frac{3}{8}}{\frac{5}{6} - \frac{3}{8}}$.

96 $\dfrac{\frac{1}{4} + \frac{11}{13}}{\frac{2}{7} + \frac{9}{11}} \div \dfrac{\frac{1}{5} + \frac{3}{4}}{\frac{1}{3} + \frac{2}{7}}$

97. $\dfrac{4\frac{1}{7} - 2\frac{1}{4}}{6\frac{1}{2} + 2\frac{1}{7}} \div \dfrac{\frac{3}{7} + \frac{1}{2}}{\frac{5}{7} - \frac{1}{2}}$.

98 $\dfrac{7\frac{1}{5} \times 5\frac{1}{7}}{7\frac{1}{5} - 5\frac{1}{7}} \div \dfrac{5\frac{1}{3} \times 3\frac{1}{5}}{5\frac{1}{3} + 3\frac{1}{5}}$.

99 $\dfrac{3\frac{1}{2} - 1}{15\frac{1}{3} + 5\frac{1}{2} - 3\frac{1}{3}} \div \dfrac{1 - \frac{3}{4}}{1 + \frac{3}{4}}$

100 $\dfrac{4\frac{1}{2} - 3\frac{3}{4}}{4\frac{1}{2} + 3\frac{3}{4}} \div \dfrac{3\frac{3}{4} + 2\frac{5}{8}}{3\frac{3}{4} - 2\frac{5}{8}}$

Continued Fractions

101. $\dfrac{2}{2 + \frac{2}{3}}$. 102 $\dfrac{3}{3 + \frac{3}{4}}$ 103 $\dfrac{1}{2 + \dfrac{1}{3 + \frac{1}{4}}}$ 104. $\dfrac{1}{2 + \dfrac{1}{3 + \frac{1}{5}}}$

105 $\dfrac{2}{3 + \dfrac{3}{4 + \frac{4}{5}}}$ 106. $\dfrac{2}{3 - \dfrac{3}{4 - \frac{4}{5}}}$ 107 $\dfrac{1}{1 + \dfrac{1}{1 + \dfrac{1}{1 + \frac{1}{2}}}}$.

108. $\dfrac{2}{2 + \dfrac{1}{3 - \dfrac{2}{5 + \frac{1}{2}}}}$ 109. $1 + \dfrac{3}{1 + \dfrac{3}{1 + \frac{1}{2}}}$ 110. $1 - \dfrac{2}{3 + \dfrac{4}{5 - \frac{6}{7}}}$

Various

111. $\frac{2}{3} + \frac{3}{4} + \frac{4}{5}$. 112 $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$ 113 $\frac{2}{3} + \frac{3}{4} - \frac{4}{5}$

114 $\frac{2}{3} - \frac{3}{4} + \frac{4}{5}$. 115 $\frac{2}{3} + \frac{3}{4}$ of $\frac{4}{5}$ 116 $\frac{2}{3}$ of $\frac{3}{4} + \frac{4}{5}$.

117 $\frac{4}{5} - (\frac{3}{4} - \frac{2}{3})$ 118. $\frac{2}{3} - \frac{3}{4} \times \frac{4}{5}$ 119 $\frac{2}{3}$ of $\frac{3}{4} \div \frac{4}{5}$

120. $\frac{2}{3} \div \frac{3}{4}$ of $\frac{4}{5}$ 121. $\frac{2}{3} + \frac{3}{4} \div \frac{4}{5}$ 122 $\frac{2}{3} \div \frac{3}{4} + \frac{4}{5}$

123 $\frac{2}{3}$ of $(\frac{3}{4} + \frac{4}{5})$ 124. $(\frac{2}{3} + \frac{3}{4}) \times \frac{4}{5}$ 125 $(\frac{2}{3} + \frac{3}{4}) \div \frac{4}{5}$

126. $\frac{2}{3} \div (\frac{3}{4} + \frac{4}{5})$ 127. $\frac{2}{3} \div \frac{3}{4} \times \frac{4}{5}$ 128. $\frac{2}{3} \div \frac{3}{4} \div \frac{4}{5}$

129 $3\frac{1}{2} + 2\frac{1}{12} + 3\frac{3}{16} - 7\frac{19}{24}$. 130. $11\frac{1}{9} - 10\frac{26}{27} + 9\frac{1}{6} - 8\frac{17}{18}$.

131. $17 - 3\frac{1}{3}$ of $4\frac{1}{5} + 5\frac{1}{4}$ of $2\frac{2}{3}$. 132. $\frac{1}{4} \times 6\frac{2}{3} + \frac{7}{8} \div 2\frac{1}{3} - \frac{1}{24}$.

133. $\dfrac{1\frac{1}{2}}{2\frac{1}{2}} \times \dfrac{1\frac{1}{4}}{2\frac{1}{4}} \times \dfrac{1\frac{1}{6}}{2\frac{1}{6}} \times \dfrac{1\frac{1}{8}}{2\frac{1}{8}} \times \dfrac{1\frac{1}{10}}{2\frac{1}{10}}$.

134 $\dfrac{\frac{1}{2} - \frac{1}{4}}{\frac{1}{3} - \frac{1}{4}}$ of $\dfrac{\frac{1}{4} - \frac{1}{6}}{\frac{1}{4} - \frac{1}{5}}$ of $\dfrac{\frac{1}{6} - \frac{1}{8}}{\frac{1}{5} - \frac{1}{6}}$.

135. $\dfrac{2\frac{1}{2}}{3\frac{2}{5}} + \dfrac{3\frac{1}{3} - 1\frac{1}{6}}{5\frac{1}{3} - 1\frac{1}{6}} + \frac{1}{22}$ of $4\frac{9}{11}$.

136. $\dfrac{5\frac{2}{5}}{6\frac{4}{5}} + \dfrac{3\frac{1}{2} + 4\frac{1}{5}}{5\frac{1}{2} + 7\frac{1}{3}} + \frac{9}{16}$ of $\frac{1}{10}$.

137. $\dfrac{12\frac{1}{4}}{1\frac{1}{8}} + \dfrac{12\frac{1}{4} - 3\frac{1}{5}}{12\frac{1}{4} + 3\frac{1}{5}}$ of $34\frac{1}{3} - 15\frac{1}{2}$.

138. $\dfrac{2\frac{5}{9}}{8\frac{1}{3}}$ of $\dfrac{1\frac{3}{5} + \frac{5}{6}}{1\frac{3}{5} - \frac{5}{6}} + \frac{5}{8}$ of $\dfrac{1\frac{5}{6} - \frac{2}{3}}{\frac{5}{6} + \frac{2}{3}}$.

139. $\dfrac{1\frac{1}{2}}{4\frac{1}{3}} + \dfrac{3\frac{1}{3} - 2\frac{5}{8}}{4\frac{1}{2} + 5\frac{9}{10}} + \frac{3}{26}$ of $5\frac{1}{4}$.

140. $1\frac{1}{6}$ of $5\frac{1}{4} + \dfrac{4\frac{1}{2} - \frac{3}{10}}{1\frac{1}{15}} - \dfrac{6\frac{3}{4}}{\frac{1}{2} \text{ of } 2\frac{2}{3}}$.

141. $\dfrac{2\frac{1}{4}}{2\frac{2}{5}} + \dfrac{2\frac{1}{2} + 5\frac{1}{5}}{3\frac{1}{3} + 9\frac{1}{2}} + \frac{1}{2} + \frac{3}{8}$ of $\frac{3}{20}$.

142. $\dfrac{1\frac{1}{2}}{8\frac{1}{2}}$ of $\dfrac{1\frac{1}{2} + 1\frac{1}{3}}{2\frac{3}{8} - 1\frac{1}{6}} + \dfrac{2\frac{1}{5} + 4\frac{1}{3}}{1\frac{1}{5} - \frac{1}{9}}$ of $\frac{1}{9}$.

143. $\frac{1}{4}$ of $\frac{7}{8}$ of $\left(\dfrac{2\frac{1}{3}}{3\frac{1}{2}} + \dfrac{4\frac{1}{2}}{5\frac{1}{4}}\right)$

144. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{7} \div \left(\dfrac{1\frac{2}{3}}{1\frac{4}{5}} + \dfrac{2\frac{1}{3}}{4\frac{1}{5}}\right)$.

145. $\dfrac{3\frac{1}{2} \times 3\frac{1}{2} \times 3\frac{1}{2} - 3}{3\frac{1}{2} \times 3\frac{1}{2} - 3}$.

146. $\dfrac{2\frac{1}{3} \times 2\frac{1}{3} \times 2\frac{1}{3} - 1}{2\frac{1}{3} \times 2\frac{1}{3} - 1}$.

147. $\dfrac{(5\frac{3}{8} - 3\frac{5}{8}) \text{ of } \frac{28}{30}}{5\frac{3}{8} - 3\frac{5}{8} \text{ of } \frac{20}{29}}$.

148. $\dfrac{(5\frac{3}{8} + 3\frac{5}{8}) \text{ of } \frac{28}{30}}{5\frac{3}{8} + 3\frac{5}{8} \text{ of } \frac{20}{29}}$.

149. $\dfrac{1\frac{1}{2} \times 2\frac{1}{4} + 3\frac{1}{8} \times 4\frac{1}{16}}{1\frac{1}{2} + 2\frac{1}{4} \times 3\frac{1}{8} + 4\frac{1}{16}}$.

150. $\dfrac{2\frac{1}{2} + 3\frac{1}{4} \div \frac{4}{5} \text{ of } \frac{18}{20}}{(2\frac{1}{3} + 1\frac{3}{4}) \times 2\frac{4}{7} - 5\frac{1}{4}}$

151. $\dfrac{4\frac{3}{4} + 4\frac{1}{4}}{4\frac{3}{4} \times 4\frac{1}{4}} \div \dfrac{4\frac{3}{4} - 4\frac{1}{4}}{4\frac{3}{4} \div 4\frac{1}{4}}$.

152. $\dfrac{4\frac{4}{9} \text{ of } 8\frac{1}{3}}{4\frac{1}{4} + \frac{7}{36}} \div \dfrac{3\frac{1}{4} + 5\frac{1}{7}}{1\frac{1}{4} + 3\frac{1}{7}}$.

153. $\dfrac{\frac{13}{18} - \frac{8}{31}}{1 - \frac{8}{31} \times \frac{13}{18}} - \dfrac{\frac{10}{13} - \frac{23}{31}}{1 + \frac{23}{31} \times \frac{10}{13}}$

154. $\dfrac{\frac{7}{8} + \frac{2}{3} \text{ of } \frac{1}{4}}{(\frac{7}{8} + \frac{2}{3}) \text{ of } \frac{1}{4}} \times \dfrac{\frac{7}{8} \text{ of } (\frac{2}{3} + \frac{1}{4})}{\frac{7}{8} \text{ of } \frac{2}{3} + \frac{1}{4}} \times 37$.

155. $\left\{2\frac{3}{4} + 2\frac{1}{2} \text{ of } \dfrac{7}{3\frac{4}{5}} - \dfrac{1\frac{2}{3}}{2\frac{1}{2}}\right\} \div 1\frac{77}{228} + 5$.

156. $\left\{\dfrac{3\frac{1}{3} - \frac{3}{5} \text{ of } 1\frac{1}{9}}{\frac{1}{3} \text{ of } 2\frac{1}{4} + \frac{5}{8}} - \dfrac{1}{5\frac{1}{2}}\right\} \div \dfrac{2}{1\frac{4}{29}} - 1$.

157. $\dfrac{\frac{2}{3} + \frac{2}{5}}{1 - \frac{2}{3} \text{ of } \frac{2}{5}} + \frac{3}{10}$ of $\dfrac{3\frac{1}{9}}{5\frac{5}{6}}$ of $\dfrac{8\frac{1}{3}}{1\frac{1}{2}} - \dfrac{4\frac{1}{6}}{3\frac{3}{4}} \times 4\frac{1}{2}$ of $\frac{2}{5}$.

158. $\frac{7}{12} \times 4\frac{1}{3} - \dfrac{4\frac{1}{4}}{3} + \dfrac{1\frac{3}{5} - \frac{4}{15}}{\frac{2}{3} - \frac{2}{15}} \times \dfrac{5\frac{2}{5}}{9\frac{1}{3}} + \left(\dfrac{1}{1\frac{4}{11}} + 2\frac{2}{5}\right) \times \frac{5}{6}$.

159. $\dfrac{(5\frac{1}{5}-3\frac{1}{3})\div\frac{3}{5}\text{ of }1\frac{1}{6}}{18\frac{2}{3}-5\frac{1}{5}\text{ of }3\frac{1}{3}}$

160. $\dfrac{5\frac{3}{4}+2\frac{2}{35}\div1\frac{11}{25}-\frac{3}{7}\text{ of }15\frac{3}{4}}{\frac{3}{4}\text{ of }7\frac{3}{7}-5\frac{3}{5}\div3\frac{4}{15}}.$

161 $\dfrac{2\frac{1}{2}}{2\frac{1}{4}}\text{ of }\dfrac{3\frac{1}{2}}{3\frac{1}{4}}\text{ of }\dfrac{4\frac{1}{2}}{4\frac{1}{4}}\div\dfrac{2\frac{1}{2}}{4\frac{1}{2}}\text{ of }\dfrac{2\frac{1}{3}}{4\frac{1}{3}}\text{ of }\dfrac{2\frac{1}{4}}{4\frac{1}{4}}.$

162. $\dfrac{7\frac{1}{2}-10\frac{1}{3}+11\frac{1}{4}}{7\frac{1}{2}\text{ of }9\frac{1}{3}\text{ of }11\frac{1}{4}}\div\dfrac{2\frac{1}{4}-4\frac{1}{6}+6\frac{1}{8}}{6\frac{1}{4}\text{ of }8\frac{1}{6}\text{ of }10\frac{1}{8}}.$

163. $4\frac{1}{2}+\dfrac{2\frac{1}{4}}{4\frac{1}{2}}+4\frac{1}{2}\text{ of }2\frac{1}{4}+\dfrac{4\frac{1}{2}+2\frac{1}{4}}{4\frac{1}{2}-2\frac{1}{4}}\text{ of }\frac{1}{2}\text{ of }\frac{1}{4}.$

164 $2\frac{1}{3}+\dfrac{2\frac{1}{3}}{2\frac{1}{4}}+\dfrac{2\frac{1}{3}+2\frac{1}{4}}{2\frac{1}{3}-2\frac{1}{4}}+2\frac{1}{3}\text{ of }2\frac{1}{4}-2\frac{1}{4}.$

165. $4\frac{1}{2}+\left(\dfrac{2\frac{1}{4}}{4\frac{1}{2}}+4\frac{1}{2}\right)\text{ of }\left(2\frac{1}{4}+\dfrac{4\frac{1}{2}+2\frac{1}{4}}{4\frac{1}{2}-2\frac{1}{4}}\right)\text{ of }\frac{1}{2}\text{ of }\frac{1}{4}.$

166 $2\frac{1}{4}+\dfrac{2\frac{1}{4}}{2\frac{1}{3}}+\left(\dfrac{2\frac{1}{3}+2\frac{1}{4}}{2\frac{1}{3}-2\frac{1}{4}}+2\frac{1}{3}\right)\text{ of }2\frac{1}{4}-2\frac{1}{3}$

167 $\dfrac{1-\dfrac{1}{3\frac{1}{2}}}{1-\dfrac{1}{1\frac{1}{2}}}\text{ of }\dfrac{1-\dfrac{1}{2\frac{1}{2}}}{1-\dfrac{1}{5\frac{1}{2}}}\text{ of }\dfrac{1-\dfrac{1}{4\frac{1}{2}}}{1-\dfrac{1}{6\frac{1}{2}}}\text{ of }\dfrac{1-\dfrac{1}{8\frac{1}{2}}}{1-\dfrac{1}{7\frac{1}{2}}}.$

168 $1\frac{1}{11}-\dfrac{1-\frac{7}{22}}{2-\frac{1}{3}}+\dfrac{1\frac{2}{5}}{3\frac{1}{2}}-\dfrac{5\frac{5}{8}}{6\frac{1}{4}}\text{ of }\left(\frac{1}{5}-\dfrac{\frac{1}{2}-\frac{1}{3}}{4\frac{3}{4}-3\frac{2}{9}}\right).$

169. $\dfrac{6\frac{1}{2}+6\frac{5}{8}}{1\frac{5}{6}\times12\frac{7}{11}}+\dfrac{(4\frac{7}{12}-3\frac{5}{24})\text{ of }2\frac{2}{11}}{4\frac{7}{12}+3\frac{5}{24}\text{ of }2\frac{2}{11}}+\dfrac{1\frac{5}{12}}{5\frac{19}{24}}$

170. $\frac{18}{17}\text{ of }(1-\frac{64}{81})+\frac{8}{11}\text{ of }\frac{1}{6}\text{ of }(\frac{1}{2}+\frac{5}{12})+\dfrac{1\frac{1}{2}\times9\frac{1}{3}}{2\frac{3}{4}\times7\frac{7}{11}}$

171. $\dfrac{\frac{1}{3}+\frac{2}{11}+\frac{7}{24}-\frac{1}{3}\text{ of }\frac{2}{11}\text{ of }\frac{7}{24}}{1-\frac{1}{3}\text{ of }\frac{2}{11}-\frac{2}{11}\text{ of }\frac{7}{24}-\frac{7}{24}\text{ of }\frac{1}{3}}$

172 $\dfrac{2\frac{3}{25}-\frac{3}{5}\text{ of }\frac{2}{25}+3\frac{2}{5}}{2\frac{2}{5}\text{ of }(\frac{3}{5}+\frac{3}{25})\div3\frac{3}{5}\text{ of }(\frac{2}{5}-\frac{2}{25})}$

173. $(\frac{1}{2}+\frac{1}{3})\text{ of }(\frac{1}{3}+\frac{1}{4})+(\frac{1}{2}+\frac{1}{4})\text{ of }(\frac{1}{4}+\frac{1}{5})+(\frac{1}{2}+\frac{1}{5})\text{ of }(\frac{1}{5}+\frac{1}{6})$

174 $\dfrac{(\frac{1}{2}+\frac{2}{3})\text{ of }(\frac{3}{4}+\frac{4}{5})+(\frac{1}{2}+\frac{3}{4})\text{ of }(\frac{2}{3}+\frac{4}{5})+(\frac{1}{2}+\frac{4}{5})\text{ of }(\frac{2}{3}+\frac{3}{4})}{\frac{1}{2}\text{ of }(\frac{2}{3}+\frac{3}{4}+\frac{4}{5})+(\frac{1}{2}+\frac{2}{3}+\frac{3}{4})\text{ of }\frac{4}{5}}.$

175. $\dfrac{5}{7-\dfrac{9}{3-\frac{3}{4}}}+\dfrac{5}{16-\dfrac{11}{2-\frac{1}{6}}}$

176. $\dfrac{3}{8-\dfrac{7}{2-\frac{3}{4}}}+\dfrac{5}{6-\dfrac{5}{2-\frac{5}{6}}}.$

177. $\dfrac{\frac{1}{7}+\frac{1}{3}\left(\frac{1}{2}+\frac{1}{7}\right)}{1+\dfrac{1}{2-\frac{1}{4}}}.$

178. $3-3\frac{1}{3}$ of $\dfrac{3}{3-\dfrac{3}{3-\frac{1}{3}}}$ of $\dfrac{\frac{1}{3}}{11}\div 3\frac{2}{3}.$

179. $\dfrac{1\frac{2}{3}\text{ of }1\frac{1}{4}}{3\frac{2}{5}+\dfrac{5}{1\frac{19}{41}}}+\dfrac{4}{5+\dfrac{6}{7\frac{8}{9}}}.$

180. $2\frac{4}{5}-\frac{1}{5}$ of $\dfrac{5\frac{1}{7}-2\frac{1}{4}}{1+\dfrac{1}{3+\frac{1}{6}}}$ of $\dfrac{9\frac{1}{3}}{12\frac{1}{11}}\div 2\frac{1}{3}.$

181. $3\dfrac{3\frac{1}{3}}{3}\div 3\dfrac{3}{3\dfrac{3}{3\frac{1}{3}}}.$

182. $\left\{\dfrac{1}{2-\dfrac{3}{4-\frac{5}{6}}}-\dfrac{1}{2+\dfrac{3}{4+\frac{5}{6}}}\right\}\times 10\frac{5}{8}.$

183. $\dfrac{\dfrac{1}{1\frac{1}{2}}+\dfrac{1}{1\frac{1}{3}}+\dfrac{1}{1\frac{1}{4}}}{3-\dfrac{1}{1-\frac{1}{16}}+\dfrac{1}{4-\frac{8}{17}}}$

184. $\frac{13}{17}\times\dfrac{15\frac{1}{2}}{15\frac{2}{3}}\times\dfrac{\dfrac{1}{3\frac{1}{2}}+\dfrac{1}{4\frac{1}{3}}}{\dfrac{1\frac{1}{2}}{3}+\dfrac{1\frac{1}{3}}{4}}\times\dfrac{\dfrac{3}{4+\dfrac{1}{2+\frac{1}{3}}}}{\dfrac{3}{4-\dfrac{1}{2-\frac{1}{3}}}}$

185. $\frac{4}{5}$ of $6\frac{1}{3}-2\frac{1}{13}$ of $\dfrac{1\frac{6}{7}\times\dfrac{\frac{2}{3}}{\frac{3}{4}}\times 1\frac{1}{6}}{1\frac{1}{5}\times\dfrac{\frac{1}{3}}{\frac{3}{5}}\times 1\frac{7}{8}}-2\frac{1}{12}$ of $\frac{4}{5}+\frac{4}{5}.$

186. $7\frac{1}{7}$ of $\dfrac{2\frac{1}{2}+7\frac{1}{7}}{2\frac{1}{7}+7\frac{1}{2}}$ of $\dfrac{\dfrac{2\frac{1}{7}}{7\frac{1}{2}}+\dfrac{2\frac{1}{2}}{7\frac{1}{7}}}{\dfrac{2\frac{1}{7}}{7\frac{1}{7}}+\dfrac{2\frac{1}{2}}{7\frac{1}{2}}}\div\dfrac{7\frac{1}{7}}{2\frac{1}{2}}$ of $4\frac{13}{15}.$

187. $\frac{1}{2}$ of $\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)+7\times\left(\frac{1}{35}+\frac{1}{36}\right)-\frac{1}{12}\div\left(\dfrac{11\frac{1}{2}}{15\frac{1}{3}}-\frac{1}{4}\right).$

188. $\dfrac{1\frac{1}{4}-1\frac{1}{6}\text{ of }\frac{9}{14}}{5\frac{1}{2}+\frac{5}{6}\text{ of }\frac{3}{5}}\div\frac{1}{6}+\left(3\frac{1}{3}\text{ of }\frac{1}{7}+\dfrac{2}{10\frac{1}{2}}-\frac{5}{9}\text{ of }\frac{2}{7}\right)\div\frac{4}{7}.$

189. $\dfrac{3\frac{1}{3}-1\frac{1}{2}\text{ of }1\frac{1}{5}-1\frac{1}{7}}{\left(3\frac{1}{3}-1\frac{1}{2}\right)\text{ of }\left(1\frac{1}{5}-1\frac{1}{7}\right)}\times\dfrac{3\frac{1}{3}-1\frac{1}{2}\text{ of }\left(1\frac{1}{5}-1\frac{1}{7}\right)}{\left(3\frac{1}{3}-1\frac{1}{2}\right)\text{ of }1\frac{1}{5}-1\frac{1}{7}}.$

190. $\left\{\frac{1}{8}\text{ of }\left(\frac{1}{10}-\frac{1}{11}\right)\div\dfrac{\frac{1}{7}-\frac{1}{9}\div\left(\frac{4}{9}+\frac{4}{11}\right)}{\left(\frac{1}{7}+\frac{1}{9}\right)\div\left(\frac{4}{9}-\frac{4}{11}\right)}\right\}\times\dfrac{\frac{1}{3}+\frac{1}{7}\div\left(\frac{1}{7}-\frac{1}{9}\right)}{\left(\frac{1}{3}+\frac{1}{7}\right)\div\frac{1}{7}-\frac{1}{9}}.$

XXII. FRACTIONS.

Value of a Fraction of a Concrete Quantity.

Find, *mentally*, the value of—

1. $\frac{1}{2}$; $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, of a shilling
2. $\frac{1}{3}$; $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{24}$, $\frac{1}{48}$, of a shilling.
3. $\frac{3}{8}$; $\frac{5}{8}$· $\frac{7}{8}$; $\frac{3}{16}$; $\frac{5}{16}$, of a shilling
4. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{5}{24}$, $\frac{11}{24}$, $\frac{5}{48}$, of a shilling
5 $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, of £1.
6. $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{12}$, $\frac{11}{12}$, of £1
7. $\frac{3}{4}$, $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{3}{16}$, of £1.
8 $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{15}$, $\frac{19}{40}$, of £1
9 $\frac{1}{240}$; $\frac{17}{240}$; $\frac{1}{120}$; $\frac{1}{60}$, $\frac{7}{60}$, $\frac{1}{80}$, $\frac{9}{80}$, $\frac{1}{960}$, $\frac{17}{960}$, $\frac{1}{480}$, of £1
10. $\frac{23}{240}$; $\frac{37}{240}$; $\frac{53}{240}$, $\frac{11}{120}$; $\frac{31}{120}$; $\frac{7}{80}$, $\frac{7}{480}$, $\frac{13}{960}$, $\frac{23}{960}$, $\frac{37}{960}$, of £1.

Find the value of—

11. $\frac{1}{4}$ of £7, 12*s* 5*d*
12. $\frac{1}{5}$ of £11, 16*s* 9*d*
13. $\frac{1}{5}$ of £2, 7*s* 3*d*
14 $\frac{1}{8}$ of £12, 13*s*. 9*d*
15. $\frac{1}{7}$ of £1, 5*s* 3*d*
16. $\frac{1}{9}$ of £3, 2*s* 7*d*
17. $\frac{1}{5}$ of £8, 12*s* $7\frac{1}{4}$*d*
18. $\frac{1}{6}$ of £13, 2*s* $3\frac{1}{4}$*d*.
19. $\frac{5}{12}$ of £1.
20. $\frac{3}{8}$ of £2
21 $\frac{2}{5}$ of £7
22. $\frac{5}{9}$ of £4, 1*s*
23. $\frac{3}{7}$ of £10
24. $\frac{5}{11}$ of £12
25. $\frac{4}{9}$ of 18*s* 7*d*
26. $\frac{5}{12}$ of 13*s*. 5*d*
27 $\frac{11}{15}$ of 1 day
28 $\frac{17}{36}$ of 5 hours.
29. $\frac{5}{16}$ of 3 acres
30. $\frac{3}{11}$ of a mile.
31 $\frac{5}{6}$ of 4 yds 2 ft.
32 $\frac{5}{8}$ of 1 ton 2 cwts.
33 $\frac{5}{7}$ of 12 yds
34. $\frac{7}{11}$ of 3 sq yds

35. $\frac{1}{12}$ of £122, 4*s* $6\frac{7}{8}$*d*.
36 $\frac{1}{12}$ of £411, 17*s* $1\frac{5}{6}$*d*
37. $\frac{2}{7}$ of £16, 13*s*. 4*d*
38. $\frac{5}{11}$ of £25, 14*s* 7*d*
39. $\frac{7}{19}$ of £8, 12*s* $11\frac{3}{4}$*d*
40. $\frac{17}{73}$ of £7, 18*s* 2*d*
41. $\frac{31}{64}$ of £4523, 14*s* 8*d*.
42. $\frac{44}{45}$ of £119, 19*s* $9\frac{3}{4}$*d*

43. $\frac{7}{9}$ of 11 yds. 2 ft. 7 in.
44. $\frac{5}{8}$ of 3 tons 13 cwts 2 qrs.
45. $\frac{7}{15}$ of 3 tons 17 cwts. 1 qr 9 lbs
46. $\frac{1}{22}$ of 1 ac 33 po 22 sq yds.

47. $\frac{1}{2}$ of $\frac{3}{32}$ of £1
48. $\frac{3}{10}$ of $\frac{5}{24}$ of £7.
49. $\dfrac{73\frac{1}{2}}{42}$ of $6\frac{3}{4}$*s*
50. $\dfrac{17}{27\frac{3}{4}}$ of $6\frac{1}{6}$*s*.
51. $\frac{9}{17}$ of 2*s* $11\frac{1}{2}$*d*
52 $\frac{5}{19}$ of 7*s* $8\frac{1}{4}$*d*
53. £$15\frac{5}{8} \times \dfrac{4\frac{2}{5}}{9\frac{1}{6}}$.
54 £$3\frac{7}{8} \times \dfrac{22\frac{19}{20}}{34\frac{7}{8}}$.
55. $\frac{23}{31}$ of 10 tons.
56. $\frac{17}{29}$ of 17 tons
57. $\dfrac{30\frac{1}{3}}{10\frac{5}{6}}$ of $32\frac{1}{7}$ qrs.
58 $\dfrac{15\frac{1}{6}}{7\frac{1}{3}}$ of $4\frac{7}{12}$ hrs.

Multiply—

59. £3, 3*s* $3\frac{3}{4}$*d* by $3\frac{3}{4}$.
60. £4, 17*s* $6\frac{3}{4}$*d* by $2\frac{4}{7}$
61. £56, 10*s* $10\frac{1}{2}$*d* by $71\frac{1}{2}$
62. £22, 13*s* 5*d*. by $89\frac{1}{3}$.
63. £7, 12*s*. $4\frac{3}{5}$*d* by $8\frac{4}{7}$.
64. £5, 17*s*. $6\frac{5}{9}$*d* by $14\frac{2}{7}$.

Divide—

65. £3, 3*s*. $3\frac{3}{4}$*d*. by $3\frac{3}{4}$. 66. £24, 10*s* 3*d*. by $17\frac{2}{3}$

67. £11, 11*s* 11*d* by $11\frac{1}{11}$. 68. £23, 13*s* $7\frac{1}{4}$*d* by $17\frac{3}{4}$

69. £219, 9*s* 7*d* by $12\frac{2}{7}$. 70 £184, 15*s*. 9*d* by $18\frac{4}{4}$.

Find the sum of—

71. $\frac{3}{8}$ of 1*s*, $\frac{2}{7}$ of 21*s*, $\frac{1}{12}$ of 2*s*. 6*d*. and $\frac{1}{48}$ of £1

72. $\frac{7}{16}$ of £5, $\frac{2}{7}$ of £9, 13*s*. $2\frac{3}{4}$*d*. and $\frac{5}{12}$ of 2*s*. 6*d*.

73. $\frac{2}{3}$ of 5*s*, $\frac{13}{126}$ of £1, 1*s*., $\frac{1}{3}$ of 18*s*. 6*d* and $\frac{5}{12}$ of £1.

74. $1\frac{1}{3}$ of £1, 1*s*., $\frac{5}{8}$ of £1, $\frac{5}{16}$ of 2*s* 6*d* and $29\frac{19}{20}$ of $7\frac{1}{2}$*d*.

75. £$3\frac{43}{96}$ and $(9\frac{1}{12} - 2\frac{7}{8})$ of 5*s*. 76. £$7\frac{35}{96}$ and $(3\frac{1}{4} - 2\frac{7}{12})$ of 3*s* 6*d*.

77. $\frac{7}{8}$ po., $\frac{3}{8}$ yd and $\frac{3}{16}$ ft. 78. $\frac{4}{5}$ ac, $\frac{1}{36}$ ro and $\frac{8}{9}$ sq po.

Find the difference between—

79. $\frac{5}{8}$ of 14*s* $10\frac{1}{2}$*d*. and $\frac{4}{7}$ of the same sum.

80. $2\frac{3}{4}$ of 11*s*. 8*d* and $1\frac{1}{2}$ of 17*s*. $2\frac{1}{2}$*d*.

81. $\frac{7}{8}$ of £9, 11*s* 10*d*. and $\frac{5}{6}$ of £10, 3*s* $10\frac{1}{2}$*d*.

82. $\frac{8}{11}$ of £78, 16*s*. $2\frac{1}{2}$*d*. and $\frac{2}{3}$ of £35, 14*s* $8\frac{1}{4}$*d*

83. $\frac{2}{7}$ of $\frac{21}{22}$ of £15, 10*s* 9*d*. and $\frac{7}{130}$ of $3\frac{1}{3}$ of $2\frac{11}{14}$ of £2, 15*s*. 8*d*.

84. $\dfrac{2\frac{4}{5} \text{ of } 5\frac{1}{3}}{27\frac{1}{2}}$ of £1, 2*s*. 6*d*. and $\dfrac{5\frac{1}{12}}{2\frac{1}{2} + 3\frac{3}{4}}$ of 12*s* 6*d*.

85 $1\frac{2}{3}$ of 1 ton 12 cwts. 3 qrs and $\frac{3}{7}$ of 3 tons 13 cwts 1 qr.

86. $3\frac{1}{27}$ of 2 qrs 25 lbs and $\frac{1}{4}$ of 4 cwts 2 qrs 20 lbs.

Find the value of—

87. $\frac{1}{4}$ of $\dfrac{4\frac{7}{9} \text{ of } 6\frac{2}{3}}{7\frac{26}{27}}$ of $\dfrac{3\frac{2}{5} - 3\frac{1}{6}}{3\frac{2}{5} + 2\frac{1}{6}}$ of £182, 7*s*. 5*d*.

88. $4\frac{4}{7}$ of $\dfrac{4\frac{1}{10}}{4\frac{5}{9}}$ of $\dfrac{\dfrac{1}{20\frac{1}{2}} + \dfrac{1}{13\frac{2}{3}}}{\dfrac{1}{10\frac{1}{4}} + \dfrac{1}{8\frac{1}{5}}}$ of $\dfrac{6\frac{5}{6}}{5\frac{6}{7}}$ of $5\frac{1}{5}$ guineas.

89. $\left\{\dfrac{\frac{35}{36} - \frac{34}{35}}{\frac{24}{25} - \frac{23}{24}} - \dfrac{\frac{24}{25} - \frac{23}{24}}{\frac{15}{16} - \frac{14}{15}}\right\}$ of 15 cwts.

90. $\left\{1 + \frac{65}{81} \text{ of } \dfrac{1\frac{2}{3} + 1\frac{3}{5} - 1\frac{1}{6}}{1\frac{2}{3} \text{ of } (1\frac{2}{5} - 1\frac{1}{4})} \times \dfrac{(1\frac{2}{3} + 1\frac{2}{5}) \div 1\frac{1}{6}}{1\frac{2}{4} \text{ of } 1\frac{2}{5} \text{ of } 1\frac{1}{8}}\right\}$ of 3 tons.

XXIII. FRACTIONS.

REDUCTION OF ONE QUANTITY TO THE FRACTION OF ANOTHER.

Reduce, *mentally*, each to the fraction of 1*s* —

1. $6d$, $3d$; $1\frac{1}{2}d$, $\frac{3}{4}d$
2. $4\frac{1}{2}d$, $7\frac{1}{2}d$, $10\frac{1}{2}d$.
3. $4d$, $2d$; $1d$, $\frac{1}{2}d$, $\frac{1}{4}d$
4. $8d$, $10d$; $7d$, $2\frac{1}{2}d$., $1\frac{1}{4}d$

Reduce, *mentally*, each to the fraction of £1—

5. $10s$; $5s$; $2s\ 6d$, $1s\ 3d$
6. $15s$, $7s.\ 6d$; $12s\ 6d$, $17s\ 6d$.
7. $6s.\ 8d$, $13s\ 4d$, $3s\ 4d$, $1s\ 8d$.
8. $4s$, $8s$, $12s$, $16s$, $1s\ 4d$
9. $1d$, $2d$; $4d$, $3d$, $\frac{1}{2}d$, $\frac{1}{4}d$
10. $7d$, $11d$, $3\frac{1}{2}d$, $4\frac{1}{4}d$, $7\frac{3}{4}d$

Express, in lowest terms—

11.	$10d$ as a fraction of	$3s$	12	$1s\ 6d$ as a fraction of	$7s\ 6d$
13.	$7\frac{1}{2}d$	$2s$	14	$4\frac{1}{2}d$	$3s\ 3d$
15.	$1s\ 8d$ . .	$8s\ 4d$.	16	$2s\ 9d$	$9s\ 3d$
17.	$4s\ 2\frac{1}{2}d$.	$11s\ 6d$	18	$1s\ 11\frac{3}{4}d$	$14s\ 8\frac{1}{4}d$.
19.	$3s\ 5\frac{1}{4}d$ . .	$10s$	20	$12s\ 10d$	$16s\ 4d$
21.	$17s\ 6d$.	£2, $12s$	22	$13s\ 4d$	£3, $6s$
23.	£1, $3s\ 6d$.	£7, $5s$	24	£2, $1s\ 8d$	£3, $16s$.
25.	$9s\ 7\frac{1}{2}d$ . .	£1	26	$1s\ 11\frac{1}{4}d$	£1
27.	$\frac{2}{5}$ of $7\frac{1}{2}d$. . .	$2s\ 6d$	28.	$\frac{3}{11}$ of $1s\ 10d$	£2, $10s$
29.	$\frac{2}{3}$ of $7\frac{1}{2}d$. . .	$4s\ 3d$.	30.	$\frac{1}{5}$ of $7s$.	$\frac{3}{8}$ of $9s\ 6d$
31.	2 qrs 7 lbs.	1 cwt	32	19 lbs 8 ozs . .	1 cwt
33.	2 cwts 1 qr	2 tons	34.	14 cwts 3 qrs 7 lbs	1 ton.
35.	17 yds. 1 ft 6 in	$\frac{1}{2}$ mi	36	5 fur 18 po	1 mile
37.	$10\frac{1}{2}$ poles	1 acre	38	$5\frac{1}{2}$ in . ..	1 pole
39	16 hrs. 40 m 48 s	1 day	40.	4 oz 7 dwt 12 grs.	5 oz Tr. .

Express—

41. £2, $8s.\ 9d.$ in *shillings*
42. £5, $11s\ 7\frac{1}{2}d$ in *shillings*
43. £3, $3s\ 3\frac{3}{4}d$ in *shillings.*
44. £2, $17s\ 2\frac{1}{4}d$ in *shillings*
45. £14, $17s\ 4d$ in *pounds*
46. £21, $8s\ 4\frac{1}{2}d$ in *pounds*
47. £7, $8s\ 9d$ in *pounds.*
48. £7, $1s\ 10\frac{1}{2}d$ in *pounds*

49. 7 cwt 3 qr 21 lbs in *quarters*
50. 1 ton 13 cwts 3 qrs in *cwts*
51. 9 tons 12 cwts 16 lbs in *cwts*
52. 3 cwts 1 qr 7 lbs 8 ozs in *qrs*
53. 17 yds 2 ft. 9 in in *yards*
54. 3 fur 7 po $2\frac{3}{4}$ yds in *poles*
55. 12 ac 1 ro 16 po. in *acres*
56. 7 ac. 3 ro 24 po in *acres.*

57. How many times is £3, 17*s*. 6*d* contained in £178, 5*s* ?

58 How many times is £1, 18*s*. 9*d* contained in £120, 2*s*. 6*d*.?

59. What fraction of £50 is £8, 6*s* 8*d* ?

60. What fraction of £22, 5*s*. 6*d* is £1, 17*s* $1\frac{1}{2}$*d* ?

61 What fraction of $9\frac{1}{4}$ guineas is $\frac{7}{16}$ of £9, 17*s* 4*d*. ?

62 What part of £24, 8*s* $3\frac{3}{4}$*d* is $\frac{1}{3}$ of £1, 17*s*. $6\frac{3}{4}$*d*. ?

63. Reduce $2\frac{1}{8}$ of £5, 11*s* to the fraction of $2\frac{5}{16}$ of £4, 5*s*.

64. Reduce $\frac{3}{5}\frac{1}{7}$ of £2, 2*s* 9*d*. to the fraction of £2, 2*s* $7\frac{1}{2}$*d*.

65. Express the sum of $\frac{5}{7}$ of a guinea, $\frac{3}{8}$ of £1, $\frac{2}{70}$ of 25*s* and $\frac{1}{4}$ o. 1*s* as the fraction of £$4\frac{7}{20}$

66. Express the difference between $\frac{2}{3}$ of £38, 12*s* 6*d* and $\frac{4}{5}$ of £47, 16*s* 3*d*. as the fraction of £100

67. Express 2 ft 3 in. as the fraction of $4\frac{1}{2}$ yds.

68. What part of 1 mi. 30 po is 2 yds 3 in ?

69. Express $1\frac{1}{2}$ feet to the fraction of $\frac{1}{4}$ of a mile

70 What fraction of 30 miles is 1 fur 20 poles?

71. Reduce $\frac{3}{4}$ of a yard to the fraction of a mile.

72 Express 25 acres as the fraction of $1\frac{1}{2}$ sq. miles.

73. What fraction of 3 sq. po. is $\frac{1}{8}$ of 1 sq. po. 11 sq. yds.?

74. Reduce 4 ozs. 7 dwts. 12 grs. to the fraction of 5 ozs. Avoirdupois.

75. Reduce 22 days 4 hrs 35 mins. 42 secs to the fraction of 34 days 20 hrs. 56 min 6 secs.

76 Express the sum of $\frac{2}{3}$ of 1 ton 3 cwts. 1 qr and $\frac{3}{5}$ of 1 ton 1 cwt 3 qrs as a fraction of $\frac{4}{5}$ of 3 tons 1 cwt. 1 qr.

77 Reduce $\dfrac{3\frac{5}{6} - 4\frac{5}{8} + 5\frac{5}{12}}{5\frac{7}{10} - 6\frac{7}{15} + 7\frac{7}{20}}$ of 3 tons 19 cwts. to the fraction of 3 tons 14 cwts

78. Reduce $\dfrac{2\frac{2}{11} + 3\frac{3}{22} - 4\frac{4}{33}}{2\frac{2}{11} - 3\frac{3}{22} + 4\frac{4}{33}}$ of 1 fur 36 po. to the fraction of 5 fur. 37 po.

79. Reduce $\frac{1664}{1458}$ of $\left\{\frac{7}{10} \text{ of } £3\frac{3}{14} + 6\frac{2}{3} \text{ of } £3, 0s. 9d - 4\frac{1}{3}\frac{3}{1} \text{ of } £3, 2s.\right\}$ to the fraction of £100

80. Reduce $\dfrac{16\frac{1}{2} - 8\frac{5}{8} \text{ of } 2\frac{2}{23} + 2\frac{2}{69}}{(16\frac{1}{2} - 8\frac{5}{8}) \text{ of } (2\frac{2}{23} + 2\frac{2}{69})}$ of £18, 12*s* 9*d*. to the fraction of a guinea.

XXIV. MISCELLANEOUS EXERCISES.

1. Write $\frac{19}{98}$ in words
2. Write Ninety-one one hundred and forty-ninths in figures
3. Reduce $7\frac{7}{17}$ to an improper fraction.
4. Reduce $\frac{77}{17}$ to a mixed number
5. Express $\frac{7}{37}$ with denominator 777.
6. Reduce $\frac{1188}{1881}$ to its lowest terms
7. Reduce $\frac{8}{77}$ and $\frac{7}{88}$ to their least common denominator
8. Add together $\frac{2}{5}$, $\frac{7}{16}$, $\frac{7}{50}$, and $\frac{3}{2800}$
9. Subtract $5\frac{5}{63}$ from $13\frac{3}{35}$
10. A boy spent $\frac{5}{6}$ of his money and then had $4d$ left How much had he at first?

---

11. Express $\frac{91}{90}$ in words
12. Express in figures Seven hundred, and nineteen ninety-oneths.
13. Express $99\frac{9}{19}$ as an improper fraction
14. Express $\frac{999}{19}$ as a mixed number
15. Express $\frac{7}{17}$ with numerator 63
16. Reduce $\frac{1818}{8181}$ to its lowest terms
17. Which is greater, $\frac{17}{38}$ or $\frac{25}{57}$?
18. Add $8\frac{17}{18}$ to $8\frac{71}{81}$
19. From $99\frac{9}{19}$ take $\frac{999}{19}$.
20. A boy, having spent $\frac{6}{7}$ of his pocket-money, had $9d$ left What had he at first?

---

21. How many tenths are there in ten and one-tenth?
22. How many fifty-oneths make two-thirds?
23. Express $\frac{825}{2739}$ in its simplest form.
24. Find the value of $3\frac{23}{125} + 4\frac{23}{50}$
25. Find the value of $23\frac{7}{740} - 19\frac{19}{20}$
26. Find the value of $\frac{153}{308} \times \frac{77}{255}$.
27. Find the value of $365\frac{1}{4} \div 365$
28. Simplify $\frac{1}{3} + \frac{1}{5} - \frac{6}{7} + \frac{2}{35} + \frac{4}{15}$
29. What must be added to $\frac{1}{3}$ of $\frac{4}{7}$ of $\frac{3}{5}$ to make $\frac{1}{6}$?
30. A boy gave a third of his cake to one friend, half of what remained to another, and ate the rest How much did he eat?

---

31. How many units are there in Three-hundred-and-forty-three sevenths?

32. How many ninths are equivalent to thirty-six eighty-oneths?

33 Which is greater, $\frac{3663}{4477}$ or $\frac{9}{10}$?

34. Find the value of $2\frac{26}{75} \div 1\frac{31}{35}$.

35 Find the value of $95\frac{70}{143} - 74\frac{20}{91}$.

36 Find the value of $7\frac{13}{56} + 1\frac{23}{112}$.

37. Find the value of $4\frac{16}{17} \times 2\frac{61}{85}$.

38. Simplify $1\frac{1}{2} + 15\frac{7}{15} - 8\frac{5}{12} - 6\frac{4}{5}$.

39. From $2\frac{1}{3}$ of $4\frac{1}{19}$ take the sum of $2\frac{1}{3}$ and $4\frac{1}{19}$.

40. Half the arable land of a farm is planted with wheat, one-third with barley, one-twelfth with oats, and there are 5 acres of beans. How many acres of wheat are there?

---

41. Convert $13\frac{13}{33}$ into an improper fraction.

42. Find a fraction with numerator 19 which shall be equal to $\frac{171}{747}$.

43. Which is greater, $\frac{19}{81}$ or $\frac{56}{243}$?

44. Add together $\frac{10}{17}$, $3\frac{6}{7}$, $5\frac{7}{51}$ and $\frac{9}{14}$

45. Subtract $19\frac{15}{64}$ from $23\frac{11}{56}$.

46. Multiply together $3\frac{8}{9}$, $\frac{18}{49}$, $2\frac{10}{39}$ and $\frac{21}{165}$

47. Divide $2\frac{7}{39}$ by $2\frac{31}{78}$.

48. Express in simplest form the sum of $\dfrac{1}{8\frac{1}{7}}$ and $\dfrac{1}{7\frac{1}{8}}$.

49. Simplify $12\frac{1}{2} - 11\frac{5}{16} + \frac{13}{16}$.

50 A mowed $\frac{1}{3}$, B mowed $\frac{1}{4}$, and C mowed $\frac{2}{5}$, of a field What part of the field then remained unmown?

---

51. Reduce $\frac{2018}{101}$ to a mixed number.

52 Express $\frac{41}{77}$ with denominator 231.

53. Reduce $\frac{1881}{8118}$ to its lowest terms.

54. Add together $\frac{14}{39}$, 10 and $4\frac{9}{65}$.

55. Subtract $3\frac{9}{95}$ from $7\frac{2}{57}$.

56. Multiply together $\frac{88}{245}$, $8\frac{55}{64}$, $\frac{80}{121}$ and $2\frac{23}{27}$.

57. Divide $3\frac{71}{156}$ by $1\frac{169}{216}$

58. Simplify $2\frac{1}{5} + 1\frac{4}{7} - 4\frac{1}{2} - \frac{3}{5} + 2\frac{8}{35}$

59. How many bits each $1\frac{1}{5}$ inches long can be cut from a yard?

60. A boy after spending $\frac{4}{5}$ of his money had $7\frac{1}{2}d.$ left. How much had he at first?

---

61. How many seventieths are there in seven and one-seventh?

62. Reduce $\frac{1188}{8811}$ to its lowest terms

63. Add $2\frac{7}{45}$, $\frac{13}{165}$, $\frac{17}{60}$ and $5\frac{7}{9}$

64. What must be taken from $65132\frac{7}{8}$ to leave $51423\frac{11}{12}$?

65. Find the continued product of
$1\frac{1}{2}$, $1\frac{1}{3}$, $1\frac{1}{4}$, $1\frac{1}{5}$, $1\frac{1}{6}$, $1\frac{1}{7}$, $1\frac{1}{8}$ and $1\frac{1}{9}$

66 Divide the sum of $\frac{7}{8}$ and $\frac{11}{12}$ by $1\frac{19}{24}$

67 Find, in its simplest form, the value of $\dfrac{6\frac{47}{138}}{6\frac{21}{184}}$

68. Simplify $1\frac{2}{3}$ of $2\frac{1}{2}$ of $3\frac{3}{5} + 4\frac{1}{5}$ of $5\frac{5}{7} - 6$

69. Which is greater, three-fourths of 11 yds 0 ft 8 in, or half of 16 yds 2 ft 7 in?

70 How long will a journey of 130 miles take, at the rate of $33\frac{1}{3}$ miles per hour?

---

71. How many ninetieths are there in nine and one-ninth?

72. Reduce $\frac{17836}{24531}$ to lowest terms

73 Find the value of $\frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \frac{1}{4 \times 5 \times 6}$

74. Take half the sum of the greatest and least of the three fractions $\frac{8}{9}$, $\frac{5}{6}$ and $\frac{13}{15}$ from the other

75. What is the value of $4 \times (\frac{2}{3} - \frac{3}{4}$ of $\frac{8}{9})$?

76. Divide $23\frac{1}{25}$ by $3\frac{123}{175}$

77. Simplify $12\frac{3}{5} + 11\frac{2}{5} \times 13\frac{5}{9} - 3\frac{5}{9}$

78 Simplify $(12\frac{3}{5} + 11\frac{2}{5}) \times (13\frac{5}{9} - 3\frac{5}{9})$.

79. Express £$3\frac{9}{16}$ in £, *s* *d*

80. How long will a man who walks $3\frac{3}{4}$ miles per hour, be in walking $35\frac{1}{4}$ miles?

---

81. Express $\frac{27}{31}$ with numerator 999

82. Compare $\frac{5}{6}$, $\frac{5+6}{6+7}$, and $\frac{6}{7}$

83. What must be added to $9\frac{13}{14}$ that the sum may be $19\frac{1}{2}$?

84. Find the value of $450362 \div 5\frac{1}{2}$

85. Divide $\frac{1}{82} + \frac{1}{86}$ by $\frac{1}{82} - \frac{1}{86}$

86. Simplify $1\frac{2}{3} - \frac{6}{8} + \frac{4}{5} - \frac{20}{24} - \frac{2}{5} - \frac{1}{2} + \frac{1}{60}$.

87. Simplify $\frac{5}{12} \times \frac{9}{16} \times 2\frac{2}{11} + \frac{215}{440}$

88. Find $\frac{3}{7}$ of £5, 18*s* 5*d*

89. What fraction of 2 cwts is 7 ozs?

90. When 16 pages of a book of 112 pages have been read, what fraction of the book remains to be read?

61*. When three-fifths of the month is gone, how many days of April are left?

62*. A boy had half-a-crown; he spent $\frac{7}{10}$ of it; how much had he left?

63*. In a congregation of 200 persons, three-eighths of the number are women, and there are 34 men, find the number of children.

64*. A generous child gave one-third of her cake to A, one-fourth to each of B and C, and one-sixth to D; how much had she left for herself?

65*. After spending $\frac{9}{10}$ of his money a boy had $2\frac{1}{2}d.$ left; how much had he at first?

66*. A girl spent one-sixth of her money and saved half-a-crown; how much did she spend?

67*. A careless boy lost a shilling, which was $\frac{3}{4}$ of all he had; how much had he left?

68*. How much money had a girl who, after spending $\frac{5}{8}$ of it, had a shilling left?

69*. A spent $\frac{4}{5}$ of his money and then had $1s. 3d.$ left; how much did he spend?

70*. B spent $2s. 4d.$ and saved $\frac{3}{10}$ of her money; how much did she save?

71*. A man's income was £330; he spent $\frac{9}{11}$ of it; how much did he save?

72*. If a man spends $\frac{7}{8}$ of his income, and saves £65, what is his income?

73*. A, having $3s. 6d.$, spent $\frac{5}{8}$ of it at one shop, and $\frac{3}{7}$ of what remained at another; how much was left?

74*. B, after spending $\frac{3}{8}$ of her money at one shop and $\frac{4}{5}$ of what remained at another, had threepence left; how much had she at first?

75*. A boy lost half of his marbles to A, and one-third of the rest to B, won 8 from C, and then had as many as he started with; find this number.

76*. A man went $\frac{5}{6}$ of his journey by train, $\frac{1}{8}$ by tram, and walked the remaining mile and a half; how far did he travel?

77*. A pole stands with one-fourth of its length in the mud, one-third in the water, and 10 feet above the water; how long is it?

78*. A bought a box of sweets, gave a third of them to B, three-eighths of what remained to C, and ate the last ten; how many sweets did the box contain?

79*. After reading 184 pages of a book, I found that I had read a page more than three-fifths of it; how many pages were there in the book?

80*. After spending $1s\ 5d$, a boy had a penny more than $\frac{4}{7}$ of his money left; how much had he at first?

81*. A man's yearly expenditure amounted to $\frac{5}{6}$ of his income, his rates and taxes to $\frac{1}{18}$ of his income, and he saved £40: find his income.

82*. A book-case has 16 shelves each 2 ft. 11 in. long, how many books, $\frac{7}{8}$ inch thick, will it hold?

83*. A book-case, of three shelves each 1 ft 7 in long, will just hold an encyclopædia each volume of which is $2\frac{3}{8}$ inches thick: find the number of volumes

84* If 5 quires of note-paper weigh a pound, what fraction of an ounce does one sheet weigh?

85*. If 5 quires of note-paper make a packet $1\frac{1}{8}$ inches thick, what fraction of an inch is the paper in thickness?

86* How many times can a jug, holding $\frac{7}{8}$ of a pint, be filled from a cask containing 14 gallons of liquid?

87*. How many bits, each $2\frac{3}{10}$ inches long, can be cut from a yard of string, and what is the length of the remnant?

88* How much coin of the realm will pay for $2\frac{5}{8}$ yards of ribbon at $11\frac{3}{4}d.$ per yard?

89*. Find, to the nearest farthing, the cost of 7 lbs 10 ozs of mutton at $9\frac{1}{2}d$ per lb

90*. Find the number whose sixth exceeds its eighth part by 2

91*. If a girl's height were $\frac{4}{5}$ of what it is, she would be 6 feet high, how tall is she?

92*. Three years ago a boy weighed 7 st. 10 lbs, and that is $\frac{9}{10}$ of his present weight, what does he weigh?

93* When the sun rises at 5 and sets at 6 30, for what fraction of the day is it above the horizon?

94* What o'clock is it when the part of the day that is gone is one-third of what is left?

95*. A boy bought a bat, a ball and wickets for 11*s* the price of the ball was $\frac{6}{11}$ of that of the bat, and the price of the wickets $\frac{5}{6}$ of that of the ball, find the cost of each

96*. A girl bought a driver, a cleek and a putter, the cost of the cleek was $\frac{10}{11}$ of that of the driver, and the cost of the putter was $\frac{9}{10}$ of that of the cleek, the putter cost 4*s* 6*d* find the total cost

97*. There are six classes in a High School, the 1st contains $\frac{1}{9}$, the 2nd $\frac{1}{8}$, the 3rd $\frac{1}{6}$, the 4th $\frac{3}{16}$, the 5th $\frac{5}{24}$, of the total number of girls, which is 144; how many are there in the 6th class?

98*. There are six forms in a grammar school Form VI contains $\frac{1}{20}$, V $\frac{2}{15}$, IV $\frac{5}{24}$, III $\frac{1}{4}$, II $\frac{1}{5}$, of the total number of boys, and there are nineteen in Form I Find the number in the school

99*. A man left half his money to his wife, one-sixth to his son, and the remainder, in equal shares, to his four daughters, each of whom received £800, how much money did he possess?

100*. A man went out for a month's holiday, he paid, for his tourist's ticket, one-sixth of the money he took with him, for board and lodging, three-fourths of what remained, and his other expenses amounted to two-thirds of the cost of his ticket, he brought back £2, 3*s*. 2*d* how much did he set out with?

91. Express $\frac{1}{3}$ with denominator 777

92. Which is greatest. $\frac{11 \times 9}{12 \times 10}$, $\frac{11 + 9}{12 + 10}$, or $\frac{11 \div 9}{12 \div 10}$?

93. What proper fraction, added to the sum of $47\frac{3}{5}$, $83\frac{5}{8}$ and $29\frac{7}{12}$, makes the result integral?

94. Divide the sum of $\frac{1}{6}$ and $\frac{2}{9}$ by their difference.

95. Simplify $71\frac{4}{7} - 39\frac{23}{110} + 41\frac{5}{12} - 2\frac{17}{35} - \frac{4}{63} + 11\frac{15}{28}$.

96. Simplify $\frac{3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} - 35}{2\frac{1}{2} \times 2\frac{1}{2} \times 2\frac{1}{2} - 8\frac{3}{4}}$.

97. Divide £4, 3*s*. 9*d* by $2\frac{2}{3}$.

98. From $\frac{2}{7}$ of $\frac{5}{6}$ of £1, 1*s*. take $\frac{2}{5}$ of $\frac{3}{8}$ of £1

99. Express £1, 4*s* $0\frac{3}{4}$*d*. as the fraction of £33

100. A pole 25 feet long is sawn up into an exact number of lengths of 2 ft. 3 in. each: what fraction of the pole becomes saw-dust?

---

101. What integer is equal to $\frac{323}{17}$?

102. Simplify $\frac{68 \times 27 \times 45}{51 \times 81 \times 100}$.

103. Find, *by inspection*, the sum of $3\frac{3}{8}$, $7\frac{1}{6}$, $14\frac{2}{9}$, $5\frac{5}{8}$, $1\frac{5}{6}$ and $3\frac{7}{9}$.

104. Find the continued product of $\frac{5\frac{1}{22}}{6\frac{15}{16}}$, $\frac{6\frac{2}{17}}{10\frac{1}{11}}$, $\frac{7\frac{13}{14}}{15\frac{6}{7}}$ and $\frac{13\frac{7}{8}}{27\frac{3}{4}}$.

105. Divide $3\frac{5}{14} + 2\frac{4}{9} - 4\frac{19}{21}$ by $3\frac{37}{63}$

106. What number added to $2\frac{1}{3}$ of $\frac{5}{7} + 2\frac{1}{4}$ of $2\frac{2}{3}$ will make $9\frac{2}{3}$?

107. Simplify $\frac{(2\frac{1}{2} + 3\frac{1}{3}) \times (4\frac{1}{4} - 1\frac{1}{5})}{(2\frac{1}{2} + 3\frac{1}{3} \text{ of } 4\frac{1}{4}) \div 1\frac{1}{5}}$

108. Express £$7\frac{47}{960}$ + £$\frac{61}{240}$ in £, *s* *d*

109. Reduce 2 cwts. 22 lbs. 8 ozs to the fraction of 33 tons 17 cwts. 3 qrs. 14 lbs.

110. Divide £4, 11*s*. 7*d*. between A and B, so that A may have a third as much again as B.

---

111. Express $\frac{321}{428}$ with denominator 1712.

112. Divide $63 \times 64 \times 65$ by $26 \times 27 \times 28$.

113. Add $6\frac{91}{198}$, $5\frac{83}{286}$, $19\frac{23}{156}$ and $27\frac{22}{39}$

114. Find the value of $\frac{3}{10}$ of $\frac{5}{18} - \frac{2}{9}$ of $\frac{3}{8}$.

115. Divide $1\frac{59}{80}$ by the difference between $\frac{11}{24}$ and $\frac{7}{40}$.

116. What must be taken from 10 that the result may be equal to $3\frac{1}{2} - 2\frac{1}{3} + 1\frac{1}{6}$?

117. Simplify $\left(\frac{97}{194} + \frac{291}{388}\right) \div \left(\frac{485}{582} - \frac{291}{388}\right)$.

118. Express $£11\frac{97}{120} - £6\frac{107}{480}$ in £, *s* *d*

119. Reduce 10 yds 2 ft $7\frac{1}{9}$ ins to the fraction of $\frac{1}{6}$ of a mile.

120. Divide £19, 19*s* $4\frac{1}{2}d$ between A and B, so that A may have half as much again as B

---

121. Find the value of 420 lbs of sugar at $2\frac{3}{8}d$ per lb

122. Which is greater, $\frac{3}{5}$ of $\dfrac{2\frac{1}{3}}{4\frac{1}{4}}$, or $\frac{5}{7}$ of $\dfrac{1\frac{1}{3}}{2\frac{5}{6}}$?

123. What must be subtracted from the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{6}$ that an even integer may be left?

124. How many times does the sum of $63\frac{1}{8}$ and $72\frac{1}{7}$ contain their difference?

125. How many times can $\frac{2}{7}$ be taken from $4\frac{3}{5}$, and what remains?

126. Find the value of $120 \times (\frac{1}{21} + \frac{1}{35} + \frac{1}{49} - \frac{1}{28} - \frac{1}{42} - \frac{1}{56})$

127. Simplify $\dfrac{2\frac{1}{13} \text{ of } \frac{5}{9} - 1\frac{1}{8} \text{ of } \frac{2}{3}}{1\frac{1}{21} \div 5\frac{4}{7}}$.

128. By what fraction of a penny does $\frac{3}{100}$ of £1 exceed $\frac{3}{103}$ of £1, 0*s* 3*d*?

129. James has 70 marbles, he gives $\frac{2}{7}$ of them to George, and $\frac{4}{5}$ of those which remain to Charles, how many has he left then?

130. A can do a piece of work in $2\frac{1}{2}$ days, and B in $3\frac{1}{3}$ days, what portion of the work will they together do in one day?

---

131. Find the cost of $43052\frac{1}{2}$ lbs at 2*s* 6*d* per lb

132. A walks $20\frac{1}{2}$ miles in 6 hours, and B $16\frac{3}{4}$ miles in $4\frac{1}{2}$ hours Which walks at the faster rate?

133. Find the value of $9\frac{4}{13} + 14\frac{23}{132} + 3\frac{17}{24} + 7\frac{9}{143}$

134. By how much does $43\frac{44}{51}$ exceed $37\frac{33}{34}$?

135. Divide $1\frac{1}{2}$ of $1\frac{1}{2}$ of $1\frac{1}{2}$ by $1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2}$

136. Find the value of $7\frac{1}{7} + \dfrac{7\frac{1}{7}}{7} + \dfrac{7}{7\frac{1}{7}} + \dfrac{7\frac{1}{7} + 7}{7\frac{1}{7} - 7} + 7 \times 7\frac{1}{7}$

137. Simplify $\dfrac{5\frac{3}{4} - \frac{3}{7} \text{ of } 15\frac{3}{4} + 2\frac{2}{35} \div 1\frac{11}{25}}{\frac{3}{4} \text{ of } 7\frac{3}{7} - 5\frac{3}{5} \div 3\frac{4}{15}}$.

138. Find the value of $6\frac{2}{5}$ of $\dfrac{6\frac{3}{7} \text{ of } 6\frac{1}{8}}{6\frac{2}{9} \text{ of } 6\frac{3}{10}}$ of $6\frac{1}{15}$ of a guinea

139. A man left $\frac{1}{4}$ of his property to his wife, $\frac{1}{10}$ to each of his 3 daughters, and the residue to his only son What part of the property did the son receive?

140. A alone can reap a field in $2\frac{1}{2}$ days, B alone in $2\frac{1}{3}$ days If they work together for one day, what fraction of the field will then remain unreaped?

---

141. Multiply £7, 11*s.* 4*d* by $37\frac{1}{7}$.

142. Find both the G.C.M. and the L.C.M. of $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{7}{12}$.

143 Add $\dfrac{1}{10\frac{1}{11}}$ to $\dfrac{1}{11\frac{1}{10}}$.

144. Find the value of $(\frac{1}{20}+\frac{1}{30})+(\frac{1}{20}+\frac{1}{30})^2+(\frac{1}{20}+\frac{1}{30})^3$.

145. Simplify $\dfrac{5\frac{1}{5}+3\frac{1}{3}}{5\frac{1}{5}\times 3\frac{1}{3}} \div \dfrac{5\frac{1}{5}-3\frac{1}{3}}{5\frac{1}{5}\div 3\frac{1}{3}}$.

146. Write down at sight the number of days, hrs. &c, in $\dfrac{21\frac{3\frac{1}{2}}{24}}{365}$ of a year.

147. If 40 shillings weigh 7 ozs. 5 dwts. and contain 10 dwts. 21 grs. of alloy, how much pure silver is there in a shilling?

148. A surveyor's chain had one link bent, and was consequently $\frac{3}{8}$ of an inch too short, what was the error in measuring a mile?

149. If a man went $\frac{13}{18}$ of his journey by train, $\frac{5}{54}$ of it by tram, and walked the rest, what fraction of his journey did he walk?

150. Divide £11, 3*s.* $11\frac{1}{2}$*d.* among 44 men, giving one of them $\frac{2}{5}$ of the whole and the rest equal shares.

---

151. Multiply £2, 19*s* 9*d.* by $1\frac{81}{259}$

152. Find the G.C.M. and the L.C.M. of $2\frac{2}{5}$ and $3\frac{3}{7}$.

153. Take $\dfrac{7\frac{7}{8}}{17\frac{17}{18}}$ from $\dfrac{14\frac{2}{3}}{28\frac{2}{25}}$.

154. Find the value of $\dfrac{1}{5\times7\times11}+\dfrac{1}{3\times7\times11}+\dfrac{1}{3\times5\times11}+\dfrac{1}{3\times5\times7}$.

155. Express the product of $\frac{221}{629}$ and $\frac{72816}{85248}$ in simplest form.

156. Simplify $\dfrac{(11\frac{3}{7}-7\frac{3}{11})\div 11\frac{3}{7} \text{ of } 7\frac{3}{11}}{7\frac{3}{7} \text{ of } 11\frac{3}{11}\div(7\frac{3}{7}+11\frac{3}{11})}$

157. Write down the number of cwts., qrs., &c, in $\dfrac{\frac{17\frac{3\frac{5}{14}}{4}}{1}}{20}$ of a ton

158. If a penny stamp is $\frac{15}{16}$ of an inch long, how many placed in a row would extend a mile?

159. If $\frac{7}{15}$ of a guinea be taken from $\frac{3}{10}$ of $\frac{5}{8}$ of £5, what fraction of £3, 8*s* 8*d* will remain?

160. A and B start together on bicycles for a place 45 miles distant A goes $7\frac{1}{2}$, and B $6\frac{1}{4}$ miles an hour, how much sooner will A arrive than B?

---

161. Express $\frac{23}{35}$ as a complex fraction having numerator 1.

162. Multiply 17 tons 10 cwts. 1 qr 5 lbs. by $13\frac{1}{3}$.

163. How many times does the square of 1001 contain the product of 91 and 143?

164. Compare $\frac{7}{11}$, $\frac{7-4}{11-4}$ and $\frac{7+4}{11+4}$.

165. Find, as shortly as possible, the product of $999\frac{358}{359}$ and 287

166 What must be added to $\frac{4\frac{1}{5}-3\frac{1}{6}}{4\frac{1}{5}+3\frac{1}{6}} - 3 \div 24\frac{5}{9}$ that the sum may be 2?

167. Simplify $1 - \cfrac{2}{7 - \cfrac{3}{4-\frac{1}{2}}} + \left(\frac{3\frac{1}{3}}{7} + \frac{2}{10\frac{1}{2}} - \frac{5}{18} \times \frac{4}{7}\right) \times \frac{13}{43}$ of $1\frac{31}{32}$.

168. Find the average of the fractions $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{9}{10}$ and $\frac{7}{15}$

169. By how many inches is 3 fur 22 po $1\frac{1}{3}$ yds less than $\frac{4}{9}$ of a mile?

170. What fraction of the day is the sun above the horizon when it rises at 7 h. 3 m. and sets at 5 h 24 m?

---

171. Express $\frac{1}{7}$ with denominator consisting entirely of *threes*

172. Divide 3964 tons 17 cwts 1 qr 11 lbs by $27\frac{1}{2}$

173 How many times does the continued product of 72, 73, 74 and 75 contain 555?

174. Arrange $\frac{31}{91}$, $\frac{41}{119}$ and $\frac{51}{143}$ in ascending order of magnitude

175. Multiply $99\frac{273}{274}$ by 167, shortly

176 In a book on Arithmetic a question was printed thus —
"Add $3\frac{7}{8}$, $4\frac{\ }{12}$, $5\frac{8}{9}$", the answer given was $14\frac{13}{72}$, find the missing figure

177 Take $\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1\frac{1}{5}}}}$ of $\frac{1\frac{1}{11}}{11\frac{1}{3}-5\frac{1}{11}}$ from $1 + \frac{1}{11}$ of $1\frac{1}{4}$ of $2\frac{3}{7}$

178 Find the product of $\frac{\frac{108}{111}+\frac{1}{37}}{1\frac{46}{1113}-\frac{53}{371}}$ and $\frac{\frac{97}{101}+\frac{4}{101}}{1\frac{97}{1001}-\frac{14}{143}}$

179. How many feet are there in
$\frac{3}{35}$ of a mile $+ \frac{16}{21}$ of a furlong $- \frac{2}{77}$ of a pole?

180. During what fraction of the day was there sunshine, if the sun rose at 6 46, was obscured by clouds from 11 30 to 1 17, and set at 5 39?

---

181 The numerators of two equal fractions are 323 and 901, the denominator of the former is 437, find the denominator of the latter

182. Find the smallest integer which exactly contains each of the following mixed numbers $9\frac{5}{7}$, $8\frac{3}{14}$, $7\frac{11}{28}$

183 Arrange the fractions $\frac{7}{22}$, $\frac{8}{25}$, $\frac{9}{28}$, $\frac{10}{31}$ in descending order of magnitude.

184. Add together $\frac{1}{25\frac{1}{2}}$, $\frac{1}{12\frac{3}{4}}$, $\frac{1}{10\frac{1}{5}}$, $\frac{1}{7\frac{2}{7}}$, $\frac{1}{6\frac{3}{8}}$, $\frac{1}{5\frac{1}{10}}$, $\frac{1}{4\frac{7}{11}}$ and $\frac{1}{3\frac{12}{13}}$.

185. Find, with as little work as possible, the sum of $9\frac{1}{3} \times 94\frac{3}{8}$ and $7\frac{1}{2} \times 83\frac{8}{9}$

186. How many lengths of $2\frac{2}{3}$ feet can be cut from 1001 yards, and how many inches will be left over?

187. Simplify $\frac{(\frac{1}{2}+\frac{1}{5})(\frac{1}{2}+\frac{1}{6})}{(\frac{1}{2}-\frac{1}{3})(\frac{1}{2}-\frac{1}{4})} - \frac{(\frac{1}{3}+\frac{1}{5})(\frac{1}{3}+\frac{1}{6})}{(\frac{1}{2}-\frac{1}{3})(\frac{1}{3}-\frac{1}{4})} + \frac{(\frac{1}{4}+\frac{1}{5})(\frac{1}{4}+\frac{1}{6})}{(\frac{1}{2}-\frac{1}{4})(\frac{1}{3}-\frac{1}{4})}$.

188. What fraction is that from which if $\frac{2}{3}$ of $\frac{3-1\frac{1}{4}}{2\frac{2}{5}}$ be taken and the remainder be divided by $\frac{5\frac{1}{8}}{13\frac{2}{3}}$, the result is $\frac{1}{3}$?

189. Express the difference between 1 oz Troy, and 1 oz. Avoirdupois, as the fraction of their sum

190. A boy, instead of dividing a certain fraction by $\frac{5}{7}$, multiplied it by $\frac{5}{7}$ in mistake, and obtained the result $\frac{20}{29}$ What was the correct answer?

---

191. The denominators of two equal fractions are 11907 and 7497: the numerator of the first is 5103; find the numerator of the second.

192. Find the smallest integer which contains $19\frac{1}{4}$, $19\frac{4}{5}$ and $19\frac{5}{8}$ each an exact number of times.

193. Arrange the fractions $\frac{5}{41}$, $\frac{2}{17}$, $\frac{3}{25}$, $\frac{4}{33}$ in ascending order of magnitude

194 Find the sum of $\frac{4141}{5151}$, $\frac{3131}{3434}$ and $\frac{5757}{6868}$

195. Find, with as little labour as possible, the difference between $53\frac{2}{3} \times 2\frac{1}{3}$ and $32\frac{1}{5} \times 3\frac{1}{2}$

196. How many times can $3\frac{1}{5}$ be taken from $29\frac{7}{8}$, and what remains?

197. Find the value of $\left\{1 + \frac{1}{\frac{1}{2\frac{1}{3}} + \frac{1}{3\frac{1}{2}} + \frac{1}{4\frac{1}{2}}} \div [(\frac{1}{11}+\frac{1}{13})(\frac{1}{3}+\frac{1}{4})]\right\}$ of £4.

198. What fraction is that from which if $\frac{3\frac{3}{4}}{4\frac{4}{5}}$ be taken, and the remainder be divided by $\frac{4\frac{3}{4}}{3\frac{4}{5}}$, the result is $\frac{3}{40}$?

199. Find the number of grains in 1 cwt. 1 qr 2 lbs. $11\frac{3}{7}$ ozs.

200. If there is three times as much sea as land on the surface of the globe, and if three-fourths of all the land is in the northern hemisphere, what fraction of the surface of the southern hemisphere is water?

XXV. SIMPLE PRACTICE.

Write down the value of the following aliquot parts—

1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{16}$, of a shilling
2. $\frac{1}{2}$ $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{12}$, $\frac{1}{15}$, $\frac{1}{16}$, $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{40}$, $\frac{1}{60}$, $\frac{1}{80}$, of £1

What aliquot part

3. of £1 is 5*s*, 4*s*, 2*s*, 6*s* 8*d*, 3*s* 4*d*, 1*s* 8*d*, 2*s* 6*d*, 1*s* 3*d*?
4. of 10*s* is 5*s*, 2*s* 6*d*, 1*s* 3*d*, 3*s* 4*d*, 1*s* 8*d*, 10*d*, 2*s*, 1*s*, 6*d*?
5. of 5*s* is 2*s* 6*d*, 1*s* 3*d*, 7½*d*, 1*s*, 6*d*, 3*d*., 5*d*, 4*d*, 2*d*; 1*d*?
6. of 2*s* 6*d* is 1*s* 3*d*; 7½*d*, 3¾*d*, 6*d*, 3*d*, 1½*d*, 10*d*, 5*d*, 2½*d*?
7. of 1*s* 3*d* is 7½*d*, 3¾*d*, 5*d*, 2½*d*, 1¼*d*, 3*d*, 1½*d*, ¾*d*?
8. of a shilling is 6*d*; 3*d*, 1½*d*, ¾*d*, 4*d*, 2*d*, 1*d*?
9. of 6*s* 8*d* is 3*s* 4*d*; 1*s*. 8*d*, 10*d*, 5*d*, 1*s* 4*d*, 8*d*, 4*d*, 2*d*, 1*d*?
10. of 3*s* 4*d* is 1*s* 8*d*, 10*d*, 5*d*; 2½*d*, 8*d*, 4*d*, 2*d*, 1*d*?

Find, using only *one* aliquot part, the cost of—

11. 462 at 5*s*	17. 545 at 2*s* 6*d*	23 1541 at 6*s* 8*d*
12. 387 at 5*s*	18 673 at 2*s* 6*d*	24 1253 at 6*s* 8*d*
13. 742 at 4*s*	19. 1751 at 2*s* 6*d*	25 89 at 3*s* 4*d*
14. 635 at 4*s*	20. 1903 at 2*s* 6*d*	26 97 at 3*s* 4*d*
15. 1423 at 2*s*	21. 430 at 6*s* 8*d*	27 385 at 1*s* 8*d*
16 1978 at 2*s*	22 532 at 6*s* 8*d*	28. 769 at 1*s* 8*d*
29 1705 at 6*d*	33. 871 at 4*d*	37 1432 at 1½*d*
30 2371 at 6*d*	34 473 at 4*d*	38. 2453 at 1½*d*
31. 234 at 3*d*	35 1267 at 2*d*.	39 978 at 1½*d*
32. 179 at 3*d*	36. 2578 at 2*d*	40. 1023 at 1½*d*

Find, using only *two* aliquot parts, the cost of—

41 782 at 11*s*	45 93 at 12*s*	49. 4266 at 12*s* 6*d*
42. 1013 at 11*s*	46 107 at 12*s*	50 5711 at 12*s* 6*d*
43 987 at 6*s*	47 809 at 7*s* 6*d*	51 87 at 3*s* 9*d*
44. 1761 at 6*s*	48. 1205 at 7*s* 6*d*	52 191 at 3*s* 9*d*
53 8432 at 7*d*	59. 999 at 7½*d*	65 2431 at 6½*d*
54. 7065 at 7*d*	60. 1111 at 7½*d*	66 4315 at 6½*d*
55. 1653 at 5*d*.	61. 523 at 1¼*d*	67 237 at 3¾*d*
56 2089 at 5*d*	62. 684 at 1¼*d*	68 549 at 3¾*d*
57 345 at 4½*d*	63. 133 at 2¼*d*	69 810 at 6¾*d*
58. 543 at 4½*d*	64. 267 at 2¼*d*	70. 1103 at 6¾*d*

Find, using only *one* aliquot part, the cost of—

71. 622 at £1, 10*s*
72 903 at £1, 5*s*
73. 733 at £1, 4*s*.
74. 271 at £1, 2*s*
75 423 at £1, 2*s* 6*d*
76. 547 at £1, 2*s* 6*d*
77. 94 at £1, 6*s*. 8*d*
78. 83 at £1, 3*s* 4*d*
79. 152 at £1, 1*s* 8*d*
80. 205 at £1, 1*s*. 8*d*
81. 47 at £3, 5*s*.
82. 68 at £2, 4*s*
83. 156 at £5, 2*s*. 6*d*.
84. 171 at £11, 2*s*. 6*d*.
85. 344 at £7, 2*s*.
86. 287 at £6, 10*s*.
87. 1631 at £2, 6*s*. 8*d*.
88. 2350 at £4, 3*s*. 4*d*.

Find, using only *one* aliquot part, the cost of—

89 7353 at 1*s* 6*d*
90. 8649 at 1*s*. 3*d*
91 5732 at 1*s* 4*d*
92. 4642 at 1*s* 2*d*.
93. 784 at 1*s*. 1½*d*.
94. 809 at 1*s*. 1½*d*.
95. 40863 at 1*s* 1*d*
96. 59791 at 1*s*. 1*d*
97. 144 at 6*s*. 3*d*.
98. 288 at 5*s* 4*d*.
99 96 at 11*s* 1½*d*
100. 435 at 9*s* 1½*d*.

Find, using only *one* aliquot part, the cost of—

101. 137 at 15*s*
02. 215 at 15*s*.
03. 78 at 16*s*.
04 82 at 18*s*
05. 429 at 18*s*.
06. 683 at 17*s* 6*d*.
107. 514 at 17*s* 6*d*.
108. 2103 at 13*s* 4*d*
109. 1635 at 17*s* 6*d*
110. 831 at 16*s* 8*d*.
111. 34 at £1, 17*s*. 6*d*
112. 129 at £1, 18*s*
113. 65 at £1, 16*s*. 8*d*.
114. 73 at £1, 18*s*. 4*d*.
115. 138 at £3, 15*s*.
116. 182 at £4, 17*s*. 6*d*.
117. 29 at £9, 16*s*.
118. 45 at £9, 13*s*. 4*d*.

Find, using only *one* aliquot part, the cost of—

119 1434 at 9*d*
20. 2067 at 8*d*
121 536 at 10*d*
122. 487 at 10½*d*
123 158 at 11*d*
24. 87 at 1*s* 11*d*
125 105 at 1*s* 9*d*.
126 235 at 1*s*. 10*d*
127 653 at 1*s* 10½*d*
128. 809 at 1*s* 10½*d*
129 444 at 3*s* 11*d*
130 156 at 9*s* 11*d*
131. 72 at 11*s* 10*d*.
132. 98 at 7*s*. 8*d*
133. 126 at 4*s*. 9*d*
134. 261 at 2*s* 11*d*
135. 1234 at 9*s* 10½*d*.
136. 2431 at 7*s*. 10½*d*.

Find the cost of—

137. 279 at £1, 6*s*
138. 217 at £1, 11*s*
139. 825 at £1, 12*s* 6*d*
140. 1473 at £1, 7*s* 6*d*
141. 1503 at £1, 13*s*
142. 2387 at £1, 11*s* 3*d*
143. 163 at £1, 13*s* 7*d*.
144. 218 at £1, 7*s* 9½*d*
145. 132 at £2, 12*s* 9*d*.
146. 147 at £5, 7*s*. 10*d*.
147. 253 at £3, 4*s*. 8*d*.
148. 183 at £4, 11*s* 8*d*.
149. 710 at £12, 6*s* 4½*d*.
150 520 at £14, 13*s* 2½*d*.
151. 1162 at £21, 3*s* 1*d*.
152. 1044 at £36, 11*s* 1*d*.
153. 1430 at £1, 3*s* 2¼*d*.
154. 1560 at £1, 2*s* 7¼*d*
155. 376 at £51, 12*s* 5½*d*.
156. 264 at £73, 1*s*. 9¼*d*.

Find the cost of—

157 7931 at $2\frac{3}{4}d$
158 2107 at $5\frac{1}{4}d$
159. 842 at $6\frac{3}{4}d$
160 956 at $5\frac{1}{4}d$
161 214 at $1s$ $3\frac{1}{4}d$
162 184 at $1s.$ $5\frac{1}{2}d.$
163 137 at $11s$ $7\frac{1}{4}d$
164 175 at $1s$ $4\frac{3}{4}d$
165. 84 at $3s$ $1\frac{1}{4}d$
166. 96 at $5s$ $2\frac{1}{4}d$
167. 55 at $8s$ $4\frac{1}{2}d$
168 34 at $11s$ $7\frac{1}{2}d$
169 103 at $9s$ $1\frac{1}{2}d.$
170 142 at $12s$ $1\frac{1}{4}d$
171 76 at $14s$ $5\frac{1}{2}d$
172 94 at $10s$ $3\frac{3}{4}d$
173. 462 at $13s$ $6\frac{1}{4}d.$
174 895 at $7s$ $7\frac{3}{4}d$

Find the cost of—

175 975 at £42, $5s$ $8\frac{1}{4}d$
176 857 at £56, $2s$ $7\frac{1}{4}d$
177 624 at £13, $11s.$ $5\frac{1}{2}d$
178 432 at £23, $5s$ $7\frac{3}{4}d$
179. 1358 at £26, $10s$ $9\frac{1}{4}d$
180 1023 at £38, $11s$ $7\frac{1}{4}d$
181. 356 at £29, $5s$ $2\frac{1}{2}d$
182. 269 at £19, $7s$ $5\frac{1}{4}d$
183. 634 at £1, $8s$ $6d$
184. 657 at £1, $8s$ $8d$
185. 325 at £2, $9s$ $4d$
186 317 at £2, $9s$ $5d$
187. 327 at £1, $14s$ $6\frac{1}{2}d$
188 123 at £3, $14s$ $7d$
189 1871 at £1, $11s$ $0\frac{1}{4}d$
190 4035 at £2, $4s$ $0\frac{1}{2}d$
191. 753 at £1, $7s$ $0\frac{1}{4}d$
192 888 at £1, $9s$ $0\frac{1}{2}d$
193. 257 at £5, $5s$ $0\frac{3}{4}d$
194 183 at £6, $14s$ $0\frac{1}{2}d$

Find, as shortly as you can, the cost of—

195 4657 at $19s$
196. 4214 at $19s$ $6d$
197. 964 at $17s$ $9d.$
198. 1022 at $18s$ $10d$
199 558 at £2, $16s$ $10\frac{1}{2}d.$
200. 427 at £3, $15s$ $8d$
201 2345 at $18s$ $7\frac{3}{4}d$
202 3333 at $17s$ $9\frac{3}{4}d$
203 226 at £2, $18s$ $8\frac{1}{2}d$
204. 144 at £3, $19s$ $2d$
205. 413 at £4, $16s$ $10d$
206 285 at £9, $17s$ $7\frac{3}{4}d$
207. 567 at $2s$ $9\frac{3}{4}d$
208 286 at $4s$ $10\frac{1}{4}d$
209 1032 at $9s$ $11\frac{1}{4}d$
210. 1796 at $9s$ $8\frac{1}{4}d.$

Find the cost of—

211. $149\frac{1}{2}$ at £2, $6s$ $6d$
212. $303\frac{1}{2}$ at £1, $4s.$ $7d.$
213 $73\frac{1}{4}$ at £2, $2s$ $4d$
214 $87\frac{1}{4}$ at £3, $7s$ $3d$
215 $2132\frac{1}{4}$ at £2, $12s$ $5d$
216 $1467\frac{1}{2}$ at £2, $17s$ $9\frac{1}{2}d$
217. $38\frac{3}{4}$ at £1, $3s$ $1d$
218. $42\frac{3}{8}$ at £1, $10s$ $10d.$
219. $153\frac{1}{3}$ at £2, $2s.$ $3d.$
220 $137\frac{2}{3}$ at £2, $3s.$ $3d.$
221 $38\frac{3}{4}$ at £1, $5s$ $5d$
222 $97\frac{1}{6}$ at £1, $18s$ $10\frac{1}{2}d$
223 $245\frac{3}{10}$ at £1, $13s$ $9d$
224 $743\frac{1}{12}$ at £1, $9s$ $9d$
225 $721\frac{13}{20}$ at £1, $11s$ $8d$
226 $531\frac{5}{6}$ at £2, $14s$ $9d$
227 $423\frac{1}{2}$ at $17s$ $10\frac{1}{2}d.$
228. $532\frac{3}{8}$ at $18s.$ $10\frac{1}{4}d.$

229. $49\frac{1}{2}$ at 3*s* $5\frac{1}{2}d$
230. $73\frac{1}{4}$ at 2*s* 4*d*
231. $423\frac{1}{2}$ at 1*s* $2\frac{1}{2}d$
232. $752\frac{1}{3}$ at 1*s* $1\frac{1}{2}d$
233 $142\frac{5}{8}$ at 4*s* 6*d*.
234. $235\frac{1}{12}$ at 4*s*. 3*d*.
235. $743\frac{1}{12}$ at 9*s*. 9*d*
236. $1531\frac{5}{8}$ at 4*s*. 9*d*.
237. $2489\frac{1}{2}$ at 1*s* $1\frac{1}{2}d$.
238. $1487\frac{1}{2}$ at 1*s*. $3\frac{1}{2}d$.
239. $17463\frac{3}{4}$ at 7*s* 3*d*.
240. $26974\frac{7}{8}$ at 5*s* 4*d*

Find, in £, *s* *d* and a fraction of a penny, the value of—

241. $134\frac{1}{4}$ at 2*s* $7\frac{1}{4}d$
242. $129\frac{1}{4}$ at 15*s* $9\frac{1}{2}d$
243. $434\frac{5}{8}$ at 6*s*. 9*d*
244. $447\frac{3}{8}$ at 13*s*. $11\frac{1}{2}d$.
245. $812\frac{7}{8}$ at 2*s* 7*d*
246. $401\frac{7}{10}$ at 16*s* $9\frac{3}{4}d$.
247. $420\frac{7}{12}$ at 6*s* $1\frac{1}{2}d$
248. $986\frac{11}{13}$ at 13*s*. $3\frac{3}{4}d$.
249. 37417 at $5\frac{5}{8}d$.
250. 38643 at $4\frac{7}{8}d$.
251. $417\frac{3}{7}$ at 4*s*. $2\frac{7}{8}d$.
252. $333\frac{3}{13}$ at 3*s*. $3\frac{1}{3}d$.

Find the dividend on—

253. £5725 at 4*s* 7*d*. in the £.
254. £8790 at 8*s* 6*d* in the £
255. £4736, 5*s* at 2*s*. 8*d*. „
256. £1342, 12*s*. at 2*s*. $3\frac{1}{2}d$. „
257. £856 at 13*s*. 8*d* in the £.
258. £3469 at 15*s* $6\frac{1}{2}d$ in the £.
259. £473, 18*s* at 8*s* 4*d* in the £
260. £1723, 16*s*. 8*d*. at 7*s* 3*d*. in the £.

Find the cost of—

261. 456 tons 12 cwts. at 15*s*. 6*d*. per ton.
262. 981 qrs. 7 lbs. at 8*s*. $4\frac{1}{2}d$. per qr.
263. 73 yds. 1 ft. 6 in. at 3*s*. 2d per yd.
264. 87 yds. 0 ft. 9 in. at 2*s*. 10*d*. per yd
265. 13 tons 13 cwts. 3 qrs. of sugar at 28*s* per cwt.
266. 2 tons 3 cwts. 1 qr. 17 lbs. at 1*s* $7\frac{1}{4}d$ per lb.
267. 2 ro 13 po 24 sq yds of land at 3*s* $8\frac{1}{2}d$ per sq. yd.
268 4 ac 3 ro 27 sq po of land at 3*s*. $7\frac{5}{8}d$ per sq. yd.
269. 47 ozs 5 dwts of silver at 3*s* $10\frac{1}{2}d$. per oz.
270. 3 cwts 2 qrs 16 lbs 8 ozs. at 1*s* $10\frac{1}{2}d$. per lb.
271. 56472 young trees at £1, 2*s*. 9*d*. per dozen.
272. 4787 things at £11, 8*s* 4*d* per score.

273. Find, by Practice, 1437 times 8 lbs. 10 ozs. 7 drs.
274. Find, by Practice, 6748 times 9 yds. 2 ft. $7\frac{1}{2}$ in.
275. Find the rent of 317 ac. 2 ro. 16 sq. po. at 25*s*. per acre.
276. Find the rent of 212 ac. 3 ro. 30 sq. po at £2, 3*s*. 8*d* per acre.
277. Find the rent of 230 ac. 1 ro. 10 sq. po. at £1, 15*s* 6*d* per acre.
278. Find the rent of 1635 A. 2 R. 24 P. at £1, 12*s*. 8*d* per acre.
279. Find the weight of 132 hogsheads of tallow averaging 12 cwts. 2 qrs. 17 lbs. each.
280. Find the weight of 2468 bales of goods averaging 4 cwts 2 qrs. 6 lbs. per bale.

XXVI. COMPOUND PRACTICE.

Write down the value of the following aliquot parts —

1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$ of a ton, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{7}$, $\frac{1}{8}$ of a cwt
2. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{7}$ of a qr, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ of a lb
3. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$; $\frac{1}{16}$ of an acre, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$ of a rood
4. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{1}{12}$ of a yd, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{12}$ of a foot.

What aliquot part

5. of a ton is 10 cwts, 5 cwts, 4 cwts, 2 cwts?
   of 1 cwt. is 2 qrs, 1 qr; 14 lbs, 16 lbs?
6. of 10 cwts is 5 cwts, 1 cwt?
   of 2 qrs is 14 lbs, 7 lbs, 8 lbs, 4 lbs?
7. of 1 acre is 2 ro, 1 ro, 20 po, 32 po, 16 po?
   of 2 roods is 1 ro; 20 po, 16 po, 8 po?
8. 1 oz. Tr is 10 dwts.; 5 dwts, 4 dwts, 2 dwts?
   of 1 dwt is 12 grs; 8 grs, 6 grs, 4 grs, 3 grs, 2 grs.?
9. of 1 yd is 1 ft 6 in, 1 ft, 9 in, 6 in, 4 in, 3 in.?
   of 1 ft 6 in. is 9 in, 6 in, 3 in, $1\frac{1}{2}$ in?
10. of 1 fur is 20 po, 10 po, 8 po, 5 po? of 2 fur is 10 po, 8 po.?
    of 1 pole is 2 yds 2 ft 3 in, 1 ft 6 in?

Find the cost of—

11. 1 cwt. 3 qrs. 14 lbs. at £3, 12*s* 6*d* per cwt
12. 1 cwt. 3 qrs. 7 lbs at £2, 17*s* 8*d* per cwt
13. 2 cwts 1 qr 21 lbs at £66, 4*s* per cwt
14. 3 cwts. 3 qrs. 21 lbs. at £44, 2*s* per cwt.
15. 13 cwts. 3 qrs. 4 lbs. at £2, 18*s* 4*d* per cwt
16. 17 cwts 1 qr 5 lbs at £2, 13*s* 8*d* per cwt
17. 1 ton 6 cwts. 2 qrs. 14 lbs at £3, 13*s* 4*d*. per ton.
18. 1 ton 16 cwts 1 qr. 7 lbs at £2, 6*s* 8*d* per ton
19. 18 galls. 2 qts. 1 pint at £1, 6*s* 4*d* per gallon
20. 21 galls. 3 qts. 1 pt at £1, 2*s* 8*d* per gallon
21. 8 ac 2 ro 30 sq po at £65 per acre
22. 3 ac 3 ro 25 sq po at £120 per acre
23. 17 ac. 2 ro 25 sq. po at £76, 10*s* per acre
24. 19 ac 2 ro 24 sq po at £60, 5*s* per acre
25. 7 cwts. 1 qr 12 lbs. at £1, 19*s* 8*d* per cwt
26. 12 cwts. 3 qrs. 12 lbs at £1, 10*s* 4*d* per cwt
27. 5 cwts. 2 qrs. 16 lbs at £113, 3*s*. 4*d* per ton
28. 1 ton 10 cwts. 2 qrs. 22 lbs. at £1, 13*s*. 10*d* per cwt.

29. 18 yds. 1 ft. 9 in at £1, 16*s*. per yard.
30. 17 yds 2 ft. 6 in at £5, 7*s*. 6*d* per yard.
31. 7 yds. 1 ft 11 in. at 9*s* 9*d* a yard.
32. 11 yds 2 ft. 9 in at 7*s* 3*d* a yard
33. 16 ozs 6 dwts 12 grs. of gold at £3, 17*s* 6*d* per oz.
34. 15 ozs 11 dwts. 8 grs. of gold at £3, 17*s*. 6*d* per oz.
35. 6 lbs. 13 ozs. 11 drams at £1, 1*s*. 4*d*. per lb
36 5 lbs. 11 ozs. 12 drams at £1, 2*s*. 8*d* per lb
37. 7 cwts. 3 qrs 26 lbs. at £1, 10*s*. 4*d*. per cwt.
38. 7 cwts 2 qrs 24 lbs. at £1, 7*s* 5*d* per cwt.
39. 3 qrs. 18 lbs. 8 ozs at £8, 12*s*. 8*d* per cwt.
40. 3 qrs. 9 lbs 4 ozs at £10, 5*s* 4*d* per cwt.
41. 63 tons 15 cwts. 40 lbs at £1, 12*s* 8*d*. per ton.
42. 56 tons 15 cwts 105 lbs. at £1, 5*s* 4*d* per ton.
43. 1 ton 17 cwts. 49 lbs. 12 ozs at £65, 6*s* 8*d* per ton.
44. 1 ton 13 cwts. 38 lbs. 8 ozs. at £52, 13*s*. 4*d* per ton.
45 9 cwts. 3 qrs. 14 lbs. at 30*s* a ton.
46. 3 cwts. 3 qrs. 14 lbs. at 16*s*. 8*d* per ton.
47. 2 cwts. 1 qr. 20 lbs. 4 ozs. at £1, 17*s*. 4*d*. per quarter.
48. 1 cwt. 3 qrs. 19 lbs. 8 ozs. at £1, 18*s*. 6*d*. per quarter.
49. 1 ton 14 cwts. 1 qr. 11 lbs. 12 ozs. at £4, 13*s*. 4*d*. per cwt.
50 1 ton 18 cwts 1 qr 11 lbs 6 ozs at £6, 10*s* 8*d* per cwt
51. 5 yds 22½ in. at £2, 1*s* 2*d* per yd.
52. 3 yds 20¼ in at £3, 9*s* 8*d* per yd.
53 3 lbs 11 ozs 18 dwts 4 grs at £3, 17*s* 6*d* per oz.
54 6 lbs 8 ozs 12 dwts 12 grs at £4, 4*s* 6*d* per oz.

Find, in £, *s* *d* and a fraction of a penny, the value of—

55. 3 cwts 2 qrs 21 lbs at £55, 10*s*. 6*d* per cwt.
56 11 cwts 3 qrs 8 lbs at £3, 4*s* 6*d*. per cwt.
57. 17 cwts 2 qrs 22 lbs at £4, 6*s* 7½*d* per cwt.
58 25 cwts 3 qrs 8 lbs at 9*s* 5½*d* per cwt.
59. 7 yds 2 ft. 7 in at 7*s* 9*d* a yd
60 15 yds 2 ft 11 in at 10*s* per yard.
61. 19 yds 2 ft 4½ in at £2, 15*s* 4*d* per yard.
62. 17 yds 1 foot 7 in. at 6*s* 9½*d* per yard.
63. 4 ac 2 ro 25 po at £111, 13*s* 3*d* per acre
64. 44 ac. 2 ro. 25 po at £55, 16*s* 7½*d* per acre

65. 3 qrs 14 lbs 12 ozs at £12, 16*s* 8*d* per cwt
66. 14 cwts 2 qrs 24 lbs at £4, 1*s* per cwt
67. 24 cwts 3 qrs 16 lbs 10 ozs at £2, 0*s* 8*d* per cwt
68 21 cwts 2 qrs 9 lbs 4 ozs at £2, 1*s* 10*d* per cwt
69. 80 lbs at £5, 15*s* per cwt 70. 66 lbs at £1, 5*s* per cwt.
71. 3 tons 5 cwts 1 qr 16 lbs at £2, 15*s* per cwt
72. 1 ton 12 cwts 0 qrs 22 lbs at £4, 16*s* 8*d* per cwt
73. 8 mi. 6 fur 8 po at £15, 6*s* per mile.
74. 7 mi 7 fur 7 po at £16, 5*s* per mile
75. 2 mi 3 fur 26 po $2\frac{3}{4}$ yds at £10 per mile
76. 1 mi 5 fur 32 po 3 yds at £12 per mile.
77. 2 ac 3 ro 24 po 11 sq yds at £50 per acre
78. 7 ac 1 ro 31 po $5\frac{1}{2}$ sq yds at £60, 12*s* per acre
79. 11 cub yds 4 cub ft 144 cub in. at 8*s* 6*d* per cub yd.
80. 9 cub yds 7 cub ft. 216 cub in at 7*s* 9*d* per cub yd

Find, *by Practice*, as shortly as you can, the value of—

81. 973 tons 16 cwts 1 qr 8 lbs at £1, 15*s* per ton
82 1452 tons 13 cwts 3 qrs 5 lbs at £1, 8*s* per ton
83. 19 tons 19 cwts 2 qrs 13 lbs at 7*s* per cwt
84 23 tons 17 cwts 2 qrs 23 lbs at 10*s* 6*d* per cwt.
85. 289 qrs 7 bush 3 pks at 17*s* 9*d* per qr
86. 372 qrs. 7 bush. 2 pks at 31*s* 6*d* per qr
87. 465 ac 3 ro 10 po at £33, 6*s* 8*d* per acre
88. 434 ac 2 ro 32 po $2\frac{3}{4}$ sq. yds at £44 per acre.
89. 216 tons 12 cwts 2 qrs at £32, 17*s* 2*d* per ton
90. 319 tons 3 cwts 2 qrs 15 lbs 8 ozs at £9, 6*s* 8*d* per ton.
91 54 tons 17 cwts 3 qrs 18 lbs at 16*s* 4*d* per cwt
92. 65 tons 3 cwts 1 qr 20 lbs 9 ozs. at £1, 12*s* per cwt
93. 1213 ozs 6 dwts. 17 grs at 5*s* 10*d* per oz.
94. 35 lbs 16 dwts 12 grs at 2*s* $1\frac{1}{2}$*d* per oz
95. 99 mi 7 fur 70 yds at 3*s* 8*d* per furlong
96 9 mi 7 fur. 121 yds at £671 per mile.
97 345 ac 2 ro 15 per. at 42*s* 6*d* an acre.
98 779 ac 3 ro 32 po at £4, 3*s* 8*d* per acre.
99. 1759 ac. 2 ro 31 po at £1, 8*s*. 6*d* per acre.
100. 1215 ac 1 ro $27\frac{1}{2}$ po at £19, 6*s*. 8*d*. per acre

XXVII. INVOICES.

Make out bills with names, dates, &c., for—

1. 6 lbs of mutton at $8\frac{1}{2}d$ per lb; $10\frac{1}{2}$ lbs. of beef at $8d.$ per lb, and $1\frac{1}{4}$ lbs. of suet at $8d$ per lb
2. $8\frac{1}{2}$ lbs of mutton at $9d.$ per lb; 7 lbs. of mutton at $9\frac{1}{2}d$ per lb; and $\frac{1}{2}$ lb of suet at $8\frac{1}{2}d$ per lb
3. 3 lbs. of tea at 1*s* 8*d* per lb.; 3 lbs. of coffee at 1*s.* 6d. per lb.; and 7 lbs. of sugar at $2\frac{1}{2}d$ per lb.
4. 4 lbs of tea at 1*s* 10*d* per lb; 14 lbs. of sugar at $2\frac{1}{2}d.$ per lb.; and 6 lbs. of candles at 8*d.* per lb.
5. 12 yds. of calico at $8\frac{1}{4}d.$ per yard, 2 doz. reels of cotton at $2\frac{1}{2}d$ per reel, and 4 packets of needles at $1\frac{3}{4}d$ per packet.
6. 12 yds. of muslin at $7\frac{1}{2}d.$ per yard, 3 doz. skeins of thread at $1\frac{3}{4}d.$ per skein; and 5 knots of braid at $2\frac{1}{4}d$ per knot.
7. 4 packets of note-paper at $9\frac{1}{2}d$ per pkt.; 3 quires of blotting-paper at 1*s.* 9*d.* per qre; and 500 envelopes at $1\frac{1}{2}d.$ per pkt. of 25.
8. 3 reams of foolscap paper at 4*s* 9*d.* per ream, 2 boxes of pens at $10\frac{1}{2}d.$ per box; and $\frac{1}{2}$ gallon of ink at 5*s* 9*d.* per gallon.
9. 5 lbs. of American cheese at $8\frac{1}{2}d$ per lb; 6 lbs. of butter at 1*s* 5*d.* per lb.; and $8\frac{1}{2}$ lbs. of bacon at $10\frac{1}{2}d.$ per lb.
10. 8 lbs. of lard at $9\frac{1}{2}d$ per lb., 4 lbs. of butter at 1*s.* 2*d.* per lb.; and $3\frac{1}{2}$ lbs. of Gorgonzola cheese at 1*s.* 1*d* per lb.
11. $7\frac{1}{2}$ lbs. of salmon at 1*s* 8*d.* per lb.; 1 doz. kippered herrings at 2 for $1\frac{1}{2}d$; and 2 tins of preserved lobster at $10\frac{3}{4}d.$ per tin.
12. 13 lbs of cod at $5\frac{1}{4}d$ per lb, $\frac{1}{2}$ a score of oysters at 3*s.* 9*d.* per score, and $3\frac{1}{2}$ lbs of dried haddock at $5\frac{1}{2}d$ per lb
13. 5 loaves at $4\frac{1}{2}d.$ each, $\frac{1}{2}$ a stone of flour at 2*s* $10\frac{1}{2}d$ per stone; and 8 lbs of biscuits at $7\frac{1}{2}d$ per lb
14. 7 loaves at $4\frac{1}{4}d$ each, $3\frac{1}{2}$ lbs. of flour at 2*s* 10*d.* per stone; and 12 lbs. of biscuits at $6\frac{3}{4}d.$ per lb
15. 2 bottles of cod-liver oil at 1*s.* 2*d* per bottle; 3 ozs. of jujubes at $3\frac{1}{2}d$ per oz.; and 2 doz. tablets of soap at $2\frac{3}{4}d.$ per tablet.
16. 4 ozs of cough lozenges at $2\frac{1}{2}d.$ per oz.; $\frac{1}{4}$ oz. of quinine at 2*s* 3*d* per oz.; and 5 jars of meat extract at 1*s.* $10\frac{1}{2}d$ per jar.
17. 3 doz. bottles of sherry at 28*s* 6*d.* per doz., 6 doz. of bottled beer at 3*s.* 3*d* per doz; and $\frac{1}{2}$ doz. of whisky at 39*s* per doz.
18. 4 doz pints of bottled stout at 3*s* 9*d* per dozen, 4 bottles of brandy at 63*s.* per doz.; and 6 doz. of claret at 18*s* 6*d* per doz.
19. $5\frac{1}{2}$ lbs of cake at $10\frac{1}{2}d$ per lb, 1 doz. pots of marmalade at $6\frac{3}{4}d.$ per pot; and 1 pork-pie, weighing $2\frac{1}{2}$ lbs., at $9\frac{1}{2}d$ per lb.
20. $7\frac{1}{4}$ lbs. of cake at 11*d.* per lb; 3 doz. ices at $4\frac{1}{2}d$ each; and $3\frac{1}{2}$ lbs. of cooked ham at 1*s.* $2\frac{1}{2}d.$ per lb.

21. Leg of mutton, $9\frac{1}{2}$ lbs, at $9\frac{1}{2}d$ per lb, quarter of lamb, $15\frac{1}{4}$ lbs, at $11d$ per lb; steak, 5 lbs, at $11\frac{1}{2}d$ per lb, and chops, $3\frac{1}{2}$ lbs, at $10\frac{1}{2}d$ per lb

22. Beef, $16\frac{1}{2}$ lbs, at $8\frac{1}{2}d$ per lb, mutton, $6\frac{3}{4}$ lbs, at $8d$ per lb; suet, $2\frac{1}{4}$ lbs, at $8d$ per lb, and pork, $8\frac{1}{2}$ lbs, at $7\frac{1}{2}d$ per lb.

23. $14\frac{1}{2}$ yds. of calico at $10\frac{1}{2}d$ per yd, $9\frac{1}{4}$ yds of flannel at $1s\ 10d$. per yd, 16 reels of cotton at $2s\ 3d$ per doz reels, and 8 yds. of ribbon at $5\frac{1}{4}d$ per yd

24. $13\frac{1}{4}$ yds of calico at $9d$ per yd, 8 pairs of socks at $1s.\ 6\frac{1}{2}d$. per pair, $4\frac{3}{4}$ yds of ribbon at $9d$ per yd., and $2\frac{1}{2}$ dozen collars at $6\frac{1}{2}d$. each

25. $16\frac{1}{2}$ lbs of cheese at $10d$ per lb, $8\frac{1}{4}$ lbs of butter at $1s\ 5d$. per lb, $24\frac{1}{2}$ lbs of bacon at $9\frac{1}{2}d$ per lb; and 3 shillings worth of eggs at 14 for $1s$

26 $7\frac{1}{4}$ lbs. Stilton cheese at $1s\ 2d$ per lb, 3 lbs of butter at $1s\ 5d$. per lb, $13\frac{3}{4}$ lbs of bacon at $9d$ per lb, and 10 lbs. of lard at $9\frac{3}{4}d$ per lb

27. 7 pairs of blankets at $18s\ 6d$ per pair, 47 yards of calico at $9\frac{1}{4}d$ per yard, $\frac{1}{2}$ dozen pairs of stockings at $2s\ 11d$ per pair; and 3 pairs of gloves at $1s\ 10\frac{1}{2}d$ per pair

28. 5 pairs of gloves at $2s\ 11\frac{1}{2}d$ per pair, 47 yds of chintz at $9\frac{3}{4}d$ per yd, 1 gross of buttons at $3\frac{1}{4}d$ per dozen, and 60 yards of fringe at $1s\ 3\frac{3}{4}d$ per doz yds

29. 2 stones of sugar at $2\frac{1}{2}d$ a lb, 10 lbs of tea at $1s\ 10d$ a lb, 8 lbs of coffee at $1s\ 4d$ a lb, 12 lbs of currants at $5\frac{3}{4}d$ a lb., 20 lbs of rice at $2\frac{1}{4}d$ a lb, and 9 lbs of candles at $11d$ a lb

30. 7 lbs of raisins at $6\frac{1}{2}d$ per lb; 4 lbs of figs at $8\frac{1}{2}d$ per lb; 6 lbs of treacle at $3\frac{1}{2}d$ per lb, $\frac{1}{4}$ lb of pepper at $1s\ 7d$ per lb, 6 lbs. of tea at $1s\ 10\frac{1}{2}d$ per lb, and 3 doz bars of soap at $8\frac{3}{4}d$ per bar.

In the following bills, when the exact value of any item cannot be expressed in current coin, charge the farthing next above it:—

31 $\frac{1}{4}$ lb of butter at $14\frac{1}{2}d$, $\frac{3}{4}$ lb of cheese at $6\frac{1}{2}d$, per lb

32. 3 ozs. of butter at $15d$ per lb, 2 eggs at 14 for $1s$

33 $\frac{1}{4}$ lb of tea at $2s\ 1\frac{1}{2}d$, $\frac{1}{2}$ lb of treacle at $2\frac{1}{4}d$, per lb

34 2 ozs of coffee at $1s\ 7d$, 1 oz of pepper at $7d$, per lb.

35. 1 lb 2 oz of beef at $7\frac{1}{2}d$, $3\frac{1}{4}$ lbs of mutton at $8\frac{1}{2}d$, per lb

36. 3 lbs 6 ozs of pork at $6\frac{1}{2}d$, 5 ozs of suet at $8d$, per lb

37. $4\frac{3}{8}$ yds of calico at $5d$ a yd, $\frac{1}{2}$ doz buttons at $2s\ 6d$ per gross

38. $\frac{5}{8}$ yd of velvet at $6s\ 9d$ per yd., 6 yds of braid at $9\frac{1}{2}d$ per 3 doz yds

39. $3\frac{1}{4}$ lbs. of beef at $8\frac{1}{2}d$. per lb, $5\frac{1}{4}$ lbs. of mutton at $9\frac{1}{2}d$. per lb, and $7\frac{3}{4}$ lbs of pork at $7\frac{1}{2}d$. per lb

40. $\frac{1}{4}$ lb of tea at 1*s* $10\frac{1}{2}d$. per lb.; $1\frac{3}{4}$ lbs of bacon at $7\frac{1}{2}d$ per lb.; and $\frac{3}{4}$ lb. of cheese at $8\frac{1}{2}d$ per lb

41 $\frac{5}{8}$ yd of serge at 1*s* 10*d* per yd, $2\frac{1}{2}$ dozen buttons at $2\frac{3}{4}d$. per dozen, and $3\frac{3}{4}$ yards of fringe at $6\frac{1}{4}d$. per yard.

42. $5\frac{3}{8}$ yards of silk at 4*s*. 11*d* per yard; 2 dozen and 8 buttons at $7\frac{1}{2}d$ per dozen; and $7\frac{1}{2}$ yards of lining at $11\frac{3}{4}d$ per yard.

43. 7 lbs 5 ozs of mutton at 9*d* per lb.; $11\frac{3}{4}$ lbs. of beef at $8\frac{1}{4}d$. per lb; and 11 ozs. of suet at 8*d* per lb.

44. 6 lbs. 1 oz. of veal at 9*d*. per lb., $7\frac{1}{4}$ lbs of lamb at $11\frac{1}{2}d$ per lb, and $1\frac{1}{4}$ lbs. of suet at $8\frac{1}{2}d$ per lb

45 Beef, 11 lbs 6 ozs, at 8*d* per lb; mutton, 7 lbs 10 ozs, at $9\frac{1}{2}d$ per lb; and veal, 10 lbs. 2 ozs, at $8\frac{1}{2}d$ per lb.

46. 13 lbs. 5 ozs. of bacon at $6\frac{1}{2}d$ per lb; 5 lbs 11 ozs. of cheese at $9\frac{1}{2}d$ per lb, and 8 lbs. 7 ozs of lard at 10*d*. per lb.

Make out and receipt detailed accounts for—

47. 6 lbs. of tea at 2*s* $1\frac{1}{2}d$. per lb., 3 lbs. of coffee at 1*s* 8*d* per lb., and 3 ozs. of cloves at 1*s*. 4*d*. per lb., purchased on Jan. 7; $\frac{1}{4}$ lb. of pepper at 1*s* $1\frac{1}{2}d$. per lb., and 7 lbs. of rice at 3*s*. 10*d*. per stone, purchased on Feb. 15; and 2 doz tablets of soap at $3\frac{1}{2}d$ per tablet, 2 ozs of nutmegs at 3*s* 3*d* per lb, and 3 tins of sardines at $11\frac{1}{2}d$ per tin, purchased on Mar 4; deducting a discount of 6*d*. in the pound from the total amount.

48. $2\frac{3}{8}$ yds of ribbon at 1*s* 9*d* per yd., and 29 yds. of calico at $6\frac{1}{2}d$ per yd, purchased on May 7, $\frac{1}{2}$ doz. collars at $8\frac{1}{2}d$ each, 3 pairs of gloves at 1*s* $11\frac{1}{4}d$. per pair, and $\frac{7}{8}$ yd of velvet at 9*s* $11\frac{1}{4}d$ per yd, purchased on June 6, and 3 doz. yds of linen at 1*s* $6\frac{1}{2}d$ per yd, 20 yds of huckaback at $11\frac{1}{2}d$ per yd, and $2\frac{3}{4}$ yds. of oilcloth at 1*s*. $9\frac{1}{2}d$ per yd, purchased on June 17, deducting a discount of 6*d* in the pound from the total.

49. 3 packets of note-paper at 1*s* $1\frac{1}{2}d$ per packet, 1000 envelopes at $1\frac{1}{4}d$ per packet of 25, and 3 gross of pens at 1*s*. $3\frac{1}{2}d$. per gross, purchased on Aug 3; 30 books at 1*s*. 6*d* each, and 21 books at 1*s* 4*d* each, purchased on Sept. 12, and 2 quires of blotting-paper at 1*s* $6\frac{1}{2}d$ per quire, and 14 books at 3*s* 6*d* each, purchased on Sept. 27, allowing a discount of 2*d* in the shilling on books only

50 12 books at 2*s* 6*d*. each, 6 books at 4*s* 6*d* each, and 3 reams of foolscap paper at 4*s*. 3*d* per ream, purchased on Oct 3; 2 gross of pencils at $8\frac{1}{2}d$ per dozen, 20 books at 9*d* each, and 4 books at 3*s* 6*d*. each, purchased on Oct. 23, and 500 envelopes at 1*d*. per packet of 25, 24 books at 2*s* each, and 3 gross of penholders at 2*s*. 10*d* per gross, purchased on Nov 12; allowing a discount of 3*d*. in the shilling on books only.

XXVIII. DECIMALS.

NOTATION.

Express, in words, the meaning of—

1. ·7; ·07, ·007, ·0007; ·00007, ·000007, and ·0000007
2. ·08; ·0008; ·8, 00008, ·008, ·0000008, and ·000008.
3. ·23, ·45; ·486, 271, ·083, ·803; ·0041, and ·0632
4. 2 3; ·601; 5·03, 24·05, 17·17, ·089, 66·66, and ·6666

Express in figures, with the decimal notation—

5. Three *tenths*, Seven *tenths*, Eight *hundredths*, Four *thousandths*.
6. One *tenth*; Six *hundredths*, Nine *hundredths*, Five *thousandths*
7. Two *thousandths*, Seven *ten-thousandths*, Six *millionths*
8. Eight *ten-thousandths*; Four *millionths*, Nine *thousandths*
9. Two, and three-*tenths*, Twelve, and seven-*hundredths*
10. Seventy, and seven-*thousandths*, Ten, and one-*ten-millionth*
11. Twenty-one *hundredths*, Seventeen *hundredths*, Eleven *tenths*
12. Twelve *hundredths*, Ninety-nine *hundredths*, Thirty-three *tenths*.
13. Two-hundred-and-two *thousandths*, Fourteen *millionths*
14. One-hundred-and-nine *hundredths*; Eighty-seven *ten-thousandths*.

Write down, at sight, as a decimal, the value of—

15. ·23 × 10.	20. ·006 × 10	25 1·234 × 100	30 ·2057 × 1000.
16. ·57 × 10	21. ·701 × 10	26. 5 003 × 100	31. 2·315 × 1000.
17. 4·5 × 10	22 9·21 × 10	27. 4·21 × 100.	32. 41·03 × 1000.
18. 7·2 × 10	23. 5·34 × 10	28. 5·03 × 100.	33. 5·2 × 1000.
19. ·08 × 10	24. 72·45 × 10	29. 12·1 × 100.	34. ·008 × 1000.
35. 2·3 ÷ 10	40. 61 ÷ 10	45. 153·1 ÷ 100.	50. 236·1 ÷ 1000.
36. 5·1 ÷ 10	41. 29 ÷ 10	46. 407·6 ÷ 100	51. 400·9 ÷ 1000.
37. ·5 ÷ 10	42. 60·3 ÷ 10	47 43·7 ÷ 100	52. 21·3 ÷ 1000.
38. ·7 ÷ 10.	43. 8·02 ÷ 10	48 80 2 ÷ 100.	53 80·5 — 1000
39. ·63 ÷ 10	44. ·001 ÷ 10	49 2·3 ÷ 100	54. 1·2 ÷ 1000.

Multiply 4·728, 3045, ·00847, 23·067 and ·0000703, each

55. by 100 56. by 1000. 57. by 10000.

Divide 2340·7, 70300·57, 400·1, ·03 and 4·367, each

58. by 10. 59. by 100. 60. by 10000.

XXIX. DECIMALS.

Addition and Subtraction

Add together—

1. 3·25, 4·32, 7·04, 5·29 and 1·02.
2. 1·035, 2·503, 1·506, ·324 and ·017
3. 7·5, 3·42, 1·354, 4·706 and ·83
4. 1·073, ·457, 8·213, 11·007 and ·05
5. 17·2, 1·35, ·043, ·245 and 20·8
6. 4·07, ·435, ·009, 23·07 and 41·1
7. 71, ·0341, 810·2, 53·245 and ·0003
8. ·047, 2·7, 43·07154, 1·37036, 420
9. 172·1, 3·40532, ·00878, 30·005, 2
10. 3501, 3·501, 35·01, ·3501 and 350·1

Subtract—

11. 2·75 from 7·28.
12. 5·247 from 8·319.
13. 43·57 from 75·37.
14. 38·72 from 81·42.
15. 3·427 from 14·0239.
16. 7·59 from 11·102.
17. ·635 from 2·18.
18. ·598 from 1·23.
19. 1·6875 from 17·04.
20. 3·3078 from 25·61.

Find the sum of—

21. 4031·06, 108·304, 9·001345, 76·739 and 250·0007.
22. ·608242 ·0315044, 1·8034, ·086, ·9106 and 20.

Find the difference between—

23. ·5 and ·05.
24. ·007 and ·07.
25. ·3456 and ·35
26. ·86 and ·859.
27. ·9 and ·90
28. ·09 and ·1.
29. ·0473 and ·437.
30. 43·87 and 4·387.
31. 2·6478 and 2·60578.
32. 6·561 and 65·6132
33. ·34572 and 1·23
34. 7·4 and 6·98075.
35. 2 and ·9871.
36. 7 and 3·4956.
37. 10 and ·01.
38. 170 and ·017.
39. 5·05 and 50·5.
40. 78·9 and ·978

Find the value of—

41. $2 \cdot 7 + \cdot 031 - 1 \cdot 5638 + 40 \cdot 25 - 23 \cdot 709$.
42. $16 \cdot 8 - 3 \cdot 47513 - 7 \cdot 261 + \cdot 0083 - 3 \cdot 4541$.
43. $24 \cdot 68 - 13 \cdot 579 + 2 \cdot 468 - 1 \cdot 3579 - 10$.
44. $900 \cdot 87 - 80 \cdot 076 - 7 \cdot 0065 + \cdot 60054 - 500 \cdot 43$.
45. $20 - \cdot 02 + 2 - \cdot 2 + 200 - \cdot 002 + 22 \cdot 2 - \cdot 00222$
46. $333 - \cdot 0333 + 33 \cdot 3 - 3 \cdot 33 - \cdot 333 + \cdot 00333$.
47. Take the sum of 2·345, 23·45, 20·3045 and 30·0045 from 80·1
48. Subtract 2·29997 from the sum of 1·8, ·2304, 11·50603, and ·03417
49. By how many does the sum of ·1, ·2, ·3, ·4, ·5, ·6 and ·7 exceed the sum of ·1, ·02, ·003, ·004, ·005 and ·006?
50. By how many does the sum of 5·555 and ·5555 exceed their difference?

XXX. MULTIPLICATION OF DECIMALS.

Multiply—

1. 2·4 by 7	9. 4·17 by 230	17 ·043 by 2·05
2. ·47 by 12	10 15 6 by 3100	18 1·65 by ·67
3 1·43 by 17	11 043 by 7100	19 ·305 by 42
4. ·035 by 23	12 0018 by 18000	20 213 4 by 0085
5 5 32 by 107	13 7 3 by ·5.	21 3 25 by 1·24
6. 14·09 by 201.	14 2·8 by 6	22 7 62 by 2 35.
7. ·087 by 40	15 ·073 by 9	23 0318 by 4·05
8 ·0053 by 70	16. ·027 by 1·03.	24 ·6072 by ·0015

25. 2·145 × ·014	29 3020 × ·015	33 ·03 × ·05 × ·07
26. 315 2 × ·00107	30. 2461 × ·207	34. ·02 × ·004 × ·6
27. 3·402 × ·0203	31 ·007853 × 035	35 17 × ·07 × 1·7
28 30 635 × 1·206	32 ·3045 × 00061	36 31 × 3·1 × ·031

Find the continued product of—

37 2, ·3, 04, ·005 and ·006. 38 7, ·07, ·007, 0007 and 700.

39. 21, 2·1, ·21 and ·021 40 ·23, ·023, 2·3 and 2300.

XXXI. DECIMALS.

DIVISION BY AN INTEGER

Divide, using *short* divisions—

1 24 82 by 2	9 26·007 by 5	17. 14·1 by 16
2. 69·042 by 3	10. 17·1 by 4	18 ·137 by 25
3 2·7545 by 5	11 ·34 by 8	19 3·7 by 32
4 ·01768 by 8	12 9 by 8	20 ·0145 by 64.
5. 1246 5 by 9	13. 371·488 by 16	21 3·5 by 160
6. ·1452 by 12	14 3·8475 by 25	22 1 4 by 320
7. 1·063 by 5	15. 1·7955 by 35	23. 29 5 by 2500
8 231·3 by 4	16 2·4304 by 28	24. 831·2 by 6400

Find, as a decimal, the complete quotient of—

25. 26·64 ÷ 37.	29. 1670·4 ÷ 174	33 26·1 ÷ 2175
26. 4·983 ÷ 151.	30. 177 1 ÷ 1012	34 4006·7 ÷ 389
27 1 7388 ÷ 207	31 ·7 ÷ 175	35 7·1 ÷ 3125
28. ·01136 ÷ 71	32. 18·8 ÷ 625	36 543 2 ÷ 4096.

37 22708·8 by 3800 38. 516·1 by 39700

39. 21475·05 by 7041000. 40. 13723·226 by 8063000.

XXXII. DIVISION OF DECIMALS.

Divide, using *short* divisions—

1. 35·082 by ·3
2. ·67144 by ·8.
3. ·61705 by ·07.
4. 2·15736 by ·04.
5. 01331 by 1·1
6. 34·56 by 1·2.
7. 1·73871 by ·009
8. 4·00004 by ·0011.
9. 682·4 by ·05.
10. 35·16 by ·006.
11. 12·6 by ·0012.
12. ·5121 by ·08
13. ·01 by ·004.
14. 1 by ·05
15. 30 by ·008
16 7000 by ·0004.
17. 24·112 by 1·6
18. ·27648 by ·16
19. ·2317 by ·25
20 605·39 by 2·5
21. 4·431 by ·32.
22. ·5709 by ·032.
23. ·03 by 6·4.
24. ·9763 by ·0064
25. 2·1 by ·24.
26. 4·5 by ·036
27. 6·1 by ·0025.
28. 49·7 by ·025.
29. 1·5 by ·0064.
30. 7 by ·32.

Divide—

31. ·0506 ÷ 2·3
32. ·0507 ÷ ·039
33. ·002232 ÷ ·031.
34. ·000153 ÷ ·17.
35. 7·7811 ÷ ·037
36. 5·5811 ÷ ·067.
37. 30·5118 ÷ 50·6
38. ·358307 ÷ ·059.
39. 4·96 ÷ 15·5.
40. 4·83 ÷ 9·2.
41. ·0341 ÷ ·124.
42. 1·71 ÷ ·76.
43. ·060248 ÷ 27·2.
44. 72·5026 ÷ 7·85
45. 12·19192÷30·4
46 ·65341 ÷ ·0475
47 1028·5 ÷ ·0017.
48. 17·25 ÷ ·0023.
49. 41 ÷ ·082.
50. 800 ÷ ·00125.
51. 35 ÷ ·0175.
52. 84·375 ÷ ·00375.
53 1215013·8 ÷ 2·023.
54 343·9836 ÷ ·01605.

Divide, to *four* places of decimals—

55. 1·3 by 7
56 ·41 by 110.
57. ·43 by 8·1.
58. 5 by 8·4
59. 2·4 by 19
60. ·38 by 41
61. 6 by ·053
62. 10 by ·029.
63. 48·1 by 9700.

64. Divide 23·065 by 3·5. Hence, without further work, write down the quotient of 23·065 ÷ ·0035.

65. Divide 210720·6 by ·4206. Hence, by inspection, find ·2107206 ÷ 42·06

66. Divide 85·9625 by 26·45. Hence, find 85·9625 ÷ ·002645.

67. Divide 261 by 217·5. Hence, by inspection, find 26·1 ÷ 2·175; 2·61 ÷ ·0002175; and ·00261 ÷ ·2175.

68. Divide 51·61 by 39·7 Hence, by inspection, find ·5161 ÷ 3·97; 51610 ÷ ·397; ·5161 ÷ ·0397, and ·05161 ÷ 397.

69. Find the integral part of the quotient of 7·513 ÷ ·23 and the decimal remainder.

70. What integral number of times is ·27 contained in 140, and what decimal remains over?

XXXVI. RECURRING DECIMALS.

Express as vulgar fractions in their lowest terms —

1	$\cdot\dot{2}$	7.	$\cdot 9$	13.	$\cdot\dot{2}3\dot{4}$	19	$\dot{1}1924\dot{}$	25	$\dot{8}2417\dot{5}$
2.	$\cdot 5.$	8	$7\cdot\dot{9}.$	14	$\cdot\dot{2}7\dot{0}$	20	$\cdot\dot{0}569\dot{1}$	26	$\dot{7}5824\dot{1}$
3	$\dot{6}$	9	$\cdot\dot{2}\dot{4}$	15.	$10\cdot\dot{3}1\dot{5}$	21	$\cdot\dot{1}4285\dot{7}$	27	$\dot{4}7619\dot{0}$
4.	$\cdot\dot{8}.$	10	$\cdot\dot{4}\dot{5}$	16.	$6\ \dot{8}3\dot{7}$	22	$\cdot\dot{4}2857\dot{1}$	28	$\dot{9}2307\dot{6}$
5.	$4\cdot\dot{7}$	11	$1\cdot\dot{7}\dot{2}.$	17.	$\cdot\dot{2}07\dot{9}$	23.	$1\cdot\dot{5}7142\dot{8}$	29	$\dot{0}8641975\dot{3}$
6.	$11\cdot\dot{1}.$	12.	$4\ \dot{9}\dot{6}$	18.	$\dot{4}065$	24.	$3\cdot\dot{8}5714\dot{2}$	30	$\cdot\dot{0}11086474\dot{5}$
31.	$\cdot 8\dot{3}$	37.	$0\dot{1}\dot{5}$	43	$\cdot 2\dot{3}8\dot{7}$	49	$\cdot 8\dot{1}951\dot{2}$	55.	$9\dot{2}8571\dot{4}$
32	$\cdot 3\dot{6}$	38.	$\cdot 97\dot{2}$	44.	$\cdot 3\dot{8}3\dot{7}$	50.	$\cdot 5\dot{0}731\dot{7}$	56.	$\cdot 6\dot{5}7142\dot{8}$
33	$2\cdot 5\dot{7}$	39.	$\cdot 01\dot{0}\dot{9}.$	45.	$1\cdot 236\dot{1}.$	51	$\cdot 00010\dot{6}$	57	$\cdot 03\dot{4}28571$
34.	$1\cdot 0\dot{1}$	40.	$\cdot 02\dot{2}\dot{7}$	46.	$3\cdot 002\dot{6}$	52.	$\cdot 11458\dot{3}$	58.	$\cdot 129\dot{1}4285\dot{7}$
35.	$\cdot 2\dot{3}\dot{4}.$	41.	$1\cdot 48\dot{3}$	47.	$\cdot 10\dot{8}4\dot{6}.$	53	$\cdot 0648\dot{1}\dot{4}$	59	$839\dot{2}8571\dot{4}$
36.	$\cdot 3\dot{4}\dot{8}$	42	$2\cdot 708\dot{3}$	48	$\cdot 34\dot{4}5\dot{9}$	54.	$\cdot 1003\dot{3}7\dot{8}$	60	$05\dot{7}6923\dot{0}.$

XXXVII. RECURRING DECIMALS.

Addition and Subtraction.

Find, without reducing to vulgar fractions, the value of—

1. $\cdot\dot{2}\dot{7}+\dot{3}45\dot{6}$
2. $\cdot\dot{3}\dot{4}+\cdot\dot{5}7\dot{1}.$
3. $\cdot\dot{1}4285\dot{7}+\cdot\dot{0}7\dot{6}$
4. $\cdot\dot{1}7690\dot{4}+\cdot\dot{3}8\dot{4}.$
5. $\cdot 2\dot{3}+\cdot 7\dot{5}\dot{1}.$
6. $\cdot 40\dot{1}\dot{5}+\cdot\dot{3}\dot{2}$
7. $1\cdot 8\dot{5}\dot{4}+\cdot 5\dot{6}.$
8. $5\cdot 1\dot{4}\dot{7}+1\cdot 00\dot{8}$
9. $\cdot 0\dot{1}+\cdot 00\dot{2}+\cdot 5.$
10. $\cdot 4\dot{1}\dot{3}+\cdot 8+\cdot 0\dot{4}.$
11. $\cdot 37\dot{4}\dot{6}-08\dot{2}\dot{3}$
12. $1\cdot 48\dot{6}\dot{3}-\cdot 7\dot{3}8\dot{1}$
13. $9\cdot 78\dot{6}\dot{3}-5\cdot 7\dot{8}\dot{3}.$
14. $\cdot 247\dot{6}-10\dot{3}\dot{8}$
15. $\cdot 4\dot{2}7\dot{5}-\cdot 30\dot{2}6\dot{7}$
16. $3\cdot 4\dot{1}2\dot{7}-1\cdot\dot{0}8\dot{7}$
17. $3\cdot\dot{2}47\dot{1}-2\cdot 2471$
18. $1\cdot\dot{5}7\dot{6}-1\cdot 0576$
19. $17-\cdot\dot{1}4285\dot{7}$
20. $5-\cdot\dot{0}7692\dot{3}$
21. $4\cdot 76\dot{3}\dot{0}-1\ 0\dot{2}8\dot{5}$
22. $\cdot 20\dot{3}5\dot{1}-000176\dot{4}.$
23. $\dot{5}134\dot{2}-\cdot 3\dot{6}$
24. $\dot{7}4\dot{2}-\dot{3}16\dot{5}$
25. $\dot{6}3\dot{9}+\cdot\dot{0}2\dot{7}+\cdot\dot{3}.$
26. $\dot{8}\dot{7}+\dot{6}+\cdot\dot{4}5$
27. $\cdot 2+\dot{4}\dot{5}+\dot{5}\dot{4}.$
28. $\cdot 4\dot{0}\dot{9}+61+4\dot{7}\dot{9}$
29. $\cdot 6\dot{1}42\dot{3}+2\dot{8}57\dot{6}.$
30. $\cdot\dot{4}45\dot{5}+\cdot\dot{0}09\dot{0}-\cdot\dot{4}\dot{5}.$

Simplify—

31. $123\cdot 546\dot{2}+34\cdot\dot{0}36\dot{7}+70\cdot\dot{4}\dot{5}$
32. $4\cdot 3\dot{5}+765+7\cdot 2\dot{6}1\dot{3}+1\cdot 5\dot{4}\dot{7}$
33. $3\ 7\dot{1}\dot{3}+2\ \dot{5}671\dot{2}+5\cdot 6+\cdot 12\dot{3}\dot{7}.$
34. $\dot{2}0879\dot{1}+\cdot\dot{0}7692\dot{3}+\cdot\dot{7}1428\dot{5}$
35. $\cdot 1\dot{8}5714\dot{2}+\cdot\dot{2}8571\dot{4}+\cdot 0\dot{2}8571\dot{4}.$
36. $5\cdot 3\dot{8}5714\dot{2}+960\dot{5}7142\dot{8}$
37. $3\cdot 70\dot{4}\dot{3}+1\cdot\dot{0}56\dot{7}-2\ 513\dot{6}.$
38. $\cdot\dot{4}71\dot{6}-\cdot 2\dot{5}9\dot{0}+2\ 1305\dot{8}$
39. $\cdot\dot{6}\dot{3}+\cdot\dot{5}4\dot{0}-\cdot\dot{1}7690\dot{4}+999$
40. $\cdot\dot{9}756\dot{0}+\cdot 73\dot{4}145\dot{3}-\cdot\dot{6}097\dot{5}.$

XXXVIII. REÇURRING DECIMALS.

MULTIPLICATION AND DIVISION.

Find, without reducing to vulgar fractions, the value of—

1. $\cdot 3\dot{2} \times 4$.
2. $3\cdot\dot{4}\dot{5} \times 3$
3. $7\cdot\dot{2}\dot{4} \times 5$
4. $\cdot\dot{4}5\dot{6} \times 12$
5. $\cdot 7\dot{2}\dot{6} \times 45$.
6. $\cdot 3\dot{1}\dot{5} \times 23$
7. $3\cdot 2\dot{4}3\dot{7} \times \cdot 5$.
8. $\cdot 07\dot{1}\dot{3} \times \cdot 08$.
9. $\cdot 4\dot{7}\dot{5} \times \cdot 015$.
10. $\cdot 3\dot{8}4\dot{7} \div 5$
11. $7\cdot 3\dot{4}\dot{5} \div 4$.
12. $\cdot 372\dot{1}\dot{4} \div 16$.
13. $\cdot 70\dot{5}6\dot{1} \div 24$
14. $1\cdot\dot{3} \div 37$.
15. $14\cdot\dot{5} \div 41$.
16. $23\cdot 465\dot{3} \div \cdot 02$.
17. $15\cdot 4\dot{6} \div \cdot 21$
18. $4\cdot 8\dot{3} \div \cdot 035$.

19. Multiply $\cdot 36458\dot{3}$ by 47000.
20. Divide $\cdot 0062\dot{5}\dot{4}$ by $\cdot 00027$

Express as a decimal the product of—

21. $\cdot\dot{6}\dot{3} \times \cdot\dot{5}3\dot{9}$
22. $\cdot 0\dot{6} \times \cdot 1\dot{6}\dot{3}$
23. $\cdot\dot{2}\dot{7} \times \cdot 91\dot{6}$
24. $\cdot\dot{9}\dot{0} \times 7\cdot 8\dot{3}$
25. $\cdot\dot{8}\dot{1} \times \cdot 8\dot{1}$.
26. $6\cdot 7\dot{6} \times \cdot 0\dot{5}$.
27. $\cdot 049\dot{5} \times \cdot 30\dot{2}\dot{7}$
28. $\cdot 56\dot{7} \times \cdot 48\dot{9}6\dot{3}$.
29. $1\cdot 1\dot{6} \times \cdot\dot{4}2857\dot{1}$.

Express as a decimal the quotient of—

30. $\cdot 1575 \div 5\dot{0}\dot{9}$.
31. $\cdot\dot{2}\dot{1} \div \cdot 0\dot{1}$
32. $\cdot\dot{3}7\dot{0} \div \cdot\dot{5}$.
33. $1\cdot 6\dot{4} \div 10\cdot\dot{0}\dot{9}$.
34. $1\cdot\dot{2}3\dot{1} \div 3\cdot\dot{6}\dot{3}$.
35. $1\cdot 89 \div \cdot\dot{7}1428\dot{5}$.
36. $23\cdot\dot{6}4\dot{8} \div \cdot\dot{9}4\dot{5}$.
37. $17\cdot 61 \div \cdot\dot{5}28\dot{3}$.
38. $\cdot 8547 \div 1\cdot 3\dot{6}76\dot{5}$.

39. Multiply $6\cdot\dot{3}\dot{6}$ by $\cdot\dot{5}7142\dot{8}$
40. Divide $\cdot 2\dot{1}4285\dot{7}$ by $\cdot\dot{5}873 0\dot{1}$.

Simplify, giving results as decimals:—

41. $\cdot 065 + \cdot 75 \times \cdot 8\dot{3}$.
42. $2\cdot 4 \times 7\cdot\dot{6} - 1\cdot 84$.
43. $(12\cdot\dot{6} - 7\cdot 41\dot{6}) \times 2\cdot\dot{1}$
44. $(8\cdot 41\dot{6} + 5\cdot 58\dot{3}) \div 2\cdot 1$
45. $1\cdot 71 \div (\cdot\dot{1}4285\dot{7} + \cdot\dot{2}8571\dot{4})$.
46. $\cdot\dot{4}2857\dot{1} \times (4\cdot 3 + 3\cdot 4)$.
47. $\cdot\dot{4}2857\dot{1} \times 4\cdot 3 + 3\cdot 4$.
48. $1\cdot 71 \div \cdot\dot{1}4285\dot{7} - \cdot\dot{2}8571\dot{4}$.
49. $1\ 84 \times \cdot 8\dot{3} \div 6\cdot\dot{2}\dot{7}$.
50. $1\cdot 01\dot{9} \times \cdot 1\dot{3} \div \cdot 034$.
51. $\dfrac{53\cdot\dot{8}\dot{1}}{14\cdot 8}$.
52. $\dfrac{\cdot 019\dot{4}}{1\cdot 1\dot{6}}$.
53. $\dfrac{3\cdot 1\dot{2} - 2\cdot\dot{3}\dot{1}}{\cdot\dot{8}\dot{1}}$.
54. $\dfrac{1\cdot 8\dot{3} \times \cdot 43\dot{1}\dot{8}}{1\cdot 1875}$
55. $\dfrac{2\cdot\dot{4}\dot{5} \times 3\cdot 1\dot{6} \div \cdot 01\dot{2}\dot{6}}{615\cdot 6}$.
56. $1\cdot 6 \times \dfrac{\cdot 5\dot{3}}{\cdot 6\dot{1}} \times \dfrac{\cdot 825}{\cdot 41\dot{6}}$.
57. $\dfrac{\cdot 875 \times \cdot\dot{2}7\dot{0}}{\cdot 125 + \cdot 125\dot{6}7\dot{5}} + \dfrac{3}{53}$.
58. $\dfrac{2\cdot 8 \times 11\cdot\dot{3}\dot{6}}{5\cdot 6\dot{8}\dot{1}} - \dfrac{1\cdot\dot{6}\dot{3}}{\cdot\dot{4}\dot{5}}$.
59. $\dfrac{2\cdot 791\dot{6} \times 3\cdot 2\dot{3}\dot{7}}{1\cdot 86\dot{1} \times \cdot 809\dot{3}\dot{4}} \times \cdot 08\dot{3}$
60. $\dfrac{\cdot 041\dot{6} \div \cdot\dot{6}\dot{0}}{\cdot 02\dot{2}\dot{7} \div \cdot 0\dot{0}\dot{3}} \div \dfrac{\cdot 7\dot{3}}{\cdot 8}$.

XXXIX. DECIMALS.

VALUE OF A DECIMAL OF A CONCRETE QUANTITY.

Find the value of—

1. ·375*s*	7. 375 of 6*d*	13 1 6875 of £2
2. 4·625*s*	8 3·75 of 3*d*	14 3·0325 of £5.
3. £·85.	9 7375 of 10*s*	15. ·596875 of £1
4. £2·175.	10. 1 0625 of 5*s*	16 ·778125 of £1.
5. £·1375	11. 1·5625 of 2*s*	17. ·4790625 of £10.
6 £·9375	12. ·65625 of 4*s*	18. ·0053125 of £20

19. ·6875 of 3*s* 4*d*	29. ·8125 of 1 cwt
20. ·53125 of 6*s* 8*d*	30. ·0625 of 3 cwts
21. 1·125 of 2*s* 6*d*	31. ·07890625 of a ton.
22. 2 825 of 5*s* 10*d*	32 ·66796875 of a ton.
23 1·0625 of a guinea.	33 2·34375 miles
24. 1·0375 of £3, 10*s*	34 ·90625 of a cub yd
25. ·375 of £3, 17*s* 6*d*	35 ·8125 of 2 tons 4 cwt
26. ·1875 of £4, 2*s* 8*d*	36 4·2128 of 5 cwt 2 qr 9 lb.
27. ·0136 of £14, 6*s* 5½*d*	37. ·028125 of 1 mi 6 fur
28. ·001024 of £16, 5*s* 6¼*d*	38. ·390625 of 6 A 1 R 24 P

39. Find the value of £ 525 + 5·25 of 10*s* + 52 5*s*

40. Find the value of £1·285 − 5·115*s* + 6·92 farthings

41. Add together 2·5 of 1 ton 6 cwts, 3·125 of 2 qrs 16 lbs, and 3·75 of 1 qr

42. Add together ·9 of a pole, 241 of a yd, ·97 of a foot, and ·684 of an inch

43. From ·6409 of 1 ac 2 ro take the sum of ·1892 of 4 ac and ·1894 of 3 ro

44. Subtract the sum of 3125 of 6 cwts and ·032 of 3 cwts 2 qrs 14 lbs 4 ozs from 2 cwts

---

Find the value of—

45. $\cdot 8\dot{3}$ of 2*s* 6*d*	49 $\cdot 6\dot{1}$ of 1 yd	53 $\cdot\dot{4}2857\dot{1}$ of 1*s* 2*d*.
46 $02\dot{7}$ of 10*s* 6*d*	50 $\cdot 0\dot{1}\dot{5}$ of 1 po	54 $3\dot{5}71428\dot{8}$ of £1, 1*s*
47. $\cdot 791\dot{6}$ of a guinea	51 $\cdot 01\dot{2}\dot{5}$ of 5 mi	55 $\cdot 017708\dot{3}$ of £1
48 $\cdot 04\dot{8}$ of £1, 12*s*. 9¾*d*	52 $2\cdot 3\dot{5}$ miles	56 $01420\dot{4}\dot{5}$ of £1, 2*s*.

57. Find the difference between £ $641\dot{6}$ and $3\cdot\dot{0}69\dot{3}$ of 8*s* 5*d*

58. Subtract $\dot{7}6923\dot{0}$ of 5*s* 5*d* from $\cdot 5\dot{0}9$ of 9*s* 2*d*

59. From ·856 of 2 cwts 26 lbs take 3 $2\dot{2}\dot{7}$ of 2 qrs 10 lbs

60. Take $\cdot 3\dot{5}$ of 5·625 tons from $\cdot 7\dot{0}\dot{9}$ of 5 tons 3 cwts, 14 lbs.

XL. DECIMALS.

REDUCTION OF ONE QUANTITY TO THE DECIMAL OF ANOTHER.

Express—

1.	8s as the decimal of	£1	2.	12s as the decimal of	£1.
3.	3s	£1.	4.	7s. .	£1.
5.	5s	£1	6.	15s	£1
7.	2s 6d .	£1	8.	4s 6d. ..	£1.
9.	2s 3d.	£1	10.	1s 9d. .	£1.
11.	9d .	1s	12.	1½d. . . .	1s
13	7½d . . .	1s	14.	5¼d. .. .	1s
15.	¾d	1s	16.	11¼d.	1s
17.	15s. 6d	£5	18.	10s. 6d .	£3
19.	4s 4½d	£10	20.	14s 7½d .	£5.
21.	1 qr 7 lbs . .	10 cwts	22.	2 qrs 21 lbs.	1 ton.
23.	2 cwts 1 qr 7 lbs	1 ton.	24.	1 ton 3 cwts. 14 lbs .	5 tons.
25.	1 lb 5 ozs.	1 cwt	26.	7 ozs. . . .	2 cwts.
27.	1 chain	1 mile	28.	2 ft 7½ in.	100 yds.
29.	1 ac. 1 ro. 10 po.	12 acres.	30.	3 ac 1 ro 2 po	50 acres.

31. Reduce 2s 7½d to the decimal of 8 guineas
32. Reduce 6s 6¾d to the decimal of 4 guineas.
33. What decimal of £100 is £5, 17s. 7½d ?
34. What decimal of £20 is £7, 13s. 6¾d ?
35. Express £5, 3s 4½d as the decimal of £1.
36. Express £14, 10s. 10½d. as the decimal of £1.
37. What decimal of £1000 is $\frac{3}{5}$ of a guinea?
38. What decimal of £100000 is $2\frac{2}{5}$ of £4?
39. What decimal of a ton is 2 cwts. 3 qrs. 3 lbs. 8 ozs.?
40. What decimal of a mile is 11 po 4 yds. 4½ in ?
41 Express $1\frac{2}{7}$ of $\frac{3}{11}$ of £1, 12s. 1d. as the decimal of £100
42. Express $\frac{4}{19}$ of ·153 of 12s. 8d. as the decimal of £10
43 Reduce 2·307 of £1, 3s 2½d to pence and the decimal of a penny.
44 Reduce ·1703 of £2, 13s. 3¼d. to pence and the decimal of a penny
45. Reduce ·00125 of 2 qrs. 3½ lbs. to ounces and the decimal of an ounce.
46. Reduce ·315 of 9 lbs. 7 ozs. 6 grs to dwts. and the decimal of a dwt

47. Reduce 027 of 1 qr 3 bush 3 pks 1 gal to pints and the decimal of a pint

48 Reduce 1·0374 of $3\frac{3}{7}$ acres to sq yds and the decimal of a sq yd.

49. Subtract ·0015 of a week from ·375 of 1 hr 20 min, and express the result in seconds and the decimal of a second

50. Subtract ·0023 of 1 week 1 day from ·63 of 5 hours, and express the result in minutes and the decimal of a minute

---

Express—

51.	$8d$ as a recurring dec of	$1s$	52	$5d$ as a recurring dec of	$1s$
53	$1d$ . .	$1s$	54.	$1\frac{3}{4}d$	$1s$
55	$10\frac{3}{4}d$.	$1s$	56	$11\frac{1}{2}d$	$1s$.
57	$13s\ 4d$	£1	58	$16s\ 8d$	£1
59.	$7s\ 2d$	£1	60	$5s\ 1d$	£1

61.	Reduce $10s\ 4d$ to the recurring decimal of	£1, $7s\ 6d$
62.	$19s$.	£4, $8s$.
63	.. $8s\ 5d$	£1, $4s$
64.	$8s\ 7d$	£3
65.	$5s\ 11d$	$13s\ 8d$
66.	$3s\ 8\frac{1}{2}d$	$12s\ 4d$
67.	. $3s\ 9d$	$4s\ 8d$
68	. $4s.\ 6d.$ . . .	$5s\ 5d$
69.	. . . 7 cwts 21 lbs	2 tons 15 cwt
70	2 qrs 7 lbs .	8 cwts 1 qr
71	. 1 inch	1 pole
72.	. 1 ft 6 in	$\frac{1}{4}$ mile
73	$\frac{4}{5}$ of 1 yard	$\frac{2}{5}$ of 3 fur.
74.	$\frac{1}{8}$ of 4 ozs. 19 dwt . .	16 dwts 21 grs.
75.	. 11 sq po 11 sq yds	1 acre
76.	. 1 day 17 hrs . . .	7 days 7 hrs

77 What decimal of 2 guineas is £1, $14s\ 9\frac{3}{4}d$?

78. What decimal of 11 cwts is 11 lbs 11 ozs ?

79. Express the sum of $00892708\dot{3}$ of £100 and $\cdot 6\dot{8}5714\dot{2}$ of £2, $2s\ 7d$ as the decimal of £41, $13s\ 4d$

80. Reduce the difference between $\cdot 10\dot{1}5384\dot{6}$ of 1 qr 12 lbs. 10 ozs, and $000065104\dot{1}\dot{6}$ of 3 tons to the decimal of 9 lbs.

XLI. MISCELLANEOUS EXERCISES.

1. Which is greater. Three-thousandths, or Three-hundredths?
2. Express in figures, as a decimal. Three and three-thousandths.
3. Express ·000007 in words
4. Write $\frac{123}{10000}$ as a decimal
5. Add together 3·9064, ·08456, 327·265, and 14 00983.
6. Subtract 53·94306 from 60·1531.
7. Multiply 82·5604 by ·08425.
8. Divide 157·311 by 3405
9. Simplify 8 8 — 2·034 × ·35
10. How many times is ·0005 of a penny contained in a shilling?

---

11. Which is greater · Thirty-thousandths, or Three-hundredths?
12. Express Thirty-seven millionths in figures.
13. Write ·0047 as a vulgar fraction.
14. Find the sum of 85·0609, 4·68403, ·00689, and 529·873.
15. Find the difference between 59·98063 and 72·480527.
16. Find the product of ·98236 and 370·95.
17. Divide, using short division, 271·403 by 64.
18. Divide 2·3517 by ·005.
19. Simplify (8·8 — 2·034) × ·35.
20. A mowed ·175 of a field, and B mowed ·0175 of it: what decimal part of the field then remained unmown?

---

21. Write down, at sight, the value of 4·7031 × 100.
22. State fully in words the meaning of 2·106 inches.
23. Express $\frac{7}{10} + \frac{3}{1000}$ as a decimal.
24. Express ·0112 as a vulgar fraction in its lowest terms.
25. What must be added to 91·006705 to make 100·3?
26. Find the square of ·0217.
27. Divide, using short division, ·157035 by ·0032.
28. Simplify 2·34 × 1·025 ÷ ·0009.
29. The product of two decimals is ·135; one of them is 11·25· find the other.
30. Reduce £1·0375 to pence.

31 Write down, at sight, the value of $29 \cdot 35 \div 1000$
32 Express $3 \cdot 00671$ as an improper vulgar fraction
33. Find the value of $116 \cdot 001 - 47 \cdot 91308$
34. Find the value of $1593 \div \cdot 0472$
35 Find the value of $2 \cdot 405 \times 01308$
36. Find the value of $27 \cdot 9875 + \cdot 0059625 + 381\ 4 + 97 + \cdot 59623$.
37. Change $\frac{111}{128}$ into a decimal
38. Simplify $\dfrac{6 \cdot 3 - 4 \cdot 26}{\cdot 0017}$
39 Reduce £5·1875 to pence
40 Express 2 qrs 14 lbs as the decimal of half a ton

---

41. Add Eleven *ten-thousandths* to Ten and eleven *thousandths*.
42. Write down, at sight, the quotient of $47 \cdot 83 \div 100$.
43. Find the value of $14 \cdot 1 - 7 \cdot 568 + 1 \cdot 42325 - 5 \cdot 008 + \cdot 2504 - 3 \cdot 197$.
44. Find the cube of $\cdot 023$
45. Divide $\cdot 00567525$ by $\cdot 0805$.
46. Express $3\frac{12}{125}$ as a decimal
47. Simplify $\dfrac{1 \cdot 44 \times 0032}{7 \cdot 2}$
48. Express ·7875 of a mile in furlongs and poles
49 How many times is £·125 contained in 17·5*s* ?
50 Reduce £3, 1*s*. $1\frac{1}{2}d$ to the decimal of £5

---

51. From One hundred, and seven-*tenths* take One-hundred-and-seven-*thousandths*
52. Write down at sight the value of $57 \cdot 83 \times 1000$
53. Find the continued product of $3 \cdot 4$, $\cdot 34$, $\cdot 034$ and 3400.
54 Find the value of $24 \cdot 048 - 3\ 2735 -\ 08 - 12\ 63575 - 6 \cdot 7 - 1 \cdot 311$.
55 Divide $204 \cdot 079$ by 0437
56. Express ·71875 as a vulgar fraction in its lowest terms
57. Simplify $\dfrac{\cdot 00062}{\cdot 64}$
58. Find the value of 5·25625 of 1 oz 13 dwts 8 grs
59. Express ·42 of 4*s* $10\frac{1}{2}d$ as the decimal of £1, 11*s* 6*d*
60. A book is $1\frac{3}{4}$ inches in thickness, the covers are each $\frac{3}{16}$ in in thickness, and there are 500 pages, what decimal of an inch is the paper in thickness?

---

61. Which is greater, ·3456 or ·35?

62. Write down at sight as a decimal the value of $\frac{41\cdot23}{1000}$.

63. Without dividing, express $\frac{3}{16}$ as a decimal

64. What must be added to 87·6993 to make 392·0062436?

65. Divide the product of ·004 and 32·4 by 6·4.

66. Simplify $2\cdot345 \times \cdot062 + \cdot85461$.

67. Simplify $\frac{22\cdot4}{\cdot25} + \frac{250}{\cdot8} + \frac{1\cdot2}{\cdot0075}$.

68. Find the smallest integer which contains 7·4 and 1·11 each an integral number of times.

69. How many times is ·01 of an inch contained in a yard?

70. Express the sum of £0·1037 and £1·0713 in *£ s. d.*

---

71. What is the difference between ·700 and ·7?

72. Write down at sight as a decimal the value of $\frac{721 \div 100}{100}$.

73. Without dividing, express $\frac{407}{625}$ as a decimal.

74. Find $\cdot479 + \cdot479 + \cdot479 + \cdot479 + \cdot479 + \cdot479 + \cdot479 + \cdot479 + \cdot479 + \cdot479$.

75. Divide the difference between ·7 and ·12 by ·4.

76. Simplify $6240\cdot1 - 52\cdot104 \div \cdot0835$.

77. Simplify $\frac{\cdot23 - \cdot023}{\cdot0023 \div 23000}$.

78. Find the smallest decimal which contains ·0289 and 1·87, each an integral number of times

79. How many times is ·003 of a shilling contained in a guinea?

80. Express the sum of ·625 of £3 and ·175 of £3, 2*s.* 6*d.* as the decimal of £50

---

81 Find the value of $\cdot91 \times 19 \times 1\cdot01 \times 110$.

82. Without dividing, convert $\frac{2}{500} + \frac{5}{2000}$ into a decimal.

83. Reduce $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \frac{1}{3125}$ to a decimal.

84. Find the sum of the squares of ·17, ·017 and ·0017.

85. Simplify $\frac{\cdot0038425 - \cdot00183}{\cdot013 + \cdot022}$.

86. Simplify $(\cdot03 + \cdot016) \times \cdot1 - \cdot03611 \div 7\cdot85$.

87. Find the value of 4·2128 of 5 cwts. 2 qrs. 9 lbs.

88. Reduce ·0125 of $\frac{4}{11}$ of 1 mile 23 poles to inches and the decimal of an inch.

89 $\cdot 278$ of a certain sum of money is £1, 3*s* 2*d* Find the sum.

90. Divide the sum of £10, 2*s* 1*d* between A, B and C, so that A may have $\cdot 6$ of the whole sum, B $\cdot 06$ of it, and C the rest

---

91. Express $2\cdot 4 \times 3\cdot 15$ as a vulgar fraction in its simplest form.

92 Show that $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} = \cdot 8$

93 Find the difference of the squares of $5\cdot 4$ and $4\ 5$.

94 Simplify $22\cdot 891 + \cdot 23 \times (100\cdot 1 - 90\cdot 09) \div \cdot 013$.

95. Simplify $\frac{\cdot 00281 \times \cdot 0625}{13\cdot 39 - 11\ 985}$.

96. Find the value of $3\cdot 725$ of 25 miles 4 furlongs

97. Reduce $\frac{3}{8}$ of 69 of 1 day 3 hours to minutes and the decimal of a minute

98. What decimal of £5, 4*s* 6*d* is $\cdot 25$ of £1, 6*s* $1\frac{1}{2}$*d* ?

99. Find the value of 2 of 6*s* 8*d* $+ \cdot 4375$ of £1 $+ \cdot 25$ of £1, 12*s* 4*d* $+ \cdot 1$ of a crown

100. A man after paying away 75 of his money to one person, and then $\cdot 75$ of the residue to another, had £6, 13*s* left How much had he at first ?

---

101. Which is greater $\cdot 44$ or $\cdot\dot{4}$?

102. Find the complete quotient of $9\cdot 4 \div 11$

103. Reduce $\frac{7}{27}$ to a decimal

104. Convert $\cdot\dot{7}2\dot{9}$ into a vulgar fraction in its lowest terms

105. Add together $2\cdot 13\dot{7}$, $3\cdot 8\dot{2}\dot{3}$ and $1\cdot 29$

106. Subtract $3\cdot\dot{1}6\dot{2}$ from $5\cdot 714\dot{2}4\dot{6}$

107. Multiply $\cdot\dot{5}\dot{4}$ by $3\cdot\dot{6}$

108. Divide $\cdot\dot{1}2\dot{3}$ by $\cdot\dot{1}$, and express the result as a decimal

109. Find the value of $\cdot\dot{0}\dot{9}$ of 2*s*. 9*d*

110. What decimal of 1 cwt is 16 lbs ?

---

111. Express in figures, as a decimal, Seventeen ninety-ninths.

112 Find the complete quotient of $9\cdot 1 \div 41$

113. Reduce $\frac{7}{24}$ to a decimal

114. Convert $\cdot 97\dot{3}$ into a vulgar fraction in its lowest terms.

115. Find the sum of $\cdot 0\dot{2}\dot{7}$, $1\cdot 3\dot{4}\dot{5}$, and $2\ 08\dot{3}$

116. Find the difference between $2\cdot\dot{5}3\dot{5}$ and $2\cdot 535$.

117. Express the product of $\cdot\dot{7}5\dot{9}$ and $\cdot\dot{4}\dot{5}$ as a decimal.

118. Divide $3\cdot5$ by $7\cdot\dot{0}\dot{7}$, and express the result as a decimal.

119. Find the number of square yards in $\cdot02\dot{2}\dot{7}$ of an acre

120 What decimal of a guinea is equal to $\cdot375$ of £1?

---

121. From sixty-three *hundred* take sixty-three *hundredths.*

122. Multiply $3\cdot7532$ by 1000, and divide $4\cdot27$ by 100

123. Change $4\cdot2\dot{3}\dot{6}$ into a vulgar fraction in its lowest terms.

124. Reduce $\frac{7}{480}$ to a decimal

125. Divide $\cdot00858\dot{3}$ by $\cdot024$, without changing the decimals into vulgar fractions

126. Simplify $\dfrac{\cdot203 \times \cdot003 \times 16}{\cdot008 \times \cdot0029}$

127. Simplify $\dfrac{2\cdot8 \times 11\cdot\dot{3}\dot{6}}{5\cdot6\dot{8}\dot{1}}$.

128. Find the difference between $\cdot6$ of 6*s.* and $\cdot1\dot{6}$ of 16*s.*

129. Reduce $1\cdot047$ of 14 days 5 hrs. to mins and the decimal of a minute.

130 How many pieces each $\cdot0014$ inches long can be cut from a line whose length is $2\cdot5$ inches? How long is the piece which is left over?

---

131 Write down as a decimal, Eighty-three *nine-hundred-and-ninety-ninths.*

132. Reduce $\cdot016$ and $\cdot01\dot{6}$ to vulgar fractions in their lowest terms.

133. Convert $\frac{7}{160}$ and $\frac{160}{7}$ into decimals

134. By what decimal must $\cdot000471$ be divided to give the quotient 200?

135. Simplify $\dfrac{\cdot5\dot{3}}{\cdot6\dot{1}}$ of $\dfrac{\cdot825}{\cdot41\dot{6}}$ of $1\cdot\dot{6}$.

136. Simplify $\dfrac{12\cdot4 + \cdot064 - \cdot066}{\cdot002}$.

137. Multiply $2\cdot\dot{4}0\dot{7}$ by 83 without converting the decimal to a vulgar fraction

138 Find the value of £$15\cdot125 + 17\cdot3025$*s.* + $9\cdot62$*d.*

139. Subtract $\cdot406$ of 2 acres 1 rood from $5\cdot25$ of 1 rood 13 perches, and express the answer in perches and the decimal of a perch.

140. How many bits, each $\cdot13$ of a foot long, can be cut from a length of 1 yard, and what decimal of an inch will be left over?

---

141. Write down the product of $\cdot\dot{6}7\dot{0} \times 100$

142. Which is greater $\frac{3}{17}$ or $\cdot 1\dot{7}$?

143. Express $\dfrac{\cdot\dot{2}1\dot{1}}{\cdot\dot{2}\dot{1}}$ as a vulgar fraction in its simplest form.

144. Simplify $\dfrac{(1\cdot 005 + \cdot 201)(1\cdot 005 - \cdot 201)}{1\cdot 005 \times \cdot 201}$.

145. Find the sum of $\dot{2}$, $\cdot\dot{0}\dot{2}$, $\dot{0}0\dot{2}$ and $\cdot\dot{0}00\dot{2}$

146 Multiply $3\cdot 2\dot{1}\dot{4}$ by 058, without converting the decimals into vulgar fractions

147. A book of 384 pages is exactly 1 inch thick exclusive of the cover What decimal of an inch is the paper in thickness?

148. Express the difference between £137·75 and £13·775 in £ *s* *d*

149. Find the value of $\cdot 41\dot{6}$ of $\dfrac{\cdot 015 \text{ of } 2\cdot 1}{035}$ of $\cdot 0\dot{2}8571\dot{4}$ of 11*s* 8*d*

150. Sixty-six shillings are coined from a pound Troy of a metal containing 37 parts pure silver and three parts alloy Express as the decimal of an ounce Troy the quantity of pure silver in one shilling

---

151. Which is greatest, and which least, of the decimals $\cdot\dot{0}3\dot{7}$, $\cdot 0\dot{3}\dot{7}$, $\cdot 03\dot{7}$?

152. Express $\dfrac{3}{10^3} + \dfrac{5}{10^5} + \dfrac{7}{10^7}$ as a decimal.

153. Simplify $\dfrac{3\cdot 9 \times \cdot 95 \times \cdot 051}{\cdot 085 \times \cdot 0035 \times 3\cdot 8}$

154. Reduce $\cdot 003\dot{5}1\dot{8}$ to a vulgar fraction in its lowest terms.

155. Multiply $\cdot\dot{0}2\dot{7}$ by $\cdot 2\dot{1}$, and express the result as a decimal

156. Simplify $12\cdot 8\dot{3} \div \dfrac{1\cdot\dot{2} \div 1\cdot\dot{3}}{1\cdot\dot{4} \div 1\cdot\dot{5}}$

157. How many times does £387 5 contain ·001024 of 1*s* $11\frac{1}{4}$*d*?

158. Add together ·0075 of a week, ·463 of 3 days 4 hours, and 5·643 of 1 hour 6 min, and express the answer in minutes and the decimal of a minute

159. Find the value of $\dfrac{\cdot 43 \times \cdot 43 - \cdot 34 \times \cdot 34}{43 - \cdot 34}$ of 4*s* 2*d*

160. If $\cdot\dot{7}1428\dot{5}$ of an estate is pasture, $\cdot\dot{1}4285\dot{7}$ of it is wood, and the remaining 71 acres is arable land, of how many acres does the whole estate consist?

---

161. Express $\cdot 0034\dot{5}1\dot{8}$ as a vulgar fraction in its lowest terms.

162. Multiply $6\cdot 4\dot{7}$ by 1000, and $\cdot\dot{1}4285\dot{7}$ by 100.

163. Add together $2\cdot\dot{5}\dot{4}$, $\cdot 375$, $\cdot 125$ and $\cdot\dot{3}$.

164. Simplify $\dfrac{9\cdot 1 \times 12\cdot 1 \times 17\cdot 1}{1\cdot 33 \times 1\cdot 43 \times 1\cdot 65}$

165. Divide 1 by $\cdot 0125$, and by $\cdot 0\dot{1}2\dot{5}$

166. Multiply $\cdot\dot{2}4\dot{7}$ by $2\cdot\dot{5}7142\dot{8}$, and give the result as a decimal.

167. Find the value of $\cdot 81380208\dot{3}$ of 6 cwts. without converting the decimal into a vulgar fraction.

168. Find the difference between $\cdot 76$ of 3*s.* $1\frac{1}{2}$*d.* and $\cdot 7\dot{6}$ of the same sum

169. Express the sum of $\frac{1}{16}$ ton, $\frac{1}{16}$ cwt, $\frac{1}{16}$ qr., $\frac{1}{16}$ lb and $\frac{1}{16}$ oz. in ounces and the decimal of an ounce

170 A can do $\cdot 1\dot{6}$ of a piece of work in $\cdot\dot{0}7\dot{2}$ of a day, and B can do $\cdot 008\dot{3}$ of it in $\cdot 125$ of an hour, which is the faster workman?

---

171. Express $\frac{13}{37}$ as a decimal without dividing.

172. Convert $\frac{6075}{8505}$ into a decimal.

173. Divide $1\cdot 9$ by $\cdot 0025$, and by $\cdot 1\dot{0}2\dot{7}$.

174. Divide $\cdot 674387$ by $402\cdot 35$ as far as five places of decimals.

175. Subtract $2\cdot\dot{4}2857\dot{1}$ from $3\cdot\dot{1}4285\dot{7}$, expressing the result as a vulgar fraction.

176. Simplify $\dfrac{5\cdot 6\dot{3}}{18\cdot\dot{7}} + \dfrac{5\cdot 61}{18\cdot 7}$, giving the result as a decimal

177. Simplify $\dfrac{\cdot 0013133}{2\cdot 3 \times 57\cdot 1}$, also $\dfrac{\cdot 00\dot{1}\dot{3}}{\cdot 074}$, without converting the decimals into vulgar fractions.

178. Show that $\frac{3}{5}$ of $\frac{7}{90}$ of $\frac{1}{8}$ of 3*s* 4*d.* is greater than $\cdot 6$ of $\cdot 0\dot{7}$ of $\cdot 1\dot{3}$ of 3*s.* 9*d*

179. Express the sum of $\cdot 8\dot{3}$ of 13*s* 4*d.* and $\cdot 13\dot{8}$ of £1, 4*s.* as the decimal of £5.

180. A and B go from England to America in different ships. A's ship travels without stopping at the rate of $42\cdot 5$ miles in every $\cdot 1\dot{6}$ of a day, and B's travels, also without stopping, at the rate of 44 yards every $\cdot 0025$ of an hour. Which makes the quicker voyage?

---

181. Without dividing, find the number of places of non-recurring decimal figures in the decimal which is equivalent to $\frac{31}{16400}$.

182. Find the sum of 3, $\cdot 3$, $\cdot 03$, $\cdot\dot{3}$, $\cdot 0\dot{3}$ and $\cdot\dot{0}\dot{3}$.

183 Multiply $3\ 1\dot{2}85714\dot{4}$ by $\cdot 006\dot{3}$, giving the result as a vulgar fraction

184 Divide $47\ 8\dot{1}\dot{3}$ by 3125 without converting the decimals into vulgar fractions

185. Show that $\frac{1}{29} = \cdot 03448\frac{8}{29}$, and that $\cdot\dot{0}3\dot{7} = (\ \dot{3})^3$

186. Simplify $\left(\dfrac{\cdot 63 \times 1\cdot 25}{1\cdot 83 \times 3\ 25} \div \dfrac{1\cdot 6 \times 2\cdot 25}{6\cdot 1 \times 2\frac{1}{7}}\right) - \cdot 58\dot{3}$

187. Find, without converting the decimals into vulgar fractions, the value of £$\cdot 473614\dot{2}$ + £ $506594\dot{1}$

188. What decimal of £1 must be added to $\cdot\dot{0}\dot{9}$ of £6, 13*s* 4½*d*. that the result may be 15*s* ?

189. How many times could $\cdot 0007$ of a shilling be taken from £41 and what decimal of a penny would be left over?

190. What time is it when the part of the day that has gone is $\dot{7}$ of the part that remains?

---

191. Without dividing, find the number of non-recurring decimal places in the decimal equal to $\frac{1}{192}$

192 Express $\dfrac{35(\frac{3}{7} + \cdot 8) + 99(\cdot\dot{5} + \frac{6}{11})}{63(\frac{3}{7} + \cdot\dot{5}) + 55(\cdot 8 + \frac{6}{11})}$ as a vulgar fraction in its simplest form

193. Reduce $\dfrac{1}{2 \times 3 \times 4} + \dfrac{1}{3 \times 4 \times 5} + \dfrac{1}{4 \times 5 \times 6}$ to a decimal

194 Prove that $\frac{1}{17} = \cdot 05882\frac{6}{17}$, and hence without further division obtain the value of $\frac{1}{17}$ to ten places of decimals

195 Divide the square of $\cdot 017$ by the square of 68

196. The quotient of $66420666 \div 7358$ is 9027 hence write down at sight the quotients of $6642\cdot 0666 \div 7358000$ and $\cdot 066420666 \div \cdot 007358$

197 Find the sum of $1020\cdot\dot{3}$, $102\cdot 0\dot{3}$, $10\ \dot{2}0\dot{3}$, 10203, $\cdot 10203$ and $1\cdot 0\dot{2}0\dot{3}$

198. Simplify $\frac{7}{12}$ of $5\ 8 + 2\frac{4}{7}$ of $\cdot\dot{9}0476\dot{1} - 4\frac{1}{3}$ of $5\dot{6}9230\dot{7}$

199. Express the difference between $\cdot\dot{3}7\dot{8}$ of 13*s* 10½*d* and $\cdot 3\dot{7}\dot{8}$ of 16*s* 6*d* as the decimal of $\cdot 42\dot{6}$ of £1, 17*s* 6*d*

200. What day of the week is it when at noon on that day $3\dot{5}7142\dot{8}$ of the week is gone?

XLII. THE UNITARY METHOD,

OR SINGLE RULE OF THREE

1. If 8 lbs. cost 10*s*, what will 12 lbs. cost at the same rate?
2. If 15 lbs cost 20*s*, what will 21 lbs. cost at the same rate?
3. If 14 tons cost £8, what will 35 tons cost at the same rate?
4. If 10 tons cost £6, what will 45 tons cost at the same rate?
5. If 6 yds. cost 15*s*, what will 22 yds. cost at the same rate?
6. If 18 yds cost 21*s*, what will 81 yds. cost at the same rate?
7. If 3 lbs cost 5*s*, what will 2 lbs cost at the same rate?
8. If 4 lbs. cost 7*s*., what will 3 lbs cost at the same rate?
9. If 8 tons cost £6, what will 5 tons cost at the same rate?
10. If 10 tons cost £8, what will 7 tons cost at the same rate?
11. If 9 yds cost 12*s*, what will 2 yds cost at the same rate?
12. If 18 yds. cost 27*s*, what will 8 yds cost at the same rate?
13. If 8 lbs. cost 12*s*, how many lbs cost 15*s* at the same rate?
14. If 6 lbs. cost 10*s*, how many lbs. cost 35*s* at the same rate?
15. If 9 tons cost £6, how many tons cost £14 at the same rate?
16. If 12 tons cost £8, how many tons cost £18 at the same rate?
17. If 18 lbs cost 15*s*, how many lbs. cost 10*s*. at the same rate?
18. If 12 lbs cost 39*s*, how many lbs cost 26*s* at the same rate?
19. If 28 yds cost 21*s*, how many yards cost 15*s*. at the same rate?
20. If 9 yds cost 57*s*., how many yards cost 38*s* at the same rate?
21. If a train travel 8 miles in 15 minutes, how many miles will it travel in 10 minutes at the same rate?
22. If a train travel 20 miles in 24 minutes, how many miles will it travel in 16 minutes at the same rate?
23. If a watch gain 25 seconds in 10 days, how many seconds will it gain in 18 days at the same rate?
24. If a watch lose 15 secs. in 6 days, how many seconds will it lose in 7 days at the same rate?
25. If a journey of 78 miles take 13 hours, how many hours would a journey of 60 miles take at the same rate?
26. At the rate of 42 miles in 4 hours, how many hours would it take to travel 91 miles?
27. If in 4 hours a man travel 26 miles, how far would he travel in 6 hours at the same rate?
28. If a steamer travel 161 miles in 14 hours, how far would it go in 8 hours at the same rate?

29. If in 15 weeks a labourer earn £12, in how many weeks would he earn £28?

30. If in 51 weeks a labourer earn £34, in how many weeks would he earn £10?

31. If 2 men hoe a field in 6 days, how long would 3 men take?

32. If 7 men hoe a field in 3 days, how long would 21 men take?

33. If 5 men can do a piece of work in 4 days, how long would 4 men take to do as much?

34. If 8 men can do a piece of work in 27 days, how long would 12 men take to do it?

35. If 4 men can mow a field in 5 days, how many days would 10 men take to mow it?

36. If 8 men can mow a field in 3 days, how long would 12 men take to mow it?

37. If 3 men can do a piece of work in 6 days, how long would 2 men take to do it?

38. If 49 men can do a piece of work in 32 days, how long would 28 men take to do as much?

39. If 18 men in 10 days do a certain amount of work, how long would 45 men take to do it?

40. If 24 men can finish a piece of work in 45 days, in how many days would 60 men finish it?

41. If 4 men can do a piece of work in 90 days, how many men could do the same in 36 days?

42. If 123 men can finish a certain work in 18 days, how many men would be required to do as much work in 2 days?

43. If 30 men took 14 days for a certain task, how many men could have done it in 20 days?

44. If 171 men take 12 days to perform a certain task, how many men would be able to complete the same in 19 days?

45. How long would food, sufficient to feed 35 men for 20 days, supply 50 men at the same rate?

46. How long would a supply of food, sufficient to feed 16 men for 27 days, supply 36 men with the same rations?

47. If a stack of hay will feed 8 horses for 18 weeks, how many horses would it feed for 16 weeks?

48. If a quantity of corn will feed 20 horses for 45 days, how many horses will it feed equally well for 36 days?

49. If I lend a man £40 for 3 months, for how many months should he lend me £60 in return?

50. If I lend a man £100 for 39 weeks, for how long ought he to lend me £75 in return?

51. If 4 lbs of sugar cost $10d$, what will 7 lbs. of the same kind of sugar cost?

52. If 3 lbs of tea cost $5s.$, what will 8 lbs of the same kind of tea cost?

53. If 8 lbs of sirloin cost $6s\ 4d$, what will 14 lbs. cost?

54. If 7 yards of calico cost $4s\ 8d$, what will 13 yards of the same kind cost?

55. If 12 yards of ribbon cost $7s\ 6d$, what will 40 yards of the same ribbon cost?

56. If 6 lbs of lard cost $4s\ 3d$, what is the cost of 10 lbs.?

57. If 28 cows cost £308, what, at the same rate, would 13 cows cost?

58. If 54 horses cost £3600, what, at the same rate, would 42 horses cost?

59 When 4 lbs of fresh butter cost $5s$, what is the cost of 3 lbs.?

60. When a 9-lb leg of mutton costs $6s\ 4\frac{1}{2}d$, what should be the price of a 7-lb. leg?

61. If 40 articles are worth £1, $16s$, what is the value of 35 of them?

62. Find the value of 56 articles, any 16 of which are worth £2, $10s$.

63 If 26 gross of pens cost £1, $4s$, what would 65 gross of the same kind of pens cost?

64. When 51 score of cabbages cost $8s\ 6d$, what should be given for 85 score?

65. If 17 sheep are worth £48, $9s$, what is the value of 11 similar sheep?

66. If 15 lambs are worth £32, $5s$, what is the value of 52 equally fine lambs?

67. If 5 copies of a book cost $5s\ 7\frac{1}{2}d$, what is the cost of 23 copies?

68. If 41 copies of a book cost $15s\ 4\frac{1}{2}d$, what is the price of 3 copies?

69. When 3800 fleeces are worth £950, what is the value of 8300 similar fleeces?

70. If 2640 fleeces sell for £660, what would 6240 fleeces sell for at the same rate?

71. What is the cost of 5 lbs. of tea, if 3 lbs. of the same tea cost $5s\ 7\frac{1}{2}d$?

72. What is the cost of 9 lbs. of coffee at the rate of $7s.\ 8\frac{1}{4}d.$ for 5 lbs.?

73. Find the cost of 91 oranges, at 7 for sixpence

74. Find the cost of 156 oranges, at 13 for a shilling.

75. Find the value of a dozen articles, any five of which are worth £1, 18*s* 9*d*

76 Find the value of 3 dozen articles when any seven of them are worth £1, 1*s* 10½*d*

77. Find the cost of 39 lambs at the rate of £5, 18*s* for four

78. Find the cost of 85 sheep at the rate of £16, 15*s* for five

79. What should be the rent of 7 acres, if the rent of 145 acres of land of the same kind is £228, 7*s* 6*d* ?

80 Find the rent of 111 acres, if the rent of 10 acres of the same kind of land is £18, 6*s* 8*d*

81. If a labourer earns £5, 5*s* in 7 weeks, in how many weeks does he earn £8, 5*s* ?

82. If a labourer earns £7, 4*s* in 9 weeks, in how many weeks does he earn £4 ?

83 If 6 sheep cost £13, 10*s*, how many, at the same rate, can be bought for £33, 15*s* ?

84 If 7 sheep cost £22, 8*s*, how many, at the same rate, can be bought for £60, 16*s* ?

85. How many eggs, at 14 for a shilling, can be bought for £1, 2*s* 6*d* ?

86 When eggs are sold at 24 for 1*s*, how many can be bought for £1, 11*s* 5½*d* ?

87 If a pigeon fly 17½ miles in 15 minutes, how long will it take to go 14 miles?

88 If an express train travel 4½ miles in 5 minutes, how long will it take to travel 36 miles at the same rate?

89 How far will a train go in 35 minutes at the rate of 6 miles in a quarter of an hour?

90. How far will a train go in 21 minutes at the rate of 50 miles per hour?

91. How long would 40 men take to do the work which 25 men can do in 24 days?

92 How long would 27 men take to do the work which 36 men can do in 12 days?

93. How many miles ought 22 tons to be carried for the money which pays the carriage of 10 tons for 33 miles?

94. How far should 14 cwts be carried for the money which is paid for the carriage of 3 tons for 21 miles?

95. How long can 13 horses be kept on the food which lasts 7 horses for 52 days?

96 If a chest of tea last a family of 10 persons for 22 weeks, how long should an equally large chest last a family of 8 persons?

97. How far should 48 tons be carried for the same sum as will pay for the carriage of 36 tons for 144 miles?

98. If the carriage of $5\frac{1}{4}$ cwts for 114 miles cost 4*s* 8*d*, how far ought $3\frac{1}{2}$ cwts to be carried for the same money?

99. If the first-class railway fare for a journey of 57 miles be 9*s* 6*d*., what should be the first-class fare for a journey of 122 miles?

100. If the second-class fare for 102 miles is 10*s* $7\frac{1}{2}d$, what should be the second-class fare for 272 miles?

101. If 17 yards of silk cost £4, 8*s* $6\frac{1}{2}d$, what would 120 yards cost?

102. If 13 yards of cloth cost £7, 11*s* 8*d*, what would 73 yards cost?

103. If 18 gallons of beer cost £1, 1*s*, what is the cost of 3 pints?

104 If 18 gallons of beer cost £1, 1*s*, what is the cost of 11 pints?

105. If 3 cwts cost 7*s* 9*d*, what will 4 tons 4 cwts. cost?

106 If 3 lbs 3 ozs cost 2*s*. 10*d*., what will 17 lbs 7 ozs. cost?

107. If 1 cwt 2 qrs. 14 lbs of sugar cost £3, 0*s*. 8*d*, what is the cost of 7 lbs.?

108. If 1 cwt 3 qrs 4 lbs. of sugar cost £1, 17*s* 6*d*, what is the cost of 11 lbs?

109. If 14 cwts. 8 lbs of steel cost £54, 3*s* 6*d*, what is the cost of 17 cwts 16 lbs?

110. If 3 cwts. 12 lbs. cost £13, 13*s*. 6*d*, what is the cost of 5 cwts 20 lbs?

111. If 9 gallons of beer cost 10*s*, what is the value of a pint and a half?

112. If 3 lbs of tea be worth 10*s*, what is the value of an ounce and a half?

113. If 17 yards of calico cost 12*s* 9*d*, what is the cost of $12\frac{1}{4}$ yards?

114. If 15 yards of flannel cost £1, 13*s* 9*d*, how much will $20\frac{1}{3}$ yards cost?

115. If $4\frac{5}{8}$ yds. of cloth cost £4, 14*s* 3*d*, what is the cost of $5\frac{1}{12}$ yards?

116. If $6\frac{1}{4}$ tons of coal cost £6, 15*s*. 5*d*, what is the price of 5 tons 3 cwts.?

117. If 2 tons 3 cwts. 3 qrs. of sugar cost £61, 5*s* 0*d*., what will 5 cwts. 1 qr 16 lbs. cost?

118. If 4 cwts. 2 qrs. 7 lbs. of coffee cost £36, 3*s* 11*d*., what is the cost of 3 qrs. 14 lbs.?

119. If 2 roods 15 poles of land cost £59, 7*s*. 6*d*., what would 8 ac 0 ro 17 poles of the same land cost?

120. When the value of 47 lbs. 1 oz. Troy of silver was £184, 16*s*. $0\frac{1}{2}d$., what was the value of 84 lbs 9 ozs.?

121. If $1\frac{1}{4}$ lbs. of butter cost 1*s* $5\frac{1}{2}d$., how much can be had for $5\frac{1}{4}d$.?

122. If $2\frac{1}{2}$ lbs. of butter cost 2*s*. $8\frac{1}{2}d$., how much can be bought for $3\frac{1}{4}d$.?

123. If 12 bushels of wheat cost £2, 0*s* 9*d*., how many bushels can be bought for £55, 0*s* 3*d* ?

124. How many bottles of wine can be bought for £26, 3*s* 3*d*. if 15 dozen cost £34, 2*s* 6*d* ?

125 If £3, 10*s* is the wages of a servant for 18 weeks, for how many weeks would her wages be £8, 15*s* ?

126. If 2 qrs 5 bush of wheat be worth £3, 11*s* 9*d*, how much can be bought for £85, 8*s* 4*d* ?

127 If 45 yards of silk cost £29, 18*s* 3*d*, how many yards can be bought for £9, 19*s* 5*d* ?

128. If 288 yards of calico cost £4, 14*s* 10½*d*, how many yards can be bought for £1, 11*s* 7½*d* ?

129. If the scale of a map is 25 miles to an inch, what length on the map represents 415 miles?

130. A map is made on the scale of 6 miles to an inch, what distance on the map represents 375 miles?

131. How many sheep worth £2, 17*s* 6*d* each should be given in exchange for 69 pigs worth £2, 12*s* 6*d* each?

132. How many horses worth £21, 2*s* 6*d* each should be given in exchange for 52 cows worth £11, 7*s* 6*d* each?

133 If 7 francs are worth 5*s* 8¼*d*, what is the value of 20570 francs?

134. If 3 dollars are worth 12*s* 6*d*, what is the value of 73025 dollars?

135 If 6 ac 3 ro 18 po of land be let for £21, 9*s* 9*d*, what, at the same rate, is the rent of 34 ac 1 ro 10 po ?

136 If 3 ac 2 ro 12 po of land grow 15 qrs 3 bush 2 pks of wheat, what should be the produce of 7 ac 3 ro 36 po of the same land?

137 If 44 yds 1 ft 9 in of cloth cost £42, 7*s* 9*d*, what is the cost of 74 yds. 11 in ?

138. If 7 cwts 2 qrs 11 lbs of flour cost £7, 1*s* 10*d*, what would 5 tons 17 cwts of the same kind of flour cost?

139. If 13 cwts 2 qrs 16 lbs of rice cost £15, 17*s* 6½*d*, what will 3 tons 1 cwt 1 qr 16 lbs cost?

140. If 1 ton 13 cwts 1 qr 9 lbs 5 ozs cost £186, 13*s* 3¾*d*, what will 17 cwts 7 ozs cost?

141. If 1000 square yards of meadow produce a load of hay, how many loads should 25 acres yield?

142. How many miles will a coach, travelling at the rate of 7½ miles an hour, go between 10 15 A M and 5 45 P M ?

143. If 4 tons are carried 193 miles for 17*s* 4*d*, what will it cost to carry 6 tons 3 cwts 3 qrs the same distance?

144. If, travelling at the rate of 12½ miles an hour, I require 15½ hours to complete a journey, in how many hours shall I complete it if I increase my rate of travelling by 7½ miles an hour?

145. A map is made on the scale of 12 miles to an inch; how far apart are two towns the names of which on the map are $7\frac{1}{2}$ inches apart?

146. Edinburgh is 330 miles from London; how far apart should these names be placed on a map the scale of which is 30 miles to an inch?

147. What should be the price of 4 lbs. 7 ozs. 18 dwts. of gold when 2 lbs 11 ozs. 15 dwts. is worth £116, 17*s.* 6*d.*?

148. If gold is worth £4, 3*s.* 4*d.* per ounce, what is the value of a bar weighing 4 lbs 5 oz. 5 dwts. 21 grs.?

149. The shadow of an upright staff 6 ft. high measures 7 ft. 6 in.; how high is a tower whose shadow, at the same time, measures 40 yards?

150. What is the height of a tower whose shadow measures 25 yards 10 in., when an upright stick 2 ft. 8 in. long has a shadow 2 ft. 2 in. long?

151. Silver is at 3*s* 6*d* per oz., gold at £3, 17*s* $10\frac{1}{2}$*d.* per oz.; how much gold is equivalent to 7 lbs. 5 ozs. of silver?

152. If gold is £3, 17*s* 6*d* per ounce, and silver is £2, 14*s.* per lb., what is the value of a piece of silver of equal weight with a piece of gold worth £1085?

153. If 68 chests of tea, each weighing 2 qrs. 17 lbs., cost £625, 13*s.* 5*d.*, how much did 34 lbs of the tea cost?

154. If a field of $3\frac{3}{4}$ acres is let for £13, 15*s* a year, what should be the rent of land of the same quality measuring 15 acres 2 roods 4 poles 11 sq yards?

155. If the rent of 810 A. 0 R. $33\frac{1}{2}$ P. be £1620, 8*s.* $4\frac{1}{2}$*d.*, how much land, at the same rent per acre, would be rented for £1389, 15*s.* 9*d.*?

156. If 26 acres 2 roods 25 poles of land be sold for £4691, 10*s*, what, at the same rate, will 415 ac 30 po $2\frac{3}{4}$ sq yds. sell for?

157. A clock which gains uniformly 10 seconds per hour is set right at 6 P.M on April 22, when will it again denote correct time?

158. A clock which gains at a uniform rate shows the correct time at noon on May 1st, and again at noon on August 9th How much has it gained at 3 P.M. on June 7th?

159 A watch loses 23 secs. every 13 days: it is 10 minutes fast at noon on May 18th; how much fast or slow will it be at noon on Nov. 3rd?

160. A clock gains uniformly 55 seconds per day. It is 3 minutes too fast at noon on Monday; what time will it indicate at 12 minutes past 7 on the next Saturday morning?

XLIII. THE UNITARY METHOD (*continued*), OR DOUBLE RULE OF THREE

1. If 4 men in 6 days earn £5, what will 15 men earn in 8 days?*
2. If 10 men in 12 days earn £18, what will 28 men earn in 15 days?
3. If £7 is earned by 6 men in 5 days, what will 14 men earn in 12 days?
4. If £20 is earned by 8 men in 12 days, what will 9 men earn in 22 days?
5. If 8 men earn £9 in 10 days, what will 20 men earn in 12 days?
6. If 4 men earn £16 in 15 days, what will 7 men earn in 24 days?
7. If 6 men in 3 days mow 12 acres, how many acres will 7 men mow in 9 days?
8. If 8 men in 4 days mow 28 acres, how many acres will 12 men mow in 6 days?
9. If 10 men mow 15 acres in 2 days, how many acres will 18 men mow in 6 days?
10. If 6 men mow 21 acres in 5 days, how many acres will 11 men mow in 9 days?
11. If 4 men in 15 days earn £16, what will 3 men earn in 10 days?
12. If 6 men in 5 days earn £7, 10*s*, what will 4 men earn in 3 days?
13. If 10 men in 3 days mow 20 acres, how many acres will 9 men mow in 2 days?
14. If 6 men in 5 days mow 24 acres, how many acres will 5 men mow in 4 days?
15. If 4 men in 6 days earn £5, 10*s*, what will 30 men earn in 2 days?
16. If 10 men in 12 days earn £18, what will 25 men earn in 7 days?
17. If £20 is earned by 6 men in 5 days, what will 2 men earn in 9 days?
18. If 8 men earn £9 in 10 days, what will 3 men earn in 22 days?
19. If 10 men mow 15 acres in 2 days, how many acres will 6 men mow in 5 days?

---

* Here, and in all similar questions, the words "at the same rate" are implied

20 If 3 men mow 20 acres in 8 days, how many acres will 11 men mow in 6 days?

21 If £8 is earned in 5 days by 6 men, how many men would earn £4 in 3 days?

22. If £20 is earned in 12 days by 8 men, how many men would earn £15 in 9 days?

23 If 4 men earn £6 in 12 days, how many men will earn £15 in 10 days?

24. If 10 men in 12 days earn £15, how many men will earn £36 in 8 days?

25. If 8 men earn £14 in 10 days, how many men will earn £63 in 15 days?

26. If the wages of 29 men for 12 days are £87, how many men should receive £405 for 54 days?

27. If 8 men in 4 days mow 28 acres, how many men will mow 21 acres in 6 days?

28. If 6 men mow 20 acres in 5 days, how many men will mow 70 acres in 3 days?

29 If 16 men reap 76 acres in 6 days, how many men will reap 114 acres in 9 days?

30. If 7 men reap 57 acres in 9 days, how many men will reap 38 acres in 2 days?

31. If 10 men earn £18 in 12 days, in how many days will 35 men earn £21?

32. If 6 men earn £9 in 5 days, in what time will 15 men earn £4, 10*s*?

33 If 10 men mow 15 acres in 2 days, in how many days will 3 men mow 18 acres?

34. If 8 men in 4 days mow 28 acres, in how many days will 6 men mow 21 acres?

35. If 48*s* is charged for the carriage of 8 cwts. for 128 miles, how far would 4 cwts. be carried for 6*s*?

36. What is the cost of the carriage of 15 cwts. for 12 miles, if 6 cwts is carried 20 miles for 4*s*?

37. If $10\frac{1}{2}$ acres are mown by 3 men in 3 days, in what time could 112 acres be mown by 96 men?

38. If $11\frac{2}{3}$ acres are mown in 2 days by 6 men, how long will 3 men take to mow $17\frac{1}{2}$ acres?

39. If 14 men earn £9, 16*s*. in 7 days, what will 20 men earn in 4 days?

40. If 4 men earn £3, 18*s*. in $6\frac{1}{2}$ days, what will 17 men earn in $21\frac{1}{2}$ days?

41. If 17 men earn £6, 7*s* 6*d* in 2 days, how long will it take one man to earn £11, 5*s* ?

42 If 15 labourers earn £68, 12*s* 6*d* in 24 days, how long will 20 labourers take to earn £137, 5*s* ?

43. If the rent of 8 acres for half a year be £4, 16*s*, what is the rent of 23 acres for 3 months at the same rate?

44 If the rent of 9 acres be £11, 5*s* per year, find one quarter's rent of 350 acres at the same rate

45 If 4 chests, each containing 20 lbs of tea, cost £6, 6*s*. 8*d*., what is the cost of 15 chests each containing 90 lbs of the same kind of tea?

46. If the carriage of 2 tons for 60 miles be £1, 12*s* 6*d*., what, at the same rate, is the charge for the carriage of 17 tons for 14 miles?

47. If 16 horses eat 12 pecks of oats in 10 days, how long will 63 bushels last 56 horses?

48. If 7 bushels 2 pecks of corn feed 10 horses for 7 days, how many horses will 3 quarters 6 bushels feed for 10 days?

49. If 8 men dig a field of $9\frac{1}{2}$ acres in 19 days, how long will 5 men take to dig a field of 5 ac 1 ro 10 po ?

50 If the carriage of 13 cwts 2 qrs for 124 miles be £6, 9*s* 4*d*, how far can 1 ton 11 cwts be carried for £2, 8*s* 6*d* ?

51. If 4 lbs of bread cost $5\frac{1}{2}d$ when wheat is 44*s* per quarter, what should be the cost of 6 lbs of bread when wheat is 48*s* per quarter?

52. If a five-penny loaf weighs 4 lbs when wheat is 35*s* per qr, what should be the weight of a three-penny loaf when wheat is 32*s* per qr ?

53 If 6 men do a piece of work in 10 days, working 8 hours a day, how many hours a day must 5 men work to do as much in 9 days?

54. How many men, in 14 days of 9 hours each, can do as much work as 21 men in 18 days of 8 hours each?

55. In how many days of 12 hours each would 100 men do as much work as 45 men can do in 16 days of 10 hours?

56. How many hours per day must 24 men work in order to do in 5 days twice as much work as 25 men, working 6 hours a day, can do in 4 days?

57. How many days would 36 men, working 7 hours a day, require to do a piece of work which 24 men, working 9 hours 20 minutes a day, can do in 9 days?

58 How many men, working 11 hours per day, can do in 91 days as much work as 143 men, working 9 hours per day, can accomplish in 133 days?

59. If 6 men do a piece of work in 30 days of 9 hours each, how many men could do ten times as much work in 25 days of 8 hours?

60. If 25 men do a certain amount of work in 14 days of 8 hours each, in how many days of 10 hours each would 7 men do $\frac{3}{5}$ of the same amount of work?

61. If 3 men reap 8 acres in 5 days, working 8 hours a day, in how many days will 8 men, working 12 hours a day, reap 192 acres?

62. If 12 men, working 8 hours a day, could do $\frac{4}{5}$ of a piece of work in 20 days, in how many days could 15 men, working 10 hours a day, do $\frac{7}{8}$ of it?

63. If 26 men, working 8 hours a day, earn £41, 12*s.* in 6 days, how much will 17 men, working 9 hours a day, earn in 39 days, if they all are paid at the same uniform rate per hour?

64. If 12 men, working 9 hours a day, earn £37, 16*s* in 12 days, in how many days of 8 hours each would 21 men earn £102, 18*s*, if paid at the same rate per hour?

65. If 3 pumps in 4 hours, working 45 minutes per hour, can discharge 16200 gallons of water from the hold of a ship, how many gallons can 5 pumps, working 40 minutes per hour for 72 hours, discharge?

66. If 5 men dig a trench in $1\frac{1}{2}$ days, working 9 hours a day; how long would one man take to dig a trench half as long again, of the same depth and half the width, working 10 hours a day?

67. If 14 men can build a wall 19 feet long and 4 feet high in 6 days of 10 hours each; how many men would be required to build a wall of the same kind, 38 feet long and 3 feet high, in 9 days of 7 hours each?

68. How many men, working 8 hours a day, could dig a trench 8 yards long, 8 feet wide, and 8 inches deep, in the same number of days that 9 men, working 12 hours a day, could dig a trench 12 yards long, 12 feet wide, and 12 inches deep?

69. If 16 men build a wall 25 yards long and 6 feet high in 14 days of 9 hours each, in how many days of 8 hours each would 7 men build a wall 30 yards long and 5 feet high?

70. If 20 men, working 7 hours per day, build a wall 800 feet long and 10 feet high in 14 days, how many hours a day must 15 men work to build a wall 900 feet long and 15 feet high in 21 days?

71. If 6 men dig a trench 15 yards long and 2 yards wide in 3 days of 12 hours each, how many days of 8 hours each will 8 men take to dig a trench of the same depth 10 yards long and 8 yards wide?

72 If 18 men dig a trench 630 yards long, 3 yards wide and 2 ft. deep in 12 days of 10 hours each, how many men would be required to dig a trench 330 yards long, 7 yards wide and 3 feet deep in 22 days of 9 hours each?

73. If 3 compositors, in 15 days of 10 hours each, can set in type 300 pages averaging 55 lines per page and 40 letters per line, in how many days of 8 hours each will 5 compositors set up 400 pages, averaging 60 lines per page and 44 letters per line?

74. If 6 compositors, in 16 days of $10\frac{1}{2}$ hours each, can set in type 720 pages, each page containing on the average 60 lines, and each line 40 letters, in how many days of 7 hours each will 9 compositors set in type 960 pages, averaging 45 lines per page and 50 letters per line?

75. If the carriage of 9 tons 11 cwts 1 qr for 114 miles cost £3, 11*s* 3*d*, how much should be charged for carrying 5 tons 4 cwts 2 qrs 39 miles further?

76 If, when meat is 9*d* per lb, it costs £11, 16*s* 3*d* to supply a family of 12 persons for 24 weeks, what should be the cost of meat for a family of 18 persons for 14 weeks when the price has risen 3*d* per lb?

77. If 3 men working 8 hours a day can build a wall 80 feet long, 6 feet high and 2 feet thick, in 5 days, how many men working 7 hours a day will be required to build a wall of similar materials 100 feet long, 7 feet high, and twice as thick, in 25 days?

78. If 124 men in $5\frac{1}{2}$ days of 11 hours each can dig a trench $232\frac{1}{2}$ yards long, 3 ft 8 in wide, and 2 ft 4 in deep, in how many days, of 9 hours each, will 36 other men dig a trench $337\frac{1}{2}$ yards long, 5 ft $7\frac{1}{2}$ in wide, and 3 ft 6 in deep, supposing that each man of the second set is capable of doing as much work in 6 hours as each man of the first set can do in 7 hours?

79. A garrison of 3000 men is supplied with provisions for 15 weeks at the rate of 13 ounces per man per day How many men must leave in order that the provisions may last those who remain for 26 weeks, allowing each man 10 ounces per day?

80. In 11 days 250 men working 9 hours a day complete 528 yds. of an embankment which is to be $1\frac{1}{2}$ miles long How many additional men must be employed in order that, if all work 10 hours a day, the embankment may be finished in 33 days more?

MONEY.

4 farthings (f) = 1 penny (d)
12 pence = 1 shilling (s).
20 shillings, or 240 pence = 1 pound (£)

Note—A *florin* = 2*s*, a *crown* = 5*s*; a *guinea* = 21*s*
A *sovereign* = 20*s*, or 8 half-crowns, or 10 florins, or 40 sixpences or 80 threepences, or 480 half-pence, or 960 farthings

---

TIME

60 seconds (*sec*) = 1 minute (*min.*).
60 minutes = 1 hour (*hr*)
24 hours = 1 day
7 days = 1 week (*wk*)
365 days = 1 (common) year (*yr*)
366 days = 1 leap year

Note—A *common year* = 52 weeks + 1 day, a *century* contains 100 years, a *lunar* month contains about 4 weeks

The year is divided into 12 *calendar* months, of which February, in common years, contains 28 days; and, in leap years, 29 days,

"Thirty days hath September, April, June, and November;"

and the remaining seven calendar months each contain 31 days

Leap year occurs once in four years (except at the end of a century) In order to discover whether any year (not the last in a century) is a leap year, *divide the number of the year by 4, and if there is* no *remainder it is leap year.* But if the year ends a century it is *not* leap year *unless the first two figures divide by 4 without remainder*

AVOIRDUPOIS WEIGHT

16 drams (*dr*) = 1 ounce (*oz*).
16 ounces, or 7000 grains = 1 pound (*lb*).
14 pounds = 1 stone (*st*).
28 pounds, or 2 stones = 1 quarter (*qr.*).
4 quarters, or 8 stones, or 112 pounds = 1 hundredweight (*cwt.*)
20 hundredweights = 1 ton.

Note.—This weight is used for all common substances, *e g.* coal, meat, &c.

TROY WEIGHT.

24 grains (*gr*) = 1 pennyweight (*dwt*).
20 pennyweights, or 480 grains = 1 ounce Troy (*oz Tr*).
12 ounces Troy = 1 pound Troy (*lb. Tr.*).

Note.—Troy weight is only used for gold, silver and jewellery
The *grain* alone is the same in both Avoirdupois and Troy weights.

Long Measure

12 inches (*in.*) = 1 foot (*ft.*).
3 feet, or 36 inches = 1 yard (*yd.*)
1760 yards = 1 mile (*mi.*).
$5\frac{1}{2}$ yards, or 11 half-yards = 1 pole (*po.*), rod, or perch (*per.*).
40 poles, or 220 yards = 1 furlong (*fur.*)
8 furlongs = 1 mile

Note.—A *chain* = 22 yards, or 4 poles, 100 links = 1 chain
A *fathom* = 6 feet; a *cable-length* = 100 fathoms, a *knot* = 6080 feet.
A *hand* = 4 inches A *league* = 3 miles
$2\frac{1}{4}$ inches = 1 *nail*, 4 nails = 1 quarter, 4 quarters = 1 yard.

Square Measure

144 square inches (*sq. in.*) = 1 square foot (*sq. ft.*)
9 square feet = 1 square yard (*sq. yd.*).
$30\frac{1}{4}$ (*i.e.* $5\frac{1}{2} \times 5\frac{1}{2}$) square yards, or 121 square quarter-yards = 1 square pole (*sq. po.*, or P), or square perch
40 square poles = 1 rood (*ro.* or R.).
4 roods, or 4840 square yards = 1 acre (*ac.*, or A.)
640 acres or 1760×1760 square yards = 1 square mile (*sq. mi.*)

Note.—A *square chain* = 100×100 square links, or 22×22 square yards, 10 square chains = 1 acre

Cubic Measure

1728 (*i.e.* $12 \times 12 \times 12$) cubic inches (*cub. in.*) = 1 cubic foot (*cub. ft.*)
27 (*i.e.* $3 \times 3 \times 3$) cubic feet = 1 cubic yard (*cub. yd.*).

Capacity

Liquids.
2 (imperial) pints (*pt.*) = 1 (imperial) quart (*qt.*)
4 quarts, or 8 pints = 1 gallon (*gal.*)

Dry goods, *e.g.* corn
2 gallons = 1 peck (*pk.*)
4 pecks, or 8 gallons = 1 bushel (*bush.*)
8 bushels = 1 quarter (*qr.*)

Note.—A *hogshead of beer* = 54 gallons, a *hogshead of wine* = 63 gallons; a *pipe* of wine = 2 hogsheads 4 *gills* = 1 imperial pint
6 "reputed" quarts (*i.e.* common wine bottles) contain a gallon
"A pint of pure water weighs a pound and a quarter"
A cubic foot of water weighs 1000 ounces

Number.—12 units = 1 dozen, 12 dozen = 1 gross 20 units = 1 score.

Paper.—24 sheets = 1 quire, 20 quires, or 480 sheets = 1 ream

EXERCISES.—PART II.

XLIV. VARIOUS APPLICATIONS OF THE FOREGOING METHODS.

A. AGENTS OF DIFFERENT POWERS, &c.

1. If the work of 12 boys is equivalent to that of 7 men, how long will it take 15 boys to do as much work as 21 men can do in 5 days?

2. If 40 women can do a piece of work in 20 days, in how many days could 15 men do it, the work of 5 women being equal to that of 3 men?

3. If two men can do as much as 3 boys, and if 6 men can complete a certain task in 6 days, how long would 5 men and 6 boys together take to do it?

4. How long would it take 6 men and 16 women to do the work which 12 men can accomplish in 3 days; the work of 3 men being supposed to be equal to that of 4 women?

5. If 3 boys eat as much as 4 men, and it cost £1, 14*s*. 6*d* to feed 3 men for a week, how much would it cost to feed, in a similar way, 7 boys for a week?

6. If 5 men and 9 boys could do a piece of work in 17 days, in how many days would 9 men and 12 boys do it; the work of 2 men being equal to that of 3 boys?

7. If 7 oxen eat 3 tons of hay in a month, how long would 35 tons feed 49 sheep, an ox consuming three times as much as a sheep?

8. If 8 men with 5 boys can do as much as 5 men with 18 boys, how many boys can do as much as 6 men?

9. If either 4 men, or 7 boys, could perform a piece of work in 5 hours, how long would 4 men and 21 boys take?

10. If 12 sheep, or 18 lambs, eat 96 bushels of turnips in a fortnight, how long will 14 sheep and 21 lambs take to eat 64 bushels?

11. If 7 men, or 12 boys, can hoe a field of 12 acres in 6 days working 10 hours a day, in how many days of 8 hours each can 14 men and 6 boys hoe a field of 60 acres?

12. If 5 men can mow 33 acres in 5 days working 11 hours a day, how many days would 4 men take to reap 32 acres working 10 hours a day, when it takes as long to reap 5 acres as to mow 6?

13 If 6 men earn as much as 9 women in a day, and 9 women as much as 12 boys, how long will it take 6 men, 9 women and 12 boys, working together, to earn £19, 16*s*, a boy's earnings being a 1*s* a day?

14 If 14 labourers could dig a field of potatoes in 5 days, how many additional men would be required to finish the work 3 days sooner?

15. Fifty navvies are engaged to make an embankment 3 miles long After working 6 days, and completing 6 furlongs, 30 of them strike work How long will the rest take to finish it?

16. A contractor undertook to finish a certain work in 60 days After 30 men had been employed for 20 days he found that only $\frac{1}{4}$ of the work was done How many extra men must he employ in order to complete the work in the specified time?

17 A garrison of 5400 men was provisioned for 13 weeks, when a reinforcement arrived and the provisions only lasted 9 weeks, what was the strength of the reinforcement?

18. A garrison of 5985 men is provisioned for 180 days, but after 72 days it is reinforced by 171 men How long do the provisions last?

19. A number of workmen perform a certain task in 60 days of 11 hours each How many extra hours a day would they have to work to do the same amount in the same time, taking a half holiday on the last day in each week?

20. Three men can do as much work as 5 boys the wages of 3 boys are equal to those of 2 men A work on which 40 boys and 15 men are employed takes 8 weeks and costs £350 how long would it take, and what would it cost, if 20 men and 20 boys were employed?

B. Assets, Dividends, &c

21. A bankrupt's debts are £6228, his available assets £188, 2*s* 9*d*. How much can he pay in the pound?

22 A bankrupt's assets are £10,252, 4*s* 8*d* and his liabilities £13,901, 6*s* 8*d* How much can he pay in the pound?

23 A bankrupt's debts are £3427, 6*s* 8*d* and his assets are only £2184, 18*s* 6*d* How much will a creditor receive to whom he owes £375, 10*s*.?

24. A bankrupt pays a dividend of 5*s* 6*d* in the pound One of his creditors received £27, 18*s* 3*d*, what was the amount of the debt due to this creditor?

25. A creditor received 16*s* 3*d* in the pound, and thereby lost £135, 10*s* how much was due to him?

26. A bankrupt paid a dividend of 8*s* 2*d* in the pound, and one of his creditors received £376, 5*s* 7*d* what did this creditor lose?

27. On a debt of £230, the creditor lost £142, 11s. $0\frac{1}{4}d$: what dividend did the bankrupt pay?

28. A bankrupt pays 9s 10d. in the pound; his assets, after deducting the "costs" of his bankruptcy, are £1888. find his debts?

29. A bankrupt's debts amount to £800, 10s, his assets to £223, 15s 6d Of his debts, £48, 5s have a prior claim and are paid in full what dividend can he pay on the rest?

30. A bankrupt's liabilities are £2672, 17s 6d., his assets £1640. Debts amounting to £325, 12s 6d are fully secured, and the legal expenses connected with his bankruptcy amount to £140, 15s · what sum will an unsecured creditor receive whose claim is for £47, 13s 4d?

C. Rates, Taxes, &c

31. Find the income-tax on £653 at 7d. in the pound.

32. How much will the income-tax, at 8d in the pound, amount to on an income of £523, 17s 6d?

33. A man's income-tax at 5d in the pound amounts to £13, 17s. 6d.; find his income.

34. What is the income of a man who pays £60, 13s. 8d income-tax at the rate of 10d. in the pound?

35. If the income-tax on £500, 10s. is £16, 13s. 8d., what does it amount to on £510, 15s. at the same rate?

36. If £49, 4s 8d is the income-tax on £1477, what is it on £1020, 10s. at the same rate?

37. If a parish is rated at 2s. 1d. in the pound, what do the rates amount to on a house valued at £85, 10s. per annum?

38. A rate of 2s $7\frac{1}{2}d.$ in the pound on a parish produces £745, 1s. 3d, what is the rateable value of the parish?

39. If the rates on a house rented for £26 amount to £2, 14s 2d. what is the rental value of another house in the same parish on which the rates amount to £6, 15s. 5d?

40. By the reduction of the income-tax from 7d. to 5d. in the pound a man saved £7, 15s. 9d.; what was his income?

41. Find the net income which remains after deducting income-tax at 8d. in the pound from £513, 12s. 6d.

42. Find the balance which results, after deducting income-tax at the rate of 9d. in the pound, from £5173, 6s. 8d.

43. Find the gross income which is reduced to £812 net by the payment of income-tax at 8d. in the pound.

44. A man's gross income is £666, 16s 8d.; find his net income after an income-tax of 6d in the pound has been paid.

45. A man's net income after payment of income-tax at 6d. in the pound is £1755; what would it be if the tax were 10d.?

D. Fractions

46 Three-eighths of a number is 27, find the number.

47. Five-sevenths of a number is 65, what is the number?

48. What number exceeds its fifth part by 44?

49 The sum of a number and its fifth part is 42, find the number

50 What is the number whose fifth part exceeds its seventh part by 6?

51. Find the number the sum of whose fifth and seventh parts is 108.

52 What number is that the sum of the third, fourth and fifth parts of which is 47?

53 What number exceeds the sum of its fourth, sixth and eighth parts by 231?

54. Find the sum of money $\frac{19}{20}$ of which amounts to £22, 8*s* 10½*d*

55. Five-ninths of a sum of money is 11*s* 1¾*d*, find the sum

56. Two-thirds of a certain sum of money exceeds $\frac{3}{5}$ of it by £1, 1*s* 1¼*d*., find the sum

57 A lost $\frac{2}{3}$ of all the marbles he had to B, won 17 from C, and then had 30; how many had he at first?

58. A paid $\frac{4}{5}$ of his money to B, and then had 8*s*. 2*d*. left What did he pay to B?

59 A man went $\frac{16}{19}$ of his journey by train, $\frac{5}{57}$ of it by tram, and walked the remaining 8 miles What was the length of his journey?

60. A pole stands with $\frac{1}{4}$ of its length in mud, $\frac{1}{3}$ in water, and 10 feet in air How long is it?

61 A boy spent $\frac{1}{6}$ of his money at one shop, $\frac{7}{8}$ of the remainder at another, and then had 2½*d*. left What had he at first?

62. After paying $\frac{5}{12}$ of the contents of my purse to one person, $\frac{5}{7}$ of the remainder to another, and $\frac{3}{8}$ of what then remained to a third, there still remained 1*s* 3*d*. How much was there in the purse at first?

63. A boy spent $\frac{3}{4}$ of his pocket-money in oranges, $\frac{1}{4}$ of what he had left in apples, and $\frac{3}{4}$ of what then remained in sweets, and had 1½*d* left What had he at first?

64. The liquid in a cask fills $\frac{7}{9}$ of the cask, after 13 gallons have been drawn off it fills $\frac{4}{7}$ of the cask How many gallons will the cask hold?

65. A cask is $\frac{7}{8}$ full; after 10 gallons have been drawn from it, it is one gallon short of half full. How many gallons will it hold?

66. If a man can do $\frac{3}{8}$ of a piece of work in $\frac{9}{16}$ of a day, how long will it take him to do the whole?

67. If in $\frac{3}{8}$ of a day a man does $\frac{15}{28}$ of his work, how long does it take him to do it all?

E. Time and Work.

68. A alone could mow a field in 10 days B alone could mow it in 15 days How long would it take them working together?

69 A can mow a field in 12 days, B in half that time. how long would they take together?

70. A can finish a piece of work in 21 hours; B could do the same in 28 hours how long would they take working together?

71. A alone can fill a cart in 24 minutes; B alone in half an hour: how long would it take them together?

72. If the cold-water tap alone is opened a bath is filled in 9 minutes; if the hot-water tap alone is opened, in 12 minutes: how long would the bath take to fill if both taps were opened together?

73. A alone can mow a field in 6 days, B alone in 8 days, C alone in 12 days; how long would they take working together?

74. Three taps can separately fill a cistern in 10, 12 and 15 minutes respectively; how long would the cistern take to fill if all three were opened together?

75. A can do a piece of work in $2\frac{1}{4}$ days, B in $4\frac{1}{2}$ days; how long would they take together?

76. A can do a piece of work in $2\frac{1}{2}$ days, B in $3\frac{1}{3}$ days, C in $3\frac{3}{4}$ days; how long would they take to do it working together?

77. A can mow 4 acres in 3 days; B can mow 3 acres in 2 days: how long will they take together to mow 34 acres?

78. If a man eats 3 loaves in 2 days, and his wife eats 2 loaves in 3 days, how long would 26 loaves last the man and his wife?

79. If a man eats four threepenny loaves in 5 days, and his wife eats three twopenny loaves in 4 days, what does their bread bill amount to for the month of April?

80. A cistern is filled from a tap in 15 minutes; it can be emptied by a waste pipe in 25 minutes. If the cistern is empty and both are opened, how long will it take to fill?

81. One tap fills a cistern in 8 minutes, another empties it in 15 minutes. If the cistern be empty and then both taps are opened, how long will it take to fill?

82. A and B together mow a field in 8 days; A alone could do it in 18 days: how long would it take B alone?

83. A and B together dig a garden in 7 days; with C to help them they could do it in 5 days how long would C take alone?

84. A, B and C together can finish a piece of work in 8 days; A alone could finish it in 20 days, and B alone in 24 days: how long would C alone take?

85. A alone can build a wall in 30 days, which B alone could build in 25 days. After A has worked alone for 10 days, B comes to help him. How many days after this will the work be done?

86 A and B together can do a piece of work in $7\frac{1}{2}$ hours, B and C in $10\frac{1}{2}$ hours, A and C in $8\frac{3}{4}$ hours How many hours would A, B and C together take to do the work?

87 A and B together reap 5 acres in 3 days, B and C reap 3 acres in 2 days, A and C reap 7 acres in 5 days How long would A alone take to reap $23\frac{1}{2}$ acres?

88. A is twice as good a workman as B how long would each alone take to do what they can do together in 4 days?

89. A can do a piece of work in 30 days, and B can do the same in 20 days. After A has worked alone for 13 days, B helps him for 2 more days and then C joins them, and the work is completed 3 days later How long would C alone take to do the whole work?

90. A, B and C together could do a piece of work in 60 days. They work at it together for 10 days, and then A leaves off, and B and C work together for 20 days more, when B leaves off, and C, working $\frac{1}{3}$ longer each day, then finishes the work alone in 96 more days C, working at his former rate, could have done the whole in 222 days How long would B take alone?

F. Time and Distance

Express in feet per second the rates—

91. 60 miles per hour

92. 45 miles per hour

93 $10\frac{1}{2}$ miles per hour

94 $3\frac{1}{2}$ miles per hour

95. How many yards does a horse, trotting at the rate of 8 miles an hour, go in half a minute?

96 How many yards does a steamer, which travels at the rate of 18 knots an hour, go in $\frac{1}{4}$ of a minute?

Express the following rates in miles per hour—

97. 11 feet per second

98. $27\frac{1}{2}$ feet per second

99. 32 feet per second

100. 625 feet per second

101. A body moves over 100 yards in 9 seconds; express its rate in miles per hour

102 How many miles would a man go in an hour who walks uniformly at the rate of 11 yards in 5 seconds?

103. If a train 88 yards long occupies 5 seconds in passing a signal-post, at the rate of how many miles per hour is it moving?

104. If telegraph posts are 55 yards apart, and a passenger finds that he passes 12 every minute, how many miles an hour is the train travelling?

105. How many seconds will a train 120 yards long, travelling at the rate of 30 miles an hour, take to pass completely over a bridge 122 yards long?

106. A train, 88 yards long, takes 12 seconds to pass completely over a bridge 66 yards long how many miles an hour is the train travelling?

107. If a train, 80 yards long, going 42 miles an hour, takes a quarter of a minute to pass completely through a station, how long is the station?

108. If a train leaves London at 5 A.M., travelling at the rate of 40 miles an hour, at what time should it reach a town 136 miles distant, 25 minutes being spent in stoppages?

109. Two men start from the same place at the same time, one towards the north at $3\frac{1}{2}$ miles an hour, the other towards the south at $8\frac{1}{2}$ miles an hour. how far apart are they at the end of $7\frac{1}{2}$ hours?

110. A, who travels $8\frac{1}{2}$ miles per hour, and B, who travels $7\frac{1}{2}$ miles per hour, start at the same time from places 56 miles apart to meet each other· how many hours after the start will they meet?

111. Two boys start at the same time on bicycles from places 80 miles apart and ride to meet each other, travelling at 11 and $9\frac{1}{2}$ miles an hour respectively: how far apart will they be in $3\frac{1}{2}$ hours?

112. My friend, who walks 4 miles an hour, starts from a place 15 miles off at the same time that I start to meet him. When we meet I have walked 6 miles; at what rate do I walk?

113. Two persons, A and B, start at the same time from places $2\frac{1}{4}$ miles apart, A walking away from B at the rate of $3\frac{1}{4}$ miles per hour, and B following him at the rate of 4 miles an hour. How long will it take B to catch A?

114. A starts from a certain place and travels uniformly at the rate of $7\frac{1}{2}$ miles per hour Two hours later B follows, travelling at the uniform rate of 12 miles per hour How long will it take B to catch A?

115. An express train leaves London for York at 10 A.M travelling at the rate of 42 miles an hour A slow train leaves York at 8.30 for London and travels at the average rate of 24 miles an hour How far from London will they be when they pass each other, the distance between London and York being 200 miles?

116. A train 88 yards long travelling at the rate of 45 miles per hour overtook a man running by the side of the line and passed him in 5 seconds. At what rate was this man running? The same train met another man walking at the rate of 3 miles per hour; how long did it take to pass him?

117. How long will it take a train 76 yards long travelling at the rate of 45 miles an hour to completely pass another train 78 yards long, travelling on a parallel line, at the rate of 35 miles per hour, when they move (i) in opposite directions, (ii) in the same direction?

118 An express train travelling at the rate of 52 miles an hour overtakes a coal train half as long again as itself, travelling on a parallel line at the rate of 16 miles an hour, and passes it completely in $12\frac{1}{2}$ seconds, find the length of each train

119. In a game at hare and hounds the hare had a start of 10 minutes and ran at the uniform rate of $5\frac{3}{4}$ miles per hour, the hounds followed at the rate of $6\frac{1}{2}$ miles an hour The length of the course was 8 miles Was the hare caught?

120. A starts 3 minutes after B for a place $4\frac{1}{2}$ miles distant B, on reaching his destination, immediately returns, and after walking a mile meets A If A's speed be $3\frac{1}{3}$ miles per hour, what is B's speed?

121. A thin candle 8 inches long which burns at the rate of 3 inches in 2 hours, and a thick candle $5\frac{1}{2}$ inches long which burns at the rate of 2 inches in 3 hours, are lighted at the same time. After what interval will (i) both be the same length; (ii) the thick candle one inch longer than the thin one?

122. If the current of a river flow at the rate of 2 miles per hour and it take a man 3 hours to row 9 miles up the stream, how long will it take him to return?

Clocks

At what time do the hands of a clock point in the same direction—

123. Between 3 and 4? 124. Between 7 and 8?

125. Between 5 and 6? 126. Between 4 and 5?

At what time are the hands in opposite directions—

127. Between 2 and 3? 128 Between 10 and 11?

129. Between 8 and 9? 130 Between 3 and 4?

At what times are the hands at right angles—

131. Between 6 and 7? 132. Between 1 and 2?

133. Between 12 and 1? 134. Between 9 and 10?

135. How soon after 4 o'clock will the hands of a clock be separated by an interval of 6 minute-spaces on the face?

136. At what time between 9 and 10 will the minute hand be exactly 8 minute-spaces in advance of the hour hand?

137. It is between 4 and 5 o'clock, and the minute hand is 10 minute-spaces in advance of the hour hand What is the exact time?

138. It is between 11 and 12 o'clock, and the minute hand is 5 minute-spaces behind the hour hand What is the exact time?

G. RACES.

139. A can give B 5 yards' start in a race of 100 yards, and B can give C 4 yards in the same distance. How many yards can A give C?

140. A can give B 12 yards in 100, and B can give C 35 yards in $\frac{1}{4}$ mile How many yards can A give C in 200?

141. A can score 100 points in a game while B scores 80, and B can score 100 while C scores 75. how many can C score while A scores 50?

142. A can give B 1 point in 10 in a certain game; B can give C 1 in 10, and C can give D 1 in 10: how many points can A give D in 1000?

143. A can give B 20 yards and C 41 yards start in a quarter of a mile. B can give C a start of 3 seconds over the same distance How long does C take to run a quarter of a mile?

144. A can give B 10 yards and C 19 yards in 100 yards: B can give C $1\frac{1}{4}$ seconds in the same distance How long does A take to run 100 yards?

H. CHAINS.

145. If 3 geese are worth 10 ducks, and 3 ducks are worth 4 chickens, and a couple of chickens cost 4*s*. 6*d*., what is the value of a goose?

146. If £1 is worth $25\frac{1}{2}$ francs, and 20 francs are worth $9\frac{3}{8}$ Dutch florins, how many Dutch florins are equivalent to £50?

147. If 35 napoleons are worth 192 thalers, 10 thalers worth 7 dollars, and 2 dollars worth 5 florins, how many napoleons are worth 144 florins?

148. If 6 pears are worth as much as 9 apples, 5 apples as much as 12 plums, 15 plums as much as 2 apricots, 4 apricots as much as 3 peaches, and 1 peach as much as 25 strawberries, how many strawberries should be given in exchange for 11 pears?

149. A clerk A can copy 5 lines while B copies 4, B can copy 2 while C copies 3, C copies 7 while D copies 8, and D copies 14 while E copies 17. how many lines can E copy while A copies 490?

150. If $\frac{1}{2}$ lb of tea costs as much as $\frac{3}{4}$ lb. of coffee, $\frac{1}{2}$ lb. of coffee as much as $\frac{3}{4}$ lb of chocolate, $\frac{1}{2}$ lb. of chocolate as much as $\frac{3}{4}$ lb of biscuits, $\frac{1}{2}$ lb of biscuits as much as $\frac{3}{4}$ lb. of jam, and $\frac{1}{4}$ lb. of jam costs 4*d*., what is the cost of a pound of tea?

XLV. RATIO AND PROPORTION.

Ratio

Simplify the ratios—

1. 58 : 87
2. 803 73
3. $11\frac{1}{11}$ $8\frac{5}{7}$
4. $\cdot\dot{7}1428\dot{5}$ $7\cdot5$
5. 15 51 18·81
6. £5, 6*s* 8*d* . £8, 13*s* 4*d*
7. 3 tons 12 cwts 7 tons 16 cwts
8. 7 hrs 30 min 1 day
9. $10\frac{1}{2}$ chains 3 poles
10. 5 fur 121 yds 1 mi 121 yds.
11. Which is greater 11 17, or $1\frac{3}{4} \cdot 2\frac{5}{8}$?
12. Which is greater $1\cdot3 . 2\cdot\dot{3}$, or 5*s* 9*d* 11*s* 8*d*.?
13. Prove that the ratio of £3, 8*s*. to £7 is equal to the ratio of $3\frac{1}{11}$ to $6\frac{4}{11}$
14. Show that the ratio 2 lbs. 3 ozs : 1 qr 3 lbs 8 ozs is equal to the ratio 5 ins 2 yds
15. What sum of money has to £1, 15*s* 9*d* the ratio $3\frac{1}{3}$ $5\frac{1}{5}$?
16. What number has to ·75 the ratio of 3 tons to 9 lbs ?
17. Is the fraction $\frac{13}{15}$ increased, unaltered, or decreased, by subtracting 3 from both numerator and denominator?
18. Is the fraction $\frac{17}{18}$ increased, unaltered, or decreased, by adding 7 to both numerator and denominator?
19. If 2 be added to the numerator of the fraction $\frac{5}{11}$, what number must be added to the denominator in order that the value of the fraction may be unchanged?
20. If 3 be taken from the numerator of the fraction $\frac{9}{7}$, what number must be taken from the denominator that the value of the fraction may be unaltered?
21. If 7 be taken from the denominator of the fraction $\frac{27}{35}$, what number must be taken from the numerator that the value of the fraction may remain unchanged?
22. If 12 be added to the denominator of the fraction $\frac{56}{65}$, what number must be added to the numerator that the value of the fraction may remain unchanged?

Show by taking a numerical example that—

23. A ratio of greater inequality is diminished by adding the same number to both its terms
24. A ratio of greater inequality is increased by subtracting the same number from both terms
25. A ratio of less inequality is decreased by subtracting the same number from both terms

PROPORTION.

Find a fourth proportional to—

26. 12, 21 and 56

27. $7\frac{1}{2}$, $5\frac{1}{4}$ and $9\frac{1}{3}$

28. $\cdot 3$, $5\cdot 6$ and £2, 10s

29. 3 yds 1 ft, 2 ft 3 ins. and 5 cwts.

Find the missing term in each of the following proportions—

30. $1\frac{1}{4}$: $1\frac{1}{3}$ :: $1\frac{1}{2}$: ———.

31. 3 lbs. : 5 ozs. :: 8s. : ———.

32. 21 : 24 :: ——— : 5s. 4d

33. 2 ft. : 3 yds. :: ——— : 15.

34. $\cdot\dot{3}$: ——— $= \cdot 1\dot{6}$: $\cdot\dot{2}\dot{7}$.

35. 2 gals. 1 qt. : ——— = 13 acs 2 ro : 3 acs 2 ro.

36. ——— : $4\frac{2}{5}$ = 25 : 33

37. ——— 10 tons 10 cwts. = £6 : 5 guineas.

Find a mean proportional between—

33. 16 and 49. 39. 32 and 200 40. 68 and $1\frac{8}{9}$

41. Are 51, $42\frac{1}{2}$ and $35\frac{5}{12}$ in *continued* proportion?

42. Find a *third* proportional to $3\cdot 4$ and $5\cdot 1$.

43. Prove that—

$\cdot 15 : 4\cdot 5$ :: 8d. : £1 :: 2 lbs 3 ozs : 2 qrs. 9 lbs. 10 ozs.

44. Are £100, £700, £103 and £721 proportionals?

45. Are 17, 51, $17-2$ and $51-2$ proportionals?

46. The shadow of a tower is $16\frac{1}{2}$ yards long when the shadow of an upright stick 4 feet long is 3 feet 8 inches, what is the height of the tower?

47 On a ground-plan made on the scale of half an inch to a yard the length of a house is represented by a line $7\frac{1}{4}$ inches long; how long is the house?

48. If the circumference of a circle of radius 7 inches is 3 ft. 8 ins., what is the circumference of a circle of radius 4 yds. 2 ft.?

49. The longest side of a triangular field measures $16\frac{1}{2}$ chains. On a plan of the field the sides are represented by lines $1\frac{7}{8}$, $2\frac{1}{4}$ and $2\frac{3}{4}$ inches long respectively. Find the lengths of the other sides of the field.

50. On a plan of a field the sides measure 5, 3, $4\frac{1}{2}$ and 7 inches. The shortest side of the field itself measures 17 chains 10 links; find the length of each of the other three sides.

XLVI. PROPORTIONAL PARTS, AND PARTNERSHIP

1. Divide 308 into two parts which shall be in the ratio 9 19.
2. Divide 1602 into two parts in the ratio 35 143
3. Divide 483 into parts proportional to 3, 7, 11
4. Divide 46875 into four parts proportional to 1, 4, 4, 6.
5. Divide 22 into parts in the ratio of 86 to 68
6. Divide 28 into parts proportional to 29, 37 and 46
7. Divide 133 into two parts which shall be in the ratio $\frac{2}{3}$ $\frac{3}{5}$.
8. Divide 716 in the ratio $1\frac{3}{7}$ $2\frac{5}{6}$
9. Divide 46875 into parts proportional to $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$
10. Divide 150 into parts proportional to 2, 3·2, and 4 8.
11. Divide 2601 into parts proportional to 1·6, ·16, and 1·13
12. Divide 171 in the ratio of 2·7 to ·72.
13. Divide 36·5 into two parts, which shall be in the ratio of $5\cdot\dot{6}$ to 6·5
14. Divide £28900 into four parts proportional to 8, 5, 3, 1.
15. Divide a mile into six lengths proportional to 3, 5, 13, 17, 21 and 29
16. Divide £4, 12*s* 6*d* in the ratio 3 7
17. Divide £405, 2*s* 6*d* among three persons in the ratios 6 7 8
18. Divide £97, 18*s* 8½*d* into parts proportional to the numbers 7, 9 and 13.
19. Divide £32, 10*s* between A and B, so that A's share may have to B's share the ratio of 17 to 83
20. Divide £4, 8*s* in the ratio 53 43
21. Divide £30 into parts proportional to 230, 250 and 270.
22. Divide £154 between 4 persons, so that their shares may be proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$
23. Divide £1330 between 3 persons, in the ratios $\frac{2}{3}$ $\frac{3}{4}$ · $\frac{4}{5}$
24. The sides of a triangle are proportional to $2\frac{1}{5}$, 3, $3\frac{1}{4}$ The sum of their lengths is 338 feet Find the length of each side

---

25. A, B and C are partners in a firm A's capital is £1000, B's £2000 and C's £1500 What should each receive out of profits which amount to £3000?
26. A puts into a business £2500, B £1500 and C £1000, the profits of the business amount to £625; how should this sum be divided amongst the partners?

27. A man divided £504 amongst his three sons, whose ages were 19 21 and 23, in sums proportional to their ages Three years afterwards he similarly divided an equal sum How much did each receive in all?

28. 684 policemen are to be distributed among three towns in proportion to the population—viz., 10,944; 12,312, 25,992 How many policemen will be sent to each town?

29. A bankrupt, whose estate is worth £698, has four creditors A, B, C, D, he owes A £270, B £300, C £150, D £100. The legal expenses of his bankruptcy amount to £83 How much will each creditor receive?

30. The proceeds of a business amounting to £2689, 8*s* are divided among the four partners whose contributions to the capital were severally £543, 10*s*, £274, 10*s*, £685, 13*s*. 4*d* and £417, 6*s*. 8*d* ; how much does each partner receive?

31. On the dissolution of a partnership, £16,690, 11*s* 3*d*. was divided between the two partners in the ratio $47\frac{1}{4}$: $33\frac{3}{4}$. How much did each receive?

32. Distribute 500 policemen between three towns, of population 19,000, 8500, 23,600 respectively, as nearly as possible in proportion to population

---

33. Divide £150 between A, B and C, so that A may receive £7 as often as B receives £8, and B £4 as often as C receives £5.

34. Divide £60, 2*s*. $1\frac{3}{4}$*d*. between A, B and C, so that A may have half as much again as B, and B half as much again as C.

35 Divide £150 among A, B and C, so that B may have twice as much as A, and C twice as much as A and B together

36. Divide £3, 18*s* 4*d* among three persons, A, B, C, so that B may receive seven-eighths as much as A, and C five shillings more than B

37. Divide £520 among four persons so that the first may have twice as much as the second, the second twice as much as the third, and the first and third together as much as the second and fourth together

•38. Divide £954, 9*s*. between A, B and C, so that A's share may be to B's as 3 is to 5 and B's to C's as 10 is to 11

39. Divide £2100 among A, B, C and D, so that A's share may be to B's as 7 : 6, B's to C's as 5 : 4, and C's to D's as 3 : 2.

40. Divide 114 gallons into 4 measures, so that the first shall be to the second as 3 is to 5, the second to the third as 7 is to 9 and the third to the fourth as 15 is to 17.

41. If 7 men do as much work as 12 women, and 5 women as much as 8 children, divide £69 among 4 men, 6 women and 9 children in proportion to the work they do

42. If 12 men can do the work of 19 women, and 8 women that of 11 boys, divide £584, 15*s* among 14 men, 6 women and 10 boys in proportion to their work

---

43. Three men hire a pasture for £42, 12*s* One puts in 10 oxen for 9 months, another 12 oxen for 8 months, the third 14 oxen for 7 months What part of the rent should each pay?

44. A, B and C are partners in a business A puts in £900 for 6 months, B £1200 for 4 months and C £1800 for a year Divide profits amounting to £742 fairly among them

45. Three persons A, B, C, rent the grazing of a park for £570 A puts 126 oxen in the park for 3 months, B puts in 162 oxen for 5 months, and C puts in 216 oxen for 4 months. What part of the rent should each person pay?

46. A and B enter into partnership each with a capital of £1000 At the end of the first year the profits are £500, and A then invests his share in the business, but B withdraws £200 of his capital How should the profits, amounting to £492, be divided at the end of the second year?

47. A begins to trade with a capital of £1500 He is joined 5 months later by B with £1200 At the end of the year the profits amount to £453, 7*s* 6*d* Find, to a penny, the share of each partner.

48. A begins business with a capital of £850 Three years later he takes B, with a capital of £1100, into partnership, and two years later still they are joined by C with £1400 At the end of 8 years from the commencement the business is sold for £5500; what part of this sum is due to A?

49. Two merchants A and B trade together for a year A invests £1200 in the business for the whole time B puts in £300 at first, £300 more at the end of 3 months, and again £600 at the end of another 3 months The total profits amount to £270, how should this sum be divided between them?

50. A and B enter into partnership, A with £2000, and B with £1800, of capital Nine months later they are joined by C with £2400. At the end of the year the profits are £656, 5*s*, find, within a penny, the share of this which belongs to C

51. A began business with a capital of £550, and was joined during the year by B with a capital of £600 At the end of the year B received £75 out of profits amounting to £240 When did B enter the business?

52. Two traders, A and B, enter into partnership with capitals of £800 and £960 respectively. Two months later A increases his capital by £300, and three months later still B increases his by £560 One month after this A increases his capital by £500 Ten months from the commencement of their partnership the profits are divided How must this be done?

After a second interval of 10 months profits are again divided and each partner receives the same amount, one of them having during the interval contributed an additional £400 of capital Which did this, and when?

53. Two farmers A and B rent a field for 10 weeks and pay £5 for it. A puts in 3 horses for 5 weeks, 4 cows for 8 weeks, and 12 sheep for the whole time, whereas B has 1 horse and 30 sheep in the whole time. If the cost of the keep of a horse, a cow, and a sheep are in the ratio 5 3 : 1, find, to the nearest penny, what share of the rent each should pay.

54. A and B enter into partnership, A puts in £1800, but at the end of 4 months withdraws £450 of this, and again at the end of 9 months £200 If at the end of the year their just shares of the profits are as 5 is to 3, how much capital did B put into the business?

55. Three partners A, B and C had shares in a business proportional to the numbers 4, 5 and 6 respectively C retired, and then A and B had equal shares If, for his share, C received altogether £15,000 from A and B, what sum should each have contributed?

56. A and B enter into partnership, their capitals being as 5 : 6; at the end of two months they withdraw respectively one-third and one-fourth of their capital, and four months afterwards B increases his capital by the addition of one-third of his original capital. How are they to divide the profits, amounting at the end of the year to £379, 2*s* 6*d*?

57. A, B and C enter into partnership, their capitals being as 1 . 2 : 3. After 6 months A withdraws half his capital, but 3 months later restores it Four months from the commencement B withdraws one-third of his capital, and 2 months later C adds twice as much as B withdrew. How must profits, amounting to £658, 10*s*, be divided at the end of the year?

58. A, B and C are partners in a bank in which they invest £12,800, £8000 and £5000 respectively C, as manager, is to be paid $\frac{3}{20}$ of the gross yearly profits, while A and B are each to be allowed 2 guineas a day for attendance. At the end of the year the gross profits amount to £7650, and A has attended 6 days, and B 4 days. What is the total sum due to B?

59 The values of equal weights of three metals are as 2 : 5 : 19; volumes proportional to 3, 4 and 6 are mixed. what value of the first metal is contained in a lump of the mixture worth £7?

60. A piece of metal weighing 11 cwts. 3 qrs. has been formed by compounding three metals in quantities which, by measure, are as 5 : 3 : 2, but the weights of equal volumes of them would be as 7 : 11 : 13. What weight of each of the component metals has been used?

XLVII. AREAS.

RECTANGULAR SURFACES.

Find the area of a rectangle—

1. 8 feet long and 7 feet wide
2. 13 feet long and 11 feet wide
3. 3 ft 6 ins long and 2 ft 3 ins wide.
4. 5 ft 4 ins long and 4 ft 8 ins wide
5. 10 ft 7 ins long and 6 ft 2 ins wide
6. 3 yds 2 ft long and 2 yds 1 ft 6 ins wide.
7. $4\frac{3}{8}$ ins long and $2\frac{5}{8}$ ins wide
8. $1\frac{7}{8}$ ins long and $1\frac{5}{8}$ ins wide
9. 8 ft. $4\frac{1}{2}$ ins long and 5 ft $7\frac{1}{2}$ ins wide
10. 3 ft $7\frac{1}{4}$ ins long and 1 ft $11\frac{1}{2}$ ins wide

Find in acres, &c, the area of a rectangular field—

11. 176 yards long and 110 yards wide
12. 480 yards long and 242 yards wide
13. 550 yards long and 33 yards wide
14. 27 poles long and 19 poles wide
15. 14 chains long and 8 chains wide
16. 17 chains long and 12 chains wide

Find the area of a square, the length of a side of which is—

17. 1 ft 1 in.	20 220 yards	23. 9 chains
18. $7\frac{3}{8}$ inches	21 77 yards	24 7 chains 25 links.
19 2 yds. 2 ft 7 ins.	22 360 yards	25. 5 chains 75 links.

Find the area of—

26. A rectangular floor 18 ft 3 ins long and 16 ft 6 ins wide
27. A page of a book $7\frac{1}{2}$ inches long and $5\frac{1}{4}$ inches wide.
28. A tablecloth $3\frac{1}{4}$ yds long, $1\frac{5}{8}$ yds wide
29. A piece of tape $\frac{3}{4}$ in. wide, 12 yds long
30. A piece of wall-paper 12 yards long and 21 inches wide.
31. A straight road 187 miles long, 10 yards wide
32. A straight path $3\frac{1}{4}$ miles long and 2 ft. 9 ins wide
33. A straight road 5 furlongs in length and of the average width of 1 chain 5 links
34. An oblong field the dimensions of which are 55·5 chains and 10·125 chains.

Find the length of a rectangle*—

35. 119 sq. ft in area and 7 ft in width.

36. 195 sq. yds in area and 13 yds. in width

37. 35 square yards in area and 5 feet in width.

38. $6\frac{3}{4}$ sq ins in area and $1\frac{7}{8}$ ins in width.

39. 7 sq ft in area and 8 ins. in width

40. $5\frac{5}{8}$ sq ft. in area and $2\frac{2}{3}$ ins. in width.

41. 2 acres in area and 88 yards in width.

42. 3 ac 1 ro in area and $27\frac{1}{2}$ yds in width.

Find the breadth of a rectangle—

43 8 sq ft 36 sq ins in area and $5\frac{1}{2}$ feet in length

44 5 sq yds in area and 3 yds 2 ft. 3 ins in length.

45 $2\frac{1}{2}$ sq ft in area and 2 ft $9\frac{3}{8}$ ins in length

46. 1 sq ft in area and 13 yds. 1 ft in length.

47. 4 ac 3 ro in area and $9\frac{1}{2}$ chains in length.

48. 1 acre in area and 1 chain 25 links in length.

49. 3 ac 3 ro in area and 33 poles in length.

50. 5 ac. 1 ro. 10 po in area and $514\frac{1}{4}$ yds. in length.

Find the length of—

51. A rectangular ceiling the breadth of which is 14 feet and its area 259 square feet.

52. A strip of paper $1\frac{1}{2}$ inches wide, the area of which is 1 square foot

53. A roll of matting 18 inches wide which would cover a floor 27 feet long and 12 ft 8 ins wide.

54. A nine-inch board the area of which is 3 square feet.

Find the width of—

55. A window-blind 7 feet long and $2\frac{5}{8}$ sq. yds. in area.

56. A straight canal a mile long which covers two acres of ground.

57. A strip of paper 12 yards long and $2\frac{1}{2}$ sq. yds. in area.

58. A straight road $16\frac{1}{2}$ miles long which occupies 84 acres of ground.

59. A "piece" of wall-paper 12 yards long which covered 63 sq. ft. of wall-space.

60. 17 equal planks each 18 feet long, which formed a floor whose area was $184\frac{7}{8}$ sq. ft

* Exercises in finding the length of the side of a *square* when the area is given will be found under Square Root

How many—

61. Times is the area of a rectangle 3 ins long and 2 ins wide contained in that of a rectangle 4 ft 6 ins long and 1 ft. 10 ins wide?

62. Cards each $3\frac{1}{2}$ ins long by $2\frac{3}{4}$ ins wide would just cover an oblong table 5 ft 6 ins long by 3 ft 6 ins wide?

63. Mats each 1 yd long and 15 ins wide would cover a floor 15 ft long and 14 ft wide?

64. Bricks each 9 ins by 4 ins would pave a kitchen 24 ft by 16 ft 6 ins?

65. Postage-stamps $\frac{15}{16}$ in long by $\frac{5}{6}$ in wide would cover an envelope $7\frac{1}{2}$ ins long by $3\frac{3}{4}$ ins wide?

66. Tiles six inches square would pave a passage six feet wide and six yards long?

67. Tiles $2\frac{1}{2}$ ins square would pave a hall 10 ft square?

68 Paving-stones each having a face 6 sq ins in area would pave a straight road 5 chains long and 5 yards wide?

69 Slabs each 2 sq ft in area would pave a floor 16 yards square?

70 Slabs each 2 feet square would pave a courtyard 16 square yards in area?

71 Pieces of turf 1 ft 3 ins square would make a grass-plot 40 feet square?

72. Square yards of floor-cloth would cover a floor 29 feet long and 23 feet wide?

How many yards of patternless carpet would just cover a floor—

73 21 feet long, 14 ft wide, carpet 3 ft wide?

74. 28 feet 6 ins long, 19 ft 6 ins wide, carpet a yard wide?

75 27 ft long, 18 ft wide, carpet 2 ft wide?

76. 15 yds long, 4 yds 2 ft wide, carpet 4 ft wide?

77. 27 ft long, 16 ft wide, carpet $\frac{3}{4}$ yd wide?

78. 56 ft long, 18 ft 8 ins wide, carpet 28 ins wide?

79. 6 yards square, carpet $\frac{3}{4}$ yd wide?

80. 14 feet square, carpet $31\frac{1}{2}$ ins wide?

81 21 ft. long, 15 ft wide, carpet $\frac{3}{4}$ yd wide, leaving an uncarpeted margin 18 ins wide all round the floor?

82 20 ft 10 ins long, 16 ft wide, carpet 2 ft wide, leaving a margin 1 foot wide uncarpeted all round the room?

83 7 yds. long, 6 yds wide, carpet $\frac{3}{4}$ yd wide, leaving a margin $\frac{3}{4}$ yd wide uncarpeted all round the floor?

84. 20 ft 6 ins square, carpet 18 ins wide, leaving an uncarpeted margin 9 ins. wide all round?

Find the cost of plain carpet sufficient to just cover a floor—

85. 18 ft 6 ins. long and 12 ft 3 ins. wide at 3*s*. per sq yard.

86. 18 feet long and 15 feet broad, with carpet 30 inches wide, at 5*s* per yard

87. 20 ft 3 ins long by 13 ft 4 ins. wide, with carpet 2 feet 3 inches wide, at 5*s* a yard.

88. 10 yds 2 ft. long and 7 yds. 1 ft. broad, with carpet $\frac{3}{4}$ yd wide, at 4*s* 6*d* a yard.

89. 12 ft 9 ins by 16 ft. 6 ins, with carpet 33 ins. wide, at 3*s*. 8*d*. a yard.

90. 7 yds. 8 ins. long and $5\frac{1}{4}$ yds wide, with carpet 27 ins. wide, at 3*s* $4\frac{1}{2}d$ a yard

91. 40 feet long by 30 feet broad, with carpet at 5*s*. 9*d* per square yard, leaving uncovered a space of 3 feet wide all round the room

92. 100 ft long by 25 ft broad, with carpet at 4*s* 6*d*. per square yard, leaving uncarpeted a space 4 feet wide all round.

---

93. Find the cost of staining a border 15 inches wide all round a floor 20 ft long and 17 ft. broad at 1*s*. per square yard.

94. Find, to the nearest penny, the cost of staining a border 16 ins. wide all round the floor of a room 16 ft. 8 ins. square, at 9*d*. per square yard.

95. A room is 25 feet long by 17 feet broad What will it cost to carpet it at 6*s* 9*d* a square yard, leaving uncovered a margin one yard wide? Find the extra cost of covering this space with India matting at 1*s*. 3*d* a square yard

96. All round the floor of a room, which is 28 feet long and 22 feet wide, there is a border 2 feet wide which is left uncarpeted. Find the cost of staining the border at 1*s* $1\frac{1}{2}d$ a square yard Find also the number of yards of carpet, 27 inches wide, required for covering the rest of the floor, and the cost of this carpet at 3*s* 9*d* a yard

97. A room, 20 feet long and 16 feet wide, has a stained border 2 feet wide; the rest of the floor is covered with carpet at 27*s* per square yard; the staining costs 9*d* per square yard. Find the whole expense

98 A carpet 19 feet 6 inches by 15 feet 9 inches, which costs 16*s*. per square yard, is laid down in a room measuring 23 feet by 17 feet, and the rest of the floor is covered with floor-cloth at 8*d* per square foot Find the total cost.

99. A room 22 ft by 19 ft. has in it a Turkey carpet 19 ft 6 ins. by 15 ft 9 ins, costing 12*s* per square yard, the rest of the floor is covered with felt at 4*d*. per square foot; find the cost of the carpet and of the felt.

100. What will it cost to make a gravel walk 7 feet wide along the inner edge of each side of a square field whose side is 110 yards long, at 1*s* 6*d* a square yard?

101 In a courtyard 67 ft 6 ins long and 42 ft 9 ins wide there is a footway, 5 ft 6 ins wide, the whole length of the yard What is the cost of paving the whole, the price per square yard for the footway being 3*s* 6*d* and for the remainder 3*s*?

102. A room is 24 ft long, $19\frac{1}{2}$ ft wide What will be the total cost of painting, at 1*s* 3*d* per square yard, a border a yard wide, and a skirting-board 18 in high, all round the floor?

---

103. Carpet 2 ft wide, at 4*s* 9*d* per yd, sufficient to just cover a rectangular floor would cost £6, 10*s* $7\frac{1}{2}$*d*., find the area of the floor

104. The cost of matting, $\frac{3}{4}$ yd wide at 1*s* 10*d* per yd which just covers a floor 21 feet long, is £5, 2*s*. 8*d*, how broad is the floor?

105 Drugget at 3*s* 9*d* per yd, which just covers a floor 16 ft square, costs £4, 5*s* 4*d*, how wide is the drugget?

106. The cost of carpet 21 ins wide which just covers a floor whose dimensions are 16 ft 6 ins and 13 ft 3 ins. is £7, 5*s* 9*d*, find the price of the carpet per yard

107. A carpet for a room cost £12, 5*s*. The room was $25\frac{1}{2}$ feet long and $19\frac{1}{2}$ feet wide, and between the edge of the carpet and the walls was a margin $2\frac{1}{4}$ feet wide Find the cost of the carpet per square yard

108 The cost of patternless carpet 2 ft wide at 7*s* per yd for the middle of a floor 18 feet wide is £19, 15*s* 6*d* A margin 1 ft wide is left uncarpeted all round the floor. How long is the floor, if the value of the quantity of carpet turned under for the hem is 3*s* 6*d*?

109 The cost of staining a border 2 feet wide all round a room 30 feet long by 27 feet wide, is 17*s* 8*d*, find the price per square yard

110. In making a pavement 7 feet wide all round the inside of a square, 648 slabs, each 2 ft long and 1 ft 9 ins wide, were used Find the length of the side of the square

111. A pavement 6 feet wide all round the outside of a square grass-plot is formed of 552 slabs each 2 ft long by 1 ft 6 ins. wide What is the length of a side of the grass-plot?

112. Each of two rooms is 18 feet wide, but one is twice as long as the other, the floor of the shorter room is covered with carpet at 9*s* per yard, of the longer at 6*s* per yard, the width of the carpet in each case being $\frac{3}{4}$ of a yard Find the length of each room in feet, when the difference in the cost of the two carpets is £9.

Walls of a Room, &c.

Find in feet (i) the perimeter, (ii) the area of the walls; of a rectangular room—

113. 18 ft long, 14 ft. wide, 11 ft. high.

114 10 ft high and the floor 15 ft square

115. 20 ft long, 17 ft. wide, 9 ft. high.

116 21 ft. long, 16 ft broad, 10 ft. 6 ins high

117. 17 ft 6 ins in length, in breadth, and in height.

118. 24 ft. long, $16\frac{1}{2}$ ft wide, 12 ft. 3 ins high

119. 23 ft 8 ins long, 17 ft. 4 ins broad, 11 ft. 6 ins high.

120. 41 ft 2 ins long, 32 ft 4 ins broad, 18 ft 6 ins. high.

121. 23 ft 8 ins long, 15 ft 10 ins. wide, 11 ft. 11 ins. high.

122. 6 yds 2 ft 3 ins long, $5\frac{1}{4}$ yds broad, 9 ft. $10\frac{1}{2}$ ins. high.

---

123. Find the area of the sides and ends of a rectangular box 15 ins long, 10 ins. wide and 7 ins. high

124. Find the area of the whole external surface of a rectangular box 14 ins long, 9 ins wide and $4\frac{1}{2}$ ins. high

125. Find the total area of the walls, ceiling and floor of a rectangular room 12 ft. 6 ins. long, 11 ft 6 ins. wide and 9 ft. 3 ins. high.

126. How many square feet of sheet-lead would line an open rectangular tank which measures inside 4 ft in length, 2 ft. in depth and 2 ft 9 ins. in breadth?

127. How many square yards of paper would just completely cover the outside of a rectangular box 6 ft. long, 3 ft. wide and 2 ft high?

128. A rectangular box, without lid, is made of wood 1 inch thick; it measures outside 3 ft 8 ins in length, 2 ft. 2 ins. in breadth and 1 ft 7 ins in height, find the total area of its inside surface

129. A box, with lid, made of wood $\frac{3}{4}$ inch thick, measures outside 4 ft $4\frac{1}{2}$ ins in length, 2 ft $7\frac{1}{2}$ ins in width and 1 ft. 9 ins in height; find the total area of its inside surface

130. How many sheets of paper each 3 feet long and 2 feet wide would just completely cover the entire surface of the walls of a rectangular room 22 ft. long, 18 ft. wide and 12 ft. high?

Making no allowance for doors, &c, how many—

131. Square yards of paper would cover the walls of a room 18 ft. long, 15 ft. wide and 12 ft. high?

132. Yards of paper 2 ft. wide would cover the walls of a room 20 ft. long, 16 ft. wide and 11 ft. 6 ins. high?

Making no allowance for doors, &c, how many—

133. Yards of paper 2 ft 6 ins wide would cover the walls of a room 20 ft. 6 ins long, 9 ft 6 ins wide and 13 ft 6 ins. high?

134 "Pieces" of paper 12 yards long and 21 inches wide would cover the walls of a room 24 ft 3 ins long, 14 ft 3 ins. broad and 9 ft high?

135. Sheets of paper 9 ins square would cover the walls of a room 16 ft 7 ins long, 14 ft 2 ins wide and 8 ft 3 ins high?

136. Feet of 7-inch board would line to a height of 3 ft 6 ins the walls of a room 21 ft long and 14 ft wide?

Find the cost of paper (making *no* allowance for doors, windows, &c, or for matching the pattern), sufficient to cover the walls of a room—

137. 21 ft long, $16\frac{1}{2}$ ft wide and 10 ft high at $7\frac{1}{2}d$ per sq yd.

138. 25 ft long, 18 ft 6 ins wide and 10 ft high; paper 2 ft wide at $3d.$ a yd.

139. 21 ft 6 ins long, 15 ft 3 ins broad and 12 ft high; paper 21 inches wide at $7s$ $6d$ per piece of 12 yards

140. 27·7 ft long, 19·55 ft wide and 14 4 ft high; paper 2·7 ft. wide at $1s$ $3d$ per piece of 12 yards

---

141. What is the cost of paper half a yard wide, at $1s$ $6d$ per piece of 12 yards, which would just cover the side-walls and ceiling of a passage 12 yards long, 6 ft wide and $10\frac{1}{2}$ ft high?

142. The length, breadth and height of a covered tank are 24 ft, 17 ft. and $14\frac{1}{2}$ ft respectively Find the cost of painting it all over outside at $1d.$ per sq ft

143. An open rectangular cistern, made of sheet-iron, is 7 ft long, 4 ft wide and 2 ft 6 ins deep Find the cost of painting it all over, inside and outside, at $1\frac{1}{2}d$ per sq ft

---

How many sq ft of plaster are there on the walls and ceiling of a room,—

144 10 feet high, 20 ft long and 16 ft wide, having a door 8 ft by 4 ft, and two windows each 5 ft by 3 ft?

145 28 ft long, 20 ft. wide, 10 ft high, having a door 7 ft by 4 ft, a fireplace 5 ft by 4 ft, and two windows each 5 ft by 3 ft?

146. 24 ft long, 18 ft. broad, 12 ft high, having two doors each 6 ft 6 ins by 3 ft 4 ins, two windows, opening down to the floor, each 10 ft high by 5 ft wide, a fireplace 5 ft high by 4 ft 6 ins wide, and a skirting-board a foot high

Find the cost of paper sufficient for the following rooms, allowing for waste 2 complete pieces over and above what is saved from windows, &c, the paper being bought *by the piece* 12 yards long and 21 ins wide—

147. 28 ft 6 ins. long, 18 ft 9 ins. wide, 12 ft high; price of paper 2*s* 6*d* per piece

148. 31 ft 4 ins long, 17 ft 3 ins wide, 13 ft. 6 ins. high, price of paper 1*s* 10½*d* per piece

149 20 ft 8 ins long, 18 ft 6 ins broad, 12 ft. high; price of paper 2*s* 9*d* per piece.

150 30 ft 7 ins long, 17 ft 8 ins. wide, 14 ft. 2 ins. high; price of paper 3*s*. 3*d*. per piece.

---

151. The area of the walls of a room 13 ft 7½ ins long and 13 ft. 3 ins broad is 645 sq ft, how high is the room?

152. The area of the walls of a room 18 ft. 3 ins. long and 12 ft high is 764 sq feet, how broad is the room?

153. The area of the whole outside surface of a covered box 14 ins. long and 8 ins wide is 3 sq ft 78 sq ins., how high is the box?

154. The area of the whole inside surface of an open tank 2 ft 3 ins. wide and 1 ft 9 ins. deep is 36⅝ sq. ft.; how long is the tank?

*155. The area of the sides, ends and bottom of a tank 17 ft. long and 5 feet deep is 345½ sq ft, how wide is the tank?

156. The cost of painting at 5*d* per sq yd. the two sides and the ceiling of a dark passage 30 yds long and 10 ft. high is £5, 14*s* 7*d*., how wide is the passage?

157. If paper 30 inches wide at 7½*d* per yard sufficient to just cover the entire walls of a room 31 ft long and 23 ft. wide would cost £5, 3*s* 6*d*, how high is the room?

158. If paper at 2*d*. per yard sufficient to just cover the entire walls of a room 21 ft long, 16 ft wide, and 10 ft. high would cost 13*s* 4*d*, how wide is the paper?

159. The cost of painting the walls and ceiling of a room 17 ft. long 14 ft wide, and 10 ft 6 ins. high, having a door 7 ft. by 3 ft 9 ins, a window 6 ft by 4 ft 6 ins. and a fireplace 3 ft by 3 ft 6 ins, is £3, 8*s* 9¼*d*; what is the price per square foot?

160 A room 20 ft long has two windows each 7 ft by 4 ft, two doors each 6 ft 6 ins by 3 ft 4 ins and a fireplace 4 ft by 3 ft. 3 ins The cost of painting the walls at 1½*d* per sq ft is £4, 2*s* 2½*d*, and the cost of polishing the floor at 1*s* per sq yd. is £1, 13*s*. 4*d*. How wide and how high is the room?

XLVIII. VOLUMES;

Rectangular Solids

Find the volume of a rectangular solid of—

	Length	Breadth	Thickness
1.	9 ft,	4 ft,	3 ft
2	17 ft;	7 ft,	5 ft
3	3 yds. 2 ft,	2 yds. 1 ft,	2 ft.
4	5 yds,	7 feet,	1 yd 2 ft
5.	3 ft 9 in,	1 ft 8 in,	1 ft 2 in
6.	6 ft. 6 in,	4 ft 4 in,	3 ft 3 in
7.	3 yds;	2 ft,	11 in
8.	17 yds,	17 ft,	17 in

Find the volume of a cube the length of an edge of which is—

9 $1\frac{3}{4}$ in 10. 2 ft. 4 in. 11. 1 ft 1 in 12 3·1 in

Find the third dimension of a rectangular solid of volume—

13. 364 cub ft, breadth 7 ft., thickness 4 ft

14. 105 cub. ft, breadth 3 ft 6 in, thickness 2 ft 3 in

15. 1 cub. ft 876 in, length 4 ft 8 in, thickness 2 ft 7 in.

16. 8 cub. yds 4 ft, length 5 yds 1 ft, width 15 in

17. 57 cub. ft. 828 in, length 7 ft 5 in, breadth 2 ft 7 in.

18. 4 cub yds 1 ft 1528 in, length 16 ft 10 in, breadth 7 ft 10 in.

---

19. How many cubic feet of air are contained in a room 17 ft long, 15 ft 3 in wide and 12 ft high?

20. How many cubic feet of timber are there in a beam 24 ft long, 8 inches wide and 6 inches thick?

21. How many cubic feet of wood are there in the boards of a floor 18 ft long and 16 ft wide the boards being $1\frac{1}{4}$ inches thick?

22. What is the height of a room 18 ft 6 in long and 14 feet broad, which contains 2849 cub ft of air?

23. A room contains 2684 cub ft of space, its length is 16 ft 6 in., and its height is 10 ft 8 in, what is its breadth?

24. What length must be cut off a beam $7\frac{1}{2}$ in wide and 4 in thick, that the volume of the piece cut off may be one cubic foot?

25. What is the length of a uniform beam 36 sq inches in section, if its volume is 1 cub yd.?

26. What is the thickness of a board 9 inches wide, when the volume of a piece 21 ft 4 ins long is one cubic foot?

27. How many children will a school-room 60 feet long, 28 feet wide and $13\frac{1}{2}$ feet high accommodate, allowing 140 cubic feet of space for each child?

28. A school-room 40 feet long and 15 feet high accommodates 120 children, allowing 120 cubic feet of air for each child. how wide is the room?

29. A school-room 72 feet long accommodates 180 persons allowing 12 sq ft of floor and 200 cub ft of space per head. how wide and how high is the room?

30. If a cubic foot of lead be made into a sheet 32 feet long and 6 feet wide, what decimal of an inch is the sheet in thickness?

31. Find the value of a uniform beam 23 feet long, and 54 sq. inches in section, at *2s. 3d* per cubic foot.

32. Find the cost, at *4s 3d.* per cubic yard, of a coat of gravel 2 inches thick for a court-yard 9 yards long and 7 yards wide

33. A pond of 3 acres is covered with ice 6 inches thick, if a cubic foot of ice weigh 896 ozs, find in tons the weight of the ice on the pond

34. A level field of 4 acres is flooded to a depth of 4 inches; if a cubic foot of water weigh 63 lbs., find in tons the weight of water on the field.

35. Find in tons the weight of a block of stone containing $670\frac{5}{16}$ cub. ft., if a block of the same kind of stone which is 5 ft. long, 3 ft 9 in. broad and 2 ft. 6 in. thick weighs 7500 lbs.

36. If a cubic foot of iron weighs 480 lbs., what is the weight in tons, &c, of 14,000 iron bars each 10 feet long, $1\frac{1}{2}$ in. wide and $\frac{3}{8}$ in thick?

37. An inlaid chess-board is composed of $1\frac{3}{4}$-inch squares, $\frac{1}{16}$ in. thick, of ebony and boxwood arranged alternately. How many cubic inches of ebony are there in it?

38 A gravel walk 6 feet wide runs round a grass-plot 60 feet long and 40 feet wide. Find the cost of the gravel, 3 inches deep, at *3s* per cubic yard

39. A lawn 23 yards long and 17 yards wide is surrounded by a path 5 feet wide Find the cost of a coat of gravel, 3 inches thick, for the path at *4s. 6d.* per load (*i.e.* per cubic yard).

40. If the cost of digging a trench 2 feet wide and 18 inches deep at *9d* per cubic yard be £3, *19s 3d.*, how long is it?

---

41. How many wooden blocks, each 7 in. by 5 in. by 3 in., could be packed in a cellar 14 feet long, 10 feet wide and 8 feet high?

42. How many bars of soap, 18 in. long, 3 in. wide and 3 in. thick, could be packed in a box the internal dimensions of which are 5 ft., 3 ft. and 2 ft. 9 in.?

43 How many cubes of $\frac{3}{4}$ in edge could be packed in a box 1 ft 3 in long, 9 in wide and $3\frac{1}{2}$ in deep, inside?

44 How many gross of exercise-books, each book being 8 in long $6\frac{1}{2}$ in. wide and $\frac{1}{8}$ in thick, could be packed in a box 2 ft. 8 in long, 2 ft 2 in wide, and 1 ft 6 in deep, inside?

45. How many bricks would be required for a wall 80 yards long, 10 feet high and 18 inches thick, if a brick, with its share of mortar, measures 9 in by $4\frac{1}{2}$ in by 3 in?

46 How many tiles six inches square and 1 in thick could be packed in a cellar 12 ft long, 11 ft wide and 7 ft high?

47. In a builder's yard there are two stacks of bricks, the first stack is $18\frac{3}{4}$ ft long, $3\frac{3}{4}$ ft wide and $8\frac{1}{4}$ ft high, the second is $11\frac{1}{4}$ ft long, $9\frac{3}{4}$ ft wide and $5\frac{1}{4}$ ft high How many more bricks are there in one stack than in the other, the dimensions of a brick being 9 in, $4\frac{1}{2}$ in and 3 in?

48 Two boxes, whose inside dimensions are 2 ft 6 in, 2 ft $2\frac{1}{2}$ in., 2 ft and 2 ft $11\frac{1}{2}$ in, 2 ft, 1 ft $10\frac{1}{2}$ in respectively, are filled with copies of a book which is $7\frac{1}{2}$ in long, 6 in wide and $\frac{1}{2}$ in thick How many more books are there in one box than in the other?

49 If a brick with its share of mortar measures 9 by $4\frac{1}{2}$ by 3 inches. how many bricks would be required for a wall 100 yds long and 8 ft high, the lowest course of bricks being 18 inches wide, the second course $13\frac{1}{2}$ inches wide, and the remaining courses each 9 inches wide?

50 Supposing that a brick with its share of mortar occupies $9 \times 4\frac{1}{2} \times 3$ cubic inches of space, how many bricks are there in a wall 5 ft high and 9 inches thick which surrounds a square the side of which measures, on the inner side of the wall, 18 yards?

If a pint of water weighs a pound and a quarter, and a cubic foot of water weighs 1000 ozs, how many—

51. Gallons would a tank 6 ft long, 4 ft wide and 2 ft. 4 in. deep, hold?

52. Gallons would a cistern 6 ft 8 in long, 5 ft 4 in wide and 4 ft 6 in deep, hold?

53 Imperial quart bottles could be filled with the contents of a full tank 3 ft 8 in long, 2 ft 4 in wide and 2 ft 3 in deep, and what fraction of a pint would be left over?

54 Feet wide is a tank 24 ft long and 8 ft deep which will just hold 7200 gallons?

55 Feet long is a tank 4 ft 3 in wide and 2 ft 6 in high which holds $531\frac{1}{4}$ gallons?

56. Acres are there in a field upon which 45,375 tons of rain-water fall in the course of a year, the average rain-fall being $2\frac{1}{2}$ in. per calendar month?

How many cubic inches of wood are there in a box, with lid—

57. 2 ft long, 1 ft. 10 in wide and 1 ft 5 in high, the boards of which it is made being 1 in. thick?

58 3 ft long, 2 ft wide, 1 ft $5\frac{1}{2}$ in. high, of wood $\frac{3}{4}$ in thick?

59. 5 ft 7 in by 4 ft 7 in by 3 ft 1 in, of wood 1 inch thick?

60 4 ft by 3 ft. by 2 ft, the boards being $\frac{3}{4}$ in. thick?

---

61. How many cubic inches of iron are there in a safe which measures outside 2 ft 6 in in length, 2 ft. in width and 2 ft 9 in in height, the iron being 2 in. thick throughout?

62 How many cubic inches of wood are there in a box, without lid, made of board half an inch thick, the length being 3 ft. 2 in., width 1 ft 10 in and height 1 ft. 8 in.?

63. What length of 7-inch board, $\frac{3}{4}$ in thick, would make a packing-case 2 ft 3 in long, 1 ft 9 in wide and 1 ft $10\frac{1}{2}$ in. high, allowing a length of 3 in of board for waste in sawdust?

64 How many cubic feet of stone are there in a rectangular trough, of internal length, 4 ft 6 in; width, 1 ft. 9 in; depth, 1 foot, the stone being everywhere 3 in thick?

65. A closed tank made of wood $1\frac{7}{8}$ in thick, and lined throughout with sheet-lead $\frac{1}{8}$ in thick, measures outside 5 ft. in length, 2 ft. 4 in in breadth and 2 ft. 8 in. in depth, how many gallons of water will it hold?

66. A stone trough is 4 ft long, 1 ft 6 in. wide, 1 ft. 3 in. deep, inside, and the stone is everywhere 3 in thick If the weight of a cub ft of the stone be 175 lbs, and of water be 62 lbs., what would the trough weigh when full of water?

67. A hollow rectangular vessel, without lid, formed of material 1 in. thick, whose external dimensions are 13 ft 6 in., 6 ft 8 in. and 6 ft 11 in respectively, weighs 3 qrs 25 lbs. Find the weight of a solid mass of the same material and the same dimensions

68 Gold can be beaten out so thin that 275,625 gold leaves placed one upon another are but an inch in thickness If a cubic foot of gold weigh 10 cwt. 95 lbs, what is the weight in grains of a piece of gold leaf 8 inches long and 7 inches wide?

69. Find the cost of bricks, at £1, 17*s*. 6*d* per 1000, for a wall 5 ft high and 9 inches thick surrounding (with the exception of a doorway 6 ft wide) a rectangular garden which measures, outside the wall, 40 yards in length and 24 yards in breadth. The dimensions of a brick to be taken as 9 in. by $4\frac{1}{2}$ in. by 3 in, and no allowance to be made for mortar.

70. A safe 2 ft. 6 in long, 2 ft. wide and 2 ft. 9 in high, measured externally, is made of iron 2 in. thick, and weighs $23\frac{1}{4}$ cwt.; how high is a safe 2 ft. 10 in long and 2 ft 1 in. wide, made of iron $2\frac{1}{2}$ in. thick, and weighing $35\frac{1}{4}$ cwt.?

XLIX. DUODECIMALS.

Express, as duodecimals,

1 5 ft $10\frac{1}{2}$ in
2. 6 ft $7\frac{1}{2}$ in
3. 4 ft $3\frac{1}{4}$ in
4. $11\frac{3}{4}$ in
5. $8\frac{5}{8}$ in
6. $2\frac{1}{8}$ in
7 $6\frac{3}{8}$ ft
8. 4 yds $2\frac{3}{4}$ ft
9 5 yds $1\frac{3}{16}$ ft
10 3 sq ft. 72 in.
11 2 sq ft 36 in
12. 4 sq ft 88 in.
13 136 sq in
14. 143 sq in
15 $89\frac{1}{2}$ sq in
16 $5\frac{3}{4}$ sq ft
17 $6\frac{5}{8}$ sq ft
18 5 sq yds $1\frac{1}{16}$ sq ft
19. $10\frac{3}{4}$ cub ft.
20. $17\frac{3}{8}$ cub ft
21 $3\frac{13}{16}$ cub ft
22. $3\frac{1}{2}$ cub in
23 $86\frac{3}{4}$ cub in
24 $117\frac{1}{8}$ cub in
25 6 cub ft 436 in.
26 11 cub ft 864 in
27. 2 cub ft. 1524 in

Express in feet and inches,

28 2 ft. $5'$.
29. 1 ft. $7'.6''$
30. 5 ft $4'\ 9''$
31. 11 ft $10'\ 8''$
32. 7 ft. $0'\ 4''$
33 16 ft $2'\ 7''\ 6'''$
34 5 sq ft $9'$
35 14 sq ft $3'\ 8''$
36 6 sq ft $11'\ 9''$
37 1 sq ft $8'\ 7''\ 6'''$
38. $10'\ 11''\ 8'''$
39 $9'\ 4''\ 5'''\ 6^{iv}$.
40 12 cub ft. $2'$
41. 7 cub ft $0'\ 5''$.
42 3 cub ft $5'\ 7''\ 3'''$.
43 $10'\ 8''\ 11'''$
44 $8'\ 7''\ 10'''\ 6^{iv}$
45 $5'\ 10''\ 0'''\ 8^{iv}\ 9^{v}$

46 Add 18 ft $9'\ 8''$, 2 ft $5'\ 7''$, $11'\ 10''\ 8'''$.
47 Add 15 sq ft $3'\ 7''$, 1 sq ft $11'\ 8''.5'''$, and $6'\ 0''\ 8'''\ 6^{iv}$
48 Add 12 sq ft $5'\ 8''$, 3 sq ft $10'\ 5''\ 9'''$, and $11'\ 8''\ 7'''$
49 Subtract 2 cub ft $8'\ 7''\ 5'''\ 6^{iv}$ from 7 cub ft $2'\ 0''.3'''$.
50 Multiply 2 ft $7'\ 8''$ (i) by 12, (ii) by 2, (iii) by 13
51. Multiply 5 sq ft $6'\ 9''\ 4'''$ (i) by 12, (ii) by 72, (iii) by 47
52 Divide 16 ft $3'\ 5''$ (i) by 12, (ii) by 10, (iii) by 24
53 Find, by Practice, the cost of 5 sq ft $8'\ 6''$ at 6*s* 6*d* per sq ft.
54 Find, by Practice, the cost of 4 cub ft $6'\ 10''\ 8'''$ at £2, 12*s*. 6*d* per cub ft

Find, by duodecimals, the area of a rectangle of dimensions—

55. 3 ft $4'\ 6''$ and 3 ft
56. 4 ft $5'\ 9''$ and $3'$.
57. 1 ft $6'\ 3''$ and $9''$
58 $8'\ 4''$ and $8''$
59 3 ft $5'$ and 2 ft $3'$
60 4 ft $9'$ and 2 ft $6'$.
61 7 ft $10'$ and 5 ft $8'$
62. 12 ft $8'\ 2''$ and 7 ft $10'$
63 21 ft $9'\ 7''$ and 12 ft $9'$
64 4 ft $3'\ 9''$ and 2 ft $5'\ 6''$
65 11 ft $10'\ 5''$ and 8 ft $9'\ 7''$
66 23 ft $8'\ 9''$ and 14 ft $3'.6''$.

Find, by duodecimals, the area of a square whose side is—

67 4 ft $7'$. 68 $5'.5''$ 69. 1 ft $6'.8''$. 70. 13 ft. $9'\ 6''$.

Find, by duodecimals, the volume of a rectangular solid of dimensions—

71. 2 ft 9′, 2 ft 3′; 2 ft
72. 1 ft. 8′, 1 ft 6′, 1 ft 3′.
73. 4 ft. 7′, 3 ft 8′, 2 ft 6′
74. 3 ft 4′.6″; 1 ft 6′, 8′.
75. 16 ft. 8′, 13 ft. 9′; 3′
76. 32 ft. 9′; 1 ft. 3′, 2′.
77. 2 ft 5′.6″, 2 ft 2′; and 1 ft. 6′
78. 5 ft 7′.9″; 3 ft 8′; and 2 ft. 9′
79. 11 ft 4′.3″; 8 ft 9′.7″; and 3 ft. 10′.
80 16 ft 2′.8″; 4 ft 11′ 2″, and 2 ft. 8′ 6″.
81. 14 ft. 7 in; 13 ft $8\frac{1}{2}$ in, and 3 yds. 2 ft. 7 in.
82. 7 yds 2 ft. 9 in.; 16 ft 3 in 10 pts; and 5 yds. $11\frac{1}{2}$ in

Find, by duodecimals, the volume of a cube whose edge is—

83. 2 ft 4′. 84 3 ft. 8′. 85. 8′.8″. 86. 7 ft 7′.7″

Find by duodecimals—

87. The number of square feet of glass in 24 windows, each containing 4 panes, each pane 1 ft 7′ long and 10′.9″ wide.
88. The number of square feet of glass in 37 windows, each containing 12 panes, each pane 11′ 6″ long and 8′.3″ wide.
89. The cost of glazing 60 windows, each containing 12 panes, each pane 1 ft. 2′ 6″ long and 10′.6″ wide, at 1*s* 6*d* per sq. ft.
90 The cost of glazing 96 windows, each containing 9 panes, each pane 1 ft 5′.6″ long and 1 ft 1′ 3″ wide, at 16*d* per sq. ft.
91. The cost, to a penny, of digging out a cellar 18 ft. 4′ long, 12 ft 9′ wide, and 14 ft. 6′ deep, at 1*s* 6*d* per cubic yard.
92. In sq yds, ft, and ins the area of the entire surface of a block 11 ft 1′ 6″ long, 11 ft 8′ wide, 6 ft. 6′ 8″ thick
93. The area of the entire surface of a cube of edge 8 yds 2 ft. $6\frac{3}{4}$ in
94 The value of 3 dozen blocks of granite, each 15 ft 8′ long, 10 ft 4′ wide, and 6 ft 9′ thick, at £1, 16*s* per cub. yd.
95 How many more cubic inches of water a tank 12 ft. 6′ long, 5 ft. 3′ wide, and 3 ft. 9′ deep, will hold than a cubical tank whose edge is 6 ft 3 in.
96. The value, to a penny, of a block 11 ft. $4\frac{1}{4}$ in. long, 3 ft. $3\frac{5}{12}$ in. wide, and 2 ft $4\frac{1}{4}$ in. thick, at 8*s*. 9*d*. per cubic foot.
97. The quotient of 118 sq. ft. 10′ 6″.6‴ ÷ 21 ft 3′.6″.
98 The quotient of 390 sq ft. 5′ 3″.6‴.6$^{\text{iv}}$ ÷ 23 ft. 4′ 5″
99. The thickness of a rectangular solid of volume 17 cub ft. 1′ 11″.10‴.6$^{\text{iv}}$, length 3 ft 1′ 6″, and breadth 2 ft. 4 in.
100. The length of a rectangular solid of volume 303 cub ft. 3′.5″.7‴.4$^{\text{iv}}$, breadth 8 ft. 4 in, and thickness 2 ft. $7\frac{3}{4}$ in.

L. APPROXIMATION.

DECIMALS

Write down the nearest approximation
(i) in *three* places, (ii) in *five* places, of decimals to—

1. 4·5632765	**4** ·2789325	**7** 2·068$\dot{3}$
2. ·0326549	**5.** 6·1499325	**8** 473·$\dot{6}$
3. 15·2476256	**6** ·4233655	**9.** ·0$\dot{5}\dot{8}$

Find the nearest approximation, within the limit mentioned, to—

10. The sum of 24·175, 4·775625, 18·49636, ·1356 and ·085725, to *two* places of decimals

11 7·4632 + 12·05768 + ·0195 + 23·40585 + 301·62385 + 1·45367, to *one* place of decimals.

12 The sum of ·5$\dot{6}\dot{3}$, 18·4078$\dot{3}$, 6·$\dot{7}$, ·05975, 2·48$\dot{6}$ and 21·34625, to *three* places of decimals

13 The difference between 4·452296 and 104·235 to *three* places of decimals.

14. 12·1$\dot{6}$ — ·857425 to *four* places of decimals

15. 8·56340075 — 2·08$\dot{3}$ to *five* places of decimals

16. Find the integer nearest to the sum of 87·563, 910·2675, 7·864, 204·4793, 8·5, 47·5, 60·075 and 5675

17. Find to the nearest million the sum of 4756320, 10478630, 1562500, 8479000, 2936860 and 12075680

18. Find, within one-thousandth of the whole, the sum of 7·875, 1·20584, 12·67325, 148·8366 and 27695

19. Find, within one-hundredth of the whole, the sum of 4·$\dot{7}$, 12·6834, 3·14159, ·$\dot{0}7\dot{3}$, 42·587632 and 28·5

20 Find, within one-millionth of the whole, the sum of 54763·25, 80·4725, 6·47965, 97318·476, 293·647875 and ·086795

21. Find, within one-thousandth of the whole, the difference between 87·6187345 and 147·53225

22. Find, within one-millionth of the whole, 438·573 — 37·8$\dot{3}$

Find the nearest approximation in *three* places of decimals to—

23 14·5637525 × 8	**31** ·0814275 ÷ 7	**39** 813·576385 × $2\frac{1}{2}$
24 2·41573825 × 12	**32** 7·375284 ÷ 12	**40** 8·241675 × $3\frac{3}{4}$
25. ·13068875 × 40	**33** 32·6410565 ÷ 60	**41** 52·50291$\dot{6}$ × $2\frac{3}{4}$
26 ·8302545 × 15	**34** 45690385 ÷ 32	**42** 4·638225 × $3\frac{1}{3}$
27 6·62078$\dot{3}$ × 800	**35** 2·5069578 ÷ 5	**43** 23·7225 × 4 ÷ 100
28 26·508472 × ·4	**36** 5·2506725 ÷ 56	**44** 491·75 × $2\frac{1}{2}$ ÷ 100
29 3·55381$\dot{6}$ × ·05	**37** 56·48$\dot{3}$ ÷ 73.	**45** 76·48$\dot{3}$ × $3\frac{1}{4}$ ÷ 100
30. 8·51$\dot{6}$ × 1·6	**38.** 4·3259 ÷ 1·7.	**46** 506·41$\dot{6}$ × $3\frac{3}{4}$ ÷ 100.

Decimalize to *three* places—

47. £2, 16*s* 5*d*	51. £4, 15*s* 7*d*	55. £3, 3*s* 6*d*.
48 £13, 18*s* 2*d*.	52 £0, 9*s* 9*d*	56. £75, 19*s* 10*d*.
49. £1, 16*s*. 7*d*	53. £63, 11*s* 1*d*	57 £2, 1*s* 11*d*.
50. £43, 13*s* 1*d*	54 £10, 3*s* 8*d*	58. 17*s*. 8½*d*

Decimalize to *five* places—

59 £1, 9*s* 10*d*.	62. £0, 5*s* 8*d*	65. £4, 18*s*. 8½*d*.
60 £16, 12*s* 7*d*	63 £453, 16*s*. 2*d*	66. £87, 3*s* 3¾*d*
61. £0, 13*s*. 11*d*	64. £197, 13*s*. 5*d*	67. £142, 1*s*. 7¼*d*.

Find, to the nearest *penny*, the value of—

68. £4 816	71. £13·7028.	74. £123·88375
69. £1·584.	72. £·6363.	75 £1·5951$\dot{6}$.
70. £17·908	73. £38·3565	76. £11·0983

Find, to the nearest *farthing*, the value of—

77. £47·5465.	79 £1·519	81. £546·46
78. £·685.	80. £11·046535.	82. £3·298156

Find, to the nearest penny, the value of—

83 £4·5735 + £15·032 + £1·4723575 + £·01275 + £12·428766.

84. £13·469725 + £4·8$\dot{3}$ + £·16775 + £23·02$\dot{3}$1$\dot{6}$ + £305·970$\dot{5}\dot{7}$.

85. £52·470895 — £6·528775. 86 £17·18$\dot{3}$ — £9·241$\dot{6}$.

Find, to the nearest penny, *by Decimalized Practice*,* the—

87. Value of 1672½ things at £3, 11*s* 7½*d* each

88. Cost of 893¼ things at £1, 17*s* 5¼*d*. each

89. Rent of 956A. 3R 10P at £1, 6*s* 3*d* per acre.

90. Rent of 1857A 2R 24P. at £1, 14*s* 8½*d* per acre

91. Value of 1473 tons 13 cwts at £2, 11*s* 5*d*. per ton.

92. Dividend on £1485, 17*s* 6*d* at 5*s* 6½*d* in the pound.

93. Dividend on £884, 16*s* 8*d*. at 11*s*. 8½*d* in the pound

94 Value of 1 cwt. 2 qrs 17 lbs. at £2, 13*s*. 8*d* per cwt.

95. Cost of 3 cwts. 1 qr 16 lbs. 9 ozs. at £14, 16*s*. 6*d* per cwt

96. Rent of 2 ac. 3 ro 31 po at £1, 11*s* 8*d* per acre

97 Rent of 4 ac 2 ro. 37¾ po at £1, 8*s*. 6*d*. per acre.

98. Value of 10 Troy ozs. 397 grs. at £3, 17*s*. 10½*d* per oz.

99. 2 tons 11 cwts 2 qrs 17½ lbs at £2, 15*s* 11½*d*. per ton.

100. 39 cwts 3 qrs 24¼ lbs at £5, 13*s*. 8*d* per ton.

---

* Further exercises in this method may be obtained from pages 89 and 92, the results *to the nearest penny* being easily seen from the *exact* answers given.

Contracted Multiplication

Obtain, in the number of places of decimals mentioned within brackets, the nearest approximation to—

101 90·760125 × 8·0127 (*two*).
102. 12·345675 × 1·23456 (*two*)
103. ·9846725 × 7·90845 (*two*)
104 5436·7928 × 5·36419 (*two*)
105 ·0598765 × 2·107685 (*three*)
106 4·7603561 × 5·7140232 (*three*)
107 ·04327184 × 80·85642 (*two*)
108. 2·706315 × 85 07685 (*two*)
109 ·91843275 × 51·0826 (*three*)
110 416·087925 × ·0807685 (*two*)
111. 34·76912 × 41·2804 (*three*).
112 41 69207 × 004037 (*three*).
113. 1705 32 × ·014623 (*two*)
114 42·506 × ·00840542 (*four*).
115. ·0521685 × ·0084032 (*four*).
116 ·810901 × ·809101 (*three*).
117. 4·132167 × ·01098 (*two*)
118. 234678 × ·378426 (*six*)
119 $8\ 215\dot{7} \times \cdot\dot{8}23\dot{1}$ (*four*)
120. $456\ 8\dot{3} \times \cdot\dot{1}4285\dot{7}$ (*six*)

121. $(4\cdot1506312)^2$ (*three*)
122. $(3\cdot14159)^2$ (*three*)
123. $(\cdot57075)^2$ (*two*)
124. $(12\cdot0575)^2$ (*four*)
125. $(\cdot02138607)^2$ (*six*)
126 $(2\cdot1703)^3$ (*two*).
127 $(3\cdot1416)^3$ (*two*)
128 $(\cdot41506)^3$ (*two*)
129. $(\cdot0286038)^3$ (*five*)
130 $(\cdot\dot{5}13\dot{2})^3$ (*three*)
131. $(1\cdot0325)^4$ (*three*).
132 $(1\cdot03)^8$ (*three*)
133 $(1\cdot025)^5$ (*five*)
134 $(1\cdot04)^9$ (*five*)
135. $(1\cdot05)^{11}$ (*three*)

136 Find ·015632 × 14·0514 × ·72315 to *two* places of decimals
137. Find ·0012345 × 805·6231 × 12·14285 *three*
138. Find ·265137 × 128 23 × 9·2671 *four* .
139. Find $\cdot3141\dot{6} \times 21\ 5\dot{7}\dot{2} \times 41\cdot5\dot{3}\dot{8}$. *three*

140 Find the integer nearest to 72 96415 × 18·74625
141 Find the integer nearest to the square of 41·38716
142 Find the integral part of the product of 43·126725 and 39 425683
143 Find, to the nearest *thousand*, 76045 285 × 91·8375
144 Find, to the nearest *thousandth* of unity, 30·7865 × ·4312
145 Find, within *one-thousandth of the whole*, 143 325 × 64 495
146. Find, within *one-thousandth of the whole*, 7·056275 × 1·175875
147. Find, within *one-hundredth of the whole*, 1 92648 × ·182356
148. Find, within *one-millionth of the whole*, 45·105275 × 608·05625
149 Find, within *one-thousandth of the whole*, $(15\ 607)^3$
150. Find, within *one-thousandth of the whole*, the continued product of ·000386, ·010274 and 65·25

Find, to the nearest penny, the value of—

151. £·57325 × 2·54

152. $£75{\cdot}41\dot{6} \times {\cdot}1352$.

153. £14, 17*s* 8*d* × 203·75.

154 £138, 13*s*. 6*d* × 315·425

155. 426 tons 13 cwts. 3 qrs. 7 lbs at £815, 12*s* 6*d*. per ton

156. 273 miles 6 fur. 120 yds. at £613, 12*s*. 8*d*. per mile.

157. $£8175{\cdot}8\dot{3} \times (1{\cdot}025)^5$.

158. $£506{\cdot}75 \times (1{\cdot}05)^7$.

159. $£851,\ 10s \times (1{\cdot}04)^{12}$.

160. $£3500 \times (1{\cdot}05)^{20}$.

Contracted Division.

Find, in the number of places of decimals mentioned within brackets, the nearest approximation to—

161. 8109·75 ÷ 15623 (*three*).

162. 314210·8 ÷ 18306 (*two*)

163. ·0264508 ÷ 1·2054 (*four*).

164. 8·14623 ÷ 12·954 (*three*)

165. 8·567145 ÷ 47·632 (*three*).

166. 142·875 ÷ 603·625 (*three*).

167. 72·2416 ÷ ·2365 (*two*)

168. 32·5172 ÷ ·046325 (*two*).

169. ·046325 ÷ 32·5172 (*four*).

170. ·03765 ÷ 13·2416 (*four*)

171. ·3060875 ÷ 2·3641525 (*five*).

172. 352·4258 ÷ 23·575 (*four*).

173. 6·25040875 ÷ 124·528 (*six*).

174 324·816 ÷ 7·248057 (*six*).

175. $1{\cdot}425 \div {\cdot}0\dot{5}6\dot{7}$ (*three*).

176. $8{\cdot}2\dot{1}5\dot{7} \div {\cdot}\dot{8}23\dot{1}$ (*four*).

177. $\cdot36\dot{2}\dot{5} \div 14{\cdot}\dot{6}\dot{9}$ (*five*).

178. $17{\cdot}\dot{3} \div 219{\cdot}1\dot{5}7\dot{3}$ (*six*)

179. 1847·345 × 1·72 ÷ 2185 (*two*).

180. 241·345 × ·2075 ÷ 3·1416 (*two*).

181. $\dfrac{516{\cdot}5 \times 852}{36500}$ (*three*).

182. $\dfrac{{\cdot}8856 \times 5{\cdot}3718}{12{\cdot}6784}$ (*three*).

183. $\dfrac{7{\cdot}14765}{8{\cdot}76 \times 4{\cdot}25}$ (*three*)

184. $\dfrac{407{\cdot}6275 \times {\cdot}1568}{(3{\cdot}1416)^2}$ (*two*).

185. $\dfrac{17{\cdot}525 \times {\cdot}015745}{3{\cdot}13675 \times 21{\cdot}24325}$ (*four*).

186. Find the integer nearest to 1·002591 ÷ ·038764875.

187. Find the integral part of the quotient of 4863·375 ÷ 1·9258725

188. Find, within *one-thousandth of the whole*, 73·15688 ÷ 158·45.

189. Find, within *one-thousandth of the whole*, 3568·5 ÷ 16·517845.

190. Find, within *one-millionth of the whole*, 14·68705 ÷ ·0316283.

Find, to the nearest penny, the value of—

191. $£863{\cdot}48\dot{3} \div 617$.

192. £103·725 ÷ 318·546

193. £21769, 11*s*. 10*d*. ÷ 8576

194. £1490, 13*s*. 8*d*. ÷ 13·825.

195. £31·1426 × 12·18 ÷ 73.

196. £1156, 17*s*. 6*d*. × 4·86 ÷ 365.

Find, to *five* places of decimals, the sum of the series—

197. $\frac{1}{1\times2}+\frac{1}{1\times2\times3}+\frac{1}{1\times2\times3\times4}+\quad\ldots$

198. $\frac{1}{1\times2}-\frac{1}{1\times2\times3}+\frac{1}{1\times2\times3\times4}-\ldots$

199. $\frac{1}{1\times3}+\frac{1}{1\times3\times5}+\frac{1}{1\times3\times5\times7}+\ldots$

200. $\frac{1}{1\times3}-\frac{1}{1\times3\times5}+\frac{1}{1\times3\times5\times7}-\ldots$

Find, to the nearest penny, by contracted work in Decimals, the

201. Value of 5 tons 11 cwts 3 qrs at the rate of £41, 13*s*. 7*d* for 7 tons 9 cwts 1 qr

202. Value of 78 ozs. 210 grs of gold, if 53 ozs 300 grs. are worth £175, 6*s* 3*d*

203. Dividend paid by a bankrupt whose debts are £2957, 18*s* 8*d*, and his available assets £1025, 13*s* 6*d*

204 Dividend to be paid by a bankrupt whose assets amount to £10,223, 14*s* 8*d*, his liabilities to £13,901, 16*s* 6*d*, and the "costs" of his bankruptcy to £473, 18*s*

205 Shares of profits amounting to £473, 18*s*. 3*d* due to each of two partners whose capitals are £2756 and £1350 respectively.

206. Average cost per ton if two truck-loads of coal are bought, the first, containing 9 tons 9 cwts 3 qrs, for £5, 8*s* 6*d*, and the second, containing 8 tons 18 cwts 2 qrs, for £5, 3*s* 6*d*.

207. Find, to the nearest *penny*,

parts of (i) £56, 16*s* 4*d* proportional to 67, 47 and 29;
(ii) £147, 13*s* 9*d* . . 13, 15 5 and 19;
(iii) £1413, 12*s* . . 13, 19¼ and 22

Also, to the nearest *quarter*,

parts of (iv) 57 tons 13 cwts 3 qrs . .... 7, 17 and 37

and to the nearest *pole*,

parts of (v) 473 A 2 R 37 P. . 9, 9·5 and 9·25.

208 Find, within a thousandth of the whole, the area of a rectangle 34 ft. 5 in long and 26 ft. 7½ in wide

209 If the length of a rectangle is known to be greater than 6 135 ft and less than 6·136 ft, and its breadth greater than 4·027 ft and less than 4·028 ft, within how many square inches can its area be determined with certainty?

210. If a bicycle-wheel has a circumference of 7·145 ft. and makes, on the average, 6500 revolutions per hour, find, to the nearest hundredth of a mile, the distance travelled in 5 hrs 27 min.

LI. PERCENTAGES.

Find, *mentally*, the percentage equivalent to the fraction—

1. $\frac{1}{2}$	5. $\frac{1}{25}$	9. $\frac{7}{10}$.	13. $\frac{1}{3}$.	17. $\frac{3}{40}$.
2. $\frac{1}{4}$	6. $\frac{1}{50}$.	10. $\frac{9}{20}$	14. $\frac{1}{8}$.	18 $\frac{7}{200}$.
3. $\frac{1}{5}$	7. $\frac{3}{4}$.	11. $\frac{11}{25}$.	15. $\frac{1}{12}$.	19. ·625.
4. $\frac{1}{20}$.	8. $\frac{2}{5}$.	12. $\frac{17}{50}$	16. $\frac{1}{40}$	20. ·1275.

Find, *mentally*, in lowest terms, the fraction equivalent to—

21. 8 per cent	25. 78 per cent	29. $6\frac{1}{4}$ per cent.
22. 24 per cent.	26. 6 per cent.	30. $66\frac{2}{3}$ per cent.
23. 15 per cent.	27. $4\frac{1}{2}$ per cent.	31. $8\frac{1}{3}$ per cent.
24. 35 per cent.	28. $3\frac{1}{3}$ per cent.	32. 12·5 per cent.

Find the percentage equivalent to the fraction—

33. $\frac{3}{16}$. 34. $\frac{5}{19}$ 35. $\frac{17}{120}$. 36. $\frac{7}{17}$ 37. $\frac{11}{305}$. 38. ·375.

Find, in lowest terms, the fraction equivalent to—

39. $3\frac{1}{8}$ per cent	41. $6\frac{7}{8}$ per cent.	43. 42·5 per cent.
40. $4\frac{1}{3}$ per cent	42. $2\frac{3}{16}$ per cent.	44. 18·4 per cent.

Find—

45. 72 per cent of 375.	48. $16\frac{2}{3}$ per cent of 492.
46. 12 per cent of 3775	49. 6·25 per cent of 648.
47. $3\frac{3}{4}$ per cent of 4720	50. 8·375 per cent of 42400.

How much per cent is—

51. 19 of 475?	53. 189 of 504?	55. 23·75 of 475?
52. 235 of 750?	54. 4455 of 5445?	56. 58·8 of 3920?

57. If $7\frac{1}{2}$ per cent of a certain number is 69, what is that number?

58 Find the number $8\frac{1}{3}$ per cent of which is 48.

59 Of what number is 109·8 twelve per cent?

Find, *mentally*, 5 per cent of—

60 £94	64. £12, 10*s*	68. £36, 12*s* 6*d*.	72. £86, 8*s*. 4*d*.
61. £116.	65. £23, 5*s*	69. £4, 6*s*. 8*d*.	73. £132, 16*s* 8*d*.
62. £275.	66. £8, 15*s*.	70. £9, 13*s*. 4*d*.	74. £189, 11*s*. 8*d*.
63. £347.	67. £17, 10*s*.	71. £5., 4*s* 2*d*.	75. £537, 18*s*. 4*d*.

Find, *mentally*, neglecting fractions of a penny, 5 per cent of—

76. £10, 2*s*	**79** £14, 14*s* 8*d*	**82.** £107, 7*s* 2*d*.
77. £3, 12*s*	**80** £43, 15*s* 9*d*	**83** £181, 9*s* 8*d*
78. £26, 19*s*	**81** £62, 11*s*. 10*d*	**84** £467, 13*s* 10*d*.

Find, *mentally*, neglecting fractions of a penny, $2\frac{1}{2}$ per cent of—

85 £34.	**88** £7, 10*s*.	**91.** £10, 16*s* 8*d*.
86. £57	**89** £14, 11*s*	**92** £4, 7*s* 6*d*.
87. £12, 10*s*	**90** £28, 6*s* 8*d*.	**93.** £59, 14*s*. 3*d*.

Find—

94. 4 per cent of £7, 10*s*	**99** $3\frac{1}{2}$ per cent of £50, 16*s* 8*d*.
95 4 per cent of £71, 5*s*	**100** $2\frac{1}{4}$ per cent of £26, 13*s*. 4*d*.
96. 3 per cent of 6 tons 5 cwts	**101** $7\frac{1}{4}$ per cent of £125
97. 85 per cent of 19 ozs Troy	**102.** $4\frac{3}{4}$ per cent of £202, 10*s*.
98. 15 per cent of 193·5 grs	**103.** $5\frac{5}{8}$ per cent of £423, 13*s* 4*d*

How much per cent is—

104. 2*d* of 1*s* ?	**110** 1*s* $7\frac{1}{2}$*d* of £1, 12*s* 6*d* ?
105 2*s* 6*d* of £1 ?	**111** 12*s* 9*d* of £1, 9*s* 9*d* ?
106. $1\frac{1}{2}$*d* of £1 ?	**112** £38, 10*s* of £924 ?
107. 19*s* 6*d* of £1 ?	**113.** 42 lbs of 2 cwts ?
108 6*s* 8*d* of £5 ?	**114** 16 grs of 1 oz Troy ?
109. 8*s* 4*d* of £2 ?	**115.** 2 hrs 24 mins of 5 days ?

If a tradesman allow 5 per cent "discount for cash", find, neglecting fractions of a penny, how much ready money would pay for an article priced—

116 14*s* 10*d*	**119** £4, 3*s* 9*d*	**122.** £11, 7*s* 9*d*.
117. £2, 12*s*	**120** £6, 16*s*. 8*d*.	**123.** £22, 14*s* 6*d*
118. £1, 8*s* 6*d*	**121.** £5, 5*s*	**124.** £17, 13*s* 3*d*.

If a tradesman allow $2\frac{1}{2}$ per cent discount for prompt payment of an account, find, neglecting fractions of a penny, the ready money which would pay for goods invoiced at—

125 £4, 17*s* 7*d*	**128.** £7, 8*s* 5*d*	**131** £5, 13*s*. 10*d*
126. £3, 5*s* 10*d*.	**129.** £18, 10*s* 9*d*.	**132** £1, 19*s* 1*d*
127. £2, 11*s*. 6*d*.	**130.** £25, 12*s*. 3*d*	**133.** £29, 1*s* 6*d*

Increase—

134. 450 by 2%.	**136.** £14, 10*s*. by 5%	**138.** £43, 10*s*. 8*d*. by $12\frac{1}{2}$%.
135. 236 by 25%	**137.** £33, 15*s*. by 4%.	**139.** £93, 12*s*. 3*d* by $16\frac{2}{3}$%.

Increase, giving the result to the nearest penny—

140. £47, 10*s* by 4%.
141. £51, 13*s*. 4*d* by 3%.
142. £18, 15*s*. by $3\frac{1}{2}$%.
143 £91, 12*s*. 3*d*. by $2\frac{1}{4}$%.
144. £156, 18*s*. 8*d*. by $4\frac{1}{2}$%.
145. £284, 6*s*. 10*d*. by $3\frac{3}{4}$%.

Decrease—

146. 1890 by 10%.
147. 4620 by 15%.
148. £51, 10*s* by $2\frac{1}{2}$%.
149. £137, 13*s*. 3*d*. by $33\frac{1}{3}$%.
150. 11 tons by $7\frac{1}{2}$%.
151. 9 tons 16 cwts. by $12\frac{1}{2}$%.

Decrease, giving the result to the nearest penny—

152. £36, 11*s* 2*d* by 2%.
153. £16, 7*s*. 11*d* by 7%.
154. £73, 15*s* 7*d*. by $3\frac{1}{4}$%.
155. £105, 5*s*. 9*d*. by $2\frac{1}{3}$%
156. £272, 1*s*. 10*d*. by $4\frac{1}{4}$%.
157. £518, 16*s*. 4*d*. by $2\frac{5}{8}$%.

---

158. If a man with an income of £570 a year spends 85 per cent of it, what does he save annually?

159. The population of a town was 143,500 in 1881, and it had decreased 3 per cent at the next census; what was its population in 1891?

160. The population of a town at one census was 172,800; at the next it was 181,440; find the rate per cent of increase.

161. Standard gold contains 22 parts of pure gold to 2 parts of alloy. Find the percentage of alloy in a sovereign

162. If 17·5 per cent of a number be 1946, what is the number?

163. If $3\frac{1}{2}$ per cent of a sum of money is £22, 10*s* 11*d*, what is that sum?

164. The death-rate one week in a town of 82,000 inhabitants was 17·5 per thousand. How many persons died?

165. The population of a certain town increased during a certain interval from 24,500 to 25,250; find, to two places of decimals, the rate per cent of increase.

166. If a bankrupt pays a dividend of 12*s*. 6*d*. in the pound, how much per cent do his creditors lose?

167. If the price of coal is 16*s*. 8*d*. a ton, subject to discount of 5 per cent for cash, how much ready money must be paid for 6 tons 8 cwts.?

168. Between 1871 and 1881 the population of a certain town increased by $24\frac{3}{4}$ per cent, and in the latter year it was 206,087; what was it in the former?

169. If 8 per cent of a garrison of 4125 men desert to the enemy who number 6000, what is the gain per cent to the enemy?

170. In a concert hall there are 600 reserved seats and 1300 unreserved seats; when $14\frac{1}{2}$ per cent of the former and 10 per cent of the latter are unoccupied, of how many persons does the audience consist?

171. If 11 men working 10 hours a day earn 643·5 francs in 13 days, in how many days would 25 men, working 9 hours a day, earn 2673 francs when the pay for an hour's work has risen 20 per cent?

172. In an examination, A obtains 48 per cent and B 33 per cent of full marks. Their marks, if added together, amount to 567. What are full marks?

173. An arithmetic examination paper is worked by 2500 candidates, of whom one-fifth are girls and the rest boys. If 5 per cent of the boys, and 40 per cent of the girls, fail, what percentage of the total number of candidates pass?

174. The populations of the upper and lower parts of a town were equal. After the former had fallen 20 per cent, and the latter risen 15 per cent, the total number of inhabitants was 39,390. What was the population of each part at first?

175. At a census three towns had populations of 17,650, 19,600 and 18,760. At the next census the first had decreased 18 per cent, the second had increased 21 per cent, and the total population of the three had increased by 4691 persons: find the change per cent in the population of the third town.

176. A man whose income is £900 a year pays an income-tax of 8*d.* in the pound; what percentage of (i) his gross income, (ii) his net income, is his tax?

177. After paying 10 per cent of his income in rates and taxes, a man has £1350 left. Find his whole income.

178. In a battle 5 per cent of an army were killed, 18 per cent wounded, 8 per cent were taken prisoners and 243 men deserted. The effective force then remaining numbered 5691. Find the number of men at first.

179. If a man can travel 360 miles in 12 days of 8 hours each, how many hours a day less need he travel, if he increase his rate 20 per cent, in order to accomplish 450 miles in 20 days?

180. If the income-tax were $3\frac{1}{3}$ per cent on a man's income, instead of 8*d.* in the £, what difference would it make to a man whose annual income is £3125?

181. A bankrupt's debts amount to £21,140, and his assets to £4832. Allowing $12\frac{1}{2}$ per cent for expenses, what would the dividend be?

182. In a forest 5 per cent of the trees are blown down by a gale, and, after 3 per cent of those remaining have been cut down, there still stand 55,290 trees. How many trees were there in the forest before the gale occurred?

183. If A pays $8d$ in the pound income-tax and other charges amounting to 5 per cent on the remainder of his income, what will he have left out of an income of £600 per annum?

184. The net rental of an estate after deducting $7d$ in the pound for income-tax, and 5 per cent on the remainder for the expenses of collecting, is £479, 11s 10d., find the gross rental.

185 A man after paying 5 per cent on his gross rental for collecting, and income-tax at the rate of $8d$. in the £1 on it, has left £672, 15s. 9d., find his gross rental

186. If the length and breadth of a rectangle were each increased 20 per cent, how much per cent would its area be increased?

187. If the length of a rectangle were increased 15 per cent and its breadth decreased 15 per cent, what would be the change per cent in its area?

188. If air is composed of 75·55 per cent of nitrogen, 23·22 per cent of oxygen and 1·23 per cent of carbonic acid, how much is there of each gas in a chamber containing 6548 cubic feet of space?

189. If air contains 23·01 per cent of its weight of oxygen, and if a cubic foot of air weigh 1·23 ounces avoirdupois; find, to the nearest ounce, the weight of the oxygen in a room 18 feet by 27 ft. 4 in by 12 feet.

190. At an examination in which full marks were 1000, A got 20 per cent more than B, B 10 per cent more than C, C 20 per cent less than D If A got 660, what percentage of full marks was obtained by D?

191. In an examination paper of 5 questions, 5 per cent of the candidates answer all, 5 per cent none, 25 per cent, and 20 per cent, of the rest answer respectively one and four; also 24·5 per cent of the total number of candidates answer two, and 200 candidates answer three, questions How many candidates are there?

192. A rate of $4\frac{3}{4}$ per cent was to be paid on a certain sum, but income-tax at $6d$. per £ of rate was to be deducted from the rate The net rate paid was £342, 14s 3d. Find the sum on which it was paid

*193. If I add 5 per cent to a certain number and deduct 5 per cent from the same number, I obtain two numbers which differ by 51 Find these numbers.

194. If an income-tax of 8 pence in the pound brings in a net revenue of 15 millions, and the cost of collection is 4 per cent of the sum collected, what is the total amount of the incomes taxed?

195 If, when the income-tax is raised from $6d$. to $10d$. in the pound, the resulting revenue is only increased 10 per cent, what is the decrease per cent in the amount of incomes taxed?

196 At the census of 1891 the population of England was 27,501,362; of Wales, 1,501,163, of Scotland, 4,025,647, and of Ireland, 4,704,750 Find to the nearest *tenth* in decimals the percentage of the total population of the British Isles then inhabiting each country.

Find the numbers required to complete the following tables, giving the percentages of the numbers for 1893, correct to the nearest *tenth* in decimals.—

RAILWAY PASSENGER TRAFFIC RECEIPTS

197.

L & N W R	1894.	1893	Increase or decrease per cent.
	£	£	
First Class,	481,248	498,469	dec
Second „	240,762	257,782	dec
Third „	2,875,012	2,813,434	inc
Total,	3,597,022	3,569,685	inc

198.

G N R	1894	1893	Increase or decrease per cent.
	£	£	
First Class,	159,837	164,349	
Second „	20,824	23,779	
Third „	1,137,357	1,115,204	
Total,	1,318,018	1,303,332	

199.

G W R	1894	1893	Increase or decrease per cent.
	£	£	
First Class,	239,828	249,406	
Second „	262,810	291,980	
Third „	2,782,897	2,718,825	
Total,	3,285,535	3,260,211	

NUMBER OF PASSENGERS CARRIED ON ENGLISH RAILWAYS

200.

	1894	1893	Increase or decrease per cent.
Class	No	No	
Ordinary 1st,	29,821,010	30,048,982	
„ 2nd,	60,161,714	59,989,640	
„ 3rd,	821,430,202	783,138,430	
Season Tickets,	1,184,861	1,574,876	
Total,	912,597,787	874,751,928	

* Exs. 196 to 200 should be worked by the contracted methods of Chap L.

LII. PROFIT AND LOSS.

Find the gain, or loss, per cent of cost price in the following cases:—

1. Cost £30; sold for £36	6. Cost £10, 10s; sold for £3, 10s.
2. .... £4; . . . £4, 10s.	7. ... 3s. 9d; . 4s. $1\frac{1}{2}$d.
3. .... £36; . . . £30.	8. . . 12s. 6d; .. . . . 14s. 9d.
4. . £8; £7, 5s	9. . ... £5, 16s 8d ... £5, 14s 4d
5. ... .. £3, 2s. 6d . £3, 7s. 6d	10. ...... £1, 3s. 9d........ £1, 8s. 6d.

Find the selling price in the following cases —

11. Cost £8, 5s, and sold at a profit of	20	per cent of the cost.
12. ..... 7s 6d., ...	10	. .
13. . . £1, 15s, ...	15	. .. .
14. .... £5, 12s, ...	$12\frac{1}{2}$	. . . . . .
15. . . . 2s. 2d, ...	$37\frac{1}{2}$	.. . .
16. . . . 12s. 6d., and sold at a loss of	12	. . .. .
17. . . £12, 10s, ...	30	. .. .
18. . . . £1480, ...	$7\frac{1}{2}$	... .
19. . . £5, 12s. 6d, ...	4	. ...
20. . . . £1, 6s. 8d, ...	$6\frac{7}{8}$	....

Find the cost price of an article—

21. Sold for 14s, at a profit of	5	per cent of the cost.
22. .. .. . £1330, ...	14	. . .. .
23. ... . 9s. 9d, ...	17	.. .. ..
24. .. . . £1, 5s. $4\frac{1}{2}$d., . .	$8\frac{3}{4}$	. . . .
25. . . 88 guineas, ...	110	. ..
26. . . . 7s. 8d, at a loss of	8	.. . .
27. .. . . 480 guineas ..	10	. . ..
28. .. .. $11\frac{3}{4}$d., ...	6	. ...
29. . .. . . £4, 11s, ...	$13\frac{1}{3}$	.. . .
30. .... £16, 10s, ...	$72\frac{1}{2}$	. . . .

---

31. For how much must goods, bought for £31, 10s., be sold so as to gain $7\frac{1}{2}$ per cent of the cost?

32. What profit per cent is made if £7, 4s. 11d. be gained by an outlay of £123, 6s. 8d.?

33. If 123 yards of cloth are bought for £8, 4s., and sold at 1s. 6d. a yard, how much profit per cent on the cost is made?

34. A man sold a horse for £36, losing 4 per cent of the cost price. How much did he pay for the horse?

35. What was the cost price of goods which were sold for £95, 3s $3\frac{1}{4}d$. at a profit of $5\frac{1}{4}$ per cent of the cost?

36. If by selling an article at 19s 6d 17 per cent of the cost is gained, what did the article cost?

37. A tradesman gains 17 per cent of the cost by selling an article for £17, 11s, what is the cost price?

38. If a horse be sold for £90 at a gain of 20 per cent of the cost, what was the cost price of the horse?

39. If goods are bought at £1, 3s 4d a cwt, at what price per lb must they be retailed to bring in 10 per cent profit?

40. If sugar is bought at £20, 16s 8d a ton, at how much per lb must it be sold to realize 12 per cent profit?

41. If 10 cwts of tea are bought for £108, and sold again at 2s 6d. per lb, what is the gain per cent on the capital employed?

42. If tea be bought at 3s a lb., and sold at the rate of £19, 12s 0d. per cwt, what is the gain per cent?

43. A grocer bought 150 lbs of tea Selling it at 3s 7d a lb he gained $7\frac{1}{2}$ per cent Find what he gave for the whole

44 Eggs bought at 15 for 1s are sold at 10 for 9d What is the gain per cent on the outlay?

45. If herrings are bought at 2s 6d per 100 and sold at two for $1\frac{1}{2}d$, what is the gain per cent?

46. If eggs be bought at 10d per dozen and sold at 10 for a shilling, what is the gain per cent?

47. Find the gain per cent in buying oranges at seven for 6d and selling them at six for 7d

48. If eggs be bought at 9d per dozen, how many must be sold for a shilling so as to gain 60 per cent?

49. If eggs are bought at the rate of 10 for a shilling, how many ought to be sold for 11s 3d so as to gain $12\frac{1}{2}$ per cent?

50. By selling tea at 3s. a lb a grocer gains 4 per cent, for what must he sell it that he may gain 17 per cent?

51. If by selling tea at 2s 3d per lb I lose 10 per cent, at what price must I sell it to gain 15 per cent?

52. If by selling cloth at 9s per yard there is a gain of 5 per cent, how much per cent will be gained by selling it at 11s 3d per yd?

53. If sugar sold at 3d a lb. gives a profit of 12 per cent, at what price per cwt must it be sold to gain 15 per cent?

54. By selling goods for £120 a merchant gains 25 per cent, how much money would he have lost if he had sold them for £88?

55. By selling 8 tons of tea at 1s $6\frac{1}{2}d$ per lb a merchant loses $7\frac{1}{2}$ per cent For what must he sell the whole so as to gain 10%?

56. If a grocer gains one-eighth of the cost price by selling tea at 3s per lb., how much per cent would he gain if he sold it at 3s 4d.?

57. By selling an article for £49, 17*s* 6*d*. 5 per cent is lost, what should it be sold for in order to gain 14 per cent?

58. If I lose 5 per cent by selling a piece of land for £2337, what shall I gain or lose per cent by selling it for £2644, 10*s*.?

59. A sells his house for £1855, and gains thereby 6 per cent on the original cost. How much per cent would he have lost by selling it at £1715?

60. If I lose $6\frac{1}{4}$ per cent of the cost when I sell goods for £18, 15*s*., at what price must I sell them to gain $6\frac{1}{4}$ per cent?

61. If 6 per cent be gained by selling 340 lbs. of tea for £59, 10*s*, at what price per pound, to the nearest halfpenny, must it be sold in order to gain 10 per cent?

62. By selling 624 yds. of cloth at a loss of 4 per cent I lost altogether £15, 12*s*, what were the buying and selling prices per yd.?

63. A man sells oranges at a penny a piece, and so doing makes a profit of 44 per cent. How many does he buy for £10?

64. If $5\frac{1}{2}$ per cent be lost by selling articles at 7*s* a gross, find the gain or loss per cent by selling them at 5*s* $1\frac{1}{4}d$. a hundred.

65. Seven per cent was lost by selling a carriage for £69, 15*s*., and $4\frac{1}{2}$ per cent was gained by selling a horse for £94, 1*s* Find the total gain or loss.

66. A man bought 9 sheep for £25, and 8 oxen for £95 He sold the sheep at a gain of 12 per cent, and the oxen at a gain of 10 per cent What was his total profit?

67. A man buys two articles for £8 and £36 respectively, and sells them again, losing 40 per cent on the first and gaining 15 per cent on the second. How much does he gain or lose altogether?

68. How much is gained or lost per cent by buying a number of oranges at 5 for twopence, and selling half of them at two a penny and half at three a penny?

69. A person buys equal quantities of eggs at the rates of 2 a penny and 3 a penny What does he gain or lose per cent by selling them all at the rate of sixteen for a shilling?

70. If a milk-seller makes a profit of £438 per annum by selling milk at 4*d*. per quart, his rate of profit being 60 per cent, how many gallons does he sell on the average daily?

71. Find the cost price of 300 engravings to a dealer, who sold them all at a profit of 2*s*. 6*d* each, being $16\frac{2}{3}$ per cent profit.

72. A man sells an article for £15, less 5 per cent discount for cash. If he has bought it for £12, 10*s*., what is his gain per cent on his outlay?

73. A tradesman marks his goods at $22\frac{1}{2}$ per cent above cost price, but allows for ready money a discount of 5 per cent on the price marked. What profit does he make on a ready-money customer's payment of £279, 6*s*.?

74. A grocer mixes 28 lbs of tea at $2s\ 0\frac{1}{4}d$ per lb., with 3 at $2s\ 8d$, at what price per lb must he sell the mixture to make 20 per cent profit?

75 A tobacconist mixes together two kinds of tobacco, worth $5s$ and $6s$ per lb respectively, in such a manner as to have 3 lbs of the cheaper to 2 lbs of the dearer He sells the mixture at $6s\ 3d$ per lb, what does he gain per cent?

76. If 300 lbs of coffee worth $1s\ 10d$ per lb are mixed with the same quantity worth only $1s.\ 6d$ per lb, what should be the selling price per lb to gain 10 per cent on the outlay?

77. A grocer buys coffee at £8, $10s$ per cwt and chicory at £2, $10s.$ per cwt, and mixes them in the proportion of 5 parts chicory to 7 of coffee If he sells the mixture at £7 per cwt what is his gain per cent?

78. A grocer buys 17 lbs of tea at $2s$, 20 at $1s\ 9d$ and 54 at $1s\ 6d$, and mixes them He sells the mixture so as to make a profit of $21\frac{1}{3}$ per cent What does he charge for it per lb?

79. A grocer has two qualities of tea By selling the first at $3s\ 6d$ per lb he gains 25 per cent, and by selling the second at $3s$ he gains 20 per cent If he sell a mixture of one part of the first with two parts of the second at $3s\ 2d$ per lb., what is his gain per cent?

80. If 40 lbs of tea at $2s\ 3d$ per lb be mixed with 60 lbs at $1s\ 10d$ per lb, how must the mixture be sold so as to make a profit of 25 per cent?

81. Spirits at $25s\ 4d$, $22s\ 10d$, and $21s\ 8d$ per gallon are mixed in the proportion of the numbers 1, 2, 3 respectively, and sold at $4s\ 6d$ per $1\frac{1}{4}$ pint bottle Other expenses amounting to $2\frac{1}{2}d$ per bottle, what is the gain per cent?

82. If the prime cost of 125 gallons of wine is 100 guineas, and 20 pints are lost in bottling, what percentage of profit will be made on the whole by selling the remainder at $48s$ per dozen bottles, six bottles containing a gallon?

83. A wine merchant buys brandy at $16s$ per gallon, dilutes it with water and then sells the mixture at $15s$ per gallon His profits by the sale are 20 per cent How much water is there in each gallon of the mixture?

84. A merchant mixes 12 gallons of spirits at $10s.\ 6d$ a gallon with 15 gallons at $12s$ a gallon, and 7 gallons of water What is the cost price per gallon of the mixture, and what will be gained per cent by selling the whole at $10s\ 8d$ per gallon?

85. If 48 gallons of spirit at $12s$ per gallon, $2\frac{2}{3}$ gallons at $10s\ 6d$ and $19\frac{1}{2}$ gallons at $1s\ 4d$, be mixed with $19\frac{5}{8}$ gallons of water, and the mixture sold at $7s\ 10\frac{1}{2}d$ per gallon, what is the gain per cent?

86. A wine merchant buys 120 gallons of spirit for £96. How much

water does he mix with it if, when it is sold at 18*s.* a gallon, he makes a profit of 20 per cent?

87. A man buys 416 gallons of spirit at 10*s.* a gallon. How much water must he add to it that he may gain 10 per cent on his outlay and retail the mixture at 8*s.* a gallon?

88. A milkman buys daily 36 gallons of milk at 8*d.* per gallon. How much water does he mix with it if, by selling the mixture at 9*d.* per gallon, he clears 25 per cent profit?

89. If a person had sold his horse for £75 instead of for £72, he would have gained 5 per cent more than he did Find the cost price of the horse

90. If by selling cloth at 12*s* per yd. 5 per cent more is gained than by selling it at 11*s.* 6*d* per yd, what was the cost price per yd?

91. If 7 per cent more be gained by selling a house for £146, 13*s.* 4*d.* than by selling it for £137, 6*s* 8*d*, find the original cost.

92. A tradesman marks an article 15 per cent above cost price, but he allows his customer 5 per cent discount for ready money What rate per cent of profit does he really gain?

93. A manufacturer formerly sold an article at 11*s.*, gaining thereby 10 per cent The cost of manufacturing has lately advanced 25 per cent. Find at what price it must now be sold so as still to gain 10 per cent.

94. A wine merchant mixes his brandy with one-sixth of its own volume of water, and then sells it at such a price that if it were pure he would gain 5 per cent. What per cent is his profit?

95. A sold a horse to B, gaining $7\frac{1}{2}$ per cent on what it cost him; B sold it to C for £70, 19*s.*, gaining 10 per cent on what it cost him. What did A pay for the horse?

96. If the manufacturer makes a profit of 20 per cent, the wholesale dealer a profit of 25 per cent, and the shopkeeper a profit of 40 per cent, what was the cost of the manufacture of an article bought at a shop for 17*s.* 6*d.*?

97. A makes an article and sells it to B, B sells it to C, then C sells it to D for 3 guineas; A makes 20 per cent, B $12\frac{1}{2}$ per cent, C 40 per cent profit. What did it cost A to make it?

98. An article of commerce passes successively through the hands of three dealers, each of whom adds to the price, as his profit, 10 per cent of the price he paid for it. What did the first dealer pay for goods which the third sold for £11, 1*s.* 10*d.*?

99. If sugar is sold at £1, 7*s* $8\frac{3}{4}d$ per cwt., after passing through the hands of three dealers, each of whom makes 10 per cent profit on it, what did it cost the first dealer per stone?

100. A person buys 3 houses, for which he pays sums proportional to 4, 3 and 2. He sells them all at once, clearing 10 per cent by the first, 6 per cent by the second, but losing 4 per cent on the third. His entire gain is £101. Find what each cost.

LIII. PERCENTAGES.

COMMISSION, BROKERAGE, &c.

Find the commission on—

1. £1472, 15*s* at 5%
2. £853, 6*s* 8*d*. at 3%.
3. £128, 3*s* 4*d* at $2\frac{1}{2}$%
4. £4857, 1*s* 8*d* at 1%
5. £2091, 19*s*. 7*d* at 4%

Find the brokerage on—

6. £550 at $\frac{1}{4}$%
7. £3450 at $\frac{1}{8}$%
8. £1936, 13*s* 4*d* at $\frac{1}{8}$%.
9. £759 at $1\frac{1}{4}$%
10. £6786, 13*s* 4*d* at $\frac{3}{16}$%

Find, to the nearest penny, the commission on—

11. £492, 12*s* 9*d* at 5%
12. £2351, 18*s* 6*d* at 5%
13. £885, 9*s* 4*d* at $2\frac{1}{2}$%
14. £70,312, 8*s* 3*d* at $2\frac{1}{2}$%
15. £421, 5*s* 10*d* at 3%
16. £5791, 10*s* 8*d* at 1%
17. £1080, 15*s* at $\frac{1}{8}$%.
18. £349, 17*s* 9*d* at $\frac{3}{4}$%
19. £981, 11*s* 6*d* at $\frac{5}{16}$%
20. £3489, 6*s* 9*d*. at $\frac{7}{8}$%.

---

21. The rental of an estate is £2356, 10*s* Find the agent's commission at 5 per cent
22. Find the premium at $4\frac{1}{2}$ per cent on an insurance policy for £7250
23. A man insures his life for £2500 at £2, 17*s* per cent, find his annual premium
24. An auctioneer's charge, at £2, 12*s* 6*d* per cent, for a sale was £19, 5*s* What did the goods sell for?
25. Find the brokerage at 12*s* 6*d* per cent on £666, 13*s* 4*d*.
26. A broker buys goods to the value of £970, and his charge is $\frac{5}{8}$ per cent How much has his employer to pay altogether?
27. What is the value of a cargo if the insurance premium at 8 per cent amounts to £610?
28. After deducting 5 per cent as his agent's commission, a landlord received £8647, 17*s* Find the agent's commission
29. An agent collecting rents at a commission of $4\frac{3}{8}$ per cent deducted his commission and paid over £741, 1*s* $10\frac{1}{2}$*d*. What amount of rents did he collect?
30. A cargo worth £855 was insured for that sum, the premium being $6\frac{3}{4}$ per cent The cargo was lost, and the broker who obtained a settlement charged $\frac{1}{2}$ per cent What was the owner's actual loss?

LIV. SIMPLE INTEREST.

Find the Interest on—

1.	£721, 13s 4d	for 1 year	at 3	per cent per annum.
2.	£313, 2s 6d	.. 1 ..	.. 4	..
3	£191, 5s	.. 2 years	at 5	..
4.	£1562, 10s	.. 2 ..	.. 3	..
5.	£142, 17s. 6d	.. 4 ..	.. 5	..
6.	£141, 13s 4d.	.. ½ a year	at 4	..
7.	£2408, 6s 8d	.. ½ ..	.. 6	..
8	£351, 7s. 6d.	4 years	at 2½	..
9.	£206, 16s 6d.	5	.. 3⅓	..
10.	£3304, 11s 8d	.. 4¼	4	..
11.	£512,	.. 6⅔	.. 5¼	..
12.	£960,	.. 3¼	.. 2½	..
13.	£6750,	.. 3½	.. 2¾	..
14	£232, 13s 4d	.. 5	2¼	..
15.	£289, 17s 6d	.. 4	2½	..

Find the Amount of—

16.	£117, 10s	in 1 year	at 5	per cent per annum.
17.	£362, 10s	.. ½ a year	4	..
18	£88, 6s 8d	3 years	3	..
19.	£991, 10s	.. 1½ ..	5	..
20	£3643, 6s 8d.	.. 3½	.. 4¾	..

Find the Interest on—

21.	£175,	for 7 months	at 4	per cent per annum.
22.	£1347,	.. 15 ..	.. 3	..
23	£821, 5s.	.. 8 ..	.. 3½	..
24.	£25, 6s 8d	.. 7½ ..	.. 5	..
25.	£1226, [illegible]	.. 5 ..	.. 2½	..
26.	£37, 10s	219 days ..	3	..
27.	£2160, 12s 6d	73 ..	.. 10	..
28.	£353, 2s 6d	.. 146 ..	2	..
29.	£93, 15s.	.. 2 years 292 days	at 2½	per cent ..
30.	£7950, 18s 4d	.. 3 .. 105 ..	.. 3	..
31.	£737, 18s. 4d	from Mar. 3 to May 15,	at 5	per cent ..
32.	£210, 16s 8d.	.. Ap 7 .. Nov. 12,	.. 3½	..
33.	£105, 8s 4d.	.. June 1 .. Oct. 25,	.. 2½	..

Find the Amount of—

34	£133, 6s 8d	from Sept 27, 1892,	to Feb 20, 1894,	at	3%.
35	£215, 12s 6d	. . July 13, 1892,	May 1, 1895,		4%.

Find, *neglecting any fraction of a penny*,
(i) the Interest on, (ii) the Amount of—

36	£274, 11s 9d	for	1 year	at 3	per cent per annum
37.	£89, 18s 2d.		½ a year	4	
38.	£1703, 5s. 10d.		¼ . .	2½	
39	£26, 13s 7d		2 years	3	
40	£716, 12s 5d		3 .	4½	
41	£69, 9s 10d.		4 months	4	
42	£1025, 8s	.	11 .	2½	
43	£3383, 19s		292 days	3⅓	
44.	£10, 10s 10d		146 .	4½	
45.	£107, 13s 5d		219 . . .	3¼	. . .

Find, *mentally*, neglecting any fraction of a penny, the Interest

at 5 per cent on			at 2½ per cent on		
46	£325,	for a year	53	£267,	for a year
47	£256,	. half a year	54	£1261,	a year
48.	£143,	. . . half a year	55	£362,	half a year
49	£291,	3 months	56.	£142,	. a month.
50	£238, 6s.	. . a month	57	£13,	. a month
51	£85, 11s	a month	58	£491, 10s	. a month
52.	£63, 14s.	. . 2 months	59	£211, 5s	. a month.
			60.	£188, 12s	. 2 months.

*Find, *to the nearest penny*, the Interest on—

61.	£186, 11s	for 143 days	at 4	per cent per annum	
62	£423, 7s	229	3		
63.	£4917, 14s 6d.	.. 317	5	. . .	
64	£2038, 5s 10d.	.. 87	3	.. .. .	
65	£53, 13s	... 91	2½	. ...	
66	£2553, 8s	from Jan. 17	to May 23	at 2	per cent. ...
67.	£906, 3s. 8d.	.... Mar 11	... June 24	.. 5	....
68	£26, 7s. 5d	.... June 24	July 29	.. 3	. .. .
69.	Eleven guineas	. July 1	... Nov 30	2½	. ....
70.	£197, 17s 4d	Aug. 14	. Dec 31	3½ .	.

* Exs 61 to 70 may conveniently be worked by the contracted methods of Chap L.

At what Rate per cent per annum would the simple interest on

71.	£350,	be	£21,	for	2	years?
72.	£325,	.	£42, 5*s*	.	4	
73.	£1020,	.	£290, 14*s*.		6	.
74.	£560,		£41, 13*s*.		$3\frac{1}{2}$	.
75.	£4373, 6*s* 8*d*	.	£246,	.	$1\frac{1}{4}$	
76.	£21, 17*s*. 6*d*	..	£5, 2*s* 1*d*.	.	7	.. .
77.	£737, 10*s*	.	£19, 13*s*. 4*d*		8	months?
78.	£2500,	.	£365, 12*s* 6*d*	.	$3\frac{1}{4}$	years?
79	£1350,	.	£271, 13*s* 9*d*	.	$5\frac{3}{4}$	.
80.	£466, 13*s* 4*d*		£5, 16*s* 8*d*	.	5	months?

At what Rate per cent at simple interest would

81.	£560,	*amount* to	£679,	in	5	years?
82.	£2560,	. .	£3328,	.	5	.. .
83.	£308, 6*s*. 8*d*	. ..	£481,	..	14	.. ....
84.	£450,	. .	£470, 5*s*.	..	$1\frac{1}{2}$	. ....
85.	£537, 16*s* 8*d*	.. . .	£591, 12*s* 4*d*	.	4	..... .
86.	£657, 10*s*		£664, 1*s*. 6*d*		146	days?
87.	£285, 12*s*. 6*d*	. ..	£291, 6*s*. 9*d*.	..	6	months?
88.	£422, 10*s*	.	£435, 3*s* 6*d*		9	months?
89.	£183, 6*s* 8*d*	. .	£230, 1*s*. 8*d*.	..	$1\frac{5}{12}$	years?
90.	£6825, 10*s*	.	£7061, 11*s*. 9*d*.	.	101	days?

In what Time would the simple interest on

91.	£650,	be	£130,	at	5	per cent per annum?
92.	£2000,	..	£315,		$4\frac{1}{2}$	... .
93.	£840,	.	£1050,		5	. ...
94.	£80,		£1,	.	$2\frac{1}{2}$	. . . .
95.	£550, 10*s*	.	£220, 4*s*.		$2\frac{1}{2}$	. . . .
96.	£112, 10*s*	.	£400, 10*s*		4	. . . . . .
97.	£783, 6*s* 8*d*.	.	£14, 13*s*. 9*d*		$4\frac{1}{2}$	. . . .
98.	£953, 11*s*. 3*d*		£2, 12*s* 3*d*		4	. .. ..
99.	£484, 3*s* 4*d*		£12, 2*s* 1*d*		3	.. .. ..
100	£5062, 10*s*.		£75, 18*s*. 9*d*		$2\frac{1}{2}$	. . . . . .

In what Time, at simple interest, would

101.	£175,	*amount* to	£196,	at	4	per cent per annum?
102.	£3200,	....	£3424,	...	$3\frac{1}{2}$	....
103.	£950,	... .	£1199, 7*s*. 6*d*.	...	$5\frac{1}{4}$	. . ..
104.	£2652,	....	£3779, 2*s*.	...	5	. . . .

In what Time, at simple interest, would

105.	£1260,	*amount* to £1307, 5*s*.	at $3\frac{3}{4}$	per cent per ann.?
106	£183, 6*s* 8*d*	£230, 1*s* 8*d*	$4\frac{1}{2}$	
107.	£10055,	£11186, 3*s* 9*d*	$2\frac{1}{2}$	
108.	£221, 17*s*. 6*d*.	£226, 6*s* 3*d*	3	
109.	£740,	£750, 15*s*. 10*d*	$3\frac{1}{2}$	
110.	£133, 16*s* 8*d*	£134, 4*s* 11*d*.	$2\frac{1}{4}$	

On what Principal would the simple interest be

111.	£17,	for 1 year	at 4	per cent per annum?
112.	£237, 10*s*	2 years	5	
113.	£111, 12*s*	1 year	3	
114.	£162, 15*s*	2 years	$3\frac{1}{2}$	
115	£514, 10*s*	14	7	
116	£39, 7*s* 6*d*	$\frac{1}{2}$ a year	5	
117.	£40, 17*s* 3*d*	$3\frac{1}{2}$ years	4	
118.	£100,	73 days	$2\frac{1}{2}$	
119	£125,	5 months	$4\frac{1}{2}$	
120	4*s* 6*d*.	10 days	5	

What Principal would *amount*, at simple interest, to

121	£795,	in 2 years	at 3	per cent per annum?
122.	£234,	1 year	4	
123.	£539, 15*s*.	6 years	$4\frac{1}{2}$	
124.	£647, 10*s*.	4 yrs 2 mo	4	
125.	£423, 2*s* 6*d*.	3 yrs 8 mo	$3\frac{1}{2}$	
126.	£338, 1*s* 9*d*	$1\frac{1}{2}$ years	5	
127.	£728, 12*s* 9*d*	7 months	$4\frac{1}{2}$	
128.	£366, 1*s*	42 days	$2\frac{1}{2}$	
129.	£178, 1*s*. 6*d*.	3 yrs 73 dys	$6\frac{1}{4}$	
130	£1058, 13*s* 3*d*	6 months	$4\frac{3}{8}$	

131 Find the simple interest on £1375 for $4\frac{1}{2}$ years at £2, 16*s*. per cent

132 Find the simple interest on £953, 11*s* 3*d* for 20 days at 4 per cent per annum

133 Find, mentally, the interest on £315, 15*s* for a month at 5%

134. Find the simple interest on £2970, 16*s* 8*d* for 3 years 73 days at £3, 2*s* 6*d* per cent

135. Calculate the simple interest on £725 from 1st January to 15th March, 1897, at $3\frac{3}{4}$ per cent

136. Find to the nearest penny the interest on £536 for 124 days at 5 per cent per annum

137. Find to the nearest penny the simple interest on £147, 11*s*. 3*d*. from April 11 to September 19, at $2\frac{1}{2}$%

138. Find to a penny the interest on £425 from Jan. 1st to May 4th in Leap year, at $2\frac{1}{2}$ per cent

139. A dealer buys a machine for £525 cash and sells it, after nine months, for £423, 10*s* How much did he lose, considering money worth 6 per cent per annum?

140. If the simple interest on a certain sum of money amount to £119, 2*s* 9*d* in 2 years 3 months, what is the interest on six times the sum for one-fourth of that time at the same rate?

141. In what time will £200 amount to £232, 0*s*. 8*d* at $6\frac{1}{4}$ per cent simple interest?

142. In what time will the simple interest on £2541, 17*s*. 6*d* amount to £228, 15*s* $4\frac{1}{2}$*d*. at 3 per cent per annum?

143. What sum of money will amount to £769, 5*s*. at the end of $8\frac{1}{4}$ years at 4 per cent simple interest?

144. At what rate per cent per annum will the simple interest on £1560 amount to £245, 14*s*. in $3\frac{1}{2}$ years?

145. On what sum will the simple interest amount to £82, 10*s* in 4 years at $5\frac{1}{2}$ per cent per annum?

146. At what rate simple interest will £855 amount to £956, 10*s*. $7\frac{1}{2}$*d*. in 5 years?

147. In how many years at 4 per cent per annum simple interest will the interest be one-quarter of the principal?

148. In what time will money double itself at $3\frac{3}{4}$ per cent simple interest?

149. On what date will £650, lent on June 1st, 1896, amount to £708, 10*s*. at 5 per cent per annum simple interest?

150. The interest for one year on £2550, after deduction of income-tax at 6*d*. per £, is £82, 17*s*. 6*d*. Find the rate per cent.

151. What sum lent on a mortgage at $4\frac{1}{2}$ per cent would yield an annual income of £191, 5*s*.?

152. A man mortgages his estate for £3500, the interest at 5 per cent per annum being payable half-yearly. Find the amount of one half-yearly instalment of interest.

153. A money-lender charged £3 for the loan of £40 for a month; at what rate per cent per annum did he charge interest?

154. The rate of interest in a savings-bank is £2, 13*s* 4*d* per cent per annum. A man deposits a sum of money, his interest at the end of 11 months was 14*s*. 8*d*. What sum did he deposit?

155 What would be the interest on a loan of £10 for 3 weeks at the rate of $\frac{1}{8}$ per cent per *diem*?

156 On what sum is £1, 10*s* 5 months' interest at the rate of 1*s* 6*d* per £1 per annum?

157. Find the ratio of the interest on £247, 10*s* at 5 per cent to that on £375 at $3\frac{3}{8}$ per cent, both for two years

158 The interest on £2000 for 3 years at $3\frac{1}{2}$ per cent is to be divided between A, B, and C, A's share being four times B's, and C's being equal to A's and B's together Find the shares.

159. A question being proposed in an examination, to find the simple interest on a certain sum of money for $2\frac{1}{2}$ years at $3\frac{1}{4}$ per cent, a candidate by mistake reckoned it for $2\frac{1}{4}$ years at $3\frac{1}{2}$ per cent, and so obtained a result too little by £26, 4*s* 8*d* What ought the answer to have been?

160. A man lent £1800 on house property at a certain rate per cent, and £1200 on land at $\frac{1}{2}$ per cent less His total income from both sources was £114 Find the rates of interest

161. If £860 amounts to £963, 4*s* in 3 years, in what time will £1060 amount to £1166 at the same rate, simple interest?

162. In what time will £1455 amount to £2328, if, in 3 years, £430 amount to £494, 10*s* at simple interest at the same rate?

163. Divide £3300 into two parts such that the simple interest on the one at 3 per cent for $4\frac{1}{2}$ years would be equal to the simple interest on the other at $2\frac{3}{4}$ per cent for 5 years

164. Divide £3783 into two parts such that the interest on the first for 3 months at $4\frac{1}{2}$ per cent shall be equal to that on the second for 7 months at $2\frac{1}{4}$ per cent

165. A has £2000, he lends £800 to B, and receives as interest £18 half-yearly; he lends £650 to C, and receives as interest £4, 17*s* 6*d* quarterly, the rest of his money he lends to D at 5 per cent per annum Find the average rate of interest A makes on his capital

Find to a penny, by the "*Third, tenth, and tenth*" rule, the

166. Interest on £3259, 14*s* for 283 days at $2\frac{1}{2}\%$

167. Amount of £8563, 17*s* 9*d* from April 13 to Dec 31 at $2\frac{3}{4}\%$.

168 Interest on £837, 15*s* 7*d*, from July 7 to Aug 23, at $2\frac{1}{4}\%$

169. A merchant supplies a tradesman with goods to the cash value of £650 on May 10 The tradesman pays, on account, £160 on June 24, £175 on Sep 8, and £130 on Nov 16 Find the balance owing on Dec 31, allowing interest at 6 per cent

170 A man opens an account at a bank, on July 10, by depositing £320 On July 30 he pays in £85, on Aug 23 he draws £120, on Sep 14 he draws £56, 10*s*, and on Nov 2 he pays in £27, 15*s*. Find the interest, at 2%, due to him on Dec 31.

LV. COMPOUND INTEREST.

Find the Amount, at compound interest, of—

1.	£2500	in	2	years at	5	per cent per annum.
2.	£4250		2		4	. ..
3.	£600	.	3		5	. ....
4.	£320		2		$2\frac{1}{2}$	... .
5.	£750		4		10	....

Find the Compound Interest on—

6.	£625	for	2	years at	3	per cent per annum.
7.	£3125		3	..	3	. .
8.	£8000		3	.	$2\frac{1}{2}$	. . .
9.	£1500		2	.. .	$3\frac{1}{2}$	. ..
10.	£3150		$2\frac{1}{2}$	.	5	. .. . .

Find, neglecting any fraction of a penny,

the Amount of—

			yrs			
11.	£540 . .	in	2	at	3	%.
12.	£1500	.	3		4	
13.	£2450 ..	.	3	.	3	.
14.	£173, 17*s* 6*d*.	..	2	.	4	.
15.	£99, 19*s* 9*d*		3	...	2	..
16.	£2610		3	.	5	..
17.	£975, 15*s*	...	3	..	5	
18.	£453, 10*s* ..	.	2	.	$2\frac{1}{2}$	
19.	£1607, 11*s* 3*d*	.	3	..	$3\frac{1}{3}$	
20.	£2481, 6*s* 8*d*		3	.	$2\frac{1}{2}$	

the Compound Interest on—

			yrs.			
21.	£364, 5*s*.	...... for	2	at	$2\frac{1}{4}$	%.
22.	£7275	. . . .	2	...	$4\frac{1}{4}$	...
23.	£528, 15*s*	... . ..	2	...	$2\frac{3}{4}$	
24.	£608, 15*s*. 4*d*.	...	3	...	$2\frac{3}{4}$	..
25.	£172 .	...	3	. .	$4\frac{3}{4}$	...
26.	£1400 .	.. .	$2\frac{1}{2}$	...	4	.
27.	£623, 17*s*	.	$3\frac{1}{2}$	..	$2\frac{1}{2}$	.
28.	£482, 9*s* 8*d*	. .	$3\frac{1}{4}$	...	$3\frac{1}{4}$	. .
29.	£340	.	$3\frac{1}{2}$	...	$3\frac{1}{2}$	. .
30.	£3185, 3*s*. 4*d*. ..		$2\frac{1}{4}$	...	$3\frac{3}{4}$	...

Find, to the nearest penny, the Amount of—

31.	£825, 10*s* . . .	in	2	years at	3	per cent per annum.
32.	£428, 12*s*. 8*d*	. .	3	.	3	....
33.	£666, 13*s*. 4*d*.	...	4	. . .	5	... .
34	£4523, 17*s*. 6*d*.	..	4	...	$2\frac{1}{2}$	. ...
35	£157, 16*s* 7*d*	...	2	. .....	$6\frac{2}{3}$	. .. .
36	£1000	... .	5	. . . .	$3\frac{1}{2}$	. . ..
37.	£4765	....	3	. . . .	$3\frac{3}{4}$	....
38	£760	. . . .	3	....	$4\frac{7}{8}$	. . . . .
39.	£1234 .	... ..	$3\frac{1}{2}$	. . .	$2\frac{1}{2}$	. .
40.	£243, 10*s*.	. ...	$2\frac{1}{4}$	....	$2\frac{3}{4}$	.. ..

Find, to the nearest penny, the Amount, at compound interest, payable *half-yearly*, of—

41	£428	in 2 years at	4	per cent per annum
42	£753, 10*s*	$1\frac{1}{2}$	5	
43.	£2670	2	$2\frac{1}{2}$	
44	£115, 12*s* 8*d*	$1\frac{1}{2}$	$3\frac{1}{2}$	
45.	£3500 .	3	$3\frac{1}{3}$	
46.	£973, 18*s* .	$1\frac{1}{2}$	$3\frac{3}{4}$	

Find, to the nearest penny, the Comp Int, payable *quarterly*, on—

47. £680 for 1 year at 4 per cent per annum
48 £2130 1 5 ..
49. £517, 15*s* for $\frac{3}{4}$ of a year at 3 per cent per annum
50 £850 . . . $1\frac{1}{4}$ years $2\frac{1}{2}$.

Find, to the nearest pound, the Amount, at compound interest, of—

51.	£1250	in 8 years at 4%	54.	£1013, 10*s*	in 7 years at 5%
52.	£800	.. 10 5	55	£500 .	16 . 3
53.	£851, 10*s* ..	12 4 .	56	£7000	20 . 5

57. £675 for $6\frac{1}{2}$ years at 4% per annum, payable *half-yearly*
58. £2350 for $7\frac{1}{2}$ years at 5% per ann payable *half-yearly*

Find the difference between the simple and compound interest on—

59.	£425	for 3	years at	4	per cent per annum
60.	£1333, 6*s* 8*d*	4	.	5	. ...
61.	£7650 .	2	.	$3\frac{1}{2}$	
62.	£615, 10*s*.	3		$2\frac{1}{2}$	. ....
63.	£2517, 11*s*. 8*d* .	$2\frac{1}{2}$		$3\frac{3}{4}$	...

---

*In what Time, at compound interest, would—

64 £750 *amount* to £843 648 . at 4 per cent per annum?
65 £3000 . £3472, 17*s* 6*d* 5 . . . ?
66. £1250 gain £197, 0*s* $7\frac{1}{2}$*d* *interest* at 5 per cent?
67 £480 . £43, 7*s* 5*d* (nearly). $2\frac{1}{2}$. ?
68 £1800 . £170, 1*s*. .. .. .. $3\frac{1}{3}$. ?
69. £25000 . £1787, 14*s* 6*d*. 3 ?
70. £520, 16*s*. 8*d* amount to £552, 14*s* 3*d* at 4% payable hf -yearly

* For Exs on finding the Rate in the case of Comp Int see under Cube Root.

What Principal would, at compound interest—

71.	*Amount* to £1352	in	2 years at	4	per cent?
72.	. £4862, 0s 6d		4 . .	5	.
73.	. . . . £6892, 2s		3	$2\frac{1}{2}$	. . ?
74.	Gain £164, 14s *interest* .		2 . .	$3\frac{1}{3}$	.. . ?
75.	. £5751, 2s 6d	.	$2\frac{1}{3}$	$3\frac{3}{4}$.	... ?

76. Find, neglecting any fraction of a shilling, the compound interest on £720 for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.

77. Find, to the nearest penny, the difference between the first and third year's interest on £4735 at $3\frac{1}{4}$ per cent.

78. Find the exact difference between the simple and compound interest on £2450 for 3 years at 3 per cent.

79. Find, to the nearest shilling, the difference between the simple and compound interest on £4560 for 2 years at 4 per cent per annum, the interest being payable half-yearly

80. Find, to the nearest pound, the compound interest on £300 for 10 years at $2\frac{1}{2}$ per cent

81. If the present population of a town is 35,200, and the births average annually 23 per 1000 and the deaths 18 per 1000 of the population at the beginning of the year, what will be the population of the town at the end of 4 years?

82. The difference between the simple and compound interest on a certain sum for 2 years at 4 % is £1, 8s Find the sum.

83. The compound interest on a certain sum for 2 years at $2\frac{1}{2}$ per cent exceeds the simple interest for the same time at the same rate by £4, 7s. 6d. Find the sum.

84. The difference between the second and third year's interest on a certain sum at 5 per cent is £5, 5s Find the sum

85. If at the beginning of each year a man invests £100 at compound int. at 4 %, what will he be worth at the end of the fifth?

86. If a man borrowed £300 at 4 per cent, and repaid the principal and interest by equal annual instalments of £50, in how many years would he be out of debt?

87. £820 is borrowed for 2 years at 5 per cent, to be repaid, principal and interest, in two equal annual instalments. find them.

88. If £1700 be borrowed for 2 years, to be repaid, with interest at 4 per cent, in two equal annual instalments, what must be the amount of each instalment?

89. If £100 amounts to £127 in 5 years at compound interest, what would £2000 amount to in 10 years at the same rate?

90. If, in a certain time, at a certain rate per cent, £500 amounts, at compound interest, to £864, what would £3125 amount to in double that time at the same rate?

LVI. DISCOUNT AND PRESENT WORTH.

PRACTICAL, OR BANKER'S, DISCOUNT

Find, neglecting any fraction of a penny, the Banker's discount, at 5 per cent, on—

1. £325	due 3 mo hence	5. £400	due 71 days hence
2. £214, 12s	3 .	6 £215	123
3. £257, 10s .	6	7 £342, 7s 6d	64
4. £507, 15s.	6	8 £152, 18s 9d	101

Find, neglecting any fraction of a penny, the cash value of a "bill' for—

9. £280. if discounted, at	5 per cent,	3 months before it was due	
10. £417, 12s. 6d .	5	6	. .
11. £325 . .	. 4 .	57 days.	.
12 £253, 16s. 6d	3	163	.

Find the Banker's charge for cashing a "bill" for—

13. £410	drawn June 19, at	3 mo, discounted	July 1, at	5%
14. £224	June 20	3 .	July 24	4%
15. £2000	May 5	3	June 10	$3\frac{1}{4}$%
16. £550, 6s 8d .	Jan 3	5	Jan 11	4%
17. £847, 15s 9d	Feb 5	6 .	Ap 30 .	$\frac{3}{4}$%

Find the Commercial present worth of a "bill" for—

18 £300	drawn Mar 12, at	3 mo, discounted	Mar 24, at	5%
19 £275	Sep 24	3 .. .	Oct 4	5%
20. £1400	Ap 5	6	Ap 26	4%.
21. £670, 10s	. June 17	3	July 2	4%.
22 £375, 12s. 6d	May 8	4	July 12	$3\frac{1}{2}$%
23 £1123	Nov 7	3	Nov 28	5%.
24 £573, 14s.	Oct 30	6	Nov 14	5%
25. £2476, 15s	Feb 20	9	Mar 15	$4\frac{1}{2}$%

---

26 How much will a broker, who charges 5 per cent discount, give for a bill for £600 due 7 months hence?

27 If I take to a bank a bill for £350 which is due in 63 days, what shall I receive for it, discount being charged at the rate of 5 per cent?

28 A bill for £1000 at 90 days sight has been presented for acceptance on February 10th, 1888, and on March 10th is discounted at 5 per cent How much will the discounter pay for the bill?

29. "*London, June* 26, 1894.
"Three months after date I promise to pay to Mr. John Brown, or order, Three hundred pounds for value received
Thomas Jones"
The above promissory note was cashed on July 7, at 5 per cent, how much was received for it?

30 The banker's discount on a bill due in 2 months at $5\frac{1}{2}$ per cent is £19, 5*s* What is the amount of the bill?

31. The commercial present worth of a sum due 219 days hence, at 5 per cent, is £499, 11*s*. What is the sum?

32. A 4 months' bill drawn on 1st Feb and discounted on 23rd Mar, at 5 per cent, realized £742, 10*s*. Find the amount of the bill.

33. If £217, 16*s*. was the cash value on April 7 of a bill for £220, drawn Mar 16, at 3 months, at what rate per cent was it discounted?

34. If the banker's charge on Sep 25 for discounting, at 5 per cent, a bill drawn Sep 11, at 3 months, was £6, 3*s*, find the "face value" of the bill.

35. If the banker's charge for cashing, at 5 per cent, a bill for £520 drawn May 8, at 3 months, was £5, 4*s*, on what date was it cashed?

Theoretical, or "True", Discount.

Find the Theoretical (i) discount, (ii) present worth, of—

36.	£780 due in	1 yr	at 4 %		41.	£284, 18*s*. due in	6 mo	at $3\frac{1}{2}$ %.
37.	£4200 ..	. 1 .	. 5		42	£843, 15*s*	3 .	5 ...
38.	£664 .	15 mo	3		43	£474, 14*s* .	. 73 days	5 ...
39.	£407 .	6 .	$3\frac{1}{2}$		44	£150, 10*s* .	. 70	5 ..
40.	£1030	8	$4\frac{1}{2}$.		45.	£503, 10*s*	. 84 . .	$4\frac{1}{2}$ ..

46 Find the difference between the interest and the "true" discount on £694, 3*s* 4*d* for 11 months at $4\frac{1}{2}$ per cent

47. Find the difference between the Commercial and the Theoretical discount, at 5 per cent, on £1680, due 3 months hence.

48. The "true" discount on £405 due 6 months hence is £5 Find the rate per cent

49. The theoretically true present value of £627 due 146 days hence is £618, 15*s*. Find the rate per cent.

50. For how many months is £828 the "true" present worth of £848, 14*s* at 6 per cent?

51. For what time does the mathematical discount on £1028, 6*s* at 5 per cent amount to £6, 6*s* ?

52. The theoretical present value of a certain sum due 73 days hence at $4\frac{1}{2}$ per cent is £262, 10*s* Find the sum

53 On what sum due 1 yr hence is £12 the "true" discount at 4%?

54 The "true" discount at 4 per cent on a certain sum due 5 months hence is £13, 10*s* 10*d* Find the sum

55 The difference between the interest and the theoretical discount on a certain sum for 1 year, at 4%, is £2 Find the sum

56 If the difference between the "true" and the "banker's" present worth of a certain sum due in 3 months, at 4 per cent, is 3*s* $4\frac{4}{5}$*d*, what is the sum?

57 The interest on £110 is £11 Find the "true" discount on the same sum for the same time at the same rate

58 The banker's discount at 6 per cent on a sum due in 2 months exceeds the "true" discount by 10*s* Find the latter

59 The "true" discount on a sum due 3 months hence is £7, 10*s* The interest on the same sum for 3 months at the same rate is £7, 16*s* Find the sum and the rate per cent

60 Show, by an example, that the difference between the banker's and the "true" discount on any sum for any part of a year, at any rate per cent, is the interest on the "true" discount for that time at that rate

Long Period Present Value

Find, to the nearest penny, the present value of—

61.	£850	due	2	years hence at	5	per cent *compound* interest
62.	£250		3		4	
63	£725, 10*s*		4		5	
64	£4500	..	5	.	5	
65.	£2000	.	10		4	

66. How much ready money could be obtained in lieu of a legacy of £1000 due at the end of 3 years, allowing compound interest at 5 per cent?

67. Find the present value of an annuity of £100 for 4 years, the first instalment to be due a year hence, allowing compound interest at 5 per cent

68. Find the present value of a fellowship of £200 a year for 6 years, reckoning 4 per cent compound interest, the first payment being made a year hence

69 What annuity, to continue for 3 years, the first payment being made a year hence, can be purchased for £800, reckoning by compound interest at 5 per cent?

70. What annuity, to continue for 5 years, the first payment to be due 1 year hence, will £2000 cash purchase, supposing money to be worth 4 per cent?

LVII. STOCKS AND SHARES.

STOCKS.

Find the cost of—

1. £800 *stock* at 87.
2. £2500 . 92.
3. £650 78.
4. £525 *stock* at 108
5. £1320 105
6. £1400 . . $91\frac{1}{2}$
7. £900 *stock* at $85\frac{1}{2}$
8. £1700 .. $132\frac{1}{2}$.
9. £450 .. $96\frac{1}{4}$.
10. £8400 .. . $87\frac{1}{4}$.

How much Stock—

11. At 80 can be bought for £1000?
12. 96 . . £840?
13. 112 . . £980?
14. . 75 . £624?
15. 102 . . £1360?
16. At $97\frac{1}{2}$ would cost £1560?
17. $124\frac{3}{4}$ £2495?
18. . $139\frac{1}{2}$ . . £744?
19. . $61\frac{1}{4}$ . . £490?
20. $105\frac{5}{8}$. £3380?

What Annual Income is obtained from—

21. £3400 *stock* paying 3 p cent?
22. £750 .. .. . 4 . .
23. £1450 . 5
24. £850 . ... $2\frac{1}{2}$ . .
25. £725 . . 3 . . .
26. £8700 *stock* paying $4\frac{1}{2}$ p. cent?
27. £1730 . .. .. $6\frac{1}{2}$ ....
28. £640 . .... $2\frac{3}{4}$ .. . .
29. £666, 13*s*. 4*d*. . . 7 . .... .
30. £1412, 10*s*. . . ... . 4 . .. ..

What Annual Income is obtained by *investing*—

31. £1500 *cash* in 4 p c 's at 128?
32. £1890 .. .. 5 .... 135?
33. £910 . $2\frac{1}{2}$ 104?
34. £840 . $4\frac{1}{2}$ . . 126?
35. £1225 . $2\frac{3}{4}$. 98?
36. £486, 10*s* *cash* in 3 p c 's at 84?
37. £958, 10*s* . . . $5\frac{1}{2}$ .... 142?
38. £655 ... . 4 .. . $98\frac{1}{4}$?
39. £2300 . .. .. . $4\frac{1}{2}$ .. .... par?
40. £435, 15*s*.. . .... $2\frac{3}{4}$ ......$115\frac{1}{2}$?

Find the Half-yearly Dividend obtained from—

41. £1670 *stock* paying 4 per cent per annum.
42. £233, 6*s* 8*d* . . $4\frac{1}{2}$ . . ..
43. £730 . . 3 . .. .. .
44. *Investing* £780 in $5\frac{1}{2}$ per cent stock at 143.
45. .... £760 . 2 .. .. $71\frac{1}{4}$.
46. . . . .. £220 $3\frac{1}{2}$ par

Find the Quarterly Dividend obtained from—

47. £780 *stock* in $2\frac{1}{2}$ per cents
48. £3000 *stock* in $2\frac{3}{4}$ per cents.
49. *Investing* £6500 in $2\frac{1}{2}$ per cent consols at 104
50. .... £1520 . $2\frac{3}{4}$ .. .. 114.

Find the cost of—

51 £2600 *stock* at $91\frac{3}{4}$ (Brokerage $\frac{1}{4}$ p c).

52 £950 61

53 £2450 $106\frac{3}{8}$ (Brokerage $\frac{1}{8}$ p c)

54. £646, 13*s* 4*d* . $104\frac{7}{8}$

55. £500 80 (Brokerage 2*s* 6*d* p c)

How much cash would be realized by the sale of—

56. £360 *stock* at $95\frac{1}{4}$? (Brokerage $\frac{1}{4}$ p. c)

57. £700 $200\frac{1}{2}$?

58 £875 $112\frac{1}{8}$? (Brokerage $\frac{1}{8}$ p c)

59 £1760 $107\frac{1}{2}$? . . .

60. £1530 120? (Brokerage 2*s* 6*d* p c.)

How much Stock—

61 At $74\frac{3}{4}$ could be *bought* for £420? (Brokerage $\frac{1}{4}$)

62 .. $81\frac{1}{4}$ £815?

63 $111\frac{7}{8}$ £980? (Brokerage $\frac{1}{8}$)

64. .. $166\frac{1}{4}$ would be *sold* for £2490? (Brokerage $\frac{1}{4}$)

65. .. 65 £518?

66 . $102\frac{3}{4}$ £3284? (Brokerage $\frac{1}{8}$)

Find the Annual Income obtained by *investing*—

67 £2940 in 5 p c stock at $104\frac{3}{4}$ (Brokerage $\frac{1}{4}$)

68 £900 $2\frac{3}{4}$. $109\frac{7}{8}$ (Brokerage $\frac{1}{8}$)

Find the Half-yearly Dividend obtained by *investing*—

69 £342, 10*s* in $4\frac{1}{2}$ p c stock at $68\frac{1}{4}$ (Brokerage $\frac{1}{4}$)

70. £2250 7 $179\frac{3}{4}$

Find the Quarterly Dividend obtained by *investing*—

71. £1242 in $2\frac{1}{2}$ p c consols at $107\frac{7}{8}$ (Brokerage $\frac{1}{8}$)

72. £672 . $2\frac{3}{4}$ . . . $111\frac{7}{8}$

*Find the net Annual Income, after deducting income-tax at 6*d* in the pound, obtained from—

73 £840 *stock* paying 5 per cent

74. £1600 $2\frac{1}{2}$.

75 *Investing* £1200 in 4 p c stock at 96

76. £528 $3\frac{1}{2}$ 88

77. . £1700 5 $127\frac{1}{4}$ (Brokerage $\frac{1}{4}$)

78 £1000 $2\frac{1}{2}$ $103\frac{7}{8}$ (Brokerage $\frac{1}{8}$)

* Brokerage is to be neglected whenever it is not mentioned in the question

Find the net Half-yearly Dividend, after deducting income-tax at 8*d* in the pound, obtained from—

79. £600 *stock*, paying $4\frac{1}{2}$ per cent
80. £1600 . $2\frac{3}{4}$
81. *Investing* £540 in 5 p c stock at 108.
82. . £1715 . 4 . . . . . 98.
83. . £2460 . $4\frac{1}{2}$ . .. . 92 (Brokerage $\frac{1}{4}$)
84. . £714 $6\frac{1}{4}$ .. .... $169\frac{3}{4}$ . . ..

Find the net Quarterly Dividend, after deducting income-tax at 9*d.* in the pound, obtained from—

85. £3200 stock in $2\frac{1}{2}$ per cent consols
86. £8000 . $2\frac{3}{4}$ . .
87. *Investing* £720 in $2\frac{1}{2}$ p. c consols at 105.
88. . £480 $2\frac{3}{4}$ . . . . . 110.
89. ... £1270 $2\frac{1}{2}$ 111 (Brokerage $\frac{1}{8}$).
90. . . . £1850 . $2\frac{3}{4}$. $115\frac{1}{2}$ . . . .

What rate per cent interest on capital is obtained by investing in—

91. $2\frac{1}{2}$ p c. stock at 112?
92. 4 . . . . . $97\frac{1}{2}$?
93. $7\frac{1}{2}$. $202\frac{1}{2}$?
94. 3 p. c stock at $86\frac{1}{4}$?
95. $5\frac{1}{2}$ . . . . $137\frac{1}{2}$?
96. $3\frac{1}{2}$ . . . $61\frac{1}{4}$?

Which investment pays better—

97. 4 per cents at 104, or $5\frac{1}{2}$ per cents at 144?
98. 3 . . 80, . $4\frac{1}{2}$ 90?
99. $3\frac{1}{4}$ . . $97\frac{1}{4}$, $3\frac{1}{2}$ . . 105?
100. $3\frac{1}{2}$. $98\frac{1}{4}$, $3\frac{3}{4}$ .. . 105?
101. 3 $97\frac{1}{2}$, 5 . . $162\frac{3}{8}$?
102. $4\frac{1}{2}$ .. $162\frac{3}{8}$, $2\frac{3}{4}$ . . . par?

Find the change in annual income when—

103. £5000 stock paying $2\frac{1}{2}$ per cent is sold at 87 and the proceeds are invested in 5 per cents at 150.

104. £3000 4 p. c stock is sold at 95 and $6\frac{1}{2}$ p c. stock at 120 is bought with the proceeds

105. £5000 stock in $3\frac{1}{2}$ p. c's is sold at 81, and with the proceeds is bought 4 p. c. stock at 108

106. £4500 stock which pays 4 per cent is sold out at 114, and the proceeds are invested in $6\frac{1}{4}$ per cent stock at 150

107. £7400 stock paying 3 per cent is sold out at $99\frac{1}{8}$ (Brokerage $\frac{1}{8}$), and the proceeds are invested in 6 per cent stock at $147\frac{3}{4}$ (Brokerage $\frac{1}{4}$).

Find the change in annual income when—

108. £10,000 stock in $3\frac{1}{2}$ p c's is sold out at $94\frac{5}{8}$ (Brokerage $\frac{1}{8}$), and with the proceeds 6 per cent debentures at $104\frac{3}{4}$ are bought (Brokerage $\frac{1}{4}$)

109 £5000 stock paying 3 per cent is sold at $84\frac{1}{4}$, and with the proceeds 4 per cent stock at $95\frac{3}{4}$ is bought (Brokerage $\frac{1}{4}$ in both transactions)

110. £2350 stock in the $2\frac{1}{2}$ per cent consols is sold at 110 (Brokerage $\frac{1}{8}$), and the proceeds are invested in railway stock at $146\frac{1}{4}$, paying $4\frac{1}{2}$ per cent (Brokerage $\frac{1}{4}$)

Find, *neglecting any fraction of a penny*, the—

111. Cost of £3453, 12*s* 6*d* stock at 97.

112. Annual income from £865, 10*s* 6*d* stock paying 4 per cent.

113. Amount of stock at 79 which can be bought for £1463, 10*s*

114 Amount of stock at $94\frac{1}{2}$ which can be bought for £2000 (Brokerage $\frac{1}{4}$)

115. Cash realized by the sale of £776, 12*s* 8*d* stock at 89

116. Cash realized by the sale of £2340 stock at $111\frac{1}{2}$ (Brokerage $\frac{1}{8}$)

117. Half-yearly dividend, less income-tax at 10*d*. in the pound, on £843, 6*s* 8*d* stock paying $4\frac{1}{2}$ per cent

118. Half-yearly dividend obtained by investing £907, 10*s* in 3 per cent stock at $84\frac{1}{2}$ (Brokerage $\frac{1}{4}$)

119 Quarterly dividend, less income-tax at 7*d* in the pound, on £727, 10*s* stock paying 4 per cent

120 Quarterly dividend, less income-tax at 5*d* in the pound, obtained by investing £935 in $2\frac{3}{4}$ per cent consols at $113\frac{5}{8}$ (Brokerage $\frac{1}{8}$ p c)

SHARES.

Find the cost of—

121 Twelve £10 shares in a gas company, at 13

122 Thirty-five £5 shares in a water company, at 7.

123. A hundred £1 shares in a mining company, at $\frac{5}{8}$.

124 Seven £100 shares in a brewery, at 136

125. Forty-two £50 shares in a banking company, at $87\frac{1}{2}$.

126. 50 £1 shares in a company, at $\frac{1}{2}$ *premium*

127. 75 £5 ... $2\frac{1}{4}$

128 24 £1 $\frac{1}{4}$ *discount*

129. 60 £2 $1\frac{1}{4}$ ..

130. 85 £1 . .. . $1\frac{3}{16}$ *premium.*

Find the annual income from—

131. Eighty-two £5 shares in a company paying 7 p. c.
132. Thirty-six £10 ... ... 12 ..
133. 45 £1 ... ... $3\frac{1}{2}$ ...
134. 150 £50 .. .. 15 ..
135. 325 £5 ... ... $1\frac{1}{2}$ ...

How many—

136. £1 shares, at $2\frac{1}{2}$, can be bought for £175?
137. £1 .. ... $\frac{7}{8}$ ... . £350?
138. £5 ... .. $4\frac{1}{2}$ ... ... £270?
139. £10 $17\frac{1}{2}$ ... £157, 10s.?
140. 10s . 7s 9d . . £27, 2s 6d?
141. £1 shares, at $\frac{1}{4}$ *discount*, can be bought for £105?
142 £1 . ... $\frac{1}{2}$ *premium*, ... £81?
143. £5 . . $1\frac{3}{4}$ *premium*, ... £540?
144. £10 $4\frac{1}{2}$ *discount*, ... £27, 10s?
145. £50 . $7\frac{1}{2}$ *premium*, ... £402, 10s.?

Find the cost, allowing 1 per cent for brokerage and stamp, of—

146 100 £1 shares at $1\frac{3}{8}$
147. 200 £5 ... . $4\frac{3}{16}$.
148 25 £10 .. $12\frac{1}{2}$
149. 60 £1 shares at $\frac{1}{4}$ *premium*.
150. 10 £10 ... . $2\frac{3}{4}$ *discount*

How much cash is realized by the sale of—

151. Forty £1 shares at $2\frac{3}{4}$? (Brokerage $\frac{1}{2}$ p. c)
152 Sixty £2 ... . $1\frac{1}{4}$? . . . .
153 200 £1 . $5\frac{5}{16}$ *premium*? (Brokerage $\frac{1}{2}$ p. c)
154. 80 £10 . $4\frac{3}{4}$ *discount*? ... .
155. 25 £5 . . $1\frac{7}{8}$ *premium*?

What rate per cent interest on capital is obtained by investing in—

156. £5 shares, at $7\frac{1}{2}$, in a company paying $6\frac{1}{2}$ p. c.?
157. £1 ... ... $\frac{7}{8}$ . ... $3\frac{1}{4}$ ..
158. £10 ... .. $22\frac{1}{2}$ .. . . 12 ...

Which is the more profitable investment—

159 £10 shares, at $£12\frac{1}{2}$ each, in a gas company which pays $5\frac{1}{2}$ p c., or £5 shares at $6\frac{1}{4}$ in a water company paying 5 p c.?
160. £1 shares, at $\frac{1}{8}$ premium, paying $4\frac{3}{4}$ p c., or £1 shares at $\frac{1}{4}$ discount, paying $3\frac{1}{4}$ p. c.?

[*Note* —Unless specially mentioned, Brokerage need not be considered.]

161. How much $3\frac{1}{2}$ per cent stock must one hold in order to obtain from it an annual income of £68, 1*s* 6*d* ?

162 An income of £1000 is made up of £240 from 6 per cent stock, £340 from 8 per cent stock, and the remainder from $3\frac{1}{2}$ per cent stock How much of each stock is held?

163 What sum must be invested in $2\frac{3}{4}$ per cent consols at $96\frac{1}{4}$ in order to obtain an income of £100 a year?

164. If $4\frac{1}{2}$ per cents are at 119, how much must I invest to obtain an income of £225 ?

165 A man obtains an income of £120 by investing in 4 per cent stock at 116. How much money does he invest?

166. What sum of money invested in the 3 per cents at $97\frac{1}{2}$ will give an annual income of £150 ?

167. If the $2\frac{3}{4}$ per cents stand at 99, what sum must be invested in the stock to secure a dividend of £25 per quarter?

168. The income derived from investing a certain sum in $3\frac{1}{2}$ per cent stock at $106\frac{3}{4}$ is £120, 12*s* Find the sum invested

169. What is the amount of money which, invested in the $2\frac{3}{4}$ per cents at $98\frac{3}{4}$, will produce an annual income of £12, 2*s* 11*d* ?

170 What sum invested in $4\frac{1}{2}$ per cents at $89\frac{3}{4}$ will yield an income of £450 per annum? (Brokerage $\frac{1}{4}$ per cent)

171. How much money would have to be invested in 3 per cents, at 87, in order to produce a net income of £295, after deducting income-tax at 4*d* in the pound?

172. When certain 3 per cents are at $91\frac{7}{8}$, what sum must be invested in them in order to secure an income of £708, after paying income-tax at 4*d* in the pound and brokerage $\frac{1}{8}$ per cent?

173. What sum must be invested in 3 per cents at $90\frac{7}{8}$ to yield a net yearly income of £702, after deducting income-tax at 6*d* in the pound? (Brokerage $\frac{1}{8}$)

174 If the 3 per cents are at $90\frac{5}{8}$, what sum must be invested in them in order to obtain an annual income of £470, after paying income-tax at 5*d* in the pound? (Brokerage $\frac{1}{8}$ per cent)

175. A man holds £4560 railway stock the first half-year's dividend is at the rate of $3\frac{1}{2}$ per cent, and for the second half of the year the dividend is at the rate of 3 per cent What is his income for the year?

176. What difference is there between the incomes arising from the investment of £4850 in $3\frac{1}{2}$ per cents at 97 or in the $3\frac{1}{4}$ per cents at par?

177. I have £1000 to invest Shall I obtain more income by investing it in French 3 per cent Rentes at 78, or Swedish 4 per cents at 104, or Norwegian $3\frac{1}{2}$ per cents at 91 ?

178. How much $4\frac{1}{2}$ per cent stock must a person have who obtains from it a net yearly income of £337, 2*s* 6*d*, after deducting income-tax at 8*d* in the pound?

179. A person buys £500 stock at 66, and afterwards £500 more of the same stock at 69 He sells out the whole at 89 Find the increase in his capital

180. What income, to the nearest penny, will be derived from investing £5000 in the $2\frac{3}{4}$ per cent consols at $98\frac{1}{4}$?

181. A man obtains an income of £210 by investing £6160 in $2\frac{3}{4}$ per cent stock At what price does he buy the stock?

182. What is the price of $2\frac{3}{4}$ per cent stock when an investment of £2109, 7*s* 6*d* produces an income of £61, 17*s* 6*d* per annum?

183. What is the price of 3 per cent stock, when a sum of £4353, 4*s* 7*d* when invested will produce an income of £132, 5*s*?

184. What sum must be invested in the purchase of 5 per cent railway debentures at $130\frac{1}{2}$ so as to produce an income of £120 a year, after paying an income-tax of 8*d* in the pound?

185. I sell £4000 of a 4 per cent stock at 144 and invest the proceeds in a $2\frac{1}{2}$ per cent stock At what price must I buy the $2\frac{1}{2}$ per cent stock to obtain the same income as before?

186. A person who had 45 shares in a gas company sold them at 42 and invested his money in Russian 5 per cents at 105. What income did he then obtain?

187. What is the clear annual income derived from investing £6050 in the 3 per cents at $90\frac{3}{4}$, after deducting an income-tax of 4*d*. in the pound?

188. How much must be invested in the 3 per cents at 99 to produce the same income as would be obtained by investing £1508 in the $2\frac{1}{2}$ per cents at 87?

189. Find the change in income due to selling out at $222\frac{3}{8}$ £3600 stock paying $7\frac{1}{2}$ per cent, and investing the proceeds in $4\frac{1}{2}$ per cent stock at 111 (Brokerage $\frac{1}{4}$)

190. Which yields the larger return in income for the same amount invested,—4 per cents at $151\frac{1}{2}$ (Brokerage $\frac{1}{4}$), or $2\frac{1}{2}$ per cents at par (Brokerage $\frac{1}{8}$)?

191. What rate per cent does a man get for his money by investing in £10 bank shares selling at 25, and paying 10 per cent?

192. Certain £10 shares in a company are at $23\frac{3}{4}$ What is the yield per cent per annum upon the capital invested for a year in which a dividend of 12 per cent is paid?

193. What rate per cent of interest does one get on money invested in a 4 per cent stock, the price of which (including brokerage) is $119\frac{3}{8}$?

194. A man invests £1980 in $3\frac{1}{2}$ p. c.'s at 99, and £3220 in $4\frac{1}{2}$ p c.'s at 105 Find the average rate of interest he makes on his capital

195. A company pays a dividend of 6 per cent, and its £5 shares sell for £7, 5*s* What percentage does an investor obtain for his money?

196. What must be the price of 6 per cent stock in order that money invested in it may yield $4\frac{1}{2}$ per cent?

197. Find the price of the 3 per cents when a person receives $3\frac{1}{2}$ per cent for his money after paying 2*d* in the pound income-tax

198. What is the price of the 3 per cents when they give 3 per cent clear, after paying income-tax at 9*d* in the pound?

199. What is the price of 3 per cent stock if, after an income-tax of 8*d.* in the pound has been deducted, an investor gets 4 per cent interest on his money?

200. What is the price of a 5 per cent stock when money invested in it yields 4 per cent interest, after paying income-tax at 9*d.* in the pound? (Brokerage $\frac{1}{8}$)

201. If by selling out $2\frac{1}{2}$ per cent consols at 105, and investing in another stock at 126, a man could double his income, what dividend must the latter pay?

202 A man transfers his money from 4 per cents at 102 to 3 per cents at 95 Find the change per cent in his income

203 A person invested £630 in stock at $94\frac{1}{2}$ and sold immediately, gaining £10 At what price did he sell?

204 How much stock must be bought in 3 per cents at $89\frac{1}{8}$ in order that, by selling it at $91\frac{3}{8}$ a profit of £45 may be made?

205. If £3000 be invested in a certain 5 per cent stock, it produces annually £25 more than if it be invested in 3 per cent stock at 90 Find the price of the 5 per cent stock

206. By the conversion of consols from 3 per cent to $2\frac{3}{4}$ per cent, a person's income is diminished by £27 per annum What cash would he obtain by selling out at $97\frac{1}{8}$?

207 A person sells out of $3\frac{1}{2}$ per cents at $92\frac{3}{4}$ and realizes £18,550 If he invests $\frac{2}{5}$ of the proceeds in 4 per cents at 96, and the remainder in 3 per cents at 90, find the alteration in his income

208. A man sells twenty £10 shares in a company which pays a dividend of $5\frac{1}{2}$ per cent, at £13, 10*s* each, and invests the proceeds in £1 shares, at 15*s* each, in another company which pays a dividend of 3 per cent Find the change in his income

209. A person would obtain £22, 10*s* a year more if he invested his money in a $5\frac{1}{2}$ per cent stock at 132, than he would if he invested in $2\frac{1}{2}$ per cent consols at 105 What money has he?

210. A person bought consols at 94 and sold the same at $95\frac{5}{8}$, thereby gaining £55. What cash did he receive for the stock? (Brokerage $\frac{1}{8}$ per cent)

211. Which is the better investment—5 per cents quoted at $142\frac{1}{2}$, $\frac{5}{8}$, or 4 per cents at $113\frac{3}{4}$, $\frac{7}{8}$?

212. At what price must a person buy 4 per cent stock with the proceeds of the sale of £9600 three per cent stock at 96 if no change results in income?

213. At what price must I buy a 4 per cent stock with the proceeds of the sale of £9600 3 per cent stock at $86\frac{3}{8}$, in order to make no change in my income, allowing $\frac{1}{8}$ per cent brokerage?

214. A person invests £13,650 in a 4 per cent stock at 91 On the stock falling to 75 he sells out, and investing the proceeds in an 8 per cent stock he finds that he thereby loses in interest £60. What is the price of the latter stock?

215. How much stock in $2\frac{1}{2}$ per cent consols must be sold at 108 in order with the proceeds to purchase £4315 railway stock at 132? (Brokerage on consols $\frac{1}{8}$, and on railway stock $\frac{1}{4}$ per cent)

216 A man invests in consols at $106\frac{1}{2}$, and on their rising to £108 he sells out, thus increasing his capital by £20. How much stock did he buy, and what did he pay for it? (Brokerage $\frac{1}{8}$.)

217. What must be the price of 5 per cent stock in order that, after deducting income-tax at 8*d* in the pound, it may yield $3\frac{5}{8}$ per cent interest on capital invested in it?

218 What is the market price of the $2\frac{1}{2}$ per cent consols when, after deducting income-tax at 6*d* in the pound, 2 per cent interest on capital invested in them is made? (Brokerage $\frac{1}{8}$ per cent)

219. A man invested £560 in 3 per cents at $99\frac{1}{2}$, and sold out when they rose to $102\frac{1}{4}$. How much (neglecting fractions of a penny) did he gain after paying $\frac{1}{8}$ per cent brokerage on each transaction?

220. A person has an income of £600 a year from Victoria four per cents if he sells out at $101\frac{1}{4}$ and invests in three per cents at $91\frac{1}{8}$, what is his loss of income?

221. A man buys into a certain stock at $145\frac{3}{4}$ and at the end of six months sells out at $197\frac{1}{2}$. If, including the half-year's interest at $7\frac{1}{2}$ per cent per annum, he gains £5550, what amount of stock did he buy?

222. A man sells out £10,000 three per cents at 101, and £5000 four and a half per cents at 108, and invests the proceeds in five per cents at 124 Find his income before and after the transaction

223 A person sells stock paying $6\frac{1}{2}$ per cent at $128\frac{1}{2}$ and invests in stock paying 3 per cent at $72\frac{1}{2}$ By how much per cent will the interest of his investment be altered?

224. A and B have each £7200 to invest A buys Costa Rica stock, which pays no dividend, at 15; B buys Portuguese 3 per cents at 48 At the end of a year A sells at $15\frac{3}{4}$, B, having received the year's dividends, at $47\frac{1}{2}$. Which had made the better investment?

225. A speculator buys £1000 stock at 80 and pays for it with money which he borrows at 5 per cent At the end of a year he sells out (the stock having paid a dividend of 4 per cent), and, after repaying his loan with interest, he has £40 left. At what price did he sell out?

226. A man owns £7500 of 3 per cent stock He sells out and invests the proceeds in 5 per cent stock at 120, thereby increasing his income by £25 At what price did he sell out?

227 By the conversion of consols from 3 per cents to $2\frac{3}{4}$ per cents a person's income was reduced by £7 a year What cash would he have obtained by selling out at $102\frac{1}{4}$? (Brokerage $\frac{1}{8}$)

228. A man holding 4 per cent stock which brought him in £250 a year, sold at $120\frac{1}{4}$, and bought other stock, paying a dividend of $5\frac{1}{2}$ per cent, at 156 What gain in income did he effect? (Brokerage $\frac{1}{4}$)

229. A person holding £2450 three per cent stock sells out at $93\frac{1}{2}$, and invests the proceeds in $4\frac{1}{2}$ per cents, thereby increasing his income by £31, 13*s* 9*d* At what price did he buy?

230 A person had a certain capital, half of which he invested in 3 per cents at 90, and the other half in 5 per cents at 110 If his total income was £6883, 10*s*, what was his capital?

231. A man's income from three per cent stock is £750 He sells out half his stock at 96, and invests it in other securities at 120. If his income be increased by £50, what interest do these securities pay?

232. Starting with £2310 I buy into the 3 per cents at 99, and when they have risen to 101 I sell out and reinvest in foreign 4 per cents at par What is the change of income?

233. A person invested £2353 in 6 per cent stock at 181 a year afterwards he sold out when the stock was at 178, and invested the proceeds in a $2\frac{1}{2}$ per cent stock at 26 What difference does the change of investment make in his income?

234. A person sells out of the three per cents and realizes £18,000 Would it be more to his advantage to reinvest this money in the three and a half per cents at $101\frac{7}{8}$ or in a mining company paying a dividend of $1\frac{3}{4}$ per cent per annum, of which the £10 shares were quoted at $5\frac{3}{8}$?

235. If an income-tax of 6*d* in the pound be deducted before payment of the dividend on $2\frac{1}{2}$ per cent stock at 91, how much money must be invested in that stock to yield a net income of £208 per annum?

236. How much money must I invest in a $3\frac{1}{2}$ per cent stock at $135\frac{3}{4}$ so that, after paying $\frac{1}{4}$ per cent commission, a stamp of 10*s* per £100 stock, and 6*d* in the pound income-tax on the dividend, I may have a net income of £1000 a year?

237. The 3 per cents are at $96\frac{7}{8}$, and $3\frac{1}{4}$ per cents at 105. What amount invested in the former would produce an annual

income of £1 more than if invested in the latter, one-eighth per cent brokerage being charged on each investment?

238. A man sells out $2\frac{3}{4}$ per cent consols at $96\frac{1}{4}$, and, by investing the proceeds in shares which pay an annual dividend of £4 per share, raises his income 5 per cent. Find the price of a share.

239. What income would result from the investment of £990 in £50 shares in a company, if the shares, £20 being paid up, sell at £45, and the company pays a dividend of 12 per cent?

240. A person buys shares in a railway at $19\frac{1}{2}$ when £15 has been paid, and after paying a call of £10 per share, sells them again at £32, 9s How much per cent does he gain?

241. A man buys 50 shares, £10 paid up, at £12, 5s., and, after paying a call of £5 per share, sells them at £16, 10s What capital does he sacrifice?

242. If £2650 is invested partly in $4\frac{1}{2}$ per cent stock at 104, and partly in 4 per cent stock at 96, and the resulting incomes are equal, find how much is invested in each stock.

243. A man invests £9256, 10s, partly in 5 per cent stock at 150, and partly in $2\frac{3}{4}$ per cent stock at 99, and his income from each source is the same What sums did he invest in each stock?

244. A man invested one-third of his capital in 5 per cents at $142\frac{1}{2}$ and two-thirds in 4 per cents at $113\frac{3}{4}$, and the total resulting income was £455, 13s 4d Find his capital.

245. A man invests £7000, part in 3 per cents at 97, and the remainder in 4 per cents at 104 Find how much he invests in each if the returns from both sources are equal

246. If £7594 be invested partly in $2\frac{1}{2}$ per cents at $110\frac{1}{2}$ and partly in $2\frac{3}{4}$ per cents at $115\frac{1}{2}$, and the resulting incomes are equal, how much was invested in each stock? (Brokerage $\frac{1}{8}$)

247. A man invests £16,213 partly in £100 shares at 105, bearing a dividend (free from income-tax) of 8 per cent, and partly in a mortgage at 5 per cent interest on which he pays income-tax at 8d in the pound. His net income from each source is the same What is his whole income?

248. A man invests half his capital in the $2\frac{1}{2}$ per cents at 88, and the other half in 4 per cents at 116, and his total income from both sources is £642 What was his capital?

249. A man calls in a sum of £10,000 lent on mortgage at 4 per cent and invests the money in £3000 L. & N.W. Ry 4 p c deb stock at 133, and £3000 G W. Ry 5 p.c. stock at 167 The balance of the £10,000 he places on deposit at $2\frac{1}{2}$ per cent. What change is made in his income?

250. Which is the better investment (the security being equal), 3 per cents perpetually at par, or 5 per cents at 124, the latter stock being redeemable in 20 years at par? (Simple interest is to be reckoned)

251. A person is left a legacy on which he pays 10 per cent succession duty He invests the rest of the money in a 3 per cent stock at 75, bringing him in £270 per annum To how much did the legacy amount?

252. A man has £3000 of a 4 per cent stock, which he sells out at 120, and then invests £1900 in a $2\frac{3}{4}$ per cent stock at 95 What rate of interest does he get for the remainder, if his income is, on the whole, diminished by £14?

253. A person wishes to found a scholarship of £48 a year How much money must he for this purpose invest in $2\frac{1}{2}$ per cent stock at 87, in order that the scholar may have his £48 clear after paying income-tax at 8*d* in the pound?

254. A and B invested equal sums, A in $3\frac{1}{2}$ per cents at 120, B in $2\frac{3}{4}$ per cents at 96 After this A sold out at par and invested in the $2\frac{3}{4}$ per cents, which had fallen in price A now found his income the same as B's What was the price at which he bought the $2\frac{3}{4}$ per cents?

255. A person having bought a certain amount of $2\frac{3}{4}$ per cent stock at 95, afterwards sold it, and with the proceeds bought $3\frac{1}{2}$ per cent stock, he obtained £900 less stock than before, but his income was unchanged How much money did he originally invest?

256. £4600 was invested in consols at 92 Part of the stock was sold out at 93 and part at 88, and the original capital was thus decreased by £5 How much stock was sold out at each price?

257. A man invests £5900 in $2\frac{1}{2}$ per cent consols at $110\frac{1}{2}$, and afterwards sells out part of the stock at $108\frac{1}{4}$ and the remainder at 113, thus increasing his capital by £25 How much stock did he sell out at each price? (Brokerage $\frac{1}{8}$ p c)

258. A man invests £2730 in 3 per cents at $90\frac{3}{4}$, and sells out part of the stock when they have risen to $93\frac{3}{4}$ and the remainder when they have fallen to $85\frac{1}{4}$ He lost £10 by the transaction How much did he sell out at first? (Brokerage $\frac{1}{4}$ per cent)

He invests the proceeds in $4\frac{1}{2}$ per cents so as to cause an increase in his income of £16, 13*s* 4*d* At what price did he buy the $4\frac{1}{2}$ per cents? (Brokerage $\frac{1}{4}$ per cent)

259 Some 4 per cent stock at $115\frac{5}{8}$, and some 5 per cent stock at $134\frac{7}{8}$, are sold, and the proceeds are invested in $4\frac{1}{2}$ per cent stock at $123\frac{1}{2}$, if the change does not alter the income, what is the ratio of the quantities of stock sold?

260. The capital of a company consists of £1,100,000 in $4\frac{1}{2}$ per cent debenture stock, the same amount in 6 per cent preference shares, and the same amount in ordinary shares If the net annual profits are £209,000, calculate the amount per cent available for dividend on the ordinary shares after payment of the debenture and preference claims

261. A man has £9170 in 3 per cent consols. He sells out at $101\frac{3}{8}$ and buys Indian bonds at $98\frac{3}{4}$, and when these have risen to $106\frac{1}{16}$ he sells out and reinvests in consols at $98\frac{1}{8}$. If the interest on consols has meanwhile been reduced to $2\frac{3}{4}$ per cent, find the permanent change in his income, allowing $\frac{1}{8}$ per cent for brokerage on each transaction.

262. A person invests £4800 in 4 per cents at 80, and at the end of each year invests the dividend which becomes due, in the same stock. Supposing the stock to remain at 80 for 3 years, find his dividend at the end of the third year.

263. A person who has £10,257, 10*s* three per cent stock calculates that by selling it and investing in $3\frac{1}{4}$ per cent stock at $93\frac{1}{4}$ he can increase his annual income by £10, 9*s*, but, before he can effect the exchange, each stock rises $\frac{1}{4}$ per cent, by how much is his income really increased?

264. A man invests £1404, 16*s* in $2\frac{3}{4}$ per cents at 99, how much must he also invest in $3\frac{1}{2}$ per cents at 105, so that the rate of interest on the whole may be 3 per cent?

265. A person invests £7000 in the 3 per cents at 99, and pays income-tax at 8*d* in the pound. On the stock rising to 102 he sells out and invests the proceeds in railway stock at 163, paying 5 per cent free of income-tax. Calculate to the nearest penny the change in his net income.

266. A man purchases £1400 stock in three per cent consols at $94\frac{1}{2}$, and also invests £3150 in the purchase of Russian inscribed five per cent loan at $94\frac{1}{4}$. How much stock has he standing in his name? If he sells the consols at $95\frac{1}{8}$ and the Russians at $96\frac{1}{2}$, what does he gain or lose by the transaction? (Brokerage on consols $\frac{1}{8}$, on Russians $\frac{1}{4}$.)

267. Find, neglecting fractions of a penny, the change of income in transferring £2800 from Goschens ($2\frac{3}{4}$ per cents) to India $3\frac{1}{2}$ per cents, the former being quoted at $96\frac{5}{8}$, $\frac{7}{8}$, and the latter at 106, $\frac{1}{4}$. (Brokerage $\frac{1}{8}$.)

268. Show that ·0067 of the amount of $2\frac{3}{4}$ per cent consols will give the quarterly dividend, with sixpence in the pound income-tax deducted, with an error not greater than one penny for every £1000 stock.

269. Find, neglecting fractions of a penny, the net half-yearly dividend, after deducting income-tax at 8*d* in the pound, resulting from the investment of £16,000 in railway stock at $179\frac{1}{2}$, paying a dividend at the rate of $5\frac{1}{2}$ per cent, the charge for brokerage being $\frac{1}{4}$ per cent.

270. A bill for £15,000 at 3 months was discounted at $4\frac{1}{2}$ per cent and the proceeds invested in $2\frac{3}{4}$ per cent consols at 95, brokerage being $\frac{1}{8}$ per cent, and income-tax 5*d* in the pound. Find, to the nearest penny, the net quarterly income.

LVIII. FOREIGN MONEY.

DECIMAL COINAGES

Express at sight—

1. 103 05 *fr* in francs and cents
2. 1587 5 *fr*
3. 236·7 *M* in marks and pf
4. 76·055 *M*
5. 846·75 *l* in lire and cent
6. 48·25 *fl* in florins and kreuzer
7. 93·45 *R* in roubles and kopeks
8. \$7·45 in dollars and cents
9. \$154·7
10. \$6·0575

Express at sight—

11. 74 *fr* 25 *c* in francs
12. 8 *fr* 5 *c*
13. 105 *M* 50 *pf* in marks
14. 16 *M* $5\frac{1}{2}$ *pf*
15. 24 *l* 5 *c* in lire
16. 6 *fl* 25 *kr* in florins
17. 62 *fl* 5 *kr*
18. 150 *R* 19 *ko* in roubles
19. \$17, 35 *c* in dollars
20. \$175, 5 *c*

21. Add 96 *fr* 78 *c*, 108 *fr* 5 *c*, 2086 *fr* 25 *c*, and 6 *fr*. 80 *c*
22. Add 910 *M* 5 *pf*, 17 *M*. 50 *pf*, 4 *M* 20 *pf*, and 1050 *M* 85 *pf*
23. Subtract \$48, 5 *c*. from \$203, 70 *c*

Multiply—

24. 7 *fr* 35 *c* by 29
25. 4·50 *fr* by 234

Multiply—

26. 6 *M* 60 *pf* by 63
27. \$4·75 by 147

Divide—

28. 162 *fr* 61 *c* by 23
29. \$747·90 by 54

30. How many times is 23 *M* 50 *pf* contained in 1574 *M* 50 *pf*?

If a franc $= 9\frac{1}{2}d$, a mark $= 11\frac{3}{4}d$, a lira $= 9\frac{1}{2}d$, a florin $= 20d$, a rouble $= 37\frac{1}{2}d.$, and a dollar $= 50d$, express in £, *s* *d* to the nearest penny—

31. \$81, 17 *c*
32. \$473·56
33. 78 *fr* 50 *c*
34. 456·75 *fr*
35. 4 *M* 85 *pf*
36. 63·50 *M*
37. 27 *fl* 15 *kr*
38. 104 *fl* 25 *c*
39. 18 *l* 50 *c*
40. 350 *R*
41. 47 *R* 80 *ko*.
42. 17·25 *R*
43. 18560 *M*
44. 22550 *fr*
45. 225·50 *fr*
46. 185 60 *M*

Express approximately—

47. £7, 12*s* 6*d* in francs and cts
48. £37, 14*s* 8*d*
49. £21, 3*s* 5*d*
50. £405, 12*s*
51. £15, 4*s* 9*d* in marks and pf
52. £2, 5*s* 2*d*
53. £93, 15*s*. 6*d*.
54. £3, 13*s* 4*d* in dollars and cents.
55. £216, 10*s*
56. £107, 12*s* 9*d* in florins and kr
57. £31, 13*s* 7*d*
58. £2, 11*s* 10*d* in lire and cent
59. £53, 4*s* in roubles and kopeks
60. £105, 14*s*. 9*d*

LIX. THE METRIC SYSTEM

Length

Read, mentioning the special denomination of each separate figure—

Km	Km	m	m	cm
1. 4·573	2. 45·73	3. 4·573.	4. 457·3.	5. 457·3.

Read as a length expressed in *kilometres and metres*—

Km	Km.	Km	Km	Km
6. 4·573.	7. 45·73	8. 457·3.	9. 20·534.	10. 2·0534.

Read as a length expressed in *metres and centimetres*—

m	m	m.	cm	mm.
11. 3·65.	12. 36·5	13. 1·375	14 846	15. 7305.

Express in *kilometres and decimals of a kilometre*—

16. 2 *Km* 3 *Hm* 4 *Dm* 7 *m*	18. 90 *Km* 125 *m*	20. 7 *Km.* 500 *m*
17. 5 *Mm* 1 *Km* 7 *Dm* 2 *dm*	19. 13 *Km.* 25 *m.*	21. 4 *Km.* 5 *m.*

Express in the denomination *kilometres*—

22. 8750 *m* 23. 43·7 *m* 24. 4 *Dm.* 25. 4 *dm.* 26. 20 *cm.*

Express in the denomination *metres*—

27. 6 *Km.* 28. 13·5 *Km.* 29. 7 *Dm.* 30. 7 *dm* 31. 45 *mm.*

Express in the denomination *centimetres*—

32. 3 *m* 33. 7·5 *m* 34 6 *dm.* 35. 3 *mm.* 36. 3 *Km*

37. Find (in kilometres) the sum of 47 *Km*, 58 *Dm.*, 85 *m*, 2·45 *Km*, 1375 *m* and 9 *Km* 80 *m*

38. Find (in metres) the sum of 3 *Dm*, 17·5 *Dm.*, 6 *dm.*, 48 *cm.*, 152 *cm*, 10·5 *m.* and 5 *Hm.*

39. Find (in centimetres) the sum of 1 *m* 25 *cm*, 10 *cm.* 5 *mm.*, 3 *dm* 3·5 *mm.*, 2 *m.* 5 *cm* and 7 *m* 50 *cm.*

40. Find (in *Km*) the difference between 8 *Km* 250 *m* and 18 *Km* 25 *m.*

41. Find (in metres) the difference between 9 *Dm.* 7 *m.* and 9 *m* 7 *dm.*

42. Find (in *cm*) the difference between 1 *m* 5 *cm.* and 86 *cm.* 5 *mm.*

Multiply*—

43. 6 *m* 25 *cm* by 8	45. 1 *Km.* 725 *m* by 34	47. 3 *m.* 5 *cm.* by 308
44. 2 *Km* 45 *m* by 12	46. 13 *m* 50 *cm* by 600.	48. 1 *Km.* 5 *m* by 760.

Divide, giving the remainder (if any) in centimetres—

49 18 *Km* 140 *m* by 4	51 43 *m.* 20 *cm.* by 73	53 2 *Km* 860 *m.* by 571.
50. 369 *Km* by 180	52. 7 *m* 5 *cm* by 11	54. 128 *m* by 42.

* These products should be given in the highest denomination mentioned in the question.

How many times is—

55	3 *Km* 50 *m*	contained in	143 *Km* 350 *m* ?
56	17 *m* 50 *cm*		315 *m* ?
57.	13 *Km* 5 *m*		9129 *Km* 510 *m* ?
58.	7 *cm* 4 *mm.*		22 *m* 57 *cm* ?

59 How many times can 12 *m* 35 *cm* be taken from 3 kilometres, and what remains?

60. How many times can 7·5 millimetres be subtracted from 3·25 decimetres, and what is left over?

Find the cost of—

61. 24 metres 8 centimetres of silk at 4 francs 25 cents per metre.

62 18 metres 5 centimetres of cloth at 3 francs 80 cents per metre

63. 8 kilometres 25 metres of fencing at 5 francs 75 cents per metre

64. 6 kilometres 850 metres, at 28 60 francs per kilometre

65. 43 metres 25 centimetres, at 3 marks 8 pfennigs per metre

66 3 kilometres 640 metres, at 4 marks 50 pfennigs per metre

67 6 centimetres 5 millimetres, at 7 75 francs per centimetre

68. 14 metres 5 decimetres, at 2 florins 25 cents per metre

69. How many revolutions will a wheel, whose circumference is 2 metres 15 centimetres, make in travelling a distance of 4 kilometres 42 metres?

70 How many metres of calico, at 1 franc 5 centimes per metre, can be bought for 175 francs 35 centimes?

71. If 7 metres 5 centimetres of cloth cost 22 francs 56 centimes, what will 5 metres 70 centimetres of the same cloth cost?

72. If 24 metres 50 centimetres of linen cost 19 florins 60 cents., how much can be bought for 49 florins 20 cents?

73. If a bicyclist travel at the average rate of 13 *Km* 750 *m.* per hour, how far will he go in 3 hours 45 minutes?

74 How many times can a piece 3 *cm* 4 *mm* long be cut from a rod 9 *dm* long, and what is the length in millimetres of the remnant?

Find to the nearest five cents (or five pf) the value of—

75.	13 *m* 45 *cm*	at	74 *fr* 85 *c*	per metre
76	7 *m* 4·25 *cm*	at	2 *fr* 35 *c*	per centimetre
77.	8 *Km.* 17 *m*	at	6 *fr* 85 *c*	per kilometre
78	2 *Km* 326 *m*	at	7 *M* 68 *pf*	per kilometre.
79.	16 *m.* 32·5 *cm*	at	1 *M* 45 *pf*	per metre
80.	23 *m.* 88 *cm.*	at	9 *fl* 95 *c.*	per metre.

WEIGHT.*

Find the cost of—

81. 13·5 kilogrammes of sugar at 60 centimes per kilogramme
82. $28\frac{1}{2}$ kilogrammes of butter at 1 franc 20 cents per kilogramme
83. 23·8 kilogrammes of copper at 2 francs 5 cents per kilogramme
84. 14 kilogrammes 750 grammes of coffee at 3 francs 80 centimes per kilogramme
85. 82 kilogrammes 125 grammes, at 3 marks 44 pfennigs per kilogramme.
86. 17 kilogrammes 250 grammes, at 1 mark 24 pfennigs per kilogramme

87. If a kilogramme of copper is worth 1 franc 29 centimes, of what weight is 723 francs 69 centimes the value?
88. Find to the nearest centigramme the quotient of $8 \cdot 4\, g \div 53$.
89. How many times is 3 *Kg* 500 *g* contained in 845 *Kg.* 25 *g.*, and how many grammes are left over?
90. If 74·5 *Kg* of sugar cost 56·2 francs, what is the cost of 96·85 *Kg.*?
91. If 56 *Kg.* 500 *g* of lead are worth 64 marks 41 pfennigs, find the value of 18 *Kg*

Find, to the nearest five cents. (or five pf.), the value of—

92. 7 *Kg.* 385 *g* at 1 *fr* 30 *c.* per *Kg.*
93. 3 *Kg.* 465 *g* at 1 *M.* 75 *pf* per *Kg.*
94. 7·25 quintal at 25 *fr* 50 *c* per tonneau.
95. 4·75 tonneaux at 3 *fr.* 75 *c.* per quintal

CAPACITY.*

Find the cost of—

96. 3·25 litres of milk at 20 centimes per litre
97. 7 litres 5 decilitres of vin ordinaire at 70 *c* per litre.
98. 3·25 hectolitres of wine at 54 francs per hectolitre

99. If 12 hectolitres of wine cost 576 francs, what is the cost per litre?
100. Find the cost of 85·25 hectolitres at 24 francs 50 centimes per decalitre

---

* Exs 1–60 may be made to apply to Weight or Capacity, by merely substituting the word "gramme", or "litre", for "metre".

AREA.

Express in square metres—

101. 3 *sq. Dm.*
102. 3 *sq dm*
103. 3 *sq Km*
104. 23 ares
105. 23 hectares.
106. 23 centiares

Express in square centimetres—

107. 7 *sq dm* 108. 7 *sq m* 109. 485 *sq mm* 110. 3·4 ares.

Express in ares—

111. 580 *sq m* 112. 43 *sq Dm.* 113. 198 *ca* 114. 6·3 *Ha*

115. Express in square centimetres the sum of 53 *sq dm*, 63 5 *sq dm*, 8 *sq m*, 7 4 *sq m* and 654 *sq cm*

116. Express in hectares the sum of 6 *Da* 3 *a*, 5 *Ha* 17 *a*, 13·25 *Ha*, 157 *a.*, 80·5 *a* and 246 *ca*

117. A field is divided into 17 allotments, each containing 3·75 ares, find the size of the field in decares

118. How many times is 13 *sq cm* 50 *sq mm* contained in 3 *sq dm* 10·5 *sq cm.*?

119. Find the rent of a farm of 89 *Ha* 25 *a* at 57 *fr.* 50 *c* per hectare

120. Find the cost of 2 *Ha* 72 *a* of land at 2 *fr* 25 *c* per sq metre

121. Find the area of a floor 5 *m* 75 *cm* long and 4 *m* 20 *cm* wide.

122. Find the area of a rectangular field 126 *m* long and 43 *m* 50 *cm.* wide

123. Find the cost of paving a floor 7 *m* 5 *dm* long and 4 *m* 8 *dm* wide, at 3 *M* 80 *pf.* per square metre

124. Find the cost of matting, 75 *cm* wide, to cover a floor 5 *m* 5 *dm* long and 4 *m* 4 *dm* wide at 4 *fr* 50 *c* per metre

Find, to the nearest five cents (or five pf), the cost of—

125. Paving a passage 25 *m* long and 2 *m* 40 *cm.* wide, with tiles 15 *cm* square, at 1 *fr.* 50 *c* per dozen

126. Electroplating a metal box 21 *cm* long, 12 *cm* wide, and 8 *cm.* high, all over outside, at 2 *fr* 75 *c* per square decimetre

127. Lining an open tank 4 *m* 60 *cm* long, 2 *m* 25 *cm* wide, and 1 *m* 50 *cm.* deep, at 4 *M* 75 *pf* per square metre.

VOLUME

128. Express 3 *cub m* in cubic centimetres

129. Express 4850 *cub dm* in cubic metres

130. Express 15 decasteres in cubic metres.

131. Express 15 *cub dm* in steres.

VOLUME (*continued*).

132. Find, in cubic metres, the volume of a rectangular block 3 *Dm.* long, 8 *m.* wide, and 7 *dm* thick

133. How many cubic decimetres of air are there in a room 4 *m.* 20 *cm.* long, 3 *m.* 75 *cm* wide, and 3 *m* 5 *cm.* high?

134 Find the cost of gravel ·75 of a decimetre thick for a path 86·5 metres long and 2·2 metres wide, at 4 francs per cubic metre.

135. Given that a cubic centimetre of water weighs a gramme, how many kilogrammes of water would cover the floor of a skating rink, 20 metres long and 15 metres wide, to a depth of one decimetre?

136 A cask containing a hectolitre of wine bursts in a cellar 4 metres square to what depth is the floor flooded?

137. How many hectolitres of water will a tank, 3 metres long, 2·8 metres wide, and 1·25 metres deep, hold?
Find also, in kilogrammes, the weight of the water

138. Find, in tonneaux, the weight of water in a reservoir having an area of 2·35 hectares, the average depth of the water being 2 metres 35 centimetres

CONVERSION OF METRIC TO ENGLISH MEASURES, AND VICE VERSA *

Given that 1 metre = 39·37079 inches, and 1 gallon = 277·274 cubic inches, convert, correct to *five* significant figures—

139. 3 *cm.* into *inches*
140. 1 *Km* ... *yards*
141. 8 *Km.* *miles*
142 41 *m.* ... *feet*
143. 1 *sq m* . . *sq in.*
144. 36 *sq cm* ... *sq in*
145. 1 *Ha* . *acres*
146. 1 *cub. m* ... *cub in*
147. 1 *cub m* . . *cub yds.*
148. 1 *litre* ... *pints*
149. 1 *gramme* ... *grains*
150. 1 *Kg.* ... *lbs Av*

151. 1 *inch* into *cm*
152. 5 *feet* . *dm*
153. 1 *yard* .... *m*
154. 3 *miles* .... *Km.*
155 1 *sq. in* .... *sq. cm.*
156. 5 *acres* ..... *Ha.*
157. 3 *R.* 24 *P.* .... *ares*
158. 1 *cub in.* . . *cub cm.*
159. 7 *cub yds* .... *cub m.*
60. 1 *gallon* .... *litres.*
161. 1 *oz Troy* ... *grammes.*
162 25 *lbs Av.* ... *Kg.*

* These Exercises should be worked by the contracted methods of Chap L.

163. Taking 8000 metres to be equal to 5 miles, how many square metres are there in an acre?

164. Find the difference in yards between 5 miles and 8 kilometres (A kilometre = 39370 8 inches)

165. A cubic centimetre of water weighs one gramme Find the volume of 350 kilogrammes of a liquid which is twice as heavy as water

166 A metre contains 39·3708 inches Find, to the nearest integer, the number of metres in a mile

167. A kilometre being 1093 638 yards, find to four places of decimals how many kilometres there are in 100 English miles

168. Given that a metre is 3 3708 inches longer than a yard, find which is greater 10 square metres, or 12 square yards

169 Express 39 chains in metres, correct to the nearest centimetre.

170 Calculate whether posting at a shilling per mile is less or more expensive than posting at 65 centimes per kilometre, having given that 1 metre = 39·37 inches, and 25 francs = £1

171. Find the number of square centimetres in the area of a rectangle 5 ft 3 in long by 2 ft 6 in wide

172. If £1 = 25 2 francs and 1 acre = 40467 hectare, find to the nearest franc the rent per hectare equivalent to 25*s* per acre

173. A metre being equal to 39·371 inches, and a franc being equal to 9·38 pence, what is the value in English money of a yard of silk worth $7\frac{1}{2}$ francs a metre?

174. If the railway fare in France for a distance of 384 *Km* be 25·28 francs, find, to the nearest farthing, the rate per mile (1 metre = 39 3708 inches, £1 = 25 2 francs)

175. Given that 1 oz Troy = 31 1035 grammes, express 25 tons 2 cwt 27 lbs in the metric system

176. Given that 1 gramme = 15·4323 grains, express 2 cwt 3 qrs. 11 lbs in the metric system

177. Determine the value of $2\frac{1}{4}$ cwt in marks, if a ton be equivalent to 1015 kilos, and 50 75 kilos cost 95·8 marks

178. I bought 40 metres of velvet at 12 francs 60 centimes a metre. I sold 25 yards of it at 9*s* $11\frac{1}{4}$*d* a yard, and the rest at 12*s* a yard What was my gain per cent? (1 metre = 39·37 inches, £1 = 25·22 francs)

179. If the cost of the carriage of goods in England be 1*d* per ton per mile, find the equivalent cost of carriage in France in centimes per 1000 kilogrammes per kilometre, assuming that a kilometre is equal to 5 furlongs, a kilogramme to 2·2 lbs, and 100 centimes to one-twenty-fifth of a pound sterling

180. A metre being equal to 39 37 inches, and a gramme being equal to 15·43 grains, find the weight in grammes of a cubic metre of air, when 100 cubic inches of the air weigh 31 grains.

LX. FOREIGN EXCHANGES.*

Exchange—

1.	£530	into francs,	at	25·20 *fr.*	for £1
2.	£285	marks,		20·35 *M*	£1.
3	£364	lire,		27·15 *l*	£1.
4.	£1080	kroner,		18·31 *kron.*	£1.
5.	£635 ...	gulden,		12·12 *gul*	£1.
6	£186	Dutch florins,		12 *fl* 2 *st.*†	£1.
7.	£242 ...	American dollars,		49*d* .	$1.
8	£1230	roubles,		25¼*d* .	1 *R.*
9.	£470	Spanish dollars,		37½*d.*	1 *dol.*
10	£345	milreis,		39¼*d* ...	1 *mlr.*

Exchange—

11.	8570 *francs*	into sterling, at	25·16 *fr*	for	£1.
12.	3250 *marks*		20·36 *M*	.	£1.
13.	1875 *pesetas*	.	30·50 *pes*		£1.
14.	656 *florins*		12·09 *fl*	..	£1.
15.	4870 *lire*	.	26·84 *l.*	.	£1.
16.	2895 *roubles*		93·70 *R*	.	£10
17.	1680 *dollars*		$4·86̄	.	£1.
18	64850 *rupees*	.	1*s* 2⅛*d*		1 rupee.
19	498 *tael*		2*s.* 11*d*	.	1 tael
20.	564 *yen*	.	2*s* 1½*d.*	.	1 yen

21.	Exchange	£483, 16*s* 8*d*	into francs	at	25·21.
22		£176, 12*s* 6*d.*	. marks	..	20·54
23.		£1053, 10*s*	kroner	.	18·32
24.	. ...	£251, 7*s* 9*d*	Dutch florins .		12 2¾.
25	. .	£873, 15*s*	. rupees	.	1*s.* 2⅞*d.*
26	.	£531, 6*s.*	. .. milreis	..	39⅜*d*

27.	Exchange	15000 kroner	into sterling at	18·19.
28		3875 marks		20·38.
29.		14850 francs		25·16½.
30.	.	973 florins		12·09.
31.	..	1584 roubles	.. .	93·72.
32.	.... .	10860 rupees	. .	1*s* 2$\frac{5}{32}$*d.*

* *Approximate* results required † 20 stivers (*st*) = 1 *fl*

33 Find the cost in London of a bill on Paris for 5000 francs, when the short exchange, London on Paris, is 25·24

34. A merchant in London purchases goods in New York, the price of which is 3475 dollars, and remits bills in payment Find the cost, exchange in London on New York being 4 83

Find the "sight" quotations corresponding to the following "long" rates, supposing that the rate of discount is 4 p c —

35 London on Paris, 3 mo, 25 32

36. London on Brussels, 3 mo, 25 $37\frac{1}{2}$

37 London on New York, 60 days, $48\frac{3}{4}$.

38. Berlin on London, 3 mo, 20 26

39 St Petersburg on London, 3 mo, 93 70

40. Calcutta on London, 4 mo, 1*s* $2\frac{5}{16}d$

Find, when the rate of discount is 3 p c, the quotation for 3 months' bills corresponding to the following "short" rates —

41 London on Paris, cheques, 25·$21\frac{1}{2}$

42 London on Amsterdam, sight, 12 $1\frac{7}{8}$

43. Amsterdam on London, sight, 12·$09\frac{1}{2}$

44 Lisbon on London, sight, $40\frac{1}{4}d$

Find the rate of discount when the quotations are—

45 London on Paris, cheques, 25·$15\frac{1}{2}$, 3 mo, 25 $31\frac{1}{4}$

46 Berlin on London, 8 days, 20 $35\frac{1}{4}$, sight, 20 37

Find the cost in London of the bills described below, the rate of discount being 4 p c, allowing for brokerage $\frac{1}{10}$ p c and stamp 1*s* p c —

	Bill	Course of Exchange
47	1890 florins, sight	London on Vienna, 3 mo, 12·13.
48	4896 marks, sight	London on Hamburg, 3 mo, 20·35
49	1693 florins, sight	London on Rotterdam, 3 mo, 12 $3\frac{3}{4}$
50.	\$8550, sight	London on New York, sight, $48\frac{1}{2}$

Find the cost of the following bills on London, payable at sight, supposing the rate of discount to be 3 p c —

51.	£423, 12*s* 8*d*	Paris on London, 25·18, cheques
52	£857, 10*s*	St Petersburg on London, 93·72, 3 mo
53	£238, 15*s* 6*d*	Rome on London, 26·84, 3 mo
54	£192, 13*s* 9*d*	Calcutta on London, 1*s*. $2\frac{1}{8}d$ T T

55. If the cheque exchange in London on Paris is 25·30, and the rate of discount for 3 months' bills in London is at 5 per cent per annum, what debt in Paris can be discharged by a person in London who holds a 3 months' bill on London for £1000?

56. A merchant wishes to pay a debt of 5000 roubles in St. Petersburg, when the course of exchange is—London on St Petersburg, 3 mo, $25\frac{1}{8}$, St Petersburg on London, 3 mo, 93·70. Will it be better for him to remit bills to St Petersburg, or for his agent there to draw upon him, supposing the rates of discount to be 3 p c. in London and 4 p c in St Petersburg?

57. A London merchant having to pay a Berlin merchant for goods received, finds the rates of exchange as follows —London on Berlin, at 3 months, 20·53 marks for £1, Berlin on London, at 3 months, 20·27 marks for £1. Will it be better for him to remit direct to Berlin, or for his correspondent in Berlin to draw upon him in London? (Disc. 4 p c. in either case.)

Find the arbitrated rate of exchange at—

58. London on Vienna, through Berlin, when the direct rates are—London on Berlin, £1 = 20·54 marks; Berlin on Vienna, 170·30 marks = 100 florins

59. Paris on Berlin, through London, when the direct rates are—Paris on London, £1=25·18 fr., London on Berlin, £1=20·58 marks

60. New York on London, through Hamburg and Paris, when the direct rates are—New York on Hamburg, \$95 = 400 marks; Hamburg on Paris, 80·50 marks = 100 francs; Paris on London, £1 = 25·18 francs

61. A New York merchant owes 20,000 marks in Berlin, the exchange at New York on Berlin being 4·85 dollars per 20 marks, and on London 4·875 dollars per pound sterling. If the exchange at London on Berlin is 20·75 marks per £1, will the merchant find it better to remit direct from New York, or through London, and what difference will it make?

62. A merchant in America wishes to send to London £7000, when exchange on London is at a premium of 5 per cent. He may send it through France or through Hamburg. (1 dollar = 5·20 francs = 2·85 marks, £1 = 25·60 francs = 13·5 marks) Which course will be most to his advantage?

*63. How much English money will it cost a London merchant to remit \$2545·20 to the United States by way of Paris and Madrid, when the course of exchange between London and Paris is 25 fr 60 c for £1, Paris and Madrid, 1 fr. for 1 peseta, Madrid and New York, 5 pes 60 c. for \$1?

64. If £1 contains 113 grains pure gold, 15·432 grains = 1 gramme, and 20 francs contains 5·8 grammes pure gold; calculate the par of exchange between English and French currency.

65. Find the par of exchange between the pound sterling and the rupee, having given that the price of silver is 40*d* the ounce of standard fineness (37 parts pure in 40 standard), and that a rupee contains $\frac{3}{8}$ oz. of standard silver (11 parts pure in 12).

LXI. THE METHOD OF NINE MULTIPLES.

Form a table of the products of 32·185 by each of the numbers from 1 to 9, and use it in obtaining the product of 32·185 and—

1. 34 **2.** 92 **3.** 187. **4** 556 **5.** 7 2 **6** 46 35

Make a similar table of the products of ·38265, and use it in obtaining, correct to four places of decimals, the product of 38265 and—

7. 48·25 **8.** 2·065. **9.** ·3975 **10** 38265

Given that the price of one article is £ 1375, construct a table from which the price of any number of these articles may be found by addition, and use it in obtaining, in £ *s* *d*., the price of—

11 47 **12** 183 **13** 258 **14** 462 **15** 2967.

Given that the price of 1 ton is 13*s* 8*d*, make a table of the decimalized prices of 1 to 9 cwts, and use it in finding, correct to the nearest penny, the price of—

16. 14 cwts	**19.** 36 cwts	**22.** 2 tons 14 cwts 2 qrs.
17. 17 cwts	**20.** 2 tons 13 cwts	**23** 7 tons 9 cwts 3 qrs
18. 12 cwts 2 qrs	**21.** 5 tons 8 cwts	**24.** 28 tons 12 cwts 1 qr

---

Given that 1 metre = 1·093633 yards, continue the table up to 9 metres, and use it in converting into yards, correct to the nearest tenth—

25. 85 *m*	**27.** 672 *m*	**29** 94·8 *m*	**31** 87 5 *m*
26. 246 *m*.	**28.** 1703 *m*.	**30.** 306 5 *m*	**32** 593·64 *m*

Make, also, a similar table for converting yards into metres, and then express in metres, correct to the nearest centimetre—

33 208 *yds* **34** 497 *yds* **35** 75½ *yds* **36.** 1643¾ *yds*

Given that 1 *Kg* = 2 204621 *lbs Av*, continue the table up to 9 *Kg*, and use it in converting into English weight, correct to the nearest ounce—

37. 174 *Kg* **38** 3468 *Kg*. **39.** 392·8 *Kg*. **40.** 16·075 *Kg*

Make a table of nine multiples for use in changing English into French money when the exchange is 20 17½ francs for £1, and use it in finding the value in French money of—

41 £228.	**43.** £2095, 10*s*	**45.** £38, 8*s* 9*d*
42. £1376	**44** £436, 15*s*	**46** £653, 11*s* 4*d*.

Make, also, a similar table for use in changing francs into £ *s*. *d*. at the same rate, and use it in the case of—

47 8079 *fr*. **48.** 42650 *fr*. **49.** 354·5 *fr*. **50.** 1583·65 *fr*.

LXII. SQUARE ROOT.

Find, *by inspection*—

1. $\sqrt{900}$	4. $\sqrt{10000}$	7. $\sqrt{25 \times 9}$.	10. $\sqrt{144 \times 36}$.
2. $\sqrt{2500}$	5. $\sqrt{360000}$	8. $\sqrt{49 \times 16}$.	11. $\sqrt{4 \times 8 \times 8}$.
3. $\sqrt{6400}$.	6. $\sqrt{1210000}$.	9. $\sqrt{81 \times 121}$	12. $\sqrt{7 \times 9 \times 9 \times 7}$.

13. $\sqrt{7^2}$	17. $\sqrt{3^2 \times 2^4 \times 11^2}$	21 $\sqrt{5 \times 29 \times 4 \times 29 \times 5}$.
14. $\sqrt{3^4}$.	18. $\sqrt{23^2 \times 5^2 \times 2^2}$	22 $\sqrt{3 \times 4 \times 11 \times 6 \times 11 \times 2}$.
15. $\sqrt{5^2 \times 7^2}$.	19. $\sqrt{13 \times 49 \times 13}$.	23. $\sqrt{8 \times 16 \times 6 \times 3}$
16. $\sqrt{2^6 \times 12^2}$	20 $\sqrt{4 \times 17 \times 25 \times 17}$	24. $\sqrt{57 \times 5 \times 19 \times 2 \times 30}$.

Find, by factors, the square root of—

25. 576.	29. 4356	33. 16384.	37. 540225.
26. 1024.	30. 6561.	34. 59049	38. 40960000.
27. 1296.	31. 9216.	35. 105625.	39. 1234321.
28. 3969.	32. 108900.	36. 390625.	40. 81162081.

Extract the square root of—

41. 1849	51 167281.	61 10975969.	71. 404130609
42. 3364	52 173889.	62. 12432676.	72 845355625.
43. 7396	53. 277729.	63. 19175641.	73. 1095874816.
44. 11449	54. 552049	64 22297284.	74 1156136004.
45 43681	55. 603729	65 27489049	75. 1383542416.
46. 56169	56. 1164241	66 36180225	76 8617223241.
47. 66049	57 1461681	67. 81144064	77. 8674873321.
48. 84681	58. 3108169	68. 94264681	78 10856806416.
49 103041	59. 3452164	69 189475225	79. 9000426005041.
50. 106929	60. 5322249	70. 231496225	80. 50085018863929.

Find, as a vulgar fraction, the square root of—

81. $\frac{1}{10000}$	87. $\frac{36}{361}$.	93. $1\frac{9}{16}$	99. $1\frac{165}{196}$.	105. $8\frac{289}{784}$.
82. $\frac{16}{25}$.	88. $\frac{324}{961}$.	94. $1\frac{25}{144}$	100. $1\frac{72}{289}$.	106. $19740\frac{1}{4}$.
83. $\frac{49}{400}$.	89. $\frac{225}{729}$	95. $1\frac{64}{225}$.	101. $2\frac{167}{529}$.	107. $72002\frac{7}{9}$.
84 $\frac{64}{121}$	90. $\frac{1225}{4356}$	96. $2\frac{46}{49}$.	102 $8\frac{17}{169}$.	108. $301675\frac{9}{16}$.
85. $\frac{81}{625}$	91. $\frac{1024}{6561}$	97. $32\frac{1}{9}$.	103. $33\frac{16}{25}$.	109. $1\frac{223}{12321}$.
86. $\frac{4}{441}$.	92. $\frac{9}{14641}$	98. $1\frac{29}{196}$	104. $65\frac{64}{81}$	110. $134\frac{155}{169}$.

Find as a vulgar fraction—

111. $\sqrt{22\frac{11}{49}}$.	114. $\frac{7}{8}\sqrt{441}$	117. $\sqrt{\frac{4}{25}}$ of $\frac{25}{49}$	120. $\sqrt{\frac{12}{27}}$
112. $\sqrt{371\frac{46}{49}}$	115. $28\sqrt{3\frac{1}{16}}$.	118. $\sqrt{2\frac{1}{4}} \div \sqrt{1\frac{9}{16}}$.	121. $\sqrt{\frac{125}{320}}$.
113. $\sqrt{1\frac{1261}{1764}}$.	116. $\sqrt{\frac{2}{5} \text{ of } \frac{10}{49}}$.	119. $\sqrt{2\frac{1}{4}} \div 1\frac{9}{16}$.	122. $\sqrt{2\frac{84}{150}}$.

Find, by inspection, the square root of—

123	1·21	126	·09	129.	·0001	132.	·000004.
124	1·44	127.	64	130	·0036	133.	·000025
125	·04	128	81.	131.	·0049.	134.	·000144

Extract the square root of—

135.	4·41	148.	2·832489	161	125·686521.
136.	·0289.	149	901 8009	162.	14636·1604
137.	·5329	150	70702 81	163.	1830879 61
138.	·0000000484	151.	·01595169	164.	·03598609
139.	213·16	152	·00819025	165	3 54455929
140.	9 7969	153	1227·8016	166	58095 4609
141.	091809	154.	1280·9241	167.	30·86358025
142.	1274·49.	155	18·593344	168.	429496·7296.
143.	20 8849	156	315956 41	169	60691890 25.
144	25·5025	157	4281·0849	170	63·84169801
145	1·002001	158.	0·08450649	171	240398 012416
146	121·2201.	159	6248·9025	172.	4281·35971041
147	19740·25	160	64·128064	173	3601207301·2036.

Find the first *six* significant figures of the square root of—

174.	2	177	70	180	0 51	183	·4	186.	·00001
175	3	178.	2·5	181.	·051	184	3·1416	187.	4032·24
176.	5	179	48 4	182.	·3	185.	00056	188.	66·13531715

Find, correct to three places of decimals, the square root of—

189.	$1\cdot\dot{6}$.	191	$7\cdot08\dot{3}$	193	$00\dot{4}2\dot{3}$.	195	$\cdot0\dot{1}4285\dot{7}$
190.	$\cdot\dot{3}$	192.	$\dot{7}\dot{2}$	194	$20\ 41\dot{6}$	196.	$\cdot\dot{6}\dot{4}$

Find, *as a vulgar fraction*, the square root of—

197	$\cdot\dot{4}$	199.	$\cdot02\dot{7}$	201	$3\cdot36\dot{1}$	203	$8\cdot02\dot{7}$.
198.	$\cdot00\dot{1}$	200	$13\dot{4}$.	202.	$4\cdot98\dot{7}$	204	$\cdot0802\dot{7}$.

Find, correct to three places of decimals—

205.	$\sqrt{\frac{7}{9}}$	209.	$\sqrt{\frac{3}{5}}$	213	$\sqrt{2\frac{1}{6}}$	217	$\sqrt{\frac{53}{150}}$
206.	$\sqrt{\frac{5}{18}}$	210	$\sqrt{\frac{1}{11}}$.	214.	$\frac{\sqrt{29}}{\sqrt{24}}$	218	$\sqrt{\frac{465}{37}}$
207.	$\sqrt{1\frac{8}{25}}$	211	$\sqrt{\frac{5}{12}}$.	215.	$\sqrt{\frac{22}{7}}$	219.	$\sqrt{\frac{128}{12\cdot5}}$
208.	$\sqrt{4\frac{9}{64}}$	212.	$\sqrt{1\frac{1}{8}}$.	216.	$\sqrt{\frac{23}{32}}$	220.	$\sqrt{\frac{\cdot00125}{\cdot18}}$

Having given that $\sqrt{2} = 1\cdot414213\ldots$, $\sqrt{3} = 1\cdot732050\ldots$, $\sqrt{5} = 2\cdot236067\ldots$, $\sqrt{6} = 2\cdot449489\ldots$, $\sqrt{7} = 2\cdot645751\ldots$; find, correct to four places of decimals—

221. $7\sqrt{2}$
222. $40\sqrt{3}$
223. $13 - 4\sqrt{5}$
224. $4\sqrt{2} - 2\sqrt{3}$
225. $4\sqrt{3} \times \sqrt{2}$
226. $\sqrt{2} \times \sqrt{10}$
227. $\sqrt{2 + \sqrt{3}}$
228. $3\sqrt{4 - \sqrt{7}}$
229. $\dfrac{5}{\sqrt{5}}$
230. $\dfrac{3}{\sqrt{3}}$
231. $\dfrac{12}{\sqrt{6}}$
232. $\dfrac{2 + \sqrt{3}}{\sqrt{2}}$.
233. $\dfrac{11 - \sqrt{5}}{\sqrt{3}}$.
234. $\dfrac{1 + \sqrt{7}}{2\sqrt{2}}$.
235. $\dfrac{1}{\sqrt{2} + 1}$
236. $\dfrac{1}{\sqrt{5} - 1}$
237. $\dfrac{1}{\sqrt{6} - 1}$.
238. $\dfrac{1}{\sqrt{7} + 1}$.
239. $\dfrac{1}{8 - 3\sqrt{7}}$
240. $\dfrac{3}{3\sqrt{3} - 5}$.
241. $\dfrac{\sqrt{2}}{2 - \sqrt{3}}$.
242. $\dfrac{4\sqrt{3}}{10 + 7\sqrt{2}}$.
243. $\dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$.
244. $\dfrac{3 + 2\sqrt{2}}{3 - 2\sqrt{2}}$.
245. $\dfrac{11\sqrt{3} - 7\sqrt{7}}{11\sqrt{3} + 7\sqrt{7}}$
246. $\dfrac{6\sqrt{5} + 5\sqrt{6}}{6\sqrt{5} - 5\sqrt{6}}$.

Calculate to five places of decimals the value of—

247. $\sqrt{\left(\dfrac{\sqrt{7} - \sqrt{3}}{\sqrt{7} + \sqrt{3}}\right)}$
248. $\sqrt{\left(\dfrac{\sqrt{13} + 3}{\sqrt{13} - 3}\right)}$
249. $\dfrac{1}{\sqrt{7} + \dfrac{1}{\sqrt{7} + \dfrac{1}{\sqrt{7} + \dfrac{1}{\sqrt{7}}}}}$

250. Find the value of $\dfrac{7 + \sqrt{5}}{6 - \sqrt{5}} - \dfrac{7 - \sqrt{5}}{6 + \sqrt{5}}$ to three places of decimals.

---

251. The area of a square floor is 53 sq yds 7 sq. ft., find the length of a side
252. If the area of a square floor is 30 sq. yds. 5 sq. ft. 1 sq. in. what is the length of a side?
*253. The length of a rectangular floor is double its breadth, and its area is 59 sq yds 8 sq ft 2 sq in; find its length.
254. Find the perimeter of a square floor the area of which is 51 sq. yds 3 sq ft 36 sq in
255. A square field contains $2\frac{1}{2}$ acres; find the length of its side.
256. The area of a square falls short of 10 acres by 439 square yards; find the length of each side
257. A square lawn contains 1 acre 6 perches $19\frac{1}{2}$ sq. yds.; find the length of its side.

258 Find the perimeter of a square field of 10 acres

259. What is the breadth of a rectangular field three times as long as it is broad, if its area is 18 A 0 R 3 P?

260. Find the length of the side of a square equal in area to the sum of two squares whose sides are 28 yards, and 11 yds 2 feet

261. How long will it take a man to walk round a square field, whose area is 40 acres, at the rate of 4 miles an hour?

262. A square field contains 21 ac 3 ro 1 po how long will it take a man to run round it at the rate of $7\frac{1}{2}$ miles an hour?

263. Find the cost of fencing a square field containing 10 acres at 4*s* 6*d* per yard

264. How much will it cost to surround a square field, whose area is 3 acres 3 roods 619 sq yards, with netting at $1\frac{1}{2}d$ a foot?

265. If the cost of making a square lawn be £351, 18*s* $4\frac{1}{2}d$, at the rate of 3*s* $4\frac{1}{2}d$ per square yard, how many feet long is it?

266. A square field contains $2\frac{1}{2}$ acres What is the cost of making a path three yards wide inside the field round the boundary at 1*s*. 6*d*. per sq yard?

267. If it cost £643, 10*s*. 9*d* to level a square cricket ground at 9*d* per square yard, what will it cost to enclose it with a fence at 7*s* 6*d* per yard?

268. A sum of £252, 1*s* was divided among a number of persons, and each person received as many shillings as there were persons How much did each person receive?

269. If a piece of silk costs £15, 15*s* $2\frac{1}{4}d$, and the number of yards in its length is the same as the number of pence in the cost of one yard, what is the length of the silk, and its price per yard?

270. A tourist found that he had spent on the average daily half as many shillings as there were days in his holiday He spent altogether £57, 12*s*, how many days did his tour last?

271. When a regiment of 962 men is drawn up in a solid square, one man is left out, find the number of men in the front rank.

272. A company of men can be formed into a hollow square 9 deep, having 970 men in the front rank of each side How many men would there be in the front rank of each side if the company were formed into a solid square?

273. A beam, the section of which is square, is 24 feet long and its volume is $8\frac{1}{6}$ cubic feet, find its width

274. A square chess-board contains 64 equal squares, and the area of each square is 1·3225 sq in The rim round the board is two-fifths of an inch wide Find the length of the board

275. The sum of the areas of three squares is 1 sq yd 6 sq ft 94 sq in The first is four times as large as the second, and the second is nine times as large as the third Find the lengths of the sides of the squares.

276. Find the square root of the sum of the squares of $3\cdot9$, $5\cdot2$, and $15\cdot6$

277. Subtract the number whose square root is $\cdot01$ from the number whose square is $\cdot01871424$

278. Find, within one millionth of the complete result, the square root of $37\cdot7$

279 Find the first six significant figures in the square root of $\cdot005$.

280 Show that $\sqrt{7}$ is intermediate in value between $\frac{66}{25}$ and $\frac{53}{20}$

281. Find, within an inch, the length of the side of a square floor the area of which is 195 square feet

282. Find, within an inch, the length of one of the sides of a square field whose area is 3 acres

283. Find, approximately, the length of the side of a square which is equal in area to a rectangle 660 yds long and 376 yds. broad.

284. Find the least integer by which 290304 must be either multiplied or divided, so that the result may be a perfect square.

285. Find the least integer which (i) added to, (ii) subtracted from, 47962 makes the resulting number a perfect square.

286. Find the least integer by which 2008008 must be multiplied so as to make the product a perfect square.

287. Evaluate $\sqrt[4]{81 \times 256}$ and $\sqrt[4]{23\frac{266}{625}}$

288 Find the fourth root of 112550881.

289 Find the fourth root of 53 1441.

290. Find the fourth root of 17 to four places of decimals.

291. Find a mean proportional between 651 and 2604

292. Find correct to four places of decimals a mean proportional between 21 437 and $437\cdot21$.

293. A rectangular schoolroom is 21 yds 1 ft long and 16 yds. wide, find the distance between opposite corners.

294. The side of a square is 17 feet, find, to the nearest inch, the distance between opposite corners.

295 A rectangular field is 6 chains 80 links in length and 5 chains 10 links in breadth; find the distance between two opposite corners of the field.

296 The diagonal of a square is 7 inches long; find, within a thousandth of an inch, the length of a side of the square

297. A ladder 41 feet long, placed with its foot 9 feet from a wall, just reaches to the top of the wall How high is the wall?

At what rate per cent compound interest will—

298. £625 amount to £676 in 2 years?

299. £1500 gain £101, 13*s* 4*d* in 2 years?

300. £1024 amount to £1305, 0*s*. $3\frac{3}{4}$*d*. in 4 years?

LXIII. CUBE ROOT.

Find, *by inspection*, the cube root of—

1. 8000	3 64000	5. 8×27	7. $4^3 \times 6^3$.
2. 27000	4 125000	6 1728	8 $7^3 \times 3^6$.

Find, by inspection—

9 $\sqrt[3]{11 \times 11 \times 11 \times 7 \times 7 \times 7}$	11 $\sqrt[3]{700 \times 2 \times 49 \times 5}$
10. $\sqrt[3]{13 \times 13 \times 13 \times 8}$	12 $\sqrt[3]{121 \times 64 \times 2 \times 44}$

Find, by factors, the cube root of—

13 729	15. 13824.	17. 35937	19. 1953125
14 3375	16 21952	18 884736	20. 182284263

Extract the cube root of—

21. 4913.	28. 571787	35. 284890312
22. 12167	29 2248091	36. 397065375
23. 39304	30 8365427	37 1334633301
24. 50653	31 857375000	38. 24414238701
25 110592	32 1191016000	39. 83568086848
26 274625	33 12812904	40 411166897856
27. 493039	34 26463592	41 1027243729000

Find, as a vulgar fraction, the cube root of—

42 $\frac{27}{8000}$	45 $1\frac{91}{125}$	48 $41\frac{301}{1331}$	51 $2\cdot\dot{3}7\dot{0}$
43. $\frac{125}{1728}$	46 $1\frac{217}{512}$.	49 $2345\frac{22}{343}$	52. $\cdot000\dot{2}9\dot{6}$
44 $\frac{64}{343}$.	47 $20\frac{51}{64}$	50. $423987\frac{457}{512}$	53. $\cdot004\dot{6}2\dot{9}$

Find, by inspection—

54. $\sqrt[3]{\cdot008}$.	55. $\sqrt[3]{\cdot027}$.	56. $\sqrt[3]{1\cdot728}$	57. $\sqrt[3]{\cdot000001}$.

Extract the cube root of—

58. 2·197	61. 6434 856	64. 29993·266043
59. 29·791.	62. 34·965783	65 135400835·375.
60. ·438976.	63. 768575·296	66. 2079995·797125

Find, correct to two places of decimals, the cube root of—

67 2	70. $\frac{5}{8}$	73. 3·5	76 $\cdot1\dot{6}$	79. $3\frac{1}{7}$
68. 3.	71. $\frac{4}{27}$.	74 ·6315	77. $\frac{1}{3}$	80 $\frac{17}{24}$
69. 7.	72 $1\frac{9}{64}$.	75. ·08	78. $\frac{5}{11}$	81 $5\frac{104}{105}$

---

82 Find the integral part of the cube root of 7777777

83. The volume of a cubical block is 2248091 cubic inches; find the length of its edge

84 The volume of a cubical block is 2248·091 cubic feet, find the area of one of its faces

85. Find the area of the total surface of a cube whose volume is 91 cub ft 216 cub in

86. Prove that the length of a side of a square, the area of which is 15876 square yards, is to that of an edge of a cube the contents of which are 94818816000 cubic inches as 189 190.

87. The sides of a rectangular vessel are 10 ft 1 in, 11 ft 11 in., and 14 ft 1 in long respectively Find the length of a side of a cubical vessel of equal volume.

88. Given that a cubic metre is equal to 35·316581 cubic feet, find the length in feet of a linear metre, correct to four places of decimals.

89. The volume of wood in a cubical box of uniform thickness is $732\frac{3}{8}$ cubic inches If the capacity of the box is a cubic foot, what is the thickness of the wood?

90. Two cubical tanks contain together 6,133,248 cubic feet, and the edge of one is twice as long as the edge of the other; find the edge of each.

91. A beam, the section of which is square, is 27 times as long as it is wide and its volume is 8 cubic feet, find its length.

92. Three numbers are to one another as 2 3 · 4. The sum of their cubes is 33957 Find the numbers.

93. A room contains 4394 cubic feet of air; its length is double its width and its width is equal to its height Find the cost of carpeting it at 4*s* 6*d* per square yard

94. Find the smallest integral multiplier of 16435188 which will convert it into a perfect cube

95. Find the least integer which added to 33333 makes the result a perfect cube

Find, *by inspection*, the cube root of each of the following *perfect cubes* —

96. 1331.	98. 5832.	100 85184.	102. 571787.
97. 4096	99. 15625.	101. 205379	103. 912673.

104. Find, by factors, $\sqrt[5]{164916224}$.

105. Find the sixth root of $1838\frac{17}{64}$

106. Find the sixth root of 2 to three places of decimals.

107. Find the ninth root of 2357947691.

108. Find, approximately, the length of the side of a cubical cistern which holds 1000 gallons.

At what rate per cent, compound interest, will—

109. £3125 amount to £3515, 4*s.* in 3 years?

110. £1562, 10*s.* gain £95, 12*s.* 9*d.* in 3 years?

LXIV. SCALES OF NOTATION.

Add together, in

1 *Scale Five*, 4321, 203, 1304 and 32

2. *Scale Eight*, 14735; 20664, 176 and 5640

3 *Scale Seven*, 6354, 142503, 6554, 253 and 465

4. *Scale Four*, 3120, 223, 1032, 2303 and 321

5. *Scale Eleven*, 12879, 46t3, t52, 48t and 1t369

6. *Scale Twelve*, 4578; 3607t; 14e75, te7 and 864

Subtract, in

7 *Scale* 6, 35042 from 51200

8 *Scale* 3, 10212 from 11020

9. *Scale* 7, 40546 from 152364

10. *Scale* 9, 785614 from 1230203.

11. *Scale* 11, 5t34t2 from 942t30

12. *Scale* 12, t98te from 17e0t2

Multiply, in

13. *Scale* 5, 20431 by 34

14 *Scale* 8, 5746 by 75

15. *Scale* 6, 3452 by 243

16. *Scale* 7, 16054 by 1056

17 *Scale* 11, 2t385 by 72t

18 *Scale* 12, 64et9e by t8e

Divide, in

19 *Scale* 9, 7134504 by 8.

20. *Scale* 11, 362t076 by 11

21. *Scale* 4, 30321 by 23

22 *Scale* 5, 4203312 by 34

23 *Scale* 8, 7642435 by 507.

24 *Scale* 12, 5e78783 by 2t7

25 Change 5784 from the common scale to scale 7.

26 12957 . . 3

27. 64021 from scale 10 to scale 11

28 79463 12

29 3441 from scale 5 to the common scale

30 . 2463 . 8 .

31. 5342 . . . 6 to scale 4

32 . 25361 . 7 9

33. 30t5e 12 3

34. 32103 4 11

35 Express 142857 in the Octenary scale

36. Transform 31415 from the Senary to the Nonary scale.

37. Transform 687 from the Denary to the Binary scale

38 Change 33201 from the Quaternary to the Undenary scale.

39. Change 4t3e0 from the Duodecimal to the Quinary scale

40 Change 66666 from the Septenary to the Decimal scale.

41. Change 82·125 from the common scale to scale 6
42. 417·3125 12
43. 1201·12 from scale 3 to the common scale
44. 55·6 . . . ... 7 . . .
45. 675·24 .... 8 11
46. $3130\cdot0\dot{2}$ .. .. 4 . . 2

Extract the square root of—

47. 21652 in the scale of 7
48. 373444 9
49. 148115 in the scale of 11.
50. 3106571 . . . . 8.

51. Find the square root of t2t1 in the duodenary scale.
52. Express in the common scale the greatest and least numbers which can be expressed with four digits in scale 6
53. How many times is the greatest number of three figures in scale 4 contained in the greatest number of four figures in scale 8?
54. The numbers 345 and 303 are in scale 7; find the cube root of their product in that scale.
55. In what scale of notation is 357234 expressed by 3015333?
56. Express the decimal ·5625 as a duodecimal
57. Transform 275·9375 from the decimal to the duodecimal scale.
58. Transfer 355·41 from scale 6 to scale 10
59. 234·521 is in scale 6, express this number in scale 12
60. In what scale does the duodecimal number 3tte become 15143?
61 50010 is in scale 7, find its prime factors in that scale.
62. Express $\frac{1}{3}$ as a radix fraction in the scale of 5
63. Express the decimal $51\cdot2\dot{7}$ in the duodecimal scale.
64. Change $37\cdot2291\dot{6}$ from scale 10 to scale 12.
65. When can a vulgar fraction in the senary scale be converted into a terminating radix fraction? Will the denary fraction $\frac{12}{432}$ produce a terminating or a recurring radix fraction in the senary scale?
66. $\cdot14\dot{6}$ is in scale 9, convert it into a vulgar fraction in its lowest terms in that scale
67 $\cdot2\dot{5}\dot{3}$ is in scale 6; express it as a vulgar fraction in that scale.
68. $\cdot5\dot{2}\dot{4}$ is in scale 12, express it as a vulgar fraction in that scale
69. Find which of the series of weights 1 lb, 2 lbs., 4 lbs, 8 lbs., 16 lbs &c, must be taken in order to weigh 233 lbs.
70. How can weights of 1, 3, 3^2, 3^3, 3^4, &c lbs. be used in a balance so as to weigh (i) 334 lbs.: (ii) 574 lbs.?

TABLES, ETC.

Money

4 farthings (*f*) = 1 penny (*d*)
12 pence = 1 shilling (*s*)
20 shillings, or 240 pence = 1 pound (£)

Note.—A *florin* = 2*s*, a *crown* = 5*s*, a *guinea* = 21*s*
A *sovereign* = 20*s*, or 8 half-crowns, or 10 florins, or 40 sixpences, or 80 threepences, or 480 half-pence, or 960 farthings.

Time

60 seconds (*sec*) = 1 minute (*min.*)
60 minutes = 1 hour (*hr*)
24 hours = 1 day
7 days = 1 week (*wk*)
365 days = 1 (common) year (*yr.*).
366 days = 1 leap year

Note.—A *common year* = 52 weeks + 1 day; a *century* contains 100 years, a *lunar* month contains about 4 weeks

The year is divided into 12 *calendar* months, of which February, in common years, contains 28 days, and, in leap years, 29 days;
"Thirty days hath September, April, June, and November;"
and the remaining seven calendar months each contain 31 days

Leap year occurs once in four years (except at the end of a century). In order to discover whether any year (not the last in a century) is a leap year, *divide the number of the year by 4, and if there is* no *remainder it is leap year* But if the year ends a century it is *not* leap year *unless the first two figures divide by 4 without remainder*

Avoirdupois Weight

16 drams (*dr*) = 1 ounce (*oz*)
16 ounces, or 7000 grains = 1 pound (*lb*)
14 pounds = 1 stone (*st*)
28 pounds, or 2 stones = 1 quarter (*qr*)
4 quarters, or 8 stones, or 112 pounds = 1 hundredweight (*cwt.*).
20 hundredweights = 1 ton

Note.—This weight is used for all common substances, *e g* coal, meat, &c.

Troy Weight.

24 grains (*gr*) = 1 pennyweight (*dwt*).
20 pennyweights, or 480 grains = 1 ounce Troy (*oz Tr.*)
12 ounces Troy = 1 pound Troy (*lb Tr*).

Note.—Troy weight is only used for gold, silver and jewellery.
The *grain* alone is the same in both Avoirdupois and Troy weights.

Long Measure.

12 inches (*in.*) = 1 foot (*ft.*)
3 feet, or 36 inches = 1 yard (*yd.*)
1760 yards = 1 mile (*mi.*)
$5\frac{1}{2}$ yards, or 11 half-yards = 1 pole (*po.*), rod, or perch (*per.*)
40 poles, or 220 yards = 1 furlong (*fur.*)
8 furlongs = 1 mile

Note.—A *chain* = 22 yards, or 4 poles, 100 links = 1 chain
A *fathom* = 6 feet, a *cable-length* = 120 fathoms, a *knot* = 6080 feet.
A *hand* = 4 inches A *league* = 3 miles
$2\frac{1}{4}$ inches = 1 *nail*, 4 nails = 1 quarter, 4 quarters = 1 yard.

Square Measure

144 square inches (*sq. in.*) = 1 square foot (*sq. ft.*).
9 square feet = 1 square yard (*sq. yd.*).
$30\frac{1}{4}$ (*i.e.* $5\frac{1}{2} \times 5\frac{1}{2}$) square yards, or 121 square half-yards = 1 square pole (*sq. po.*, or P.), or square perch.
40 square poles = 1 rood (*ro.* or R.).
4 roods, or 4840 square yards = 1 acre (*ac.*, or A.).
640 acres or 1760×1760 square yards = 1 square mile (*sq. mi.*)

Note.—A *square chain* = 100×100 square links, or 22×22 square yards, 10 square chains = 1 acre

---

Cubic Measure

1728 (*i.e.* $12 \times 12 \times 12$) cubic inches (*cub. in.*) = 1 cubic foot (*cub. ft.*).
27 (*i.e.* $3 \times 3 \times 3$) cubic feet = 1 cubic yard (*cub. yd.*).

Capacity.

2 (imperial) pints (*pt.*) = 1 (imperial) quart (*qt.*) } Liquids.
4 quarts, or 8 pints = 1 gallon (*gal.*) } Liquids.
2 gallons = 1 peck (*pk.*) } Dry goods; *e.g.* corn.
4 pecks, or 8 gallons = 1 bushel (*bush.*) } Dry goods; *e.g.* corn.
8 bushels = 1 quarter (*qr.*) } Dry goods; *e.g.* corn.

Note.—A *hogshead of beer* = 54 gallons, a *hogshead of wine* = 63 gallons
a *pipe* of wine = 2 hogsheads 4 *gills* = 1 imperial pint
6 "reputed" quarts (*i.e.* common wine bottles) contain a gallon.
"A pint of pure water weighs a pound and a quarter"
A cubic foot of water weighs 1000 ounces.

Number.—12 units = 1 dozen, 12 dozen = 1 gross. 20 units = 1 score.

Paper.—24 sheets = 1 quire; 20 quires, or 480 sheets = 1 ream.

Decimal Coinages

France..	1 franc	= 100 centimes
Germany .	1 mark	= 100 pfennige
United States	1 dollar ($)	= 100 cents

Note—Roughly, a franc is equivalent to $9\frac{1}{2}d$, a mark to $11\frac{3}{4}d$, and a dollar to $50d$

The Metric System

Multiples of any principal unit are denoted by the Greek prefixes

deca- (10), **hecto-** (100), **kilo-** (1000),

parts of the unit by the Latin prefixes

deci- ($\frac{1}{10}$), **centi-** ($\frac{1}{100}$), **milli-** ($\frac{1}{1000}$)

Thus, a *kilo*-metre = 1000 metres, a *milli*-metre = ·001 of a metre.

The units most commonly employed are for

Length, the *kilometre*, metre, and *centimetre*,

Weight, the **kilogramme**,

also, for very small quantities, the *gramme*, and for very large quantities, the *tonne* (or *tonneau*) = 1000 kilogrammes

Note—A *gramme* is the weight of a *cubic centimetre of water*

Capacity, the litre,

Note.—A *litre* is 1000 cubic centimetres in volume, hence a *litre of water* weighs 1000 *grammes*

Area, the **square metre** and *square centimetre*,

also, for large areas, the *hectare*, an *are* being 100 square metres

Volume, the **cubic metre** and *cubic centimetre*

Connection between Metric and English Units

Correct to five places of decimals—

1 metre = 39·37079 inches

Note.—Roughly, a kilometre = $\frac{5}{8}$ mile, a centimetre = $\frac{2}{5}$ inch, a kilogramme = $2\frac{1}{5}$ lbs., a hectare = $2\frac{1}{2}$ acres

MENSURATION

In a **circle**, the length of the radius being denoted by r,

the length of the *circumference* is $2\pi r$,
and the *area* of the circle is πr^2,

the symbol π denoting a constant multiplier whose value, correct to four places of decimals, is

3·1416 (or, roughly, $3\frac{1}{7}$).

Thus, if the radius is 5 in in length, the length of the circumference is $2 \times 3{\cdot}1416 \times 5$ inches, and the area of the circle is $3{\cdot}1416 \times 25$ square inches.

In a circular **cylinder** of radius r, and length l, the *volume* is $\pi r^2 \times l$

Thus, the volume of a cylinder 6 inches in *diameter* and 2 feet in length is $3{\cdot}1416 \times 9 \times 24$ cubic inches.

And, in the case of any other solid of *uniform cross section*,

the *volume* = *area of cross section* × *length*

The *area* of a **trapezium** (a four-sided figure in which two of the sides are *parallel*) is found by multiplying

half the sum of the lengths of the parallel sides by *their* (perpendicular) *distance apart*

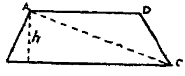
Thus, if the parallel sides are 3 ft and 7 ft, and their distance apart (*i e* the *height* of the trapezium) is 4 ft, the *area* of the figure is

$\frac{1}{2}$ of $(7 + 3) \times 4$ square feet *

---

* This rule follows from the geometrical fact that *the area of a triangle is half that of the rectangle having the same base and height*,

$$\begin{aligned} \text{for trapezium ABCD} &= \triangle ABC + \triangle CAD \\ &= \tfrac{1}{2}BC \times h + \tfrac{1}{2}AD \times h \\ &= \tfrac{1}{2}(BC + AD) \times h \end{aligned}$$



LXV. MISCELLANEOUS EXERCISES.

A.

1 Shakespeare was born in the year MDLXIV Bacon, who was 3 years older than Shakespeare, died in the year MDCXXVI, at what age did he die?

2. A railway company issues week-end tickets at the single fare and a third, what is the single fare when the week-end ticket costs 10*s* 10*d* ?

3. Find the difference between twice the sum, and half the product, of $7\frac{1}{3}$ and $8\frac{1}{7}$

4. The quotient of 5781 — 47 is 123, deduce the quotient of

   (i) 57·81 — 4·7, (ii) 578·1 — ·047,
   (iii) 5 781 — 47, (iv) 578·1 — 47000

5. How many pieces, each 1 metre 3 centimetres in length, can be cut from a kilometre of string, and what is the length of the remnant?

6. A monoplane flew 17 miles in 21 minutes, 15 seconds, find its speed in miles per hour

7. An estate is let to three tenants, the first occupies two-fifths, and the second three-eighths, of the estate, the third occupies 25 acres 2 roods 24 poles, find the size of the estate

8. A cricketer's average for twelve innings stands at 34, and he has one more innings to play, what score must he make to bring his average up to 40?

9. A typist A can copy a MS of 320 pages in 70 hours, but with B to help her it would be done in 40 hours, how long would B alone take to copy it?

10. A occupies a house from the beginning of the year 1913 to the end of August, and B occupies it for the rest of the year, the rent is £73 per annum, what share of this should each of them pay?

B.

1. Arrange the work of finding the product of 47 and 35 in six different ways

2 Find 276 times £4, 18*s* 2*d* by the shortest method you know.

3. Find all the factors, each consisting of two digits, which are common to the numbers 324 and 504

4. Find the difference between five and six-fifths, and six and five-sixths, of half-a-crown

5. Calculate, correct to two places of decimals,

$$3{\cdot}054 \times {\cdot}038 \div {\cdot}0613$$

6. The President of the United States receives a salary of $100,000;

if a dollar is worth 49·32 pence, express the salary in English money.

7. A six months' season ticket between two stations A and B costs 2 guineas, the return fare is 4*d.*; how many times a week, on the average, must I have occasion to go from A to B and back, to make it worth while to take a season ticket?

8. In a journey of 40 miles by motor-car, the first 10 miles occupied 40 minutes, and the last 4 miles 14 minutes, for the rest of the journey the car ran at the rate of 24 miles per hour, how long did the journey take?

9. The lid of a box is 2 ft 9 in. in length and 1 ft 8 in in width, and the box is 1 ft 3 in. high; how many yards of cord will be needed to cord it both ways across the lid, allowing 14 inches of cord for knots?

10. Which is dearer, French butter at 3 francs 25 centimes per kilogram, or English butter at 14 pence per lb, if £1 is equivalent to 25 francs, and a kilogram to 2·2 lbs?

C.

1. Find, by three subtractions, and without making any use of the multiplication table, the quotient and the remainder of 1958 ÷ 157.

2. Four and a half tickets are bought for a journey and the change out of 3 sovereigns is 9 pence; find the cost of a ticket for one adult person

3 Find, correct to the nearest tenth, the sum of $3\frac{7}{16}$, $5\frac{13}{24}$, $7\frac{19}{36}$, $9\frac{23}{40}$

4. If 29 metres 75 centimetres of cloth cost 190 francs 40 centimes, what is the cost, to the nearest centime, of 22 metres 5 centimetres of the same kind of cloth?

5. Two clocks strike ten together on Friday evening; on Saturday evening one strikes eleven when the other indicates ten minutes to eleven, how much must the slow clock then be put on, so that they may strike nine together on Sunday morning?

6. From the following table, calculate

(i) the average number of pupils in a class,
(ii) the average age of the whole school.

Class	VI	V	IV	III	II	I.
Number in class	10	17	23	24	25	15
Average age in class	17	16	15·5	14	13 5	11

7. A and B hire a motor-car for 3 months for £60, and agree that A shall have the use of it for 2 days in each week; what share of the hire should he pay?

8 A agrees to walk from P so as to meet B, who walks from Q, at R. P is 22 miles from Q and 8 miles from R B starts at 10 o'clock and walks $3\frac{1}{2}$ miles an hour A walks 3 miles an hour, at what time should he start?

9 Find the percentage profit when a commodity which costs £10 per cwt is sold at $2\frac{1}{4}d$ per oz

10. A tablecloth 8 ft 6 in long and 5 ft 6 in wide is spread over a table 6 ft. 6 in long and 4 ft 3 in wide, what fraction of the cloth hangs over the edges of the table?

D.

1 On a certain golf-course a tee is 414 yards from the next hole If a player, in four successive strokes, played his ball 229, 167, $22\frac{1}{2}$, and $3\frac{1}{2}$ yards, always in the direction of the hole, how far was the ball then from the hole?

2 Divide (i) £264, 11*s* by 37, (ii) £264, 11*s* by 37*s*.

3. Evaluate, correct to two places of decimals,

$$\frac{21}{365} \times \frac{3}{100} \times 254{\cdot}6$$

4. A bankrupt owes £1537, 3*s* 4*d*, his assets are £840, find, to a farthing, how much he can pay in the pound

5. On April 1st the sun rises at 5 38 A M and sets at 6 30 P M; express the time that the sun is above the horizon as (i) the fraction, (ii) the decimal, (iii) the percentage, of a day

6. Between two consecutive milestones a motor-car was timed to take a hundred seconds, at the rate of how many (i) miles per hour, (ii) feet per second, was it travelling?

7 Find the simple interest on 12550 francs for 2 years 3 months at $3\frac{1}{2}$ per cent per annum

8. If 155 men do one-third part of a certain task in 23 days, how many extra men will be required to help them to finish in 31 days more?

9. How many boards each 24 feet long and 5 inches wide would be sufficient to make a platform 32 feet long and 15 feet wide?

10 A and B start at the same time, from the same place, along the same road, in the same direction, A walks for $2\frac{1}{4}$ hours at 4 miles an hour, rests for 25 minutes, and then returns at 3 miles an hour, B walks at 3 miles an hour without resting, how far from the starting place do they meet?

E.

1. Find, as shortly as you can,

(i) $365 - 76 - 75 - 74 - 73$,
(ii) $497 \times 25 \times 26 \times 8 - 52$,
(iii) 73 times 7*s* $5\frac{1}{2}d$ – 75 times 5*s*. $7\frac{1}{2}d$.

2 How many common multiples are there, each less than 2000, of the numbers 24, 36, 42, and 56?

3. Evaluate $\frac{7s.}{£1} + \frac{11 \text{ inches}}{5 \text{ feet}} + \frac{14 \text{ decimetres}}{.3 \text{ metres}}$.

4. Find, to the nearest hundred, using no unnecessary figures,

   (i) $450 \cdot 375 \times 63 \cdot 45$; (ii) $827 \cdot 605 \div \cdot 0875$.

5. A cog-wheel which has 30 teeth fits into another which has 56 teeth; if the former turns 16 times in 15 seconds, how often does the latter turn in 14 seconds?

6. If a sum of £720, put out to interest on March 20, amounted to £725, 8*s* on June 1 following, what was the rate per cent per annum at which the interest was reckoned?

7. The 1010 mile aeroplane race round Britain, which was flown in the year 1911, was won by Lieutenant Conneau in 22 hours 28 minutes, and J Védrines finished second in 23 hours 37 minutes 54 seconds. Find, to the nearest foot, the average number of feet per second flown by each.

8. In a country with a population of 32,526,000, the number of deaths in a year was 552,000, find, to the nearest unit, the death-rate per thousand

9. Four dozen picture-frames are to be glazed; each requires a piece of glass 9, by 7, inches, the glass is cut from sheets each 30, by 18, inches, how many sheets must be used? If the sheets cost 2*d* per square foot, and 3*d.* is charged for the glass of each frame, how much does the glazier get for his work?

10. If iron is $7 \cdot 8$ times as heavy as water, find, to the nearest ounce, the weight of an iron bar 17 ft 3 in long, 1 in wide, and 1 in. thick

F.

1. Give five ways of reading the number 1077

2. When 4356 is divided by 74, the quotient is 58 and the remainder is 64; hence, without performing the divisions, find the quotient and the remainder when 4356 is divided by (i) 58; (ii) 37; (iii) 29

3. A housekeeper buys a pint and a half of milk on each of three days in the week, and a pint on each of the other days; her weekly milk bill is 1*s.* $0\frac{3}{4}d$, what is the price of milk per quart?

4. Find, without waste of labour,

   (i) the value of 3675 lbs at $9\frac{7}{8}d$ per lb;

   (ii) the difference between $\frac{39}{40}$ and $\frac{28}{29}$;

   (iii) the product of $76 \cdot 834$ and 495.

5. The Viceroy of India receives a salary of Rs 20833 per mensem; if a rupee is worth 1*s* 4*d*, find in English money his salary per annum.

6. A boy lives 550 yards from the railway station; he can get there in 2 mins. 30 secs if he runs, and in 4 mins 10 secs if he walks; at the rate of how many miles per hour does he (i) run, (ii) walk?

7. The population of a town is 71,860; if the number of dogs kept is

5 per cent of the population, find the revenue from dog-licences at 7*s* 6*d* each

8 A contractor undertook to complete a piece of work in 120 days, and employed 140 men upon it At the end of 66 days only half of the work was done, so he put on 25 extra men By how much did he exceed the specified time?

9. The average age of a class of 27 boys is 16 years; if their master be included the average is raised by half a year, find his age

10. How many wood blocks each 8 in by 3 in are required to pave a passage 4 feet wide, and in shape like the letter L, the outside length being 21 + 14 feet?

G.

1. Find by adding together five numbers, and without making any use of the multiplication table, the product of 279 and 32

2. Find the difference between the two expressions—

$$16 + (36 - 4) + (4 \times 3) \text{ and } (16 + 36) - (4 + 4) \times 3.$$

3. Find, correct to two places of decimals, the sum of $\frac{119}{220}$ and $\frac{359}{400}$

4. State what you regard as a reasonable degree of approximation in each of the following cases —
   (i) the distance of the earth from the moon,
   (ii) the distance from London to York,
   (iii) the length of a street,
   (iv) the length of a room,
   (v) the length of a post-card.

5. Crewe is 158 miles from Euston, a non-stop train, travelling at the rate of 45 miles per hour, left Euston for Crewe at 11 A M, and another, at 35 miles per hour, left Crewe for Euston at 11 40 A M At what o'clock did they pass each other?

6 Find, to the nearest penny, the premium payable on the insurance of property for £1436, at $\frac{1}{8}$ per cent

7. If the manufacturer's profit is 25% on gramophones, and 15% on the music-rolls, what total profit does he make on the sale of a £20 instrument with £8 worth of rolls?

8 The average height of a class of 21 girls is 5 feet 2 inches, if their mistress were included the average would be 5 feet $2\frac{1}{4}$ inches, find her height

9. A person buys 150 metres of ribbon in Paris, at 65 centimes per metre, and offers 4 sovereigns in payment, how much change should be given when £1 is worth 25·15 francs?

10. Divide £192, 1*s* among 16 men and 13 women, so that any three men receive as much as any eight women

H.

1. Find the sum of the products of every pair of the numbers 3, 7, 11, 15.

2. Find the amount paid in wages in 8 weeks to 8 bricklayers and 8 carpenters, working 8 hours a day, at 8 pence an hour

3. Find the greatest number below 100 which exactly divides both 1746 and 2037.

4. Find the value of 9 yards 2 feet 7 inches of gold wire at £3, 8*s* 3*d*. per yard

5. Divide $5\frac{1}{3} - 4\frac{3}{7} + \frac{20}{21}$ by $2\frac{1}{75} + 3\frac{3}{40} - 5\frac{1}{12}$.

6. Calculate to the nearest tenth

$$34{\cdot}8 \times \frac{56{\cdot}2}{18} \times \frac{345}{28 \times 20}.$$

What rough estimate would you make by which to check your result?

7. Two rulers, one graduated in inches, the other in centimetres, are placed with their edges and zero points coinciding, and the division which shows $5\frac{1}{2}$ inches appears to coincide with that which shows 14 centimetres. To what degree of approximation is this correct?

8. The sum of the ages of a family of five persons is 104 years; the ages of the two youngest members are 9, and 5, years, what was the sum of the ages of this family seven years ago?

9. A man, when cycling along a road at the rate of $8\frac{1}{2}$ miles an hour, met a procession, three-quarters of a mile long, moving at the rate of $2\frac{3}{4}$ miles an hour, how long was he in passing it?

10. A square pavement is formed of square tiles of equal size, and the number of tiles which form the boundary is 168, how many tiles are there altogether?

I.

1. A boy was 8 years 122 days old on August 1st, 1912; find the date of his birth

2. The materials required for a sewing-class of 29 girls were $1\frac{1}{4}$ yds of calico at $5\frac{3}{4}d$ per yard, $2\frac{1}{4}$ yds. of lace at $1\frac{3}{4}d$. per yard, and 3 buttons at $4\frac{3}{4}d$ per dozen, for each girl; together with tape, cotton, needles and pins which together cost about $\frac{1}{2}d$. a head, what sum should each girl contribute in order to cover the total cost?

3. The average weight of the crew of an eight-oared boat was 12 st. $5\frac{1}{2}$ lb, including the coxwain the average was 11 st. 13 lb., find his weight

4. Find, to the nearest hundredth,

$$31{\cdot}416 \times {\cdot}4812 \times {\cdot}9 \div {\cdot}151.$$

5. If a man paid 100 marks for goods which he bought at 12 marks 50 pfennige per kilogramme, and sold at 1 mark 40 pfennige per hectogram, what profit did he make?

6 At the last census the population of Scotland was 4,759,445, of Ireland 4,381,951, and the number of persons in receipt of old age pensions in Scotland was 94,243, in Ireland 202,810 Find, to one place of decimals, without waste of work, the percentage of the population in receipt of these pensions in each country

7 Two trains, travelling in opposite directions, pass each other in $2\frac{1}{4}$ seconds, their rates are 48, and 42, miles per hour, one of the trains is 155 feet in length, find the length of the other

8. By reducing the price of a book from 4*s* 6*d* to 3*s* 6*d* the sale was increased by 30 %, was the change advantageous to the publisher? If the author's royalty was $7\frac{1}{2}$ % on the price, what difference did it make to him per 1000 copies sold?

9. A rectangular plot of building land with a frontage of 34 feet, and a depth of 125 feet, was let on lease at £60 per acre, find, to the nearest penny, the rent of the plot

10. A labourer borrowed five shillings on Monday on the understanding that he paid back 5*s* 2*d* on the following Saturday; at what rate per cent per annum was he charged interest on the loan?

J.

1. Find the sum of all the numbers which can be formed by differently arranging the three digits 5, 7, 9.

2. Find, shortly,
   (i) the total cost of 83 yards at 6*s* 7*d* per yard, and 166 yards at 1*s* 2*d* per yard,
   (ii) the product of 3761 and $9\frac{7}{8}$

3 Calculate, correct to three places of decimals, 16·785 × ·076543.

4. The railway fare, third class at a penny a mile, from A to B, via Q, is 4*s* 3*d*, from A to C, via Q, is 3*s* 2*d*, from B to C, via Q, is 2*s* 1*d*, how far is Q from A, from B, and from C?

5. A cricketer in one year played 29 innings in South Africa with an average of 24·6, and 36 innings in England with an average of 31·3, what was his average for the year?

6 Two motor-cyclists A and B rode from P to Q, a distance of 150 miles A's time was 4 hours 10 minutes, B rode 2 miles an hour faster than A, but he lost 4 minutes an hour in stoppages, which got first to Q?

7 The front seats at a concert cost 2*s* 6*d* each, and the back seats 1*s* each, if 400 tickets were sold, and the receipts were £30, 10*s*, how many front seats were taken?

8. The late Cecil Rhodes founded 60 Colonial, and 104 American, scholarships at Oxford of £300 a year each, also 15 German scholarships of £250 a year each What capital sum, at 3 per cent per annum, will produce the money for these scholarships?

9 Find, to the nearest farthing, the value of 1 lb of a commodity worth 1 franc 25 centimes per kilogramme, assuming that 1 kilogramme = 2·204 lb and £1 = 25·16 francs

10. Find the cost of a coat of gravel 2 inches thick for a path 6 feet wide surrounding an oblong lawn 30 yards in length, and 20 yards in breadth, at 6*s* 9*d* per cubic yard

K.

1. If our system of numeration were based upon the number eight instead of ten, what number would be represented by the figures 226, and how would the number seventy-three be expressed in figures?

2 Evaluate $3\frac{6}{7} + 5\frac{32}{35} - 7\frac{11}{70} - 2\frac{47}{85}$

3 Find the number of complete pounds in ·6345 of 5 tons 8 cwts, and the number of complete feet in ·017 of a mile

4. A can give B ten points, and B can give C ten points, in a game of 50 at billiards; how many points should A give C?

5. Find the cost of making a road 644 metres in length, and 6·4 metres in width, at 1 franc 35 centimes per square metre

6. In order to catch a train a boy has to go 2 miles 528 yards in 26 minutes; how far must he run at the rate of 8 miles an hour if, by walking the rest of the way at $4\frac{1}{2}$ miles an hour, he is just in time?

7. Find the length of a side of a square courtyard, if 1352 bricks, each 9 in. by $4\frac{1}{2}$ in, were used in paving it.

8. A draper sold nine-tenths of a roll of carpet at a profit of 20 per cent, and the remnant at a loss of 20 per cent. He sold the remnant, $5\frac{1}{2}$ yards, for 13*s* 9*d*; how much money did he gain on the whole?

9. A man is 44, and his son is 8, years old, in how many years will the man be (i) five times, (ii) four times, (iii) three times, (iv) twice, as old as his son?

10. A packing-case is made to hold a number of small boxes, each 9, by 6, by 3, inches, the inside dimensions of the packing-case are 6 ft 3 in, 2 ft 6 in, and 1 ft 6 in, how many boxes will it hold?

L.

1 Add CCLXVIII to MCXCIV without expressing these numbers in arabic figures

2. A, B, and C, have a two days' tour together and share expenses A makes all payments the first day, and spends £1, 8*s* $1\frac{1}{2}$*d*, B pays for everything on the second day, and spends £1, 9*s*. 7*d*. How can they best settle their accounts, no farthings being used?

3. A question was printed thus "Simplify $2\frac{3}{4} \times 3\frac{1}{8} \times 4_{\overline{11}} \times 5\frac{3}{8}$ Answer $204\frac{1}{4}$." Find the missing figure.

4. Divide 31·2815 by 473·205, correct to five significant figures

5. The sum of the ages of four boys is 51 years, the first is 3 years older than the second, the second is 2 years older than the third, and the third and fourth are twins; find the age of each

6. In a company of 27 Boy Scouts, 3 are 16, 7 are 15, 9 are 14, 6 are 13, and the rest are each 12, years old, 6 are 4 ft 10 in.,

5 are 4 ft 11 in, 8 are 5 ft, 4 are 5 ft 2 in, 3 are 5 ft 4 in, and the other is 5 ft 7 in in height, 11 are 8 st 2 lbs, 6 are 7 st 12 lbs, 6 are 7 st 6 lbs, one is 9 st 1 lb, and the rest are each 6 st 13 lbs in weight Find, approximately, the average age, height, and weight of the company

7. In a paper-chase the foxes had 18 minutes' start, and ran at the rate of 7 miles an hour, the foremost hounds ran at the rate of 8 miles an hour, and reached the end of the course 3 minutes after the foxes, find the length of the course

8 In the year 1910 the United States produced 56,889,000 tons of iron ore, valued at $140,735,000, and 27,303,000 tons of pig iron, valued at $425,115,000, find, to the nearest cent, without waste of figures, the value of a ton of each kind

9. Find, to a penny, the compound interest on £1600 for 3 years at 3 per cent per annum

10. Find the largest and smallest square numbers, each consisting of six digits.

M.

1. Subtract "one, 0, double six" from "nineteen, thirteen", and express the result in Roman numerals

2 Find the number nearest to 10,000 which is exactly divisible by 54, by 63, and by 72

3. Indicate half a dozen different methods by which you could find 184 times £2, 15*s* 8*d*, and obtain the result by that method which requires the use of the fewest figures

4. Find, to the nearest unit,

$$35 \cdot 705682 \times 581 \cdot 35823 \div 82 \cdot 05972$$

5. In a congregation of 285 persons, three-fifths of the number were women and there were 35 men, find the number of children

6. A bankrupt's debts amount to £3764, 8*s* 3*d*, his assets to £923, 15*s* 6*d*, and the legal expenses connected with his bankruptcy to £254, 13*s* 9*d*; what dividend do his creditors receive?

7 A clock which was right yesterday at 9 A M is 5 minutes slow to-day at 3 P M, how much must it now be put forward so that it may be right at noon to-morrow?

8. The white population of the British empire is about $59\frac{1}{2}$ million, and the coloured population about $354\frac{1}{2}$ million, find, to one decimal place, the percentage of the total population that are white

9. If a roller is 4 feet $1\frac{1}{2}$ inches in width, how many miles must it travel to roll an acre of ground?

10. How many square inches of tin are there in a cubical biscuit box 8 inches wide, the edges of the lid which overlap the sides being half an inch deep?

Also, how many round biscuits 2 inches across, and $\frac{1}{4}$ inch thick, will the box hold?

N.

1. Find the greatest and least numbers, each of four figures, which are exactly divisible by 37.

2. A kilometre of cotton is wound on a reel, if a seamstress cuts off, on the average, 1 metre 2 centimetres each time she threads her needle, which she does 35 times a day, how much cotton is left on the reel after 28 working days?

3. Find, correct to five places of decimals, the difference between

$$\frac{22}{7} \text{ and } \frac{355}{113}.$$

4. Find, to a shilling, the rent of a farm of 253 ac. 3 ro. 20 po. at 18*s* 9*d* per acre

5. An insurance stamp, measuring $\frac{7}{8}$, by $\frac{3}{4}$, inch is stuck upon a servant's card weekly, within the prescribed space, which is a rectangle $3\frac{1}{2}$, by $2\frac{5}{8}$, inches, what fraction of this space is left vacant at the end of the quarter?

6. If £50 was borrowed on 11/11/11, what sum would repay the loan, with interest at 3%, on 12/12/12?

7. I started for the station that is half a mile from my home in time to catch my train, walking at the rate of 3 miles an hour; but 2 minutes later, remembering that I had left a parcel behind, I returned for it, quickening my pace; at what rate did I then walk, if I still succeeded in catching my train?

8. A rectangular lawn, 31 yards in length, and 17 yards in width, is surrounded by a path 4 feet wide In order to give the path a coat of gravel not less than an inch thick, how many cartloads of gravel must be bought, a cartload being one cubic yard, and not less than a load being obtainable?

9. What is gained per cent in selling by the dozen at the price paid per score?

10. A and B, with their families, share a furnished house at the seaside for six weeks at a rent of 5 guineas a week; A's family consists of 3 persons, and B's family of 6 persons, what is A's share of the rent?

O.

1. If all the integers from 100 to 1000, both included, were written down, how many figures would there be?

2. A man who was walking at the rate of 5·5 kilometres an hour found that he took 32 steps in a quarter of a minute, find the length of his step to the nearest centimetre.

3. Find, correct to three places of decimals, the sum of

$$\frac{70}{71}, \frac{71}{72}, \text{ and } \frac{72}{73}.$$

4. A boy spent a quarter of his money at one shop, a quarter of what he had left at a second shop; he bought a top at a third shop, for

which he paid a quarter of the number of pence he spent at the second shop, and then he had 1*s* 4½*d* left, what did the top cost?

5. P, Q, R, S, are four stations in order on a railway. the single fare, third class, is a penny a mile, but the company issues week end return tickets between any two of these stations at the single fare and a third, the week-end tickets, third class, between P and R, Q and S, P and S, cost 11*s*, 9*s* 8*d*, 13*s* 8*d*, respectively, how many miles is R distant from Q?

6. I bought some wallflowers to plant in a row along a border in my garden, I found that, if I planted them a foot apart, I had too few by 9, if 18 inches apart, too many by 14, how many plants did I buy?

7. The present population of a town is 21,540, if the population increases 6 per cent yearly, what will it be three years hence?

8 A round table, a yard across, is covered by a cloth four feet square; show that more than half of the cloth hangs over the edge of the table

9 A tank is 5 ft 3 in in length, 2 ft 8 in in width, and 2 ft 6 in in depth, how many gallons of water will it hold?

10 A motor-car started for a station 12 miles off at 11 15 A M, and 5 minutes later it overtook a cart which was travelling at the rate of six miles an hour The car reached the station at 11 55, waited there for 10 minutes and then returned at the same rate as before, at what o'clock did it meet the cart, and how far from the station?

P.

1. How many numbers are there, each less than 1000, which exactly contain each of the first six even numbers?

2 Find, to the nearest penny, the value of 23 yds 2 ft 10 in of velvet at £1, 5*s* 4*d* per yard

3 An aeroplane flew 2 miles 240 yards in 3 mins 9 secs, find, to one decimal place, its speed in miles per hour

4 Calculate, correct to the nearest hundred, without waste of figures

$$\frac{287\frac{3}{8} \times 523\cdot 4}{\cdot 09 \times 136\cdot 425}$$

By what rough check would you test your result?

5. How much per cent is gained when pheasants are bought at "three and four", and sold at "four and three"?

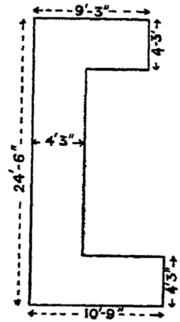
6 Find, to the nearest cent, the interest at 2¼ % on $1438·75 from 14/4/13 to 30/6/13

7. A sum of £60 is to be spent upon 100 yards of road, and the cost is to be divided between the owners, A, B, C, of the frontage, if A owns 31, and B 27, yards of the frontage, how much ought C to contribute?

8. A man who possessed £3600 of stock which paid 3 per cent, sold out at 83, and bought $4\frac{1}{2}$ per cent stock at 120 with the proceeds; find the change in his income (No brokerage to be reckoned)

9. The accompanying sketch shows the ground-plan of a passage which is 10 feet high, find (i) the area of the floor, and the number of tiles 3 inches square required to pave it, (ii) the area of the walls; (iii) the number of cubic feet of space, in the passage

10. Find in litres the capacity of a rectangular tank of the internal dimensions 2 metres 24 cm, 1 metre 50 cm, and 98 cm



Q.

1. Find the number which exceeds the product of the sum and difference of 83 and 38 by 110.

2. Find, without waste of figures, how many thousands of times 4278 is contained in 162908765

3. How much per annum provides £1, 19*s.* 6*d* per diem?

4. The following table gives the average price of $2\frac{1}{2}$ per cent consols for the year named, find, to the nearest sixteenth, the average price for the whole period—

Year	1906	1907	1908	1909	1910
Price	$88\frac{5}{16}$	$84\frac{1}{8}$	$81\frac{1}{16}$	$83\frac{7}{8}$	$81\frac{3}{32}$

5 A metre is 39·37079 inches; find, correct to the nearest tenth, the number of square feet in a square metre

6. If a news-boy buys half-penny papers, thirteen to the dozen, at 5*d.* per dozen, and receives a penny instead of a half-penny from one out of every ten customers, what percentage of profit does he make on sales?

7 A ran a quarter-mile in 53 seconds, and beat B by 1 yard; C ran that distance in $52\frac{1}{2}$ seconds, and beat D by 4 yards. If B and D have a quarter-mile race, which is likely to win?

8 Find (i) the simple interest, (ii) the compound interest; (iii) the theoretical, or "true", discount, on £322 for 2 years at $2\frac{1}{2}$ per cent per annum

9 In a cricket match K scored at the rate of 3 runs in 5 minutes, and J at the rate of 5 runs in 3 minutes; K's score was 32 when J went in, and when stumps were drawn their scores were equal, how many runs had each made?

10. The river at P is 14 yards wide and 4 feet deep, and it flows at the rate of half a mile an hour; how many tons of water pass by P in 24 hours?

R.

1. Find the difference between the largest and smallest numbers which can be represented by the six numerals X, V, L, I, D, C

2. A paymaster had 26 sovereigns, 19 half-sovereigns, 27 half crowns, 16 two shillings pieces, 183 shillings, 59 sixpences, and 196 pence in his cash box After paying the wages of a number of workpeople at 27*s* 3*d* each, the cash left amounted to £9, 13*s* 7*d*, how many people did he pay?

3. Divide the sum of the greatest and least of the fractions $\frac{7}{15}$, $\frac{9}{20}$, $\frac{13}{30}$, $\frac{16}{35}$, by the difference of the others

4. Calculate $\dfrac{12 \cdot 3754 \times \cdot 07843}{(\cdot 2945)^2}$ correct to within one-thousandth part of the whole expression

5. Find, by the method of Practice, the value, to the nearest penny, of 7·56 of £5, 13*s* 9*d*

6. If a litre is $1\frac{3}{4}$ pints and a franc is worth $9\frac{1}{2}d$, find, in English money, the value of a 36-gallon cask of claret at 1 franc 75 centimes per litre

7. A box 20 inches long, 15 inches wide, and 10 inches high is to be wrapped in a sheet of brown paper, find the least dimensions of a sheet which will cover the box

   Find also the length of string required to go once round the parcel the longest way, and twice round each of the other ways, allowing 10 inches for knots

8. A company earns a profit of 32 % on its capital, out of which it pays 13 % to the shareholders in dividends, and £5000 as bonus; there is left a balance of £3550, what is the amount of capital?

9. In the heart of a city 35,480 sq feet of land sold for £403,585, find the price (i) per sq foot, (ii) per acre

10. In a concert-hall the space available for chairs is a rectangle 43 feet wide, and 56 ft 6 in. in length, if 21 inches of width, and 2 ft 3 in from back to back, is allowed for each chair, how many persons will the hall seat?

    Also, if the first 4 rows are priced at 5*s* per chair, the next 8 rows at 3*s*, and the rest at 1*s* 6*d*., how much money will be taken when all the seats are sold?

S.

1. Resolve 51975 into two factors whose difference is as small as possible

2. A has £13, 14*s* 7*d*, B has £4, 3*s* 2*d*, but after A has paid B some money that he owed to him, B has half as much as A, how much did A pay to B?

3. Find the difference between the square of $2\frac{5}{6}$ and the sum of the squares $2\frac{1}{2}$ and $1\frac{1}{3}$

4. Calculate, within one-thousandth of the whole, the value of

$$\frac{357\cdot5}{14\cdot6} \times \frac{3\frac{7}{8}}{4\frac{5}{8}} \times \frac{403\cdot24}{5\cdot125}$$

5. Apply the method of Practice to find, correct to three places of decimals, the product of 23·574 and $6\frac{11}{16}$

6. A barrel contains 36 gallons of beer, it has two taps, and from one tap a jug, holding a pint, is filled every 4 minutes, while from the other tap a jug, holding a quart, is filled every 6 minutes, if both jugs are filled for the first time at 12 o'clock, how much beer will be left in the barrel at 10 minutes past 8 P M.?

7. The average age of the 143 boys of a mixed elementary school is 10 years 3 months, the average of the whole school of 358 children is 9 years 10 months, find, to the nearest month, the average age of the girls.

8 A train travelled 35 miles in 1 hour 18 min, 23 miles on the level and the rest on an up gradient, where the speed was $\frac{2}{3}$ of that on the level, find its speed on the level

9. Find the square number nearest to 6163

10 A rectangular block 3 times as long as a page of this book, $2\frac{1}{2}$ times as long as wide, and $1\frac{1}{2}$ times as wide as thick, is divided into 64 equal smaller blocks each similar to the large block, find in centimetres the dimensions of one of the smaller blocks

T.

1. Find, shortly, (i) 437251 × 125; (ii) 437251 × 98.

2 The sum of two fractions is $\frac{43}{105}$, and their difference is $\frac{1}{15}$; find the fractions.

3 How many bolts each 3·4 inches in length can be cut from an iron rod 7 feet in length, and what length is left over?

4 Divide 3·758 by 72·358, accurate to five significant figures, and verify your result by multiplication, using no more figures than are necessary.

5. The Navy estimates for 1913 were £44,085,000, for 1912, £42,858,000; find, correct to one decimal place, the increase per cent.

6. If a mark is worth $11\frac{3}{4}d$., find the value in £, *s* *d*, to the nearest penny, of 3478 marks 28 pfennige.

7. The Severn tunnel, the longest in the British Isles, measures 4 miles 636 yards. If an express train travels through the tunnel at the rate of 56 miles per hour, find, to a second, the time taken to pass through it by (i) a passenger seated in the train; (ii) the guard walking at the rate of 3 miles an hour along the corridor towards the engine

8. Find, to the nearest yard, the length of a side of a square field of 7 acres

9. An exhibition of motor cars was held in a hall 215 feet long and 70 feet wide A gangway 6 feet wide was kept open all round the hall and the rest of the floor-space was divided into four equal

rectangles by gangways parallel, respectively, to the sides and ends of the hall Each car was allotted a space at least 14 feet by 8 feet, and stood with its length in the direction of the length of the hall How many cars could be accommodated?

10. A man finds that if he invests his money in a 4 per cent stock at 120, he will receive £6, 13*s* 4*d* more in dividends annually than he would receive if he invested it in a 3 per cent stock at 95 How much has he to invest?

U.

1. Making use of the signs of addition and multiplication, arrange the ten digits so as to express the number one hundred

2. I bought the *Daily Mail* ($\frac{1}{2}d$) every week day, and the *Observer* (1*d*) every Sunday, from Jan 1, to March 25, inclusive, in the year 1912, what was the total cost?

3. Find, as shortly as you can, the difference between the square of the sum, and the sum of the squares, of $3\frac{3}{4}$ and $5\frac{5}{8}$

4. In each of the following cases, give from inspection a rough estimate of the result, and then obtain the result correct to the first place of decimals

   (i) $20{\cdot}058 \times 49\ 936$,
   (ii) $(29{\cdot}76 \times 30{\cdot}32) - (9{\cdot}985)^2$;
   (iii) $(39\ 7 \times 124\frac{27}{32}) - (1{\cdot}98 \times 4{\cdot}01)$.

5. If eleven sheets of paper each 24 in by 19 in are used for a book of 352 pages, and the page is 6 inches long, how wide is it?

6. If a train 77 metres in length, travelling at the rate of 56·4 kilometres per hour, overtakes another train, travelling at the rate of 48 kilometres per hour, how long will the first, take to pass by a passenger in the second, train?

7. If a tradesman sells 126 yards of silk for as much as he paid for 147 yards, what is his gain per cent?

8. A square pavement is formed of 5329 equal square tiles, find the number of tiles which form the boundary

9. Find, neglecting any fraction of a penny in the result, the amount, at compound interest payable half-yearly, of £1256, 14*s* in 2 years at 4 per cent per annum

10. A vessel A contains 8 gallons of spirit, and a vessel B contains 8 gallons of water, if the operation of taking a gallon out of each vessel and pouring it into the other be performed twice, how much spirit will be left in A?

V.

1. The sum of the ages of three boys is 43 years, what was the sum of their ages 4 years ago, and what will it be 5 years hence? Also, in how many years will the sum of their ages be 100 years?

2. What sum would be realized by the sale of 300 pairs of gloves at 1*s*. 11*d* per pair? And what profit, if they were bought at 15*s* 9*d*. per dozen pairs?

3. Find, shortly,

$$\text{(i) } \left(99 - \frac{117}{444}\right) - \left(44 - \frac{117}{999}\right); \quad \text{(ii) } 136{\cdot}5 \times {\cdot}988.$$

(iii) the average of 8·5635, 8·5632, 8·5628, 8·5627, 8·5633

4. Half of a man's income is spent in the payment of household expenses, $\frac{3}{20}$ in rent, $\frac{1}{15}$ in rates and taxes; sundry other expenses amount to £10 more than $\frac{1}{8}$ of his income, and he saves £25, what is his income?

5. A labourer and his two sons are employed on a farm for $10\frac{1}{2}$ hours a day, the man is paid four times as much as one of the boys, and three times as much as the other, and their wages per week amount to £1, 13*s* 3*d*, at the rate of how much per hour is each paid for his work?

6. If a news-boy buys 7 dozen half-penny papers at 4*d* per dozen, and receives back a farthing for each copy unsold, how many must he sell before he makes any profit, and how many to make a profit of one shilling?

7. Divide the sum of £12, 1*s* 6*d* between A, B, and C, so that for every sixpence A receives, B receives a shilling, and for every half-crown B receives, C receives a florin.

8. A racing-track for motor-cars is 2000 metres in circuit. When two cars are racing, the first is observed to pass a certain spot at 51 min. $15\frac{1}{2}$ secs past 3 o'clock, and the second at 51 min. 57 secs past 3, the next time round they pass the same spot at 52 min 9 secs, and 52 min $48\frac{1}{2}$ secs past 3, respectively. Find, in kilometres per hour, to the nearest tenth, the difference of their speeds

Also, taking 8 kilometres as equivalent to 5 miles, find the speed of the faster car in miles per hour

9. A man sold out $3\frac{1}{2}$ per cents at 105, realizing £16,590, two-thirds of this money he invested in 4 per cents at 112, and the rest in 3 per cents at 90, what alteration in his income resulted?

10. Find each of the integers whose square is less than 8000 and greater than 7500.

W.

1. The sum of seven consecutive odd numbers is 301, find the least of them.

2. Without performing the division, show that 1564875 is exactly divisible by 1125.

3. The G.C F of two numbers, each of which consists of three digits, is 31, and their L C M is 2387, find the numbers.

4. The sum of the third and fourth, exceeds the sum of the fifth and sixth, parts of a number by 1001, find the number

5. Find the whole number nearest to the product of 23·20567 and 57·87315

6. There are 24 stations on a railway; single, and return, tickets are printed from each station to each of the others, for first, and third,

class passengers; find the cost, at $3d$ per 100, of a thousand tickets of each kind

7. The value of an estate is £6957, 10s Legacy duty and legal expenses together amount to £3, 5s. per £100 After these deductions, the estate is equally divided among five legatees Find, neglecting any fraction of a penny, the sum which each receives

8. A tobacconist makes a mixture of tobaccos in the proportion of 5 lbs. which cost him $5s\ 4d$ per lb, 4 lbs which cost him $6s\ 3d$ per lb, and 1 lb which cost him $8s$, if he sells the mixture at $5\frac{1}{2}d$ per oz, find, to the nearest unit, his percentage of profit

9. Find the number of cubic yards of soil excavated in making a ditch 20 yards long, 2 feet deep, 1 foot wide at the bottom, and 18 inches wide at the top

10 A closed rectangular box, made of wood one inch thick, measures inside 10 in by 8 in, by 6 in, show that just as much wood is used as would fill the box

If the board used were 6 in wide, what length of board would be needed?

X.

1. Without performing the division, find the remainder when 3658427 is divided by (i) 4, (ii) 5, (iii) 9, (iv) 125

2. A householder's coal bill for a winter quarter, when 5 fires are burning for 15 hours a day on the average, amounts to £6, $7s\ 6d$ What should it amount to for a summer quarter when only one fire is burning for 12 hours a day, and 6 tons can be bought for the winter price of 5 tons?

3. Find the average of the fractions—

$$\frac{7+4}{9+5}, \quad \frac{7-4}{9-5}, \quad \frac{7\times 4}{9\times 5}, \quad \frac{7\div 4}{9-5}$$

4. A metre is 39·37079 inches, show that the difference between 35 yards and 32 metres is less than one-fifth of an inch

5. What integral number of centimes per decagram corresponds most nearly to the price 5 farthings per ounce, if £100 = 2515 francs, and 100 kilogrammes = 220 pounds?

6. If the manufacturer marks the prices of motor-cars 40 per cent above prime cost, but allows a trade discount of 20 per cent off list prices, what is his profit on a car which he sells for £350?

7. In the year 1911 the fastest express in England was the 1 9 P M train from Darlington to York, which made the journey in 43 minutes at the average speed of 61·7 miles per hour, and the longest non-stop run was made by the 10 30 A M train from Paddington to Plymouth, a distance of 225·75 miles, in 4 hours 7 minutes; find, to the nearest tenth of a mile, the distance from Darlington to York, and the average speed per hour of the Plymouth express

8. Two enclosures have the same perimeter, 50 yards; one of them is a square, and the other is a rectangle half as long again as it is wide; find the area of each enclosure

9. Find, neglecting any fraction of a penny, the net half-yearly dividend, less income-tax at 1*s* 2*d* in the pound, which results from £11,356, 14*s* of stock which pays 4 per cent per annum.

10. A page of the *Daily Mail* measures $23\frac{1}{2}$, by $17\frac{1}{2}$, inches; if one copy consists of ten pages, and a million copies are printed, show that the quantity of paper used for one day's edition would cover a farm of more than 320 acres

Y.

1. Given that the product of 73 and 37 is 2701, deduce the product of (i) 73 and 27, (ii) 83 and 370

2. If 52 casks of sugar, each containing 7 cwts 2 qrs, are bought at £31, 10*s* per ton, and the sugar is sold at $3\frac{3}{4}d.$ per lb, what total profit is made?

3. Find the product of

$$\frac{3}{13}+\frac{5}{17}+\frac{7}{19} \text{ and } \frac{2}{11}+\frac{16}{31}+\frac{51}{121}.$$

4. Sound travels at the rate of about 1100 feet, and light at the rate of about 200,000 miles, per second, how far off is a thunder-cloud when we hear the thunder twelve seconds after we have seen the lightning?

5. A metre = 1·09363 yards, and a hectare is 10,000 square metres; express a hectare in acres, correct to two places of decimals.

6. A cyclist rode at a uniform rate from A to B in $2\frac{1}{4}$ hours He started back at the same rate, but, after riding for an hour, he was delayed by a puncture for 20 minutes, and for the rest of the way his rate was only 6 miles an hour, consequently the return journey occupied 3 hours altogether. Find the distance from A to B

7. A man's gross income is £720 After deducting the income-tax it is £687, 5*s* 5*d* The tax is 9*d.* in the pound on earned income, and 1*s* 2*d* in the pound on unearned income What portion of his income was earned?

8. If a box of the internal dimensions 5 ft, 2 ft 8 in, and 2 ft. 2 in. will hold a quarter of a million sheets of paper each 8 in. by 10 in., what decimal of an inch is the paper in thickness?

9. On a map made to the scale of 1 inch to 3 miles, a rectangular park is represented by a green patch $\frac{9}{16}$ in long and $\frac{3}{8}$ in. wide; find the number of acres in the area of the park.

10. Divide, to a penny, £327, 16*s.* 8*d* between three persons in the proportion of the numbers 13, 35, 41.

Z.

1. Find the sum of all the prime numbers below 100.

2. In the year 1911 the number of males in England and Wales was 17,448,746, and there were 1,178,047 more females than males; find the total population, also the number of females to each thousand males.

3. A tailor, being asked the price of a coat, replied, "You may have the cloth for nothing, if you will pay for the buttons, a penny for the first, twopence for the second, fourpence for the third, and so on" There were 10 buttons on the coat, what did he want for it?

4. Divide twice the product of the sum and difference of the fractions $6\frac{3}{8}$ and $3\frac{5}{8}$, by half the difference of their squares.

5. Calculate

(i) $(\cdot 041762)^3$, correct to two significant figures;

(ii) $\sqrt{\dfrac{65\cdot 4328}{\cdot 019206}}$, correct to the nearest tenth

6. Evaluate

$$\frac{1 \text{ kilometre} - 11 \text{ decametres} + 34 \text{ metres}}{\frac{1}{5} \text{ of } 35 \text{ millimetres} + \frac{1}{7} \text{ of } 7 \text{ centimetres}}.$$

7. A contractor undertook to complete a specified amount of work by a specified date, he employed 90 men upon the work, but when half was done, he had to put on 20 extra men in order to fulfil his contract, how many men should he have employed at first?

8. Express the decimal 432 84 in the scale of radix five

9. A and B run a race, B has 40 yards' start, A runs 10 yards while B runs 9, and A wins by 4 yards, find the distance

Also, if A ran at the rate of $16\frac{2}{3}$ miles per hour, find how long B would have taken to run the whole distance

10. A box, whose greatest internal dimension is $9\frac{3}{8}$ inches, will just hold a thousand pieces of chalk, each piece of chalk is $3\frac{1}{4}$ inches in length, and its cross section is a square $1\frac{1}{2}$ inches in girth, find the other dimensions of the box

A A.

1. Find a square number, between 1000 and 2000, which is exactly divisible by 13

2. In a butcher's bill, when the exact cost of an item cannot be expressed in halfpence, the number of halfpence next above the exact value, is charged Make out a bill for 3 lb 11 ozs of mutton at $11d$ per lb, 9 ozs of suet at $8\frac{1}{2}d$ per lb, 6 lbs 5 ozs of beef at $10\frac{1}{2}d.$ per lb, and 4 lbs 6 ozs of pork at $9d.$ per lb.

3. Find, to the nearest million, the product of 478650 and 3942, using no unnecessary figures

4. To what degree of approximation is the following statement correct?

$$\frac{£2,\ 13s\ 7d}{17s\ 5d} + \frac{3 \text{ sq. yds } 5 \text{ sq ft}}{2 \text{ sq ft } 80 \text{ sq in}} = \frac{545 \text{ metres } 94 \text{ cm}}{35 \text{ metres}}$$

5. There are 87 links in a piece of chain, the external length of each link is 2 inches, and the internal length is $1\frac{1}{2}$ inches, what is the outside length of the chain?

6. Water is about twice as heavy as deal; find, approximately, the weight of an empty packing-case made of deal boards 1 inch thick, the external dimensions being 4 ft 6 in 2 ft 11 in. 2 ft 4 in.

7 If A's income were increased by 15 %, and B's income were decreased by $12\frac{1}{2}$ %, they would be equal, the sum of the two incomes is £810; find A's income.

8 A foot-rule is held vertically a yard away from a lighted candle; find the length of its shadow on a wall distant 10 feet from the candle and 7 feet from the foot rule

9. A paper bag when empty measures 12 in by 7 in, when loaded with sugar it forms a package 6 in long, 4 in wide, and 3 in. thick; compare the amounts of paper that can be seen in each case

10. A plan of an oblong field of 4 acres is made on the scale of half an inch to one chain, and one side of the plan is 4 inches in length; if fencing costs 2*s* 9*d* per yard, find the least cost at which the field can be divided into two equal fields

BB.

1. A boy multiplied 563 by a number and obtained the answer 249864, but the figures 9 and 8 were incorrect, find the multiplier

2. The earth is more than ninety million miles distant from the sun; estimate, roughly, the time it would take an aeroplane, flying continuously at the rate of fifty miles an hour, to reach the sun from the earth

3. Find the smallest multiplier of ·48375 which produces an integral product

4. In the year 1912 the expenditure of a certain hospital was at the rate of a penny per second, but the income for the year was sufficient only to cover an expenditure of 340 guineas a day; find the deficit for the year

5. If a cubic foot of wood weighs about 33 lbs, how many kilogrammes will a cubic metre of the wood weigh? (1 yard = ·9144 of a metre.)

6. When equal sums are invested in each stock, which yields the larger income, 3 per cents at 77, or $5\frac{1}{2}$ per cents at $131\frac{1}{4}$?

7. State why you know, by inspection, that the answers attached to the following questions must be wrong—

   (i) "Find, to the nearest penny, the value of 4·3248 of £1. *Ans.* £4, 6*s* 7*d*"

   (ii) "Find the G C M of 984 and 1148 *Ans* 132."

   (iii) "If the cost of paving a rectangle 10 ft 6 in by 13 ft is £2, 5*s* 6*d*, what is the cost of paving a rectangle 21 ft. by 26 ft? *Ans* £4, 11*s*"

8. Show that the number of revolutions made in five seconds by a bicycle wheel which is 28 inches in diameter, gives the speed in miles per hour at which the bicycle is travelling (Circumference of a circle = $\frac{22}{7}$ × diameter)

9. From a 36-gallon cask of brandy 4 gallons were drawn out, and the cask was filled up with water, then 6 gallons were drawn out, and again the cask was filled up with water; finally 9 gallons were drawn out; how much brandy was left in the cask?

10 A boy is due at a certain place at a certain time daily; on Monday he starts at 8 30 A M, and, walking at $3\frac{3}{4}$ miles an hour, he is 5 minutes late, on Tuesday he starts at 8 40 A M, and, cycling at 10 miles an hour, is 5 minutes early, how far has he to go?

CC.

1. One Saturday morning a builder cashed a cheque at his bank, and took away with him £40 in sovereigns, £12 in half-sovereigns, £12 in silver (no three-penny bits), and 10*s* in copper, out of this cash he paid the wages of four foremen £2, 5*s* 6*d* each, 23 mechanics £1, 18*s* 9*d* each, and 9 labourers £1, 1*s* 3*d* each, how much cash had he left (i) altogether, (ii) in gold, silver, and copper, respectively, if he only used copper when necessary?

2. Find three numbers, between 2500 and 3000, each of which is a common multiple of 21, 24, and 28

3. Evaluate, correct to two places of decimals,

$$\frac{76\frac{43}{77} - 3\frac{53}{76}}{764\frac{29}{110} + 31\frac{67}{210}}$$

4. Kilometres are converted into miles with a fair degree of accuracy by multiplying by 10 and dividing by 16, more accurately, by multiplying by 100 and dividing by 161 Find in miles, correct to two places of decimals, the difference of the results obtained by the two methods in the case of 628·75 kilometres

5 One clock gains 2 mins in 12 hours, another loses 2 mins in 36 hours, both are right at noon on Tuesday, on what day, at what hour, will they differ by 15 minutes, and what time will the fast clock then indicate?

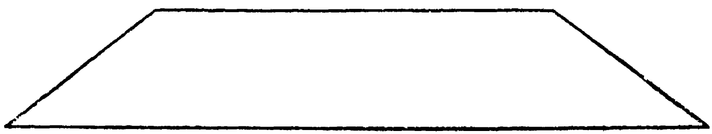
6 The inhabitants of a Swiss town speak French, or German, or both If 73 per cent can speak French, and 87 per cent can speak German, what percentage can speak both languages?

7. Find the dividend at $18\frac{1}{2}$ per cent on 72 bank shares of £50 on which £8 per share has been paid up

8. Find, correct to two places of decimals, $7\sqrt{325} - 5\sqrt{225} - 3\sqrt{125}$

9. The distance of P from Q is 7 miles An aeroplane flew from P to Q against the wind, and then back, in 22 minutes If its speed was decreased by $12\frac{1}{2}$ per cent when flying against the wind, and increased by 5 per cent when flying with the wind, how long would the flight have taken had there been no wind?

10. The diagram given is the plan of the cross section of an embankment, scale 1 to 100, find, approximately, the number of cubic metres of material in the embankment per kilometre of length



D D.

1. A grandfather clock, which struck only at the hours, went without stopping from 10 A M 10/10/01 to 10 A M 10/10/09, how many times did the hammer hit the bell during that period?

2. Divide 2225 into two parts, such that $\frac{11}{17}$ of one part is equal to 1·375 times the other part

3. A quart of water weighs 175,000 grains, and a litre of water weighs 154,000 grains, find the smallest whole number of quarts which is equivalent to a whole number of litres

4. The male population of Bulgaria is 2,200,000 The losses in the late war were—

Killed	330	officers, 29,711 men;
Wounded	950	officers, 52,550 men,
Missing.	3193	officers and men.

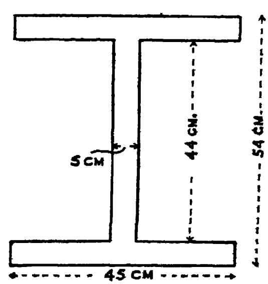
Express the total losses, in killed, wounded, and missing, as a percentage of the male population

5. If a person invests £1000 in $2\frac{1}{2}$ per cent consols at $74\frac{3}{4}$, brokerage $\frac{1}{8}$, what quarterly dividend will he receive, income-tax at 1*s* 2*d*. in the pound having been deducted?

6. A floor 11 ft 6 in square is paved with black and red tiles arranged alternately; each tile is 6 inches square, and there is a black tile at a corner, how many tiles of each colour are there?

7 The base of a rectangular cistern is 5 ft 9 in by 2 ft 11 in; to what depth, to the nearest inch, will 80 gallons of water fill it?

8. A boy, being asked his age, replied, "Twice what my age will be ten years hence is seven times what it was ten years ago" Find his age



9. A cubic centimetre of iron weighs 7·76 grams; find, to the nearest kilogramme, the weight of an iron girder 5 4 metres in length, and of the cross-sectional dimensions indicated in the diagram

10. Air consists of nitrogen and oxygen, and the oxygen is 20·8 per cent of the volume; the weights of equal volumes of oxygen and nitrogen are as 8 is to 7; what percentage of the weight of the air is oxygen?

E E.

1. By inspection evaluate

$$\frac{\cdot 4 \text{ of a decametre} + 550 \text{ centimetres}}{\cdot 7 \text{ of a metre} + 12 \text{ decimetres}}$$

2 If 18 boxes of sugar, each containing 1 cwt 2 qrs. 14 lbs, are bought at £21 per ton, and the sugar is sold at 3*d* per lb., find (i) the total cost; (ii) the cost per lb, (iii) the total profit; (iv) the percentage profit.

3. Find, shortly,

(i) the sum of $3\frac{1}{7} \times 4\frac{1}{6}$ and $2\frac{5}{6} \div \frac{7}{12}$;
(ii) the integer nearest to $(9.97)^2$,
(iii) the product of $\sqrt{176}$ and $\sqrt{275}$

4. Find, correct to four places of decimals, the sum of the series

$$\frac{1}{5} + \frac{1}{2.5^2} + \frac{1}{3.5^3} + \frac{1}{4.5^4} + \dots$$

5. If the letters P, I, r, n, are used to denote principal, interest, rate per cent, number of years, respectively, construct with them four formulæ, each formula expressing one of these four quantities in terms of the other three, and use one of the formulæ to obtain the rate at which the interest on 3 pounds for 3 months is 3 pence

6. In the Fahrenheit thermometer freezing-point is marked 32 degrees and boiling-point 212 degrees, in the Centigrade thermometer, freezing-point is 0, and boiling-point 100, degrees

What reading F corresponds to (i) 5° C; (ii) 35° C; (iii) 80° C, and what reading C corresponds to (iv) 95° F, (v) 59° F, (vi) 14° F?

7. When a man 5 ft 6 in in height stands a yard away from a lamp-post 10 feet high, what is the length of his shadow thrown by the lamp on the pavement?

8. How many posts, set 8 feet apart, and how many rails, three between each pair of posts, are required to enclose a square field whose area is 1 ac 2 ro 16 po? And what is the total cost at $10\frac{1}{2}d$ per post and $4\frac{1}{2}d$ per rail?

9. I have an appointment at B at a certain time, I find that if I go to B by the 12 50 train, which travels at the average speed of 40 miles per hour, I shall arrive at B ten minutes too late for my appointment, but if I go by the 11 35 train, which travels at the average speed of 30 miles per hour, I shall arrive at B with half an hour to spare, how far is it to B?

10. Which is greater, and by how many square feet, a square, or a circle, each 200 feet in perimeter?

F F.

1 Express in figures the number of milligrams in a tonne

2 Excluding unity, how many common factors have the numbers 5005 and 7865?

3. Find, to the nearest tenth, the results required to complete the following table of census returns —

	1901	1911	Increase, or Decrease, %
Scotland	4,472,103	4,759,445	
Ireland ..	4,458,774	4,381,951	

4. The total thickness of a book is 4 3 cm., each of the covers is 2·5 mm in thickness, the total number of pages in the book is viii + 532 + four blank, find, approximately, the thickness of the paper

5. The floor of a room 18 ft 6 in long and 13 feet wide is covered with linoleum, if the room had been 2 feet longer and 1 foot wider, the linoleum would have cost 19*s* 4½*d* more than it did cost, find its cost

6. If by a reduction of 10 per cent in the duty on tea, the consumption of tea was increased by 10 per cent, how would the revenue from the duty be affected?

7. The weights of three boys, A, B, C, are 9 st 10 lbs, 8 st. 7 lbs, 7 st 4 lb, their heights are 5 ft 6 in, 5 ft 9 in, 4 ft 6 in; and their ages are 16 yrs 3 mo, 15 yrs 2 mo, 14 yrs 1 mo. If a guinea were divided among them in proportion to their weights, another guinea in proportion to their heights, and a third guinea in proportion to their ages, how much money would each of the boys receive?

8. There are two mixtures of wine and water, in the first the ratio of wine to water is 6 · 1; in the second it is 7 : 1, if a third mixture be formed from a gallon of each of these, what will be the ratio of wine to water in it?

9. An old lady is half as old again as her son, 31 years ago she was twice as old as he was; how old is she?

10. Miss X walked up Snowdon from Llanberis, rested for an hour at the inn on the summit, and then came down the same way; she left Llanberis station at 11 15 and reached it again at 3.35, making the ascent at the rate of about 2¼ miles per hour, and the descent at the rate of about 5 miles per hour, find, to a quarter of a mile, the distance from the station to the summit

G G.

1. On and after Monday, May 5, 1913, the price of the *Times* was reduced from 3*d* to 2*d*, find the difference in the cost of that newspaper for the years 1912 and 1913, assuming that it is not published on Christmas Day

2. When sugar costs £33 per ton wholesale, what is the lowest price, expressed in our coinage, at which it can be sold per lb. without loss, and what is then the gain per cent?

3. The quickest passage from England to America has been made by the *Mauritania* in 4 days, 10 hours, 40 minutes at an average speed of 25 knots per hour A knot is 6080 feet, find, to the nearest hundred, the number of miles in the passage

4. A bill for £547, 10*s*, due on May 26, is discounted at 4 per cent on April 11, find the banker's charge, omitting days of grace.

5. A sheet of cardboard is formed into a hollow cylinder which is 10½ inches in length and 2½ inches in diameter; find, to the nearest square inch, the area of the sheet

6. If a sum of £2, 4*s* 1*d* were equally divided among a number of girls, and each girl received as many pence as there were girls, how many girls were there?

7. Find, to the nearest cubic foot, the quantity of brickwork in a wall which is 42 feet in length and 8 feet in height, the width of the wall being 9 inches, except in the case of the two lowest courses of bricks, which are 18 in and $13\frac{1}{2}$ in in width respectively, and each course is 3 inches in height, allowance is to be made for a doorway 7 ft by 2 ft 6 in

8. Two persons start from the same place at the same time, one travels at the rate of $4\frac{1}{2}$ miles per hour, and the other at the rate of $3\frac{3}{8}$ miles per hour, how far apart will they be in 4 hours (i) if both travel northwards, (ii) if one goes north and the other south, (iii) if one goes north and the other east?

9. Iron is 7·75 times as heavy as water, and a kilogramme is the weight of a cubic decimetre of water Find, to the nearest kilogramme, the weight of a solid cube of iron, the edge of which measures 2 metres 26 centimetres

10. How many equal cubical blocks, each as large as possible, can be built up so as to form a solid 15 feet 2 in in length, 10 feet 6 in in width, and 8 feet 2 in in height?

H H.

1. When 87792 is divided by a certain number, the successive remainders are 183, 104, and 1, find the divisor and the quotient

2 Find the cost of 83 tons 7 cwts of steel at £5, 8*s* 6*d* per ton, also the volume of the steel at 488 lbs per cubic foot

3. Work the following questions by short methods —

   (i) Evaluate $\dfrac{3\frac{37}{72} - 1\frac{31}{84}}{3\frac{1}{72} - 1\frac{1}{84}}$

   (ii) Find the value per lb. of a mixture formed of 454 lbs of tea at 2*s* $4\frac{1}{2}$*d*. per lb with 227 lbs at 2*s* $1\frac{1}{2}$*d* per lb

   (iii) A square of side 4·53 cm is cut out of a square of side 15·47 cm, find the area of the remnant

4. In the Fahrenheit, Centigrade, and Réaumur thermometers, freezing, and boiling, points are marked, respectively, 32°, 212°, 0°, 100°; 0°, 80°, find the readings C and R which correspond to 104° F, and the readings F which correspond to 85° C and 48° R

5. A vessel A is full of spirit, and an equal vessel B is half-full of water B is filled up from A, then A is filled up from B, and finally B is again filled up from A What percentage of the mixture now in B is water?

6. English lead is worth 2 per cent more than Spanish, if, when the price of English lead is £18, 10*s* per ton, the price of Spanish lead is 450 pesetas per tonne, what is the value in English money of a peseta? (1 kilogramme = 2·2 lbs)

7. Find the number which exactly contains 23 as many times as the number is exactly contained in 8303

8. Trams run both ways at intervals of 3 minutes on a road If a man walking along the road meets two trams, and one tram overtakes him, every quarter of a mile, at what rate, in miles per hour, is he walking?

9. If a £5 note is in circulation for 5 years, find the profit on it made by the Bank, reckoning by compound interest at 3 per cent per annum.

10 The Suez canal is 99 miles long, and, at the surface of the water, the average width is 37 metres; its cost was about £24,000,000 The Panama canal is to be 46 miles long, with an average surface width of 400 feet, and its cost is estimated at $400,000,000 Compare, roughly, the cost per square yard at the surface of the two canals

II.

1. Having given that

$$43761 \times 43981 = 1924652541,$$

deduce the product of 43761 and 45981, and then the product of 44751 and 45981.

2. If 2 litres of wine, costing 4 fr 45 c per litre, are mixed with 3 litres, costing 5 fr 5 c per litre, and half a litre of water, what is the cost of a litre of the mixture?

3. Express a mile in metres, correct to the nearest metre

4. Find, to the nearest gramme, the weight of a uniform rod of copper 31 centimetres in length and 20 millimetres in girth, the cross section of the rod being a square

Copper is 8·9 times as heavy as water, and a cubic centimetre of water weighs one gramme

5. A school of boys and girls contains 449 scholars, and the number of boys is very nearly ·39 of the number of girls, find the number of girls

6. Twenty years ago my age was double the sum of the ages of my two daughters; now it exceeds the sum of their ages by one year; what is my age?

7. Find the number which falls short of 4764 by as many tens as it exceeds 980 by units

8. In 3 years at 4 per cent per annum the compound interest exceeded the simple interest on a certain sum of money by £12, 13*s.* 4*d.*; find the sum.

9. If a crew row 3 miles down stream in 16 minutes, and 2 miles up stream in 15 minutes, at the rate of how many miles per hour do they row on still water?

10. A mile of paper ·008 inch in thickness is wound round a drum, and the diameter of the roll is 2 ft. 6 in.; find, to the nearest tenth of an inch, the diameter of the drum.

JJ.

1. Light takes more than 8 minutes to reach the earth from the sun, and more than 3 years to reach the earth from the nearest star.

The sun is more than 90 million miles distant from the earth. Express in figures the distance of the nearest star from the earth, attempting no greater degree of accuracy than is justified by the data

2 If 120 lbs of tea which costs 1*s* 6*d* per lb be mixed with other tea which costs 1*s* 9*d* per lb, and the mixture can be sold, with a profit of 20 per cent, at 1*s* 10*d* per lb, of how many pounds does the mixture consist?

3 At the last census the population of Ireland was 4,381,951, at the previous census it was 4,458,775; if the same rate of decrease continues, what will it be at the next census?

4 A litre is a cubic decimetre Find, to the nearest litre, the capacity of an open tank, made of iron 3 mm in thickness, measuring externally 2·16 metres in length, 85 cm in width, and 56 cm in depth.

5 One tenth of the contents of a bottle of wine is removed, and the bottle is filled up with water, this operation is repeated again, and again, what percentage of wine is then left in the bottle?

6 A person invested £6000 in 3 per cent stock at $98\frac{3}{4}$, he also invested in a 6 per cent stock at $102\frac{7}{8}$, on the whole he obtained interest on his capital at the rate of 4 per cent, how much money did he invest in the 6 per cents? (Brokerage $\frac{1}{4}$ in both cases)

7. A rectangular park is three times as long as it is wide, and its area is 63·48 hectares, how many metres of fencing enclose it? (100 hectares = 1 square kilometre)

8. At 12 o'clock a train 132 yards in length overtook a man, who was walking by the side of the line, and passed him in 15 seconds At 3 minutes past 12 the train overtook a second man, and passed him in $13\frac{1}{2}$ seconds At what time would the first man overtake the second?

9 The regulations for parcel post state that the combined length and girth of a parcel must not exceed six feet Find the volume of the largest cardboard box, three feet in length, which may be sent by parcel post

10. If our system of notation was based upon the number twelve instead of ten, what number would be represented by the duodecimal 127·6, and what figures would represent the decimal 43·25?

KK.

1. Given that $88 \times 88 = 7744$, deduce the product of 89 and 87

2. Work the following questions by short methods —

 (i) Evaluate $12\frac{71}{120} - 2\frac{71}{140} - 11\frac{11}{120} + 1\frac{1}{140}$,

 (ii) Simplify $\frac{3\frac{2}{7}}{4\frac{2}{5}} \times \frac{6\frac{1}{2}}{7\frac{3}{8}} \times \frac{8\frac{4}{5}}{9\frac{3}{4}} \times \frac{14\frac{3}{4}}{9\frac{6}{7}}$;

 (iii) Express £2·796 — 125 in £, *s* *d* to the nearest penny,

 (iv) Find the number of square yards in the sum of the areas of two rectangular plots, one of which measures 137 yards by 46 yds. 2 ft, and the other 253 yds 1 ft by 137 yards.

3. A coal merchant buys 3750 tons of coal for £3640; he sells 2644 tons at £1, 1*s* 8*d* per ton, and the remainder at 18*s* 9*d*. per ton; what balance results?

4. Given that 1 sq yd = ·8361 of a sq metre, express 1 decimetre in inches, correct to 3 places of decimals

5. If the manufacturer of a machine sells it to a middleman at 25 per cent above cost price, and the middleman sells it to the shopkeeper for 5 per cent more than he paid for it, and the shopkeeper sells it to a customer for £36, 11*s* 6*d*., making a profit of 10 per cent what did it cost to make?

6. In a mixture of lead and tin there is 57 per cent of tin; how much lead must be added to a hundredweight of this mixture to reduce the percentage of tin in the new mixture to 38?

7. The plan of the cross section of a railway cutting measures 90 cm. at the top, 50 cm at the bottom, and 1·5 cm in depth; the scale of the plan is 1 metre to 1 furlong, find the number of cubic yards of earth excavated per mile of the cutting

8. It was stated in a newspaper that the rainfall in one month last year, on a certain area, amounted to 1640 million tons of water with a volume of 58⅔ million cubic feet. Show that both statements cannot be right

If the first was right, what should the second have been? Hence suggest how the error arose

9. A clerk's salary was £80 for each of the first three years; then it was raised £10 a year for 12 years, after which it remained stationary. When he retired from work he found that his average salary had been £174, 10*s* How many years did he work?

10. If a sum of £5000 was borrowed at 5 per cent per annum, compound interest, and was repaid, principal and interest, in three equal annual instalments, how much was paid as one instalment?

L L.

1. Find, by resolution into factors, the cube root of 19683.

2. The railway from A to B ascends at a gradient of 1 in 300 for 25 miles from A, and then descends 1 in 200 for 15 miles to B; how many more feet above sea-level is B than A?

3. Standard gold is worth £3, 17*s* 10½*d*. per ounce Troy, and sovereigns are worth their weight, which is the heavier, a boy who weighs 6 st. 4 lb or a bag containing 5000 sovereigns?

4. There are three parties in a parliament of 670 members, the Government, the Opposition, and the Independent, party. If all the members voted, and all the Independent party voted with the Government, the Government would have a majority of 108; but if half of the Independent party voted with the Opposition, and the other half abstained from voting, the Government would be in a minority of 18, find the number of each party.

5. In firing at a target, an outer scores 3 points, an inner 4 points, and a bull's eye 5 points. If a marksman makes 11 per cent

misses, 12 per cent outers, 13 per cent inners, and the rest of his shots are bull's eyes, what percentage of the maximum score does he make?

6. A man's earned income exceeds £160 He has to pay income-tax at $9d$ in the £ on the excess of his earned income over £160, and income-tax at $1s$ $2d$ in the £ on all his unearned income The total tax he pays is £7, $9s$ $2d$, and the portion of his earned income on which he pays tax is to his unearned income in the ratio of 9 to 7. Find his total income

7. A rectangular plot of ground, which is twice as long as it is wide, was measured incorrectly, the length being taken a yard too great, and the width a yard too small, the resulting error in area was 18 square yards, find the area of the plot

8. If 7700 gallons of water are discharged per minute from a reservoir by a cylindrical pipe 3 ft 6 in in diameter, find the speed of the water through the pipe in feet per minute

9. Evaluate $\frac{\sqrt{3}+1}{\sqrt{3}-1}$, correct to four places of decimals.

10. A flagstaff was broken by the wind at a point 12 feet from the ground, so that the upper part turned down until the top rested on the ground at a point distant 9 feet from the foot of the staff Find the original height of the flagstaff.

M M.

1. If the dividend is 10 times the remainder, and the remainder is 21 times the quotient, find the divisor Find, also, the greatest possible quotient

2. A man found that his income for the year 1912 was such that he might spend at the rate of $1d$ on the first day, $2d$ on the second day, $3d$ on the third day, and so on throughout the year, and then have left $3s$ $3d$, what was his income?

3 Given that $(15334)^2 = 235,131,556$, deduce the square of 153342

4 Find a multiplier which will convert a speed expressed in centimetres per second into the corresponding number of miles per hour
Use the multiplier to find, to the nearest mile, the speed per hour of an aeroplane which flies 20 metres per second

5 Find, to the nearest million, the number of gallons of rain that fell on a square mile of country in a year when the rainfall was 40 inches

6. A veranda 8 feet wide runs along two adjacent sides of a house, and the perimeter of the floor of the veranda is 38 yards; find the cost of paving it with tiles 6 inches square at $2s$ $6d$ per dozen

7. A and B have the same number of shots and their bag consists of 245 head of game If, on the average, A kills five times in six shots, and B kills five times in eight shots, how many shots did each fire?

8. A board 21 ft 6 in in length, 7 in in width, and $\frac{3}{4}$ in in thickness, is given to a boy and he is told to make of it a packing-case to

hold 100 copies of a book which measures $6\frac{3}{4}$, by $4\frac{5}{8}$, by 1, inches. Is there sufficient board for the purpose? If so, what external dimensions would you suggest for the box in order to avoid needless sawing?

9. A flag is hoisted on a pole by a cord, twice as long as the pole, passing through a pulley at the top of the pole, and when the flag is up the cord, if stretched out, will meet the ground at a point 30 feet from the foot of the pole; find, to a foot, the height of the pole.

10. Express a million in the scale of twelve, and the duodecimal number *etet* in the common scale.

NN

1. April 8th, 1913, was Tuesday; what day of the week will April 8th, 1963, be?

2. A cubic foot of sulphuric acid weighs about 115 lbs; find, in kilogrammes, the weight of a cubic metre of the acid.

3. A contractor undertook to make a well at a charge of 1*s.* for the first foot of depth, 2*s.* for the second foot, 3*s* for the third foot, and so on. If his bill for the work was £11, 11*s*, how deep was the well?

4. A rectangular tank which is twice as long as wide, and twice as wide as deep, holds 400 gallons; find its dimensions.

5. Three cyclists, A, B, C, start together and ride the same way round a track 270 yards in circuit, their rates being 18, 14, 10, yards per second, respectively; after how many seconds will they again be together, and how far from the starting place?

6. The outer boundary lines of a lawn-tennis court are 78 feet by 36 feet; the inner boundary lines are 42 feet by 27 feet

   Find (*a*) by measurement on a plan, (*b*) by calculation, the distance between

   (i) opposite corners of the outer boundary;

   (ii) opposite corners of the inner boundary,

   (iii) a corner of the outer, and the corner nearest to it of the inner, boundary.

7. A town is supplied with water from a reservoir which is fed by a stream of uniform volume; when the reservoir is full, if 32,000 gallons are used daily, the supply fails in 50 days; if 37,000 gallons are used daily, it fails in 40 days; how much water can be used daily without the supply ever failing?

8. A rectangular metal plate has a circular hole in it; the length of the plate is 1·3 metres, the width is 80 centimetres, and the thickness is 2 millimetres, its volume is 1828·6 cubic centimetres, find the diameter of the hole

9. A party of 4 gentlemen and 3 ladies wish to be taken to a place seven miles off, but there is only one taxi-cab available, and it can carry at most but five persons in addition to the driver. The men can walk four miles an hour, so all the party start together, two walking, and the cab, travelling at the average speed of 16 miles

an hour, having deposited a part of its load at a certain point on the road, returns to pick up the others If all reach their destination together, how long does the journey take, and how far does each of the men walk?

Also, if the gentlemen share the cost, and each contributes 3s (the charge being 8*d* per mile, with an extra charge of 6*d* for the whole journey, for each additional passenger beyond two); what gratuity did the driver receive?

10 Divide 86 into two parts, such that when one part is expressed in the scale of eight, and the other part in the scale of twelve, they are represented by the same figures.

EXAMINATION PAPERS.

CAMBRIDGE LOCAL EXAMINATIONS.

[*The use of algebraical symbols and processes is permitted*]

December, 1912. PRELIMINARY (2 hours)

1. Multiply five thousand and seventeen by four thousand one hundred and seven, and state the answer in words

2. Divide £7106, 19*s*. 3*d* by 396

3. Find the Least Common Multiple of 104, 85, 136, 156.

4. Express in their simplest forms—

$$\text{(i) } \left(\tfrac{3}{4} + \tfrac{4}{5} + \tfrac{5}{6}\right) \times \left(\tfrac{11}{13} - \tfrac{7}{11}\right);$$

$$\text{(ii) } \frac{2\frac{1}{4} - \frac{7}{8}}{3\frac{5}{8} + \frac{2}{11}}$$

5 Divide 1·53 by ·068, and multiply 5·49 by ·0137.

6. (*a*) Find the value of 1·28 of £7, 16*s* 3*d*

(*b*) Reduce 22 lbs 5 ozs. to the fraction (in its lowest terms) of 17 cwts

7. What is the total cost of 16 sacks of coal, each weighing 1 cwt 3 qrs. 7 lbs., at 27*s*. 6*d* per ton?

8. Find the simple interest on £297 for 3 years 4 months at $3\frac{3}{4}$ per cent per annum

9. I pay 8 francs 25 centimes per kilogramme for tea in Paris What would this price be per pound in English money? (Take 25 francs = £1, and $2\frac{1}{5}$ lbs. = 1 kilogramme)

10. I buy a quantity of coffee at 140*s*. per cwt , and sell half of it at 1*s* 6*d*. per lb , at what price per lb must I sell the rest so as to gain 40 per cent on the whole transaction?

JUNIOR (2 hours)

A 1. Find the Greatest Common Measure of 4928 and 5852. Find also their Least Common Multiple.

A 2. (i) Divide $\dfrac{3\frac{3}{4} - (\frac{5}{6} \times 4\frac{1}{5})}{2\frac{1}{4}}$ by $\dfrac{4 - 1\frac{1}{3}}{4 + 1\frac{1}{3}}$.

(ii) Simplify $\dfrac{\cdot 27 \times \cdot 08}{\cdot 27 - \cdot 03}$.

A 3 Find, to the nearest penny, 1·45 of £21, 5*s* 1½*d*

A 4 Find the cost in francs and centimes of asphalting a footpath 752 metres long, by 2·2 metres broad, at the rate of 1 franc 50 centimes per square metre

A 5 A grocer buys 5½ cwts of tea for £38, 10*s*, and sells 300 lbs of it at 2*s* 2*d* per lb, and the rest at 1*s* 10*d* per lb What is the whole profit which he makes on the tea?

A 6 Find the simple interest on £316 for 1½ years at 3½ per cent per annum

A 7 My income from Government 2½ per cent stock is £43, 15*s* I sell out this stock at 82½ and invest in Japanese 5 per cent bonds at 105. By how much is my income increased? (Brokerage is to be neglected)

B 1 The calendar is arranged so that there are 97 leap years and 303 ordinary years in 400 years If the true length of the year is 365·2422 days, find how many whole hours the calendar would be wrong at the end of the 400 years

B 2 A sufficient number of men are engaged to make a drain in 100 working days of 9 hours each They work for 12 days, and then go on strike for 3 days, returning on the agreement that in the future they shall work only 8 hours a day, they finish the drain By how many working days is the completion of the work delayed?

B 3 Two motor-cars compete for 12 hours on a racing path One car runs at 45 miles per hour for the whole time The other runs 10 per cent quicker than its rival, but, owing to breakdowns, it loses 10 per cent of the allowed time for racing Which is the winner, and by what distance?

B 4 The owner of a 56-gallon barrel of whisky uses 7 gallons, and fills up the barrel with water He then uses 8 gallons of the mixture; and again filling up with water, he draws off 40 gallons of the mixture How much pure whisky is left in the barrel?

B 5 Eggs are imported from France at 1*s* 4*d* per score Two per cent are lost by breakage on the journey One-half of the unbroken eggs are sold at 10 for one shilling, and the other half at one penny each. What profit per cent is made?

SENIOR (2 hours)

1. How many numbers are there less than 100,000 which are divisible by each of the whole numbers from 2 to 12 inclusive?

2. Simplify—

$$\text{(i)}\ \tfrac{31}{72} - \tfrac{13}{70} + \tfrac{14}{45} - \tfrac{11}{84},$$
$$\text{(ii)}\ (3\tfrac{43}{56} \text{ of } 5\tfrac{5}{39}) - (4\tfrac{31}{36} \text{ of } 2\tfrac{8}{77})$$

3. Divide 999·99 by ·00271, and find the value of 13·86 × 17·05 ÷ 8·93 correct to two places of decimals

4 A horse ran a distance of 2¼ miles in 3 minutes 55 seconds. Find its speed in miles per hour, correct to two places of decimals

5 A farmer pays a rent of £470 for a farm containing 330 acres What should be the rent of a field containing 10 acres 2 roods 36 poles at the same rate per acre?

6 Find, to the nearest penny, the interest on £13,654 for 25 days at 7 per cent per annum

7 Assuming that a square metre contains 1550 square inches, express a centimetre as a decimal of an inch to four places of decimals

8 A path is 224 feet long and 6 feet 9 inches wide What will be the cost of the gravel required to cover it to a depth of 1½ inches at 6*s* 6*d* per cubic yard?

9 A grocer makes a profit of 33 per cent by selling tea at 1*s*. 7*d* per pound What percentage of profit would he make if he were to sell the tea at 1*s* 6*d* per pound?

10 A man invests equal amounts in 2½ per cent consols at 77½, and in Bank of England stock, which pays dividends of 9 per cent per annum at 275. From which investment does he receive the larger income?

OXFORD LOCAL EXAMINATIONS

March, 1913. JUNIOR (1¼ hours)

1 Make out a bill for the following articles· 5¾ yds of flannel at 1*s* 8*d* a yard, 9¼ yds of calico at 5*d* a yard, 64 yds. of ribbon at 9¾*d* per dozen yards, and 1½ dozen buttons at 4½*d* a dozen.

2 How many times can a jug which holds 0·625 of a pint be filled from a cask which contains 6 galls 1 qt 1 pt, and what part of a pint will be left in the cask?

3 Subtract $\frac{3}{5}$ of £35, 14*s* 8¼*d* from $\frac{3}{11}$ of £78, 16*s*. 2½*d*

4. Find the simple interest on £739, 3*s* 4*d* for 1½ years at 3½ per cent per annum

5 Find, correct to 3 places of decimals, the number of kilometres in a sea-mile, having given that 1 sea-mile = 6081 feet, and 1 foot = 0·3048 metres

6 Find the cost of the paper required for covering the four walls of a room 31 ft 9 in. long, 17 ft 3 in wide, and 13 ft 6 in high, with paper bought in pieces and costing 1*s* 10½*d* per piece, each 12 yds. long and 21 in wide

7. A man had an income of £600 a year from 4 per cent Colonial stock If he sold out at 101¼, what sum of money would he receive? And, if he invested this money in a 3 per cent stock at 91⅛, what change would there be in his income? (Neglect brokerage)

8 Find the greatest and smallest numbers, each of 4 digits, which are perfect squares.

9 A hole 8 yd 2 ft 7 in long, 5 yds 1 ft. 11 in broad, and 2 yds. 1 ft 3 in deep is to be filled up exactly with uniform cubical blocks fitting closely together. If the blocks are made as large as possible, find the number required and the number of cubic inches in each block

---

SENIOR ($1\frac{1}{4}$ hours)

1 Express $\frac{5}{8}$ of £3, 2*s* $7\frac{1}{2}d$ as the decimal fraction of £5

2 What is the annual rent of a field containing 7 ac 3 ro 15 po at £1, 10*s* 8*d* per acre?

3 What is the smallest number by which 273 must be multiplied to give a multiple of 357?

4 A grocer buys 28 lb of tea for two guineas, but 1 lb is damaged and rendered worthless, at what price per lb must he sell the remainder in order to gain $12\frac{1}{2}$ per cent on his original outlay?

5 If 25 francs 20 centimes are worth 4 84 dollars, find the value in dollars and cents of 500 metres of silk worth 12 francs 50 centimes per metre

6 A second-hand bicycle is marked for sale at £2, 10*s*, twelve months' credit being allowed for payment What should be, to the nearest sixpence, the cash price of the bicycle when money receives three per cent interest?

7 A person derived an annual income of £420 from $2\frac{1}{2}$ per cent consols He sold out at $78\frac{3}{4}$ and invested the proceeds in 5 per cent debentures, thereby increasing his income by £52, 10*s* What was the price of each £100 debenture? (Neglect brokerage)

8 Find the total area of the inside surface of a rectangular box, with lid, made of wood $\frac{3}{4}$ in thick, the outside measurements being 4 ft $4\frac{1}{2}$ in in length, 2 ft $7\frac{1}{2}$ in in width, and 1 ft 9 in in height

9 Each railway passenger is allowed to take a certain weight of luggage free, but is charged $\frac{3}{4}d$ for each lb in excess of that weight Three passengers had to pay 2*s*, 2*s* 6*d*, and 3*s* 6*d* respectively for excess luggage, but, if one passenger had taken all their luggage, he would have had to pay £1, 6*s* 9*d*, how much luggage is each passenger allowed to take free?

LONDON UNIVERSITY MATRICULATION EXAMINATION

1913

1 Find the square root of ($73\cdot5987 \div \cdot025384$), to one decimal place, and evaluate $\{0\cdot\dot{0}71432\}^3$, to three significant figures.

2 A cylindrical pipe $2\frac{1}{2}$ feet in diameter discharges 3300 gallons per minute from a reservoir Find the speed of water in the pipe, (1) in feet per minute, (2) in miles per hour [Take 1 cub ft = $6\frac{1}{4}$ gallons, area of circle = $3\frac{1}{7} \times (\text{radius})^2$]

3 If a company earns a profit of 24 % on its capital, and out of this profit pays a dividend of 7 % on the capital, places £1600 to a reserve fund, and carries forward £1375 to next year's account, find the amount of capital.

BOARD OF EDUCATION PRELIMINARY EXAMINATION FOR CERTIFICATE

1912 ($1\frac{3}{4}$ hours)

1 Find to the nearest centime the simple interest on 32,127 francs for 4 years at $3\frac{1}{2}$ per cent (1 franc = 100 centimes)

2 You are given that $271352 = 13 \times 20873\frac{3}{13}$ State to five significant figures the quotient when $\cdot0271352$ is divided by $\cdot013$.

3 A number of square tiles are arranged in one large square. How many tiles are used in making the large square if the total number along its two diagonals is 37?

4 A cricketer played 37 innings in Australia during the winter and 41 innings in England during the summer. His average number of runs in an innings was $18\cdot25$ in Australia and $20\cdot12$ in England. Calculate to two places of decimals his average for the year.

5 Find the smallest whole number by which 12096 must be multiplied that the product may be a perfect square

6 From a cask of spirits one-hundredth part is withdrawn and replaced by water From the mixture one-hundredth part is drawn and replaced by water and a similar operation is again performed. Find the percentage of spirit left

7 A pint mug weighs 7 ounces when empty and 2 lbs 3 ozs when full of treacle By how much does the weight of a gallon of treacle exceed the weight of a gallon of water?

8 A rectangular table is 5 feet 9 inches by 4 feet 3 inches A rectangular cloth 8 feet 6 inches by 3 feet 10 inches is spread unfolded on the table so that the longer sides of cloth and table are parallel. What fraction of the area of the table top remains uncovered by the cloth, and what fraction of the cloth hangs perpendicularly over the edges of the table?

9 Twenty-five children are making cardboard trays A square of side 8 inches is used for each tray. Find the total amount of cardboard used, if each child makes one tray

The cardboard was bought in sheets 18 inches long and 10 inches wide. How many sheets would be required and how much cardboard would be wasted?

10 The number of school children in a town increases by 4 per cent in a year How many children will there be at the end of two years if there are now 36,250 children at school?

11 Work the following questions as shortly as you can —

(i) Find the cost of 167 articles at 1*s* 8*d* each

(ii) Simplify $\sqrt{112} \times \sqrt{175}$

(iii) Which is the greater $\dfrac{3\frac{1}{2}}{7\frac{1}{3} + \frac{1}{11}}$ or $\dfrac{3\frac{2}{5}}{7\frac{1}{3} + \frac{1}{10}}$?

(iv) A bicyclist travels at the rate of 12 miles an hour How far will he travel in 5 hours $52\frac{1}{2}$ minutes?

12. Draw a rough diagram to illustrate the usual steps in the addition $\frac{1}{3} + \frac{1}{4}$, showing clearly that the sum is $\frac{7}{12}$.

(*From the Paper on Mathematics*)

1 A rectangular grass plot, which is 23 feet long and 18 feet wide, is bordered by a gravel walk 4 feet wide Find the cost of the gravel at 4*s* 6*d* per cubic yard, if the walk is gravelled to a depth of 3 inches.

2 A railway passes in succession through three towns, A, B, C. The fare from A to B is 1*s* 6*d*, from A to C 1*s* 11*d*, and from B to C 6*d* For all journeys up to and including 20 miles the railway company charges at the rate of 1*d* per mile, whilst for every mile thereafter it charges at a uniform lower rate What is this rate?

3 A watch chain consists of 65 circular links The outside and inside diameters of each link are 7 and 4 8 mm respectively Calculate the extreme length of the chain What would be the length if one link were cut off?

BOARD OF EDUCATION SCIENCE EXAMINATIONS, 1912

(*From the Papers in Practical Mathematics*)

From the Lower Paper

1 Without using logarithms compute by contracted methods

$$3{\cdot}207 \times 0{\cdot}01342 - 9{\cdot}415$$

2 A hollow circular cylinder of cast iron is 10 inches long and 3 inches inside diameter, what is the outside diameter if the cylinder weighs 30 lb? [One cubic inch of cast iron weighs 0·26 lb]

From the Higher Paper

Without using logarithms, compute by contracted methods so that four significant figures *shall be correct*, $10\ 32 \times 0\ 005231 - 0{\cdot}02076$

CIVIL SERVICE EXAMINATIONS, 1912

BOY CLERKS

First Paper (1 hour)

1 The population of one village is 3597 and that of another is 2909. How many people are there in the two villages together?

2 In an institution there are 540 patients How many wards are required if each ward can accommodate 36 patients?

3 A motor track is 2 miles 3 furlongs in length Express this in yards

4 The line below is the width of the cover of a book Measure its length, as shown by the distance between the two small cross-pieces, in

metric measures as accurately as you can and write it down in your answer book

5. The length of the cover in Question 4 is 1·3 times the width Find the length to the nearest millimetre

6 Using your results in Questions 4 and 5, draw a figure of the actual size of the cover of the book.

7 What is the cost of $5\frac{1}{8}$ pounds of meat at a shilling a pound?

8 How many pieces of silk each $3\frac{1}{2}$ yards long can be cut from a piece of this material 25 yards long?

9 What is $\frac{3}{7}$ of £2, 10*s* 8*d*? Give your answer to the nearest penny.

10 A grocer has a stock of $5\frac{1}{2}$ cwt of cheese If he sells 30 per cent of his stock, find the weight sold in pounds, to the nearest pound

11 A vat of liquid contained 81 gallons After some time it was found that owing to a leak only 76 gallons 2 quarts remained. Express the amount left as a percentage of the original quantity.

12 Find the cost of 17 litres 60 centilitres of spirit at 4*s* 7*d*. a litre

13 An aeroplane flies 22 miles in 27 minutes 40 seconds How far does it travel in one hour at this rate? Give your answer to the nearest mile

14 The area of the top of a rectangular box is 1 sq. metre 7 sq decimetres If the height of the box is 7 decimetres, find the volume of the box in cubic decimetres

15. The base of a rectangular cistern is 4 ft 3 in by 3 ft. 10 in To what depth will a cubic yard of water fill the cistern? Give your answer to the nearest inch

---

Second Paper ($1\frac{1}{2}$ hours)

(*You are not restricted to arithmetical methods*)

1 Two men A and B, who had previously lived alone, agree to share rooms and divide the expenses equally, and their joint expenses for board and lodging for a year are £225, 12*s*. If A's average weekly expenditure for board and lodging had previously been 52*s* 6*d* and B's 49*s*. 6*d*, find how much each saved in the year (52 weeks) by their living together.

2 A room measures 18 ft by 12 ft Taking a side of a square in your book to represent a foot, show by means of a diagram how many square yards there are in the floor of the room

The floor is covered by a carpet measuring 16 ft 6 in. by 11 ft. 3 in. which costs £7, and a surround of linoleum costing 3*s*. 3*d*. a square yard. Assuming that an exact number of square yards of linoleum must be bought, and that there is a charge of 2*d* per square yard of floor space for laying carpet and linoleum, find the total cost

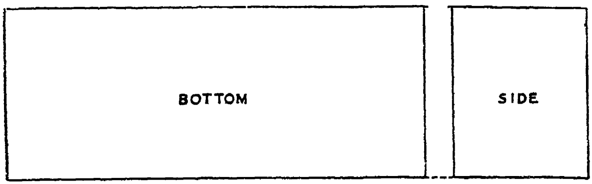
3. The tunnel recently opened at Woolwich is a tube 1635 feet long with an inside diameter of 11 ft 2 in A tube L feet long with an inside diameter of D feet has a volume of $0{\cdot}7854 \times L \times D \times D$ cubic feet.

Find the volume of the tunnel in cubic feet Give your answer to the nearest thousand

The tunnel took 25 months to complete How many cubic yards were finished on the average per month? Give your answer to the nearest integer

4 A man cycling at $7\frac{1}{2}$ miles an hour meets a body of troops which is $1\frac{1}{3}$ miles long and is marching at $2\frac{1}{2}$ miles an hour How long does he take to pass it?

5 The figures below show the bottom and side of a rectangular block of wood drawn to a scale of one-tenth, that is, each line represents one-tenth of the actual length The block is cut up into smaller rectangular



blocks, by cuts which divide each edge into three equal parts How many of these blocks will there be, and what will be the dimensions of each in centimetres? If the block weighs 19 kilograms, what is the weight (to the nearest tenth of a kilogramme) of a cubic decimetre of the wood?

WOMAN AND GIRL CLERKS (2 hours)

1 A plot of land which measures 21 acres 1 rood 14 poles is sold at £53 an acre Find the amount of the bill

2 A piece of wood is 10 ft 6 in long, $4\frac{1}{2}$ in across, and $3\frac{1}{4}$ in deep What is its volume, and how much will it weigh if a cubic foot of the wood weighs 53 lb?

3 The total amount of wheat imported into the United Kingdom in the year 1910 was 105,222,638 cwts, and of this amount 48,116,138 cwts were received from countries of the British Empire What percentage, to the nearest integer, of the total amount imported was obtained from countries of the British Empire?

Find the value, to the nearest million pounds, of the total imports of wheat in the year mentioned at the rate of 35s 6*d* per quarter, assuming that on the average the wheat weighed 62 lbs per bushel

4 A train travelled via the Severn tunnel from Newport to Paddington, a distance of $133\frac{3}{4}$ miles, in 2 hours 30 minutes The time of passing through the tunnel, which is 4 miles 636 yards long, was 6 minutes 35 seconds

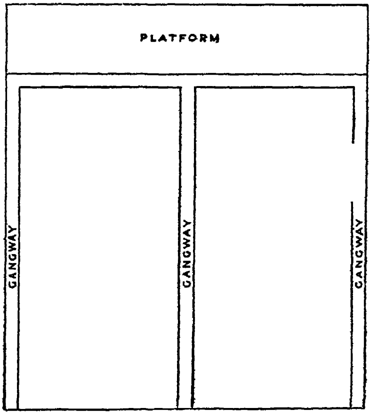
Taking the journey as a whole, what was the average number of feet the train passed over in one second? What was the average number of feet the train passed over in one second in its passage through the tunnel? Give your answer in each case to the nearest whole number

5. The number of cubic feet in a parcel of uniform circular section may be calculated from the formula: $0 \cdot 0796 \times g \times g \times l$, where g is the number of feet in the girth and l the number of feet in the length of the parcel. The dimensions of three parcels of this shape were as follows.—

(i) girth, 43 inches; length, 29 inches;
(ii) girth, 48 inches; length, 24 inches;
(iii) girth, 53 inches, length, 19 inches

Find which of the three parcels contained the greatest number of cubic feet, and find what this number was to the nearest tenth

6 The figure below is the plan of a hall, every distance in the hall being 400 times the corresponding length on the plan Two rectangular blocks, indicated on the plan, are to be filled with rows of chairs. For each chair allow a side-to-side distance (or breadth) of at least 51 cm, and for each row a back-to-front distance (or depth) of at least 65 cm From measurements of the plan find how many chairs there can be in one row of a block, how many rows each block will hold, and how many persons can be seated in the two blocks.



ANSWERS

TO THE EXERCISES OF PART I

I. **1.** Thirty *thousand*, eight hundred **2.** Fifty one *thousand*, and twenty.
3. Eighteen *thousand*, two hundred **4.** Seventy *thousand*, and twenty
5. Three-hundred-and-eighty-four *thousand*, six hundred-and ten
6. Four-hundred-and-twenty-three *thousand*, six-hundred-and fifty-four
7. Two *million* **8** Three *million*, three *thousand*.
9. Seven-hundred *thousand*, one-hundred
10. One-hundred-and-sixty *thousand*, and eleven
11. Five-hundred-and-three *thousand*, and ten
12. Six-hundred-and-seventy-five *thousand*, and thirty-two
13. Three *million*, five-hundred-and-three *thousand*, and seventy-one.
14. Four *million*, four *thousand*, and four
15 Thirty-two *million*, five *thousand*, seven-hundred-and-one
16. Sixty *million*, six-hundred *thousand*, seven-hundred-and-fifty-two.
17. Ten *million*, one-hundred-and-one *thousand*, and ten
18. One-hundred-and-twenty-two *thousand*, and twelve
19. Thirty-three *million*, three-hundred-and-thirty-three *thousand*, three-hundred-and-thirty-three. **20.** Fifty *million*.
21 Eighteen *million*, eight-hundred *thousand*, and eight.
22. Nineteen *million*, ninety-nine *thousand*, and ninety.
23. Five-hundred-and-sixty-seven *million*, eight-hundred-and-two *thousand*, and five **24.** Ten *million*, seven *thousand*, and sixty-three.
25 Three-thousand-seven-hundred-and-fifty-six *million*, four-hundred-and-twenty-one *thousand*, eight-hundred-and-seventy-one
26. Eight-thousand-three-hundred *million*, two-hundred-and-thirty-five *thousand*, and seven.
27. Three-hundred-and-eight-thousand-and-fifty-six *million*, three-hundred *thousand*, and seventy-two **28** One *billion*
29. Two-hundred-and-thirty-six-thousand-seven-hundred-and-fifty *million*, eight-hundred-and-forty-six *thousand*, three-hundred-and-seventy-four.
30. Three *billion*, eight-thousand-seven-hundred *million*, fifty *thousand*.
31. 17,020 **32.** 102,700. **33.** 650,000. **34.** 543,011.
35 3,700,070 **36.** 13,001,300 **37.** 5,694,387 **38** 23,000,110.
39. 401,040,440 **40.** 768,675,586 **41** 80,018,000 **42** 7011,060,300
43 1003,001,003. **44** 49281,550,660 **45** 100070,014,008
46. 360000,306,036. **47.** 200020,020,200 **48.** 1,000001,001,001
49 17,890045,796,016 **50** 19,001090,100,900. **51** 19. **52.** 24
53. 58 **54.** 45 **55.** 76. **56.** 68. **57.** 49. **58.** 83
59 104 **60.** 160 **61.** 89 **62** 95 **63** 112 **64** 204
65 166 **66.** 144 **67** 284. **68** 540 **69.** 390. **70.** 555
71 790. **72** 679 **73.** 1764. **74.** 1647 **75** 1885 **76.** 1566.
77. 1588 **78** 1666 **79.** 1789 **80.** 1899 **81.** XXXIV.
82. XLVI **83.** LXXIII. **84.** LXXXVII **85.** XCIX.

86 CCIII 87 DLV 88. CCCCXXIII., or CDXXIII
89. CCCXXIV 90 DLX 91 MCCXLI 92 MCCCXXXVI
93 MDCCLIX 94 MDCCCLXVII 95 MDCCCXCI
96 MC 97. MCCCCXIII, or MCDXIII 98 MDCLXXXIX
99. MDCCCLXXXVIII 100 $\overline{\mathrm{V}}$

II. 1 74040 2 258704 3 295856 4 673753
5. 676758 6 309361 7 450192 8 489952 9. 516526
10 1866945 11. 8423335 12 14492458 13 43520 14. 292112
15. 405452 16 58569 17. 1691716 18. 969785 19 7051968
20 88126046 21 181. 22. 338 23 2206 24 1172
25 334 26. 318 27 251 28 971 29. 5280
30 8689 31 1331 32 1459 33 8535 34 4127
35 1583 36 15513 37. 9077 38. 591405 39. 15293.
40 5652 41. 65711 42 48392 43 44080 44. 90195
45 24947 46 41439 47 18011 48 45116 49. 39867
50 971004 51 6271 52 2356 53 569 54. 22456
55 10270 56 16440 57 58024 58 864198 59 656546
60. 6498 61 714285 62. 666556 63 89683 64. 296355
65 882 66 15336 67 125 68. 36971 69 9628
70 11107 71 24482 72. 17224 73 2899 74 83805
75 3254 76 35647 77 634 78 9268 79. 20988
80. 1797 81 74812 82 13605 83 42287 84 38007
85 55162 86 121144 87. 214303 88 968000 89 99593496
90 2998899993 91 427 92 17 93. 2. 94. 2
95 2 96 64 97. 24 98 29 99 6 100 2
101 23 102 8 103 263 104. 9 105. 13 106 42
107 156 108. 478 109. 5271 110 5231 111. 1 112. 6
113 1633 114 848 115 359315 116 21717 117 216129
118 9278 119. 418828 120 8007 121 870 122 5600
123 19000. 124. 1750 125. 3000 126 70000 127. 15000
128 3080 129 43000 130 10020 131 170000 132 40300
133 10740 134 703000 135. 50000 136 710000 137 2700000
138. 8000000 139 1111111101. 140. 8888888889 141 11999988
142 6999993 143. 7777777707 144. 35555555556 145 682670832
146 266634058 147 103351500 148 1636529400
149 80612690400 150 26876148300 151 91132497000
152 38518802300 153 1676312100. 154. 7454185680
155. 19723000000 156. 291251870100 157 70179941000
158 80683213020 159 12014402400000. 160 606000134241
161. 7544676; 1234000. 162. 60768396, 520426776 163 27964417
164. 45808416423 165. 1700 166 24000 167 80000.
168. 75000 169. 30000 170 810000 171. 470 172 890
173 7900 174 100 175 700 176. 2300 177. 23868
178 57855 179 40905480 180 13838590116 181. 226834247640
182 375645200000 183 361 184 3025 185. 10609.
186 14100 187 25921 188 55225 189 9659664.

190	21996100.	191.	29506624.	192.	60481729	193.	4913.
194	9261	195.	262144	196	941192	197.	9938375
198.	87528384	199	247673152000	200.	1371330631.		

	quotient rem^r		quotient rem^r		quotient rem^r.
201.	42 + 3	207	350	213.	301 + 1
202.	76 + 81.	208.	700 + 6	214.	270 + 70.
203.	4 + 865	209.	8 + 46	215.	58000
204.	203.	210.	760 + 8.	216.	34 + 50
205.	51 + 40.	211.	360 + 1	217.	4 + 6321.
206.	470.	212.	2340.	218.	120 + 8

219.	62321 + 4	225.	4321 + 11	231.	1854 + 11
220.	86730 + 5.	226.	1931 + 12	232.	19026 + 13
221.	295064 + 1	227	1429 + 27	233.	7981 + 40
222.	61864 + 3	228.	6094 + 92	234.	1219 + 77
223.	705442 + 3	229.	9736 + 851	235.	657 + 6643
224.	55733399 + 7	230.	7610 + 407	236.	9704 + 3081.

237.	3800 + 1	245.	1882 + 1982	253.	411 + 72700
238	7300 + 14	246	17337 + 1288	254.	12 + 51524
239.	183003	247.	285340 + 4322	255.	30 + 51
240	80307	248.	3759 + 29556	256.	2016 + 15000
241	19759 + 2	249	160401	257.	24 + 214302.
242.	24582 + 41.	250.	14000014	258.	1263 + 1100
243.	4560	251	1680 + 51.	259.	178 + 2728
244.	30079.	252.	18 + 113	260.	788 + 2177

261.	582203 + 13	265.	26317 + 29	269.	39597 + 3
262.	161674 + 9	266	15885 + 22	270.	28957 + 38
263.	38231 + 7.	267.	187323 + 27.	271.	1721 + 13.
264	2192 + 9	268.	2219 + 1	272.	5910 + 42.

273.	8433 + 15	281.	200 + 11	288.	41 + 815
274.	124955 + 32.	282	258 + 43	289.	232 + 911.
275.	188033 + 63	283	396 + 332	290.	251 + 1341.
276.	268 + 7	284.	1089 + 212	291.	62 + 341.
277.	1128 + 53	285	127 + 245	292.	153 + 523.
278	1758 + 125	286	46 + 1206.	293.	1818 + 4887.
279.	29403977 + 74.	287	752 + 178.	294.	1447 + 2693.
280.	6730243 + 106				

295.	77	302.	207	309.	130130	315.	86
296	304.	303	15067 times	310.	7575	316.	6758
297.	34.	304	83205 times	311	609.	317	74274.
298.	72	305	8	312.	70905	318	317450
299	312 times	306.	325	313	763	319.	689902
300.	340 times	307.	15607	314.	560	320	66579051.
301.	1858	308.	6702.				

III. 1. Seven *million*, seven hundred and seven *thousand*, and seventy
2 13,050,011 3. 43795 4 33018
5. 20536900 6 1412884 + 500 rem^{r} 7 11210 + 4 rem^{r}
8 12 9 428 pages 10. 80 nuts
11. Forty-one *million*, three hundred and five *thousand*, and eighty-seven
12 17 450,302 13 2999997 14 295654
15. 3482569401472 16 1388 17 27 rem^{r}
18. 2270 19 732 years 20 63840 letters
21 One thousand seven hundred and seventy-seven 22 MDCCCLI.
23 107170. 24. 198000 25. 14395553 26 7007 times
27 226981. 28 4578 29 28 years 30 81 seats
31. 1413 32. MDCIII 33 26545 34. 550440033
35 76649903 36 3003 times 37 49885969 38 30081
39. 65 years 40. 8581 41. Five hundred thousand Five hundred
42. 12 43 742481 44 21206628450 45 1055841
46. 74000 47 3 48. 650 + 56803 rem^{r} 49 2499
50 380 51 Thirty thousand Thirty 52 26
53 74137 54 3053656476 55. 92055 56 230
57. 5 58 1677200 59 8 60. 8421
61 Two thousand three hundred *million*, twenty-five *thousand*, and fifteen 62 390 63 4227321, 385470, and 34264000
64 61146 + 92 rem^{r} 65 7903000 66 312 67 145 times
68 $40^2 = 1600$, $9^2 = 81$, $41^2 = 1681$ 69 189 figures 70 17 marbles
71 13,400,057 72 106 73 4474728 74. By 120
75. 370 + 53 rem^{r} 76. 42310, 4231, and 33848 77 43 times
79 9999 80 294 81. DCCCLXXIV
82 3,500,000 83 593496 84 5865436 + 2067 rem^{r}
85 39906 + 11 rem^{r}, 399 + 113 rem^{r} 86 15 87. 49788
88. By 4 89. 43 years 90 1080 yds 91 23,030,005
92 40 93 10675 94 2308256 95 740 + 221 rem^{r}
96 By 507 97 456 + 123 rem^{r} 98 174
99. 32 pages 100. 312 animals 101 Seven thousand three hundred and six *million*, two *thousand*, and fifteen
102 15 + 30432 rem^{r} 103 By 457327 104. 13
105 71 years old 106. $41^3 = 68921$, $236^2 = 55696$, $115^2 = 13225$
107 6113 108 107 109 867537 110 929325
111 Ten thousand eight hundred and sixty-one 112 492 figures, including 1 and 200 113 11441476 114 578 boxes
115 740480 letters 116 6 marbles 117 38 years old 118 1
119 306504 120 1080375 121 Three *sets of ten units* + four *units*
122 Seventy thousand and seven *million*, seven *thousand*, and seventy-seven 123 410921984 124 [illegible]20 125 86 years old
126 210 and 30 127 2100 128 442 129 7856136 130. 1795164
131 By 49950 132 75 and 68 133. 209 134 90 ladies
135 131 137 707594 + 770 rem^{r}. 138 58 139 42 and 55
140 992817 141 Ten thousand seven hundred and forty-nine
142 109999 143 By 888 144. Divisor 7087, rem^{r} 1569
145 6708 146 2454 147 2 more 148 86 149 37990092
150 376728 151 448847 152 2662 153 623

154 to 240. See page v.

IIIᴬ. 1. Three *tenths*, Seven *tenths*, Nine *tenths*; One *hundredth*, Four *hundredths*, Six *hundredths*, Two *thousandths*, Five *thousandths*; Eight *thousandths*

2 Two, and one tenth; Three, and seven hundredths, Five, and nine thousandths, Twelve, and six tenths, Twenty, and four tenths, Two, and four hundredths, One hundred and three, and five tenths

3 ·4; ·7; ·9	6 4·1, 7·5
4. ·03; ·08, ·05	7. 3·07, 6·004.
5 ·001; ·006, ·002.	8. 20·5; 13·09.

9. Ten; Thirty, Twenty; Thirteen, Forty-six, Thirty-seven, *tenths.*

10. Ten; Seventy, Thirty; Eighty; Fifteen; Seventy-four, *hundredths.*

11. Ten, Eighty; Twenty, Sixty, Ninety, Fourteen, Twenty-eight; Sixty-five; One hundred and twenty-three; Five hundred and eight; Four hundred and thirty; Five hundred and ninety, *thousandths.*

12. ·23, ·52	14. ·208.	16. 2·17; 5·041.
13. ·147.	15. ·034, ·019	

17. 3·4, 6·7, 21·5, 48·3, 27, 31, ·6, ·9, 4, 8, ·03, ·08.

18. 345·6, 73·1, 120·5, 3·5, 461, 7, 20, 530, ·9, 106

19. ·14, ·35, ·436, 3·72, ·09, ·03, 1·703, 1·07, ·001, ·075.

20. 2·467, 3·018, ·425, ·213, ·084, ·035, ·002, ·007, ·0015, ·0006.

21. 9·43	26. 24·795	31. 7·79.	36 6·885.
22 7·008	27. 19·92	32. 1·396	37. ·45.
23. 20·4424.	28. 44·2	33. ·346.	38. 3·16.
24. 115·2107.	29. 26	34. 8·775.	39. 7·545.
25 63·617.	30. 21·3.	35 ·175.	40. 6·3.

41 9·025. 42. 24·24. 43. 1·85 44. 3·26

45. 14·82.	48 23·828.	51. 126·28.	53 209·92.
46. 12·375.	49. 13·395.	52. 548·1	54. 15184.
47. 163·8	50. 1329·9.		

55 8 647.	**59** 1·47	**63** 3·41	**67.** 2·781.
56 ·316	**60** ·017	**64** ·403	**68** 027
57 ·088	**61** ·375	**65** 1·231.	**69** 2 067.
58. ·008.	**62.** ·012	**66** 625	**70** ·083

III^B. 1. (i) 3000, (ii) 70, (iii) 200,
2 (i) 5750, (ii) 4500, (iii) 63,
3. (i) 1685, (ii) 950, (iii) 500, (iv) 60, } metres
4. (i) ·358, (ii) 25, (iii) ·028, (iv) 005,
5. (i) ·024, (ii) ·17, (iii) ·205, (iv) ·0045, } kilometre
6. (i) 1700, (ii) 245, (iii) 20, (iv) 8,
7 (i) 130, (ii) 4800, (iii) 2500, (iv) 25, } centimetres
8. (i) ·12, (ii) 08, (iii) ·7,
9 (i) ·045, (ii) ·62, (iii) ·436, (iv) ·02, } metre
10 (i) 98·7, (ii) 6·05, metres
11 (i) 300, (ii) 4200, (iii) 1050, millimetres
12 (i) 13000, (ii) 4075, (iii) 1650, grammes
13 (i) 1·85, (ii) 4 36, (iii) 3·025, kilogrammes
14 (i) 300, (ii) 215, (iii) 3250, litres
15 (i) 4·35, (ii) 15·8, (iii) 70, hectolitres
16 22 *Km* 611 *m* **17** 25 *m* 42 5 *cm* **18.** 21 *Kg* 615 *g*
19. (i) 2 *m.* 85 *cm*, (ii) 29 *m* 30 *cm.* **20** (i) 6 *Hl* 42 *l*, (ii) 1 *Hl* 15 3 *l*

21 105 *m* 60 *cm*	**23** 35 *Km* 360 *m*	**25** 314 *Kg* 500 *g*
22 137 *m* 70 *cm*	**24** 83 *Km* 400 *m*	**26.** 98 *Hl* 40 *l*
27 29 *m* 63 *cm*	**29** 4 *m* 8 *cm*	**31** 23 *Kg* 250 *g*
28 2 *Km* 850 *m*	**30** 4 *m* 3·2 *cm*	**32.** 7·4 litres

33 453 *m*	**35** 825	**37.** 892, 12 *cm*	**39** 131
34 6·2 *cm*	**36** 265	**38** 153, 55 *cl*	**40** 477.

41. 38 *fr* 85 *c*	**47.** 97·5 *c*	**53** 5 *fr* 60 *c*
42. 72 *M* 20 *pf.*	**48** 30 *fr* 37·5 *c*	**54.** 2 *fr* 25 *c*
43. 62 *fr* 25 *c*	**49** 13037·5 *fr*	**55** 4 *fr* 30 *c*
44. 94 *fr.* 80 *c*	**50** 39 *M* 77·5 *pf*	**56** 1230 *M*
45 90 *fr* 45 *c*	**51** 29 *fr* 80 *c*	**57.** 57 *Kg*
46 11 *fr* 37·5 *c.*	**52** 144 *M* 50 *pf.*	**58** 1575

59. 60 *fr.* 19 *c.* **60.** 47 *fr* 60 *c.* **61.** 6 *fr.* 22 *c.* **62** 34 *pf.*

154. (i) 137, (ii) 5421 and 1807 155. 937 156. 93000 157. 23
158 2037 and 679 159 125 160. 116571375 161. 83952.
162 83205 163. 25000 164 19 boys 165 3974 and 3423
166. 76 and 33 167 3615 and 2892 168. 4 169. 10379355.
170 376731 171 29732 172. 412776 173. By 97
174 24 men 175 73 and 37 176 7 177. 13 and 10
178 19 179 1171000 180 1338776 181 2,000,020,000,200
182. 302 183 862 184 2375776242 185 2478 186. 6 games
187 16 188 336 189 89 190. 24429215144 191 $\overline{\text{VI}}$DCLXVI
192. 656100810001 193 61 194 337 195 8 girls
196 3034 and 2081 197 A has 36; B has 7, C has 46 198 1400
199 2362266 200 63 201 6140 202. 7700.
203 4763, 763; 63 204 6715 205 47. 206. 9
207 2449 persons 208 34635330 209. 43 210. 22653 + 2 rem^r
211 54255219 212. 2259159 213 57980 214. 650470
215 6543 and 2222 216. 33 and 18 217 56 218. 4378444
219. 375 + 857 rem^r 220 144094291992 221 4 groups of *thirty-six* + 2 groups of *six* + 3 units, *i e*, one hundred and fifty-nine units 222. 98980001 223. 3227 + 40 rem^r

224. 47) 3189 (67
282
369
329
40

225. 857142 and 142857
226 21½ years old
227 3905; 3344, and 2203

228 364893507
229. 4679 + 64 rem^r
230. 88808256828.

231. 1 group of *one hundred and twenty-five* + 2 groups of *twenty-five* + 3 groups of *five* + 4 units, *i e* one hundred and ninety-four units.

232. 10219567
233. 118 See Example on p 14 of Theory
234. 173042

235. 457
38
3656
1371
17366

236. 17 boys; 52 girls
237. 117.
238. Either 23475, or 513.
239. 35130711336
240. 181170

IV. 1. 109339 farthings 2 786926 farthings 3 1067133 farthings.
4. 959991 farthings 5. 17373 halfpence 6. 22817 halfpence.
7. 96849 halfpence 8. 106613 halfpence. 9 2399 threepences
10 3494 threepences 11. 8709 threepences. 12. 35307 threepences.
13 1073 fourpences 14. 5306 fourpences 15 8758 fourpences
16 18127 fourpences 17. 2071 sixpences 18 3437 sixpences.
19. 10303 sixpences. 20 26653 sixpences 21. 5729 florins.
22. 6317 florins 23 30518 florins 24. 42354 florins.
25. 103 crowns 26. 273 crowns 27 1890 crowns
28. 3334 crowns 29 1883 half-cr 30 5145 half-cr
31. 3661 half-cr 32. 2610 half-cr 33 5775 half-cr
34. 6644 half-cr 35. 3379 three halfpences 36. 3419 threepences.
37. 154863 half-sovereigns. 38. 4819 double florins.

39 £175, 2s 11d 40. £153, 18s 5d 41 £104, 3s 4d
42 £462, 19s 3d 43 £10, 8s 4d 44 £15, 19s 8½d
45 £125, 12s 6½d 46 £118, 15s 1½d 47 £57, 0s 10d
48 £161, 19s 8d 49. £277, 15s 6d 50 £972, 4s 3d
51 £938, 12s 6d 52 £215, 1s 6d 53. £1237, 2s
54 £1765, 18s 55 £121, 15s 56 £138, 15s
57 £154, 5s 58 £540, 2s 6d 59 £954, 15s
60 £1213, 10s 61 £1275, 17s 6d 62 £2500, 12s 6d
63 £948, 4s 64. £15410, 10s 65 £17, 6s 4½d
66. £117, 6s 1½d 67 37401s 68 97251s
69. 119196d 70. 176400f 71 11772 twopences
72 5104 sixpences 73. 14610d 74 104040 halfpence
75 148260 threepences. 76 16475 sixpences 77 844 hf -cr +2s
78. 37320f 79. 157 fl +1s 1½d 80 374 hf -cr +1s 2d
81. 116 gs 82 32 hf -sovs +2s 11d 83 9655 fl
84 184 cr +4d 85 99 gs +4s 4d 86 17 gs +13s 4d
87 £4662 88. £8757 89 £3016, 13s 90 £13027, 7s
91. 13320 gs 92 15540 gs 93 38696 gs +4s
94 34288 gs +12s 95 13143 fourpences 96 3388 sixpences+4d
97 17577s. 6d 98 7110 hf -cr +1s 99 95 hf -gs +6d
100. 7740 fourpences 101 843140 fl 102 529376 hf -cr +1s
103 7310 hf -gs 104. 2704 hf -gs 105. 796110 hf -cr 106 579065 fl
107. £2, 1s 8d 108 £6, 5s 109 1200 110 180
111 £41, 4s 112 1141 people 113 39 five-pound notes
114 76 pencils 115 £13, 15s 116 £15, 17s 8d 117. 70422 secs
118 85265 secs 119. 25307 min 120 33819 min 121 1074 hrs
122. 927 hrs 123 31505 hrs 124 22069 hrs
125 1104587 min 126 2368813 secs 127 8 hrs 43 min 43 secs
128 15 hrs 50 min 29 secs 129 12 days 11 hrs 22 min
130 14 days 10 hrs 5 min 131 3 days 17 hrs 7 min 21 secs
132 5 days 13 hrs 4 min 33 secs 133 47 wks 1 day 11 hrs 45 min
134. 45 wks 4 days 19 hrs 13 min 40 secs
135 2 yrs 223 days 4 hrs 13 min 136 3 yrs 299 days 4 hrs 3 min
137 1464 hrs 138. 131040 min 139 56 hrs 140 108 hrs
141 585 min 142 9600 secs 143. 243 days 144 274 days
145. 1826 days 146 17544 hrs 147 1945 lbs 148 1116 lbs
149. 631 ozs 150. 1021 ozs 151 23517 ozs 152 30604 ozs
153 38436 lbs 154 53439 lbs 155 1511 ozs 156. 5047 drs
157 95671 ozs 158. 272347 ozs 159 7654567 drs 160 30009615 drs
161. 31 stones 162 1242 stones 163. 3583 lbs 164. 9369 lbs
165. 10454 ozs 166 33691 ozs 167 1799000 grs 168 4431000 grs
169 2 qrs 13 lbs 7 ozs 170 3 qrs 9 lbs 15 ozs 171 17 lbs 1 oz 3 drs.
172 19 lbs 10 ozs 5 drs 173. 4 tons 15 cwt 3 qrs 5 lbs
174 21 tons 4 cwt 2 qrs 24 lbs 175. 9 cwt 0 qrs 18 lbs 1 oz
176 10 cwt. 3 qrs 7 lbs 6 ozs 177. 19 cwt 3 qrs 8 lbs 6 oz 14 drs
178 27 cwt 0 qr 24 lbs 10 ozs 3 drs 179. 12 cwt 3 qrs. 14 lbs 3 oz
180 18 cwt. 2 qrs 11 lbs 5 oz 181 1 ton 4 cwt 2 qrs 4 lbs 2 ozs
182. 2 tons 4 cwts 3 qrs 7 lbs 3 ozs 183 18 tons 1 cwt 3 qrs 4 lbs 7 ozs
184 132 tons 19 cwt. 0 qr 15 lbs. 15 oz. 185. 213 tons 13 cwts. 1 qr 17 lbs 2 oz.

186. 2 tons 0 cwts 0 qr. 1 lb 8 ozs 10 drs **187.** 9 tons 13 cwt 3 qrs 1 lb 6 ozs 3 drs. **188.** 16 tons 0 cwt. 0 qr 15 lbs. 11 ozs 9 drs.
189. 20981 grs **190.** 43855 grs **191.** 34512 grs **192.** 14155 grs
193. 18100 grs **194.** 22017 grs **195.** 1 lb Tr 3 ozs 13 dwt 13 grs
196. 4 lb Tr 5 oz 3 dwt 13 grs. **197** 15 lb Tr 3 ozs 0 dwt. 23 grs.
198. 182 ozs. Tr 416 grs **199.** 132 ozs Tr. 385 grs
200. 188 ozs Tr 431 grs **201.** 647 in **202.** 1063 in
203 2037 in **204** 3202 in **205** 7420 yds. **206.** 23936 yds
207. 18635 ft **208** 29776 ft **209** 41764 ft. **210.** 56000 ft.
211. 12 yds 0 ft 9 in **212** 19 yds 0 ft 9 in. **213.** 36 yds. 0 ft 9 in.
214. 57 yds 1 ft 7 in. **215.** 43 mi 861 yds **216** 48 mi 83 yds
217 6 mi 1633 yds. 2 ft. **218.** 9 mi 123 yds 2 ft
219. 5 mi 1153 yds 1 ft 3 in **220.** 12 mi 131 yds 0 ft 9 in
221. 132 yds **222** 209 yds. **223.** 151 yds **224** 219 yds.
225. 307 ft **226.** 380 ft **227** 341 hf -yds **228.** 407 hf -yds.
229. 217 hf -yds **230.** 314 hf -yds. **231** 7128 in **232.** 7632 in.
233 4635 in **234.** 7373 in **235** 3529 in **236** 5954 in
237. $125\frac{1}{2}$ ft. **238.** $162\frac{1}{2}$ ft **239.** 228 ft **240** 292 ft
241. 15283 yds **242.** 58741 ft **243.** 94381 ft **244.** 33779 yds.
245. 316827 in. **246.** 13278 yds. **247.** 13600 yds **248.** 36711 ft
249. 643548 in. **250.** 117851 ft **251.** 233094 in **252.** 94578 in.
253. 205862 in. **254.** 1104786 in. **255.** 1635033 in **256.** 2768645 in.
257 93 po 5 yds. **258.** 89 po. $3\frac{1}{2}$ yds **259.** 326 po $\frac{1}{2}$ yd.
260. 239 po $2\frac{1}{2}$ yds. **261.** 484 hf -yds. **262.** 1284 hf.-yds.
263. 1320 hf -yds **264.** 3268 hf.-yds **265.** 186 po. **266.** 126 po.
267. 133 po. $3\frac{1}{2}$ yds. **268.** 294 po. **269.** 155 po $3\frac{1}{2}$ yds.
270. 224 po 2 yds **271.** 8 po 4 yds. 2 ft 11 in.
272. 14 po. 3 yds 2 ft. **273.** 18 po 5 yds 1 ft. 3 in.
274. 34 po. 4 yds 1 ft **275.** 16 po 3 yds **276.** 23 po 0 yd. 1 ft 6 in
277. 30 po 0 yd 1 ft **278.** 30 po 2 yds 2 ft. **279.** 34 po. 3 yds 1 ft.
280 41 po. 3 yds 1 ft 6 in **281.** 2 mi 3 fur. 21 po 2 yds 2 ft. 3 in
282. 3 mi 6 fur 22 po 0 yd 2 ft 5 in **283** 4 mi 1 fur 36 po 1 yd. 1 ft 7 in
284. 8 mi. 2 fur. 26 po 0 yd. 2 ft. 3 in. **285.** 10 mi. 5 fur. 5 po 5 yds. 0 ft. 11 in. **286** 49 mi. 2 fur 31 po 3 yds 1 ft 11 in.
287. 147 mi 6 fur 32 po. 2 yds 2 ft 3 in. **288.** 59 mi. 0 fur. 32 po
289. 5 mi 5 fur 18 po 1 yd **290.** 25 mi. 2 fur 0 po 4 yds
291. 32 mi. 4 fur. 33 po. $1\frac{1}{2}$ yds. **292.** 701 mi. 3 fur 26 po. 4 yds.
293. 4419 mi. 1 fur 21 po $1\frac{1}{2}$ yd. **294.** 21 mi 2 fur 22 po. 1 yd.
295. 189 mi. 3 fur 6 po. 0 yd. 1 ft. **296.** 19 mi. 5 fur 20 po. 4 yds.
297. 349 mi. 7 fur 18 po 0 yd. 1 ft. **298.** 1279 mi 1 fur. 32 po. 4 yds. 1 ft
299. 1594 mi. 1 fur. 32 po 0 yd. 0 ft 5 in. **300.** 1578 mi. 2 fur. 10 po 2 yds. 2 ft 4 in **301.** 4915 sq. in **302.** 23263 sq in.
303. 2864 sq in **304.** 20013 sq in **305.** 11659 sq. po
306. 21037 sq. po. **307.** 22945 sq. po **308.** 14381 sq po
309 4 sq. yds 8 sq ft 117 sq in **310.** 6 sq yds. 5 sq. ft 11 sq in.
311. 54 sq yds 3 sq ft 122 sq in **312.** 95 sq yds. 4 sq ft 60 sq in.
313 22 ac 3 rds 11 po **314.** 46 ac 1 rd 8 po. **315** 457 ac 2 rds. 17 po
316. 566 ac 0 rds. 13 po. **317.** 726 sq yds **318** 1089 sq yds
319. 623 sq yds. **320.** 995 sq. yds **321.** $544\frac{1}{2}$ sq. yds.

322. 665½ sq yds. 323. 514¼ sq yds 324 877¼ sq yds.
325 441 sq yds 326 648¼ sq yds 327 1296 sq ft
328 2448 sq ft 329 1638 sq ft 330 2965½ sq ft
331 4680 sq ft 332 5283 sq ft 333 1176120 sq in
334 823284 sq in 335. 1058508 sq in 336. 1528956 sq in.
337 218052 sq in 338. 310716 sq in 339 37268 sq yds
340 24079 sq yds 341 10254¾ sq yds 342. 20479¼ sq yds
343 25070 sq yds 344 45563 sq yds 345 82418 sq yds
346 131130 sq yds 347 23048773 sq in 348 54650895 sq in
349 333274481 sq in 350 24820485 sq in 351. 20 sq po 11 yds
352 28 sq po 4 yds 353 8 sq po 25 yds 354 11 sq po 20¼ yds.
355. 30 sq po 9½ yds 356 35 sq po 2¼ yds
357. 3 sq po 7 yds 2¼ ft 358 4 sq po 16 yds 1 ft
359 4 sq po 17 yds 8 ft 123 in 360 6 sq po 8 yds 8 ft 99 in.
361 2A 1R 20P 0 yds 362 17A 0R 25P 24¾ yds
363. 7A 2R 39P 15¼ yds 364 15A 0R 14P 14 yds 7½ ft
365 12A 3R 27P 21 yds 6¼ ft 366 22A 3R 33P 2 yds 7¾ ft.
367 1A 1R 0P 19 yds 0 ft 8 in 368 2A 2R 8P 18 yds 5 ft 120 in.
369 4A 0R 13P 5 yds 7 ft 109 in 370 8A 2R 34P 0 yds 3 ft 87 in.
371 185A 0R 2P 26½ yds 372 76A 2R 3P 19 yds 5¼ ft
373 53A 0R 21P 0 yds 8 ft 125 in 374 15A 3R 20P 15 yds 2 ft 0 in
375 30711 cub in 376 32177 cub in 377. 88888 cub in
378 78391 cub in 379. 209093 cub in 380. 457067 cub in
381 3 cub yds 2 ft 138 in 382 16 cub yds 10 ft 815 in
383 18 cub yds 4 ft 605 in 384. 21 cub yds 4 ft 719 in
385 103 cub yds 15 ft 362 in 386. 171 cub yds 13 ft 664 in
387 111 pints 388 155 pints 389 146 pints
390 185 pints 391 109 gal 1 qt 1 pt 392. 84 gal 3 qts 0 pt
393 154 gal 0 qt 0 pt 394 225 gal 2 qts 1 pt 395 238 pecks
396 314 pecks 397. 31017 qts 398 45817 pts
399 4 qrs 7 bush. 3 pks 1 gal 3 qts 1 pt 400. 130 qrs 1 bush 2 pks 1 gal 1 qt
401 15 lbs Av + 3000 grs
402 23136 links 403 141¾ in 404 21683200 sq yds
405 864 pints 406 8483 sheets 407. 114449″ 408. 33 knots
409. 1452 acres. 410 9800 ozs Troy

V. 1 £7800, 3*s* 0*d* 2 £888, 3*s* 7½*d* 3. £22703, 0*s* 1½*d*
4 £6186, 1*s* 11*d* 5. £17625, 6*s* 8*d* 6. £28910, 13*s* 0*d*.
7 £36343, 3*s* 0*d* 8 £57447, 10*s* 11*d* 9 £42450, 5*s* 2*d*.
10. £36711, 11*s* 2*d* 11 £44375, 2*s* 8*d* 12 £48237, 16*s* 11*d*
13 £6381709, 13*s* 11*d* 14. £4278818, 3*s* 11*d* 15. £5253431, 8*s* 9*d*
16 £40418, 4*s* 0½*d* 17 £25925, 16*s* 7*d* 18 £885299, 19*s* 11*d*
19 92 days 12 hr 16 min 37 secs 20. 13 days 7 hrs 8 min 37 secs
21 33 yrs 72 days 2 hrs 22. 77 yrs 15 days 1 hr
23 22 cwt 1 qr 10 lbs 9 ozs 24 2 cwt 1 qr. 17 lbs 11 ozs 13 drs
25. 43 tons 3 cwt 2 qrs 4 lbs 26 27 tons 10 cwt 3 qrs 2 lbs
27. 22 cwt 77 lbs 6 ozs 28 24 tons 6 cwt 105 lbs 11 ozs.
29. 36 lbs. Tr 9 ozs 8 dwt 15 grs. 30. 78 ozs. Tr. 34 grs

31. 16 yds 0 ft 9 in 32. 32 yds 2 ft 7 in 33. 17 mi 774 yds
34. 25 mi 449 yds 1 ft 7 in 35 90 po 3 yds 0 ft 3 in
36. 33 mi 1 fur 38 po 5 yds 1 ft 37. 19 fur 20 po 5 yds 1 ft 9 in
38. 24 mi 30 chs 41 lks 39 30 sq yds 4 ft. 3 in
40 20 sq yds 1 ft 95 in 41. 307A 3R 2P 42 122A 1R 38P
43. 10A 1R 21P 20½ sq yds 44. 16A 1R 24P 16¼ sq yds
45. 25A 0R 39P 12 sq yds 5 ft 36 in 46. 2R 32P 8 sq yds 0 ft. 37 in
47 47 cub yds 23 ft 542 in 48. 444 cub yds 8 ft. 1259 in
49. 47 gal 3 qts 1 pt 50 84 qrs 1 bush 2 pks 0 gal 51. £8, 9*s* 2¾*d*
52 £4, 4*s* 4¼*d* 53. £3188, 8*s* 10¾*d* 54 £7599, 17*s* 9¾*d*
55. £1, 2*s* 1½*d* 56 £15, 11*s* 5¼*d* 57. £1, 18*s* 8½*d*
58 £7, 16*s* 9¾*d* 59. £91, 1*s* 4*d* 60 £21, 0*s* 10*d*
61. 8 hrs 27 min 50 secs 62. 6 hrs 27 min 44 secs
63 3 days 9 hrs 36 min 64 2 days 17 hrs 24 min 13 secs
65. 4 lbs 7 ozs 10 drs 66. 1 qr. 15 lbs 8 ozs 67. 7 tons 11 cwt. 3 qrs
68. 11 tons 15 cwt 1 qr 9 lbs 69 1 ton 16 cwt 3 qrs 23 lbs
70. 16 cwt 3 qrs 11 lbs 12 ozs 71 5 tons 4 cwt 2 qrs 3 lbs 6 ozs
72. 6 tons 2 cwt 2 qrs 8 lbs 6 ozs 3 drs 73. 7 lbs Tr 7 ozs 16 dwt 5 grs
74. 13 ozs Tr 344 grs 75. 13 yds 1 ft 7 in 76. 8 yds 1 ft 9 in
77. 7 mi 1033 yds 2 ft 78. 21 mi, 427 yds 1 ft 5 in
79. 34 m 5 fur 6 yds 80. 4 fur 24 po 2 yds 81 28 po 1 yd 2 ft 11 in
82. 3 mi 4 fur 18 po. 4 yds 1 ft 5 in 83 4 sq yds 3 ft 27 in
84. 7 sq yds 0 ft 101 in. 85 47A 2R 24P 86 106A 3R 17P
87. 37 sq po 20¼ sq yds 88. 2A 2R. 21P. 0¼ sq yd
89. 5 ac 3470 sq yds 90 1 sq mi 130 ac 2967 sq yds
91. 1A 2R 10P 2 sq yds 3 ft 36 in. 92. 1A 0R 0P 13 sq yds 2 ft 82 in
93. 1A 1R 7P 17 sq yds 5 ft 26 in 94 1A 0R 6P 26 sq yds 8 ft 3 in
95. 8 cub yds 6 ft 727 in 96 180 cub yds 16 ft 741 in
97. 19 gals 1 qt 1 pt 98 3 bush 2 pks 1 gal 99. 44 qrs 4 bush
100 67 qrs 5 bush 3 pks 101. 5*s* 2*d* 102 1*s* 8½*d*
103 £5, 18*s* 3*d* 104. 18*s* 105 £2019, 16*s* 8¾*d* 106 £8114, 11*s* 8¼*d*
107. 10*s* 3¼*d*. 108. £1, 2*s* 0¼*d* 109 £2, 1*s* 8*d* 110. 18*s* 11¼*d*
111. £3, 17*s* 1¼*d* 112 £5, 7*s* 8*d* 113 11*s* 4½*d* 114 £7, 1*s* 8¾*d*
115. £2, 9*s* 116 £15, 7*s* 8¾*d* 117 16*s* 8*d*. 118. £24, 11*s* 3*d*.
119. 14*s* 6*d* 120. £1, 8*s* 1*d* 121 £159, 1*s* 1½*d* 122. £206, 14*s* 4¾*d*.
123. £1083, 17*s* 1½*d* 124 £754, 13*s* 6*d* 125. £1402, 4*s*. 4½*d*.
126. £2695, 2*s* 1½*d* 127. £1046, 17*s* 6*d* 128. £2157, 19*s* 4½*d*.
129. £178, 12*s* 7½*d* 130. £535, 8*s* 131. £16544 132. £15851.
133. £2277, 5*s* 10*d* 134. £2411, 8*s*. 135. £2499, 3*s* 4*d*.
136. £5976, 5*s* 137. £7687, 10*s* 138. £7687, 10*s* 139. £907, 14*s* 7*d*.
140. £841, 18*s* 10½*d* 141. £201, 9*s*. 5½*d*. 142. £143, 9*s* 9¾*d*
143. £5526, 15*s* 8*d* 144. £5240, 19*s* 8*d*. 145 £974, 9*s*. 1*d*
146. £1072, 11*s* 7*d* 147. £19679, 1*s*. 10½*d* 148. £68867, 16*s*. 11*d*
149. £2568, 0*s* 10*d*. 150. £4324, 0*s*. 5*d*. 151. £1255, 3*s* 4*d*.
152. £8704, 10*s*. 153. £5655, 19*s*. 8*d* 154. £17877, 14*s* 2*d*.
155. £2363, 11*s* 2½*d* 156. £262, 16*s* 11¼*d*. 157. £11270, 10*s* 11½*d*.
158. £78697, 0*s* 10½*d*. 159. £4495, 11*s*. 1*d*. 160. £1645, 0*s*. 4*d*.
161. £28231, 5*s*. 3*d*. 162. £22240, 19*s* 4½*d*. 163. £446, 19*s*. 4*d*.
164. £600, 7*s*. 11*d*. 165. £974, 17*s*. 1*d*. 166. £1254, 13*s*. 9*d*

167 £728700 168 £89723, 2s 6d 169. £3123, 8s

170. £16340, 13s 8d 171. £9545, 4s 6d 172 £12057, 9s 9d

173 £69, 3s 4½d 174 £92, 10s 6¼d 175 £310, 13s 6¼d

176 £504, 8s 4½d 177 £8115, 16s 178 £65988

179 £4606, 17s 6d 180 £5406, 9s 181 £1753, 5s 6d

182 £5395, 16s 4d 183 £2273, 16s 10d 184 £2173, 3s 9d

185. £3602, 7s 6d 186. £12859, 15s 9d 187 £5733, 12s 3d

188 £12882, 19s 5d 189 £43955, 8s 4d 190 £36627, 11s 2½d

191. 5 dys 10 hrs 44 min 39 secs 192. 5 dys 17 hrs 45 min 45 secs

193 154 tons 7 cwts 0 qr 194 3 cwts 2 qrs 16 lbs 12 ozs

195 3 qrs 20 lbs 2 ozs 14 drs 196 10 tons 19 cwts 1 qr 20 lbs

197 73 tons 4 cwts 0 qr 1 lb 198 1 tn 1 cwt 0 qr 26 lbs 15 oz

199 418 tns 0 cwts 2 qrs 12 lbs 15 ozs 200 27 tns 6 cwt 3 qr 1 lb 8 oz

201 933 tons 9 cwts 2 qrs 1 lb 202 103 tons 7 cwts 1 qr 9 lbs 5 ozs

203 121 yds 2 ft 3 in 204 65 mi 4 fur 4 po 205 81 mi 6 fur 39 po

206 421 yds 0 ft 2 in 207 211 mi. 1 fur 13 po 3½ yds

208 3 fur 24 po 4 yds 1 ft 6 in 209. 1020 mi 2 fur 10 po 4½ yds

210 157 mi 3 fur 37 po 3 yds 1 ft 211 409 mi 2 fur 9 po 1 yd

212. 6 mi 2 fur 39 po 5 yds 213 94A 2R 0P 214 112A 0R 8P

215. 13 sq yds 6 ft 54 in 216. 13 sq yds 2 ft 108 in

217 16A 2R 34P 19 sq yd 7 ft 108 in 218. 23A 2R. 14P 8 sq yd 6 ft 72 in

219. 30A 2R 1P 7½ sq yds 220 2728A 3R 6P 8¾ sq yds

221. 119A 1R 26P 12 sq yds 2 ft 36 in 222 10A 0R 28P 10 sq yds 8 ft 96 in 223 118 cub yds 13 ft 708 in

224 15587 qrs 0 bush 225 11s 9¼d 226. £17, 17s 7¼d

227 £33, 19s 5½d + 1¼d rem^{r} 228 £225, 15s 3¾d + 1¾d rem^{r}

229 £579, 3s 8½d 230 £8118, 14s 4½d + ½d rem^{r}

231. £2607, 17s 9¼d + ¾d rem^{r} 232. £86878, 17s 6d

233. £3, 3s 3¾d 234 £4, 4s 4¼d 235. £13, 2s 0¾d + 10d rem^{r}

236. £4, 5s 9¼d + 4d rem^{r} 237 £21, 5s 5d + 2s 0½d rem^{r}

238. £27, 18s 6½d + 5¾d rem^{r} 239 £37, 13s 2½d

240 £149, 17s 10d 241 £100, 10s 10¾d 242 £147, 12s 4½d

243. £9723, 1s 2¼d 244. £12447, 5s 8¾d 245 £2, 1s 7¾d

246. £19, 19s 11¾d 247 £9, 0s 11¾d 248 £89, 16s 6d

249. £5, 5s 6¾d 250 £3, 3s 3¾d 251 £2, 15s 4½d

252 £432, 6s 2½d 253 7s 1½d 254 19s 10¼d

255. £201, 17s 6¼d + 6¾d rem^{r} 256 £1079, 6s 8¾d + 6¼d rem^{r}

257. £222, 2s. 2½d 258. £1100, 11s 9¼d 259 £79, 16s 4½d

260. £625, 10s 7d 261 £1386, 0s 5¾d. + 1s 8¼d rem^{r}

262. £2187, 16s 6¾d 263 £368, 12s 9¼d

264. £274, 18s 9¾d + 6s 2¾d rem^{r} 265 £3, 17s 4½d.

266. £29, 13s 11¼d 267 £730, 18s 0¾d + 7s 2¼d rem^{r}

268 £2534, 13s 8½d + 12s 11d rem^{r} 269 £12, 12s 7d

270 £30, 1s 11½d 271 6s 2d + £3, 7s 10d rem^{r}.

272 £8, 13s 7d + £5, 15s 11d rem^{r} 273 £1737, 3s 9d

274 £375623, 17s 11¾d 275 £1, 15s 0¾d + £3, 3s 9d rem^{r}

276 £11, 15s 10¾d + £5, 11s 1d rem^{r} 277. £19, 19s 9¼d

278 £65, 4s 3½d 279 £117, 1s 1½d 280. £320, 13s 1½d + 1s 8½d rem^{r}

281. £23, 14s 4½d + 3½d rem^{r} 282 £403, 2s 6¼d + 1s 3¼d rem^{r}

283. £1, 2s $9\frac{1}{4}d$ + 10d rem^r 284 1s $11\frac{1}{4}d$ + 18s. $6\frac{1}{2}d$ rem^r.
285. 17s $6\frac{1}{4}d$ + 11s $0\frac{1}{2}d$ rem^r 286 £279, 1s 4d.
287. 30 dys 10 hrs 29 min 4 sec 288 2 dys. 9 hrs 10 min. 10 secs
289. 11 dys 11 hrs 11 min 11 sec 290 400 dys 6 hrs 22 min 42 sec.
291 2 tons 5 cwts 2 qrs 25 lbs 6 ozs 292 16 cwts 1 qr 8 lbs. 2 ozs
293. 3 tns 11 cwts 3 qrs 10 lbs 8 ozs 294. 3 tns 3 cwts 0 qr 2 lbs 4 oz
295. 6 cwts 2 qrs 9 lbs 296 6 tons 11 cwts 3 qrs 1 lb
297. 3 cwts 0 qr 3 lbs 0 oz 3 drs 298 3 cwts 2 qrs. 12 lbs 5 ozs
299. 3 cwts 2 qrs 1 lb 300 3 tons 0 cwts 1 qr 4 lbs 15 ozs 10 drs + 154 drs rem^r 301 7 yds 1 ft 6 in. 302 224 yds 0 ft 10 in.
303 1 mi 3 fur 10 po 5 yds 0 ft 6 in 304 1 mi 4 fur 34 po 3 yds 2 ft 8 in + 5 in rem^r 305. 25 po 3 yds 1 ft 7 in.
306. 4 fur 8 po 3 yds 2 ft 307 1 mi 0 fur 1 po 0 yds. 1 ft
308 3 mi 1119 yds 2 ft 11 in 309. 2 mi 525 yds 0 ft 8 in. + 404 in. rem^r
310. 2 po 1 yd 1 ft. 311 1 sq yd 5 ft 132 in
312 2 sq ft 32 in 313. 2A. 1R 31P. 10 sq yds 8 ft. 15 in.
314. 1A 3R 15P. 11 sq yds 6 ft 92 in + 5 in rem^r
315. 51A. 2R 18P 20 sq yds 8 ft 6 in + 12 in. rem^r
316. 1A 0R 10P 4 sq. yds. 2 ft 317. 20 sq yds 4 ft. 36 in.
318. 160A 2R. 10P 1 sq yd. 319 16 cub yds 0 ft. 48 in
320. 1 bush. 3 pks. 1 gal 3 qts 1 pt 321. 84 322. 93.
323. 182 324. 1364 325. 1001 326. 15 327. 903
328 474 + 2f over 329. 13 330 15 331 6 332. 16.
333 60 334 14 335 24 336 20. 337. 71. 338. 72.
339. 16 340 18 341 1753 342. 1000 343. 16.
344 64 345. 35 346 61 347 37. 348. 103 349. 9
350. 39 + 5 yds over 351. 18026 + 7 ft over 352. 800
353. 38 + 3 in. over 354. 5 355. 119 payments 356. 483 lbs
357. 601 tons. 358. 40 persons 359. 96 bits 360. 17 plots

VI. 1. 227749f 2. 86400 sec. 3. £239, 7s $0\frac{1}{4}d$ 4. £2, 19s $11\frac{3}{4}d$.
5. £41656, 7s $2\frac{1}{4}d$. 6. £1777, 4s $11\frac{1}{2}d$ 7. £843, 17s 6d
8. £98, 2s $4\frac{1}{2}d$ 9. 48 stamps 10. 7 times 11 27283 lbs
12. 264960 mins 13. £1111, 11s. $11\frac{1}{4}d$ 14. £1, 17s $1\frac{1}{2}d$.
15 £43824, 19s 6d. 16 £2, 0s. 5d. + $2\frac{1}{2}d$ rem^r 17. £2, 19s $10\frac{1}{4}d$
18. £2, 1s $5\frac{1}{4}d$. 19. 105 eggs. 20. 26 times + 11 in. over
21. 57223 of each. 22. 7752d. 23. £120.
24. 66 tons 6 cwts 3 qrs 0 lbs. 3 oz. 25 £457, 10s 5d. 26. £621
27. 10s $4\frac{3}{4}d$. 28. $\frac{3}{4}d$ 29. 605 balls 30 2160 times.
31 2 wks 6 days 17 hrs 46 min 40 secs 32 £1055, 4s 2d
33 2694384f 34. 331 tons 16 cwt 8 oz. 35. £2, 2s $2\frac{1}{4}d$.
36 £363, 19s $6\frac{1}{2}d$ 37. $7\frac{1}{2}$ lbs. 38 £6, 5s. 39 13. 40 96
41. 1248 hf-crs 42. £13, 8s $8\frac{1}{2}d$ 43. £19, 14s. 4d
44 £1, 2s 6d 45. £6, 19s 6d 46. £13, 12s. $11\frac{3}{4}d$
47. £3447, 10s 2d 48 2s $10\frac{1}{2}d$ 49. 216 post-cards. 50 £10
51 17 hf-crs 52. £5, 14s 6d 53 £61, 12s 6d 54. £52, 13s 9d.
55. £49, 19s $11\frac{3}{4}d$ 56. £944, 0s 4d 57. £12439, 0s $6\frac{3}{4}d$
58. £4, 13s. $1\frac{1}{2}d$ 59. 777 persons. 60 £27, 8s. 4d 61. 8784 hrs.

62 474539 half-ozs 63. 57 days 13 hrs 9 min 25 secs
64 273 cwt 1 qr 21 lbs 65 1524 yds 0 ft 5 in
66. 10 oz 10 dwt 10 grs 67 2 tons 13 cwts 1 qr 23 lbs 7 oz
68 2688 pennies 69 11 yrs 4 mo 70 9216 steps 71 13 cwt
72 3553 half-mins 73 69 mi 131 yds 74 130 ac 1 ro
75 1 gross 8 doz and 9 76 137 yds 1 ft 5 in
77 2 cwt 3 qrs 1 lb 8 oz 78 2 tons 4 cwt 2 qrs 16 lbs
79 5 ft 1 in 80 72 medals 81 2316 gs + 14*s*
82 £38, 11*s* 4½*d* 83 £38936, 8*s* 8½*d* 84 £5759
85 £781328, 4*s* 4½*d* 86 £21, 0*s* 6*d* 87 £6, 9*s* 2¾*d* + ½*d* over
88 £387387, 8*s* 7½*d* 89 31 90 88 91 The latter by 1
92 £1, 5*s* 11½*d* 93 £31, 5*s* 94 £2480, 16*s* 4*d*
95 £5265, 11*s* 3*d* 96 £21, 11*s* 4½*d* 97 £93, 0*s* 6¼*d*
98 £10, 10*s* 10¼*d* 99 231 persons 100 41 persons 101 14532 ft
102 523 mi 103 19025 com yrs 319 days 10 hrs 40 min
104 22 tons 5 cwt 2 qrs 105 £1, 1*s* 1*d* 106 £72, 7*s* 11*d*
107 2 mi 2 fur 2 po 2 yds 2 ft 2 in 108 16 109 £1, 15*s*, and 5*s*
110 32640 111 980064 in 112 21 mi 3 fur 23 po 3 yds 2 ft 7 in
113 £511, 4*s* 8*d* 114 1101600 pins 115 £405, 11*s* 2½*d*
116. £199, 17*s* 11*d* 117 £1, 4*s* 118 54 allotments
119 £30, 19*s* 4*d*, £3, 17*s* 5*d* 120 12 lbs Avoir
121 54 tons 11 cwts 3 qrs 20 lbs 6 ozs 122. The farthings
123 416 acres 124 £9, 3*s* 9*d* 125. 1 fur 5 po 2 yds 2 ft 8 in + 138 in over 126 £20, 3*s* 9*d* 127 100 times
128 £3, 2*s* 1*d* for A, £1, 2*s* 3*d* for B 129 A, 6*s*, B, 7*s*, C, 8*s*
130. £49, 11*s* 8*d* 131 1718 of each 132 51 mi 1 fur 13 po 3 yds 1 ft 11 in 133 £1, 12*s* 7½*d* 134 £3, 11*s* 10½*d*
135. 9 tons 5 cwts 1 qr 4 lbs, 4 tons 12 cwts 2 qrs 16 lbs 136 7*s* 1½*d*
137 15 allotments and 4 po over 138 £2, 1*s* 10*d* for one, £1, 11*s* 10*d* for each of the others 139 Coat, £4, 4*s*, hat, 17*s*, umbrella, 19*s*
140 £6, 19*s* and £17, 7*s* 6*d* 141 £1 142 17695260 sq in
143. £25446, 8*s* 4*d* 144 £12, 14*s* 10½*d* 145 13 cwts 2 qrs 27 lbs, 1 cwt 2 qrs 3 lbs 146 £7, 2*s* 1*d*, £5, 14*s* 8*d*
147. 4½*d* per lb 148 64 times 149 13 times 150 Man, 6*s* 6*d*, woman, 4*s* 6*d*, boy, 1*s* 6*d* 151 116160 sheets, 5 cwts 3 qrs 16 lbs
152 29947000 hf-cr 153 £724, 5*s* 11¼*d* 154 £2, 10*s* 2¾*d*
155 A, £81, 3*s* 5*d*; B, £1, 1*s* 1*d* 156. £4, 17*s* 6*d* 157 720
158 1*d* per lb 159 37 persons 160 Man, 8*s* 3*d*, woman, 2*s* 9*d*, boy, 11*d* 161. 2557 days
162 17995 ozs. Tr 400 grs 163 £6, 10*s* 164 11 cwts 3 qrs 4 lbs
165 Wednesday 166. 18*s* 8*d* to one, 9*s* 4*d* to each of the others
167 1*s* 9*d* per lb 168 55 yards 169. Man, 10*s* 6*d*, woman, 7*s*, boy, 4*s* 8*d* 170 39 persons 171 4924 days
172 1092 times 173 £725, 0*s* 2*d* 174 The 11th 175 £40 and £5
176. 7*s* 9*d* per gal 177 480 yds 178 Man, 5*s*, woman, 3*s*, boy, 2*s* 179 38 men 180 1835
181 £3, 18*s* (There were 53 Sundays in that year) 182 8*s* per yd
183 £3600 184 Each man, £3, 1*s*, each woman, £2, 1*s*
185. 2742 shils, and 3 dwts. over. 186. 1 inch. 187 5*d* loss.

188. 5 lbs. 189. 12 dozen of each 190. Tuesday
191. 6*s* 2*d* (Omit 12 Sundays) 192. 1 oz Tr 6 dwts 10 grs
193 2*s* 3*d* 194. 4 gallons 195 The latter by 40 ft
196 7 horses, 7 pigs, 21 cows, 105 sheep 197 January, 1885.
198. 8 dozen of each. 199. 632 eggs 200. April 14, 1852

VII. 1 2, 4, 3, 9 2 2, 5, 10, 25, 3, 9. 3. 2.
4. 5, 25, 3, 9 5. None 6. 2, 4, 3 7 2, 4.
8 2, 4, 5, 10, 3, 9 9. 2, 3, 9. 10. 5, 25, 3 11 2, 4, 3, 9.
12 2, 5, 10, 4 13. None 14. 3 15 All 16. 5, 3.
17. 2, 5, 10, 4, 25, 3 18. 2, 4, 3 19. 2, 5, 10, 25, 3 20 3, 9
21. 38964035 22 709082649 23. 2, 11; 1; 8 24. 1, 2; 1; 2.
25. $2 \times 2 \times 3$ 26. $2 \times 3 \times 3$ 27. $2 \times 2 \times 5$ 28 $2 \times 2 \times 2 \times 3$.
29 $3 \times 3 \times 3$ 30. 2×17 31 3×13 32 $2 \times 3 \times 7$.
33. 3×17 34. 3×19 35. $3 \times 3 \times 7$ 36 5×13.
37. $2 \times 5 \times 7$ 38 $2 \times 2 \times 2 \times 11$ 39. $2 \times 3 \times 3 \times 5$.
40. $2 \times 2 \times 5 \times 5$ 41. $2 \times 2 \times 7$ 42. $2 \times 2 \times 3 \times 3$
43. $2 \times 2 \times 2 \times 5$ 44 $3 \times 3 \times 5$ 45 $2 \times 2 \times 13$ 46. 3×23.
47 $3 \times 3 \times 3 \times 3$ 48 7×13 49. $3 \times 3 \times 11$ 50 $2 \times 5 \times 11$
51 11×11 52. $5 \times 5 \times 5$ 53. $2 \times 5 \times 13$
54. $2 \times 2 \times 2 \times 13$ 55 $2 \times 2 \times 3 \times 3 \times 3$ 56. $2 \times 2 \times 2 \times 2 \times 3 \times 3$
57 $2^4 \times 7$ 58 $2^3 \times 3 \times 5$ 59. 3×37 60. 2^7. 61. $2^3 \times 17$
62 $2^2 \times 37$ 63 7×23 64. $3^2 \times 19$ 65 $2^2 \times 3^2 \times 7$.
66 $3 \times 5 \times 17$ 67. $3^2 \times 41$. 68 $2^6 \times 7$. 69. $3^2 \times 7^2$
70. $3^4 \times 7$ 71. $3^2 \times 7 \times 11$ 72 $2 \times 3^2 \times 7^2$. 73. $3^3 \times 37$.
74 $2^3 \times 5^3$ 75. 11^3 76. $2^5 \times 5 \times 11$ 77. $2^6 \times 3^3$
78. $2^6 \times 5 \times 7$ 79. $2^3 \times 5 \times 11^2$ 80. $2^2 \times 11 \times 43$ 81. 3×631
82. $2^7 \times 3^2 \times 5$ 83 $2^4 \times 3^3 \times 7$. 84. $3^2 \times 5 \times 11 \times 13$
85. $3^2 \times 11 \times 29$ 86 $7 \times 11 \times 101$ 87 $2 \times 3^3 \times 5 \times 11 \times 19$
88 $2 \times 5 \times 13^3$ 89. $11 \times 7 \times 13^2$ 90 11^4
91 $3 \times 7 \times 11 \times 13 \times 37$ 92 $2 \times 3^3 \times 11 \times 37$ 93 $3 \times 7^2 \times 101$
94. $3^4 \times 7 \times 11 \times 13 \times 97$ 95 $2^6 \times 3 \times 643$ 96 $3^3 \times 7 \times 11 \times 13 \times 37$
97 Twenty-five 98 Twenty-one 99 Yes 100 No.

VIII. 1. 7 2 10 3 5 4. 25 5. 4 6 8
7 8 8. 12 9 17 10. 13 11. 20 12. 15
13. 41 14. 23 15 19. 16. 14. 17 33 18 22
19. 8 20. 200 21 1 22 3 23. 3 24. 31.
25. 101 26 3 27. 111 28 202 29. 101 30. 102
31 35 32 85 33. 4 34. 108 35. 432 36. 96
37 27 38 283 39 77 40. 677 41. 73 42 23
43. 28 44 320 45. 440 46 119 47. 35 48. 267.
49. 729 50 693 51 113 52. 21. 53 87 54. 27
55 49 56. 51 57 1 58. 53 59 141 60. 551.
61. 495 62. 8371 63. 198 64. 61 65 741 66 71017
67 43 68 18 69 28 70. 17 71 12 72. 37
73. 207 74. 207. 75. 581 76. 1017. 77. 12 78. 17.
79. 16. 80. 23.

IX. 1. 12 2. 24 3 30 4 18 5 16 6 36
7 60. 8. 60. 9 100 10 66 11 12 12. 8
13 60 14. 60 15 40 16 18 17 42 18 24
19. 385 20 440 21 105 22 693 23. 672 24 588
25 130 26 102 27. 114 28 222 29 595 30 667
31 1207 32 91. 33 1680 34 896 35 630 36 330
37 864 38 630 39 1755 40 105 41. 112 42 2448
43 336 44 300 45 360. 46. 660 47 360 48. 180
49. 252 50. 1260 51 144 52 1260 53 1680 54 1680
55 462 56 4320 57 6720 58 2520 59 3640 60 20449
61 1680 62 240 63 2940 64 12000 65 600 66 10080
67. 554400 68 1260 69 660660 70 7140 71 17700 72 82861
73. 95040 74. 12649 75 1196421 76. 92939 77 9367 78 9702
79. 3764640. 80 1052205

X. 1. $2^3 \times 3 \times 7 \times 11 \times 17$ 2. — 3 59 4 1680 5 5031
6. $2^6 \times 3 \times 13 \times 53$ 7 — 8. 101 9 1155 10 —
11 881. 12 13 13. 707707 14 5, 7, 13, 35, 65, 91 15. 2*s* 2*d*
16 5612 17 114 18 748000 19 1575 20 1*s* 1*d*
21 $3^2 \times 7^2 \times 11^2 \times 13$ 22 Their G C F is 1 23 3 24 5863
25. 3 26 $2^2 \times 3^2 \times 5 \times 7^2 \times 13$ 27 17 28 27720 29 19
30 3 31 886 32. Yes 33 84 times 34 £21
35 76529 36 Their G C F is 1. 37. No 38 3848
39 14 ton 10 cwt 2 qrs 40 100 41 1517 42 3 hrs 35 min
43 3961 44 56700 45. 629 46 123 47 1 qr 1 lb 1 oz
48 187 49 101 50 2155 and 3017 51 999 52 4176
53. 72. 54 7 mins 55 330×330 56 984 57. 1053
58 873 59. 3 francs 60 40 grs 61 10406 62 3 yds 2 ft
63 5543 64 273 and 637, or 91 and 1911 65 84 and 42
66 999663 and 100203 67 12 days 2 hrs 16 min 68 10011
69. 137 and 822, or, 274 and 685, or, 411 and 548 70 13 71 83
72. 7, 11, 13, 77, 91, 143, 1001 73 4 cwt 2 qrs 74 678
75. 21, 22, 23

XI. 1. One-*seventh*, Two-*fifths*, Three-*eighths*, Five-*ninths*, One-*eleventh*, Four-*thirteenths*

2. One-*twentieth*, Three-*sevenths*, Nine-*tenths*, Four-*nineteenths*, Three-*fortieths*, Seventeen-*thirtieths*

3 Two-*fifteenths*, Ten-*elevenths*, Nineteen-*twentieths*, Three-*thousandths*, Nineteen-*eight-hundredths*

4 Twelve-*thirteenths*, One-*hundredth*; Sixty-nine-*seventieths*, Two-hundred-and-eleven-*three-hundred-and-fiftieths*

5 $\frac{1}{10}$, $\frac{1}{12}$, $\frac{7}{20}$, $\frac{13}{16}$ 6 $\frac{5}{9}$, $\frac{12}{19}$, $\frac{23}{100}$ 7 $\frac{2}{5}$, $\frac{73}{700}$, $\frac{9}{1000}$ 8 $\frac{3}{7}$, $\frac{19}{99}$, $\frac{8}{21}$
9 10*s*, 5*s*, 4*s*, 2*s*, 1*s* 10 6*d*, 4*d*, 3*d*, 2*d*, 1*d*
11 8 ozs, 4 ozs, 2 ozs, 1 oz 12 14 lbs, 4 lbs, 2 lbs, 1 lb
13 3*s*, 7*s*, 11*s*, 13*s*, 19*s* 14 5*d*, 7*d*, 3*d*, 11*d*, 8*d*
15 3 lbs, 5 lbs, 7 lbs, 13 lbs, 11 lbs 16 3 lbs; 11 lbs, 19 lbs; 5 lbs, 23 lbs 17 12, 8, 6, 4, 3, 2, 1, hours 18 30, 20, 15, 12, 10, 6, 5, 4, 3, 2, mins 19 7, 11, 9; 13, 23, hours

20. 17; 19, 13; 29; 51, mins. 21. 34, 31; 146 22. 16; 49; 85.
23. 15 cwts , £16, 3*d* 24. 45 min ; £27, 1*s* 6*d*.
25. 6*s* , 14*s* , 18*s* , 8*s*., 16*s* , 12*s* , 1*d* , 7*d* , 1*s* 11*d*
26. 15*s* , 2*s* , 22*s* ; 9*s* ; 18*s* , 6*d*., 11*d* ; 1*s*. 7*d* ; 2*s* 5*d*.
27 9*d* , 8*d* ; 10*d* , $\frac{1}{2}d$, $3\frac{1}{2}d$; $5\frac{1}{2}d$; $1\frac{3}{4}d$, $2\frac{3}{4}d$; $1\frac{1}{2}d$; $7\frac{1}{2}d$.
28. 45 secs , 10 hrs , 6 cwts , 18 mins ; 7 in., 40 sq in.
29 4*s* 4*d* , $3\frac{3}{4}d$, 34 min , 6 cwts 2 qrs , 10 in , 1 ro 8 sq po.
30 12*s* 8*d* , $9\frac{3}{4}d$, 146 days; 40 lbs , 800 yds ; $1\frac{1}{2}$ pints.

XII. 1. Five and three-eighths, Seven and four-fifths, Nine and one-eleventh; Twelve and three-thirteenths, Fifteen and seven-twentieths.

2. One and two-thirty-thirds, Twelve and seventeen-fiftieths; Ten and five-ninths, Twenty-eight and seven-twenty-fifths

3. $2\frac{3}{5}$, $7\frac{1}{3}$ 4. $8\frac{7}{10}$, $1\frac{1}{12}$ 5. $11\frac{3}{4}$, $17\frac{1}{4}$ 6. $20\frac{3}{20}$; $9\frac{8}{9}$.
7. 11; 3, 12; 17; 2 8 20, 4, 8, 27, 4 9. $1\frac{2}{3}$; $1\frac{3}{5}$; $5\frac{1}{2}$, $1\frac{1}{6}$; $1\frac{3}{7}$, $4\frac{1}{4}$.
10 $3\frac{1}{2}$; $1\frac{4}{5}$, $1\frac{3}{11}$; $2\frac{5}{8}$; $2\frac{2}{7}$, $4\frac{1}{3}$ 11 $3\frac{1}{7}$, $3\frac{3}{5}$, $2\frac{8}{11}$, $3\frac{5}{12}$; $8\frac{8}{9}$ 12 $6\frac{1}{6}$, $4\frac{4}{11}$; $7\frac{7}{12}$, $11\frac{1}{9}$, $12\frac{3}{10}$ 13. $\frac{6}{5}$, $\frac{7}{3}$, $\frac{15}{4}$, $\frac{39}{7}$, $\frac{53}{8}$ 14. $\frac{10}{9}$, $\frac{17}{6}$, $\frac{29}{6}$; $\frac{11}{2}$; $\frac{29}{12}$.
15. $\frac{19}{4}$; $\frac{35}{6}$, $\frac{23}{7}$; $\frac{23}{2}$, $\frac{85}{8}$ 16. $\frac{50}{7}$, $\frac{38}{9}$, $\frac{27}{20}$, $\frac{43}{2}$; $\frac{88}{9}$. 17. $\frac{88}{25}$ 18. $\frac{81}{35}$.

19. $\frac{65}{24}$	20. $\frac{79}{20}$.	21 $\frac{128}{27}$.	22 $\frac{197}{47}$.	23. $\frac{167}{13}$.
24 $\frac{144}{71}$.	25. $\frac{142}{15}$.	26. $\frac{142}{9}$.	27. $\frac{155}{12}$.	28. $\frac{83}{19}$.
29. $\frac{101}{17}$.	30. $\frac{131}{11}$.	31 $\frac{181}{13}$	32. $\frac{208}{29}$.	33. $\frac{298}{35}$.
34. $\frac{280}{13}$.	35 $\frac{157}{53}$.	36. $\frac{250}{9}$	37. $\frac{474}{41}$.	38. $\frac{425}{12}$.
39 $\frac{629}{50}$	40 $\frac{1217}{40}$.	41. $\frac{803}{8}$.	42. $\frac{643}{108}$	43. $\frac{4948}{51}$.
44. $\frac{520}{19}$	45. $\frac{10779}{200}$.	46. $\frac{42271}{831}$	47. $\frac{6286}{1197}$.	48. $\frac{13989}{2359}$.
49. $142\frac{6}{7}$	50 $111\frac{2}{9}$	51 $336\frac{2}{3}$	52. $90\frac{10}{11}$.	53. $70\frac{11}{12}$.
54 $97\frac{1}{3}$	55. $236\frac{1}{10}$	56. $28\frac{19}{20}$.	57. $42\frac{31}{100}$.	58. $53\frac{267}{1000}$.
59. $46\frac{5}{13}$	60 $15\frac{5}{14}$	61 $46\frac{11}{15}$	62. $49\frac{10}{17}$.	63. $34\frac{7}{23}$
64 $45\frac{20}{29}$	65. $19\frac{8}{35}$	66 $18\frac{17}{45}$	67. $24\frac{223}{300}$	68. $2\frac{573}{700}$
69. $7\frac{103}{110}$	70 $1\frac{3573}{5000}$	71. $1\frac{265}{552}$	72. $1\frac{281}{324}$.	73. $75\frac{47}{50}$.
74 $145\frac{11}{40}$	75 $276\frac{9}{41}$	76. $1354\frac{29}{31}$.	77. $24\frac{92}{147}$.	78. $38\frac{94}{207}$.
79. $76\frac{245}{555}$	80 $22\frac{11}{884}$			

XIII. 1. Six *ninths* 2. Three *twelfths* 3 35 *fiftieths*.
4. 27 *sixty-thirds* 5 35 *fortieths*. 6. 20 *fifty-fifths*
7. 33 *fifty-fifths* 8. 25 *thirtieths* 9 65 *seventieths*
10. 69 *sixtieths* 11. Three *fourths*. 12. Three *fifths*
13. One *sixth* 14 Two *tenths* 15 Three *fifteenths*
16 Three *sevenths* 17. Three *fifths* 18. Two *fifths*.
19. Six *tenths* 20 Five *eighths* 21. 4

22 9	23 10	24. 20	25 45	26 28	27. 7
28 35.	29 33	30. 40	31 84	32 70	33. 34.
34 44	35 72	36. 57	37. 100	38. 93	39. 85.
40 57.	41 6	42. 7	43 6	44. 11	45. $\frac{5}{6}$
46. 3	47 3.	48 4	49. 8	50. 7.	51. 10.
52. 7.	53 3.	54 6	55 11	56 8	57. 4.
58. 19.	59. 5	60 19	61 $\frac{25}{65}$.	62. $\frac{45}{65}$.	63. $\frac{38}{88}$.

64 [illegible] 65 [illegible] 66. [illegible] 67 [illegible] 68 [illegible] 69 [illegible]
70 [illegible] 71 [illegible] 72 [illegible] 73 $\frac{3}{4}$ 74 [illegible] 75 $\frac{1}{25}$
76 [illegible] 77 [illegible] 78 [illegible] 79 [illegible] 80 $\frac{4}{5}$

XIV.

1 [illegible] 2 [illegible] 3 [illegible] 4 [illegible] 5 $\frac{1}{4}$
6 [illegible] 7 [illegible] 8 [illegible] 9 [illegible] 10 [illegible] 11 $\frac{5}{6}$
12 [illegible] 13 [illegible] 14 [illegible] 15 [illegible] 16 [illegible] 17 $\frac{14}{15}$
18 [illegible] 19 [illegible] 20 [illegible] 21 [illegible] 22 [illegible] 23 $\frac{5}{8}$
24 [illegible] 25 [illegible] 26 [illegible] 27 [illegible] 28 $\frac{1}{3}$ 29 $\frac{1}{8}$
30 [illegible] 31 [illegible] 32 [illegible] 33 [illegible] 34 [illegible] 35 $\frac{1}{4}$
36 [illegible] 37 [illegible] 38 [illegible] 39 [illegible] 40 [illegible] 41 $\frac{3}{8}$
42 [illegible] 43 [illegible] 44 [illegible] 45 [illegible] 46 [illegible] 47 [illegible]
48 [illegible] 49 [illegible] 50 [illegible] 51 [illegible] 52 $\frac{2}{3}$ 53 [illegible]
54 [illegible] 55 [illegible] 56 [illegible] 57 [illegible] 58 [illegible] 59 [illegible]
60 [illegible] 61 [illegible] 62 [illegible] 63 [illegible] 64 [illegible] 65 $\frac{3}{4}$
66 [illegible] 67 [illegible] 68 [illegible] 69 [illegible] 70 [illegible] 71 [illegible]
72 [illegible] 73 [illegible] 74 [illegible] 75 [illegible] 76 [illegible] 77 [illegible]
78 [illegible] 79 [illegible] 80 [illegible] 81 [illegible] 82 [illegible] 83 $\frac{5}{8}$
84 [illegible] 85 [illegible] 86 [illegible] 87 [illegible] 88 [illegible] 89 [illegible]
90 [illegible] 91 [illegible] 92 [illegible] 93 [illegible] 94 [illegible] 95 [illegible]
96 [illegible] 97 [illegible] 98 [illegible] 99 [illegible] 100 [illegible] 101 $57\frac{1}{2}$
102 [illegible] 103 [illegible] 104 [illegible] 105 $6\frac{1}{3}$ 106 [illegible] 107 [illegible]
108 [illegible] 109 [illegible] 110 $70\frac{1}{2}$ 111 [illegible] 112 [illegible] 113. [illegible]
114 [illegible] 115 [illegible] 116 [illegible] 117 [illegible] 118 [illegible] 119 [illegible]
120 [illegible]

XV

1 [illegible] 2 [illegible] 3 [illegible] 4 [illegible]
5 [illegible] 6 [illegible] 7 [illegible] 8 [illegible]
9 [illegible] 10 [illegible] 11 [illegible] 12 [illegible]
13 [illegible] 14 [illegible] 15 [illegible] 16 [illegible]
17 [illegible] 18 [illegible] 19 [illegible]
20 [illegible] 21 [illegible] 22 [illegible]
23 [illegible] 24 [illegible] 25 [illegible]
26 [illegible] 27 [illegible] 28 [illegible]
29 [illegible] 30 [illegible] 31 [illegible]
32 [illegible] 33 [illegible] 34 [illegible]
35 [illegible] 36 [illegible] 37 $\frac{7}{9}$ 38 $\frac{13}{20}$
39 [illegible] 40 [illegible] 41 [illegible] 42 [illegible] 43 [illegible] 44 [illegible]
45 [illegible] 46 [illegible] 47 [illegible] 48 [illegible] 49 [illegible]
50 [illegible] 51 [illegible] 52 [illegible]
53 [illegible] 54 [illegible] 55 [illegible]
56 [illegible] 57 [illegible] 58 [illegible]
59 [illegible] is greatest, [illegible] is least 60 [illegible] is greatest, [illegible] is least

XVI

1 [illegible] 2 [illegible] 3 [illegible] 4 [illegible] 5 [illegible]
6 [illegible] 7 [illegible] 8 [illegible] 9 [illegible] 10 [illegible] 11 $\frac{5}{8}$
12 [illegible] 13 [illegible] 14 [illegible] 15 [illegible] 16. $\frac{5}{8}$ 17. [illegible]

18.	$\frac{11}{12}$	19	1	20	1	21	$\frac{3}{4}$	22	$\frac{3}{4}$	23	$\frac{23}{24}$
24	$\frac{31}{84}$	25	$1\frac{19}{99}$	26.	$1\frac{11}{56}$	27	$1\frac{13}{54}$	28	$1\frac{26}{105}$	29	$\frac{3}{7}$
30	$\frac{5}{72}$	31	$\frac{5}{7}$	32	$\frac{8}{15}$	33	$1\frac{1}{3}$	34	$1\frac{1}{4}$	35	$\frac{5}{6}$.
36	$\frac{7}{12}$	37	$\frac{13}{48}$	38	$\frac{23}{24}$	39	$\frac{15}{16}$,	40	$1\frac{3}{20}$	41	$1\frac{1}{3}$
42	$2\frac{23}{60}$	43.	$1\frac{1}{60}$	44	$1\frac{11}{84}$	45	$1\frac{17}{60}$	46	$1\frac{13}{30}$	47	$\frac{227}{1000}$
48.	$\frac{3333}{10000}$	49	$1\frac{9}{20}$	50	1	51	$1\frac{59}{80}$	52	$1\frac{267}{280}$	53	$4\frac{11}{40}$
54	$4\frac{57}{140}$	55	$1\frac{189}{200}$	56	$1\frac{17}{168}$	57	$1\frac{9}{17}$	58	$\frac{43}{102}$	59	$1\frac{1}{78}$
60.	$\frac{158}{195}$	61	$9\frac{47}{60}$	62	$5\frac{13}{60}$	63	$39\frac{2}{3}$	64	$20\frac{2}{105}$	65	5
66	$9\frac{10}{21}$	67.	$924\frac{3}{4}$	68	$1000\frac{7}{24}$	69	$270\frac{329}{800}$	70	$129\frac{103}{300}$		
71	$8\frac{11}{18}$	72	$45\frac{31}{60}$	73	$10\frac{62}{81}$	74	$22\frac{19}{32}$	75	$27\frac{37}{114}$		
76	$6\frac{41}{132}$	77	$10\frac{1}{12}$	78	11	79	$4\frac{29}{50}$	80	$28\frac{9}{10}$		
81	$900\frac{13}{2400}$	82	$1408\frac{74}{225}$	83	$48294\frac{17}{77}$	84.	$9255\frac{7}{24}$				
85	$11\frac{8}{21}$	86	$8\frac{296}{315}$	87	$6\frac{53}{56}$	88	$7\frac{17}{40}$				
89	$2085\frac{9}{50}$	90	$19\frac{43}{90}$	91	$3\frac{1265}{13104}$	92.	$123\frac{573}{825}$				
93.	$6\frac{3}{25}$	94	$4\frac{1931}{4380}$	95	$37\frac{33}{272}$	96	$30\frac{480}{9849}$				
97.	$11\frac{5}{8}$	98.	$28\frac{55}{63}$	99.	$1222666\frac{1}{6}$	100	$24\frac{13}{15252}$				

XVII.

1.	$\frac{5}{11}$	2	$\frac{4}{17}$	3	$\frac{13}{25}$	4	$\frac{5}{49}$	5	$\frac{1}{8}$		
6	$\frac{5}{24}$	7.	$\frac{1}{18}$	8	$\frac{7}{30}$	9	$\frac{27}{32}$	10	$\frac{31}{56}$	11	$\frac{9}{28}$
12.	$\frac{7}{64}$	13	$\frac{1}{2}$	14	$\frac{5}{48}$	15	$\frac{99}{112}$	16	$\frac{73}{80}$	17	$6\frac{1}{8}$
18.	$3\frac{3}{8}$	19	5	20.	8	21	$2\frac{1}{6}$	22	$2\frac{5}{12}$	23	$4\frac{3}{7}$
24	$6\frac{5}{8}$	25	$10\frac{13}{16}$	26.	$14\frac{11}{20}$.	27.	$11\frac{3}{14}$	28	$16\frac{6}{25}$	29.	$4\frac{3}{5}$
30.	$5\frac{1}{6}$	31	$5\frac{6}{7}$	32	$1\frac{7}{9}$	33	$9\frac{10}{17}$	34	$17\frac{18}{19}$	35	$15\frac{13}{28}$
36	$8\frac{31}{40}$	37	$5\frac{1}{4}$	38	$7\frac{1}{8}$	39	$5\frac{3}{5}$	40	$5\frac{17}{28}$	41	$3\frac{1}{12}$
42	$4\frac{3}{20}$	43	$\frac{43}{180}$	44	$\frac{5}{126}$	45	$4\frac{1}{96}$	46	$5\frac{1}{84}$	47	$\frac{1}{96}$
48	$\frac{1}{70}$	49	$\frac{1}{156}$	50	$\frac{6}{455}$	51	$\frac{5}{99}$	52	$\frac{1}{84}$	53	$1\frac{5}{72}$
54	$1\frac{7}{90}$	55	$1\frac{33}{35}$	56	$4\frac{31}{40}$	57	$3\frac{13}{20}$	58	$8\frac{16}{21}$	59	$13\frac{5}{8}$
60	$12\frac{5}{8}$	61	$3\frac{54}{55}$	62	$8\frac{51}{56}$	63	$74\frac{11}{12}$	64	$3\frac{119}{180}$	65	$3\frac{29}{78}$
66	$4\frac{67}{90}$	67	$95\frac{211}{360}$	68.	$471\frac{17}{30}$	69	$\frac{127}{154}$	70	$\frac{241}{280}$	71	$4\frac{29}{42}$
72	$110\frac{152}{165}$	73	$28\frac{59}{228}$	74	$1\frac{157}{306}$	75	$103\frac{385}{403}$	76	$18\frac{349}{432}$.		
77.	$14\frac{550}{561}$	78	$10\frac{10}{11211}$	79	$4\frac{551}{1200}$	80	$507\frac{27}{35}$	81	$\frac{5}{14014}$.		
82	$\frac{7}{1881}$	83	$111\frac{1037}{1210}$	84	$8568\frac{19}{120}$	85.	$9875\frac{1}{9875}$				
86.	$12344\frac{1}{12345}$	87	$\frac{3}{46189}$	88.	$\frac{1}{56}$	89	$\frac{17}{54}$	90	$\frac{1}{42}$		
91	$\frac{13}{252}$	92	0	93	$\frac{1}{28}$	94.	$2\frac{71}{80}$	95	$\frac{1}{255}$	96	$\frac{5}{24}$
97	0	98	$\frac{1}{154}$	99	1	100	$\frac{1}{3}$	101	$3\frac{27}{40}$	102	$2\frac{1}{15}$
103.	5	104	$10\frac{6}{7}$	105	$\frac{1}{24}$	106	$\frac{9}{10}$	107	$5\frac{23}{60}$	108	$1\frac{1}{312}$
109.	$1\frac{1}{48}$	110	$\frac{33}{35}$	111	$16\frac{19}{20}$	112	$7\frac{12}{85}$	113	$\frac{439}{4096}$	114	$\frac{41}{243}$
115	$1\frac{13}{72}$	116.	$2\frac{14}{75}$	117	$11\frac{61}{72}$	118	$3\frac{491}{512}$	119	$34\frac{51}{70}$	120.	$336\frac{1}{343}$

XVIII.

1	$\frac{6}{7}$	2	$\frac{10}{11}$	3.	$\frac{14}{15}$	4.	$\frac{16}{17}$	5.	$\frac{8}{15}$		
6	$\frac{9}{16}$	7	$\frac{7}{11}$	8	$\frac{20}{23}$	9.	$\frac{96}{97}$	10	$\frac{91}{100}$	11	$\frac{3}{4}$
12.	$\frac{5}{6}$	13	$\frac{1}{5}$	14	$\frac{3}{7}$	15	$\frac{5}{7}$.	16	$\frac{2}{5}$	17	$\frac{7}{25}$
18.	$\frac{11}{20}$	19	$\frac{2}{5}$	20	$\frac{3}{4}$	21	$\frac{3}{11}$	22	$\frac{4}{15}$	23	$\frac{5}{17}$
24	$\frac{7}{19}$	25	$\frac{4}{19}$	26	$\frac{5}{16}$	27	$\frac{4}{21}$	28	$\frac{7}{45}$	29	$\frac{5}{26}$
30	$\frac{2}{41}$	31.	$\frac{7}{32}$	32	$\frac{3}{20}$	33	$\frac{5}{52}$	34	$\frac{11}{68}$	35	$\frac{7}{64}$
36.	$\frac{23}{75}$	37	$\frac{5}{96}$	38.	$\frac{19}{56}$	39	$\frac{11}{120}$	40	$\frac{19}{77}$	41	$3\frac{1}{3}$

42. $2\frac{2}{3}$	43 $5\frac{3}{5}$	44 $14\frac{2}{3}$	45. $14\frac{2}{3}$	46 $9\frac{6}{7}$	47. $7\frac{14}{15}$
48. $30\frac{10}{17}$	49. 10	50 $40\frac{1}{2}$	51 $13\frac{1}{2}$	52 $22\frac{2}{3}$	53. $39\frac{3}{4}$
54 $34\frac{2}{3}$	55 $95\frac{1}{3}$	56 $116\frac{2}{3}$	57 $376\frac{1}{2}$	58 $297\frac{1}{3}$	59 $155\frac{1}{3}$
60 $703\frac{1}{2}$	61 $317\frac{1}{2}$	62 $154\frac{4}{5}$	63 22875		64 $13368\frac{5}{6}$
65 $\frac{4}{51}$	66 $\frac{2}{63}$	67. $\frac{2}{125}$	68 $\frac{2}{99}$	69 $\frac{20}{63}$	70 $\frac{17}{100}$
71 $\frac{2}{217}$	72. $\frac{4}{285}$	73 $\frac{17}{650}$	74 $\frac{23}{625}$	75 $\frac{1}{6}$	76 $\frac{2}{7}$
77 $\frac{5}{16}$	78 $\frac{2}{9}$.	79 $\frac{3}{16}$	80 $1\frac{7}{12}$	81 $\frac{3}{7}$	82 $\frac{4}{13}$
83 $\frac{11}{72}$	84. $\frac{2}{27}$	85 $1\frac{5}{33}$	86 $\frac{5}{16}$	87 $1\frac{7}{18}$	88. $1\frac{68}{135}$
89. 16173		90 $112\frac{5}{16}$			

XIX.

	1 $\frac{7}{24}$	2 $\frac{6}{35}$	3 $\frac{6}{35}$	4 $\frac{9}{20}$	5. $\frac{117}{187}$
6 $\frac{128}{189}$	7 1	8 $\frac{1}{2}$	9 1	10 $\frac{1}{24}$	11. $3\frac{1}{3}$
12 $\frac{7}{12}$	13 $\frac{32}{39}$	14 $2\frac{5}{14}$	15 10	16 1	17 $1\frac{6}{7}$
18. 53	19 $\frac{16}{25}$	20 $8\frac{1}{3}$	21 $\frac{2}{15}$	22 $\frac{15}{64}$	23. $\frac{2}{9}$
24 $\frac{16}{63}$	25. $\frac{2}{3}$	26 $\frac{1}{3}$	27 $\frac{5}{9}$	28 $\frac{1}{5}$	29 1
30. 1	31 $2\frac{1}{2}$	32 $5\frac{1}{7}$	33 30	34 $12\frac{8}{9}$	35 15
36 $61\frac{4}{9}$	37. $14\frac{11}{25}$	38. $51\frac{1}{49}$	39 42	40 12	41 2920
42 $41\frac{1}{2}$	43 $43\frac{1}{2}$	44 $32\frac{1}{2}$	45 $\frac{1}{25}$	46 $\frac{15}{28}$	47. $\frac{11}{4418}$
48. $\frac{49}{165}$	49 $\frac{6}{5}$	50 $9\frac{1}{3}$	51 $4\frac{1}{5}$	52 $2\frac{170}{231}$	53. $2\frac{1}{3}$
54 90	55 $140\frac{1}{4}$	56 392	57 $13\frac{1}{3}$	58 3	59. $27\frac{1}{2}$
60 $\frac{33}{169}$	61 1	62 $1\frac{7}{9}$	63 $46\frac{4}{5}$	64 70	65. 1
66 $8\frac{1}{4}$	67 1	68 $\frac{2}{7}$	69 $\frac{3}{121}$	70 $\frac{1}{12}$	71 $1\frac{7}{8}$
72 4	73 $\frac{920}{1353}$	74 $4\frac{2}{3}$	75. $40\frac{1}{3}$	76. $\frac{5}{8}$	77. 6.
78. 1.	79 $\frac{2}{125}$	80 $\frac{220}{279}$			

XX.

	1. 12	2. 10	3 15	4 39	5. 84.
6. 160	7. 1080	8. 378	9 600	10. 1015	11. $\frac{8}{9}$.
12. $\frac{16}{25}$	13. $1\frac{1}{6}$	14 $2\frac{4}{9}$	15 $1\frac{1}{3}$	16 22	17. 1.
18 1	19 $1\frac{1}{3}$	20. $1\frac{32}{45}$	21 $9\frac{1}{3}$	22. $16\frac{2}{3}$	23 $17\frac{1}{2}$
24. $17\frac{1}{3}$	25 40	26 $\frac{1}{40}$	27. $\frac{7}{40}$	28 $\frac{8}{35}$	29 95
30 $17\frac{1}{2}$	31 $2\frac{4}{5}$	32 $1\frac{1}{3}$	33 $1\frac{5}{21}$	34 $7\frac{6}{19}$	35. $\frac{3}{8}$
36. $6\frac{2}{3}$	37 $4\frac{1}{2}$	38 $9\frac{9}{80}$	39 $\frac{99}{896}$	40 $\frac{203}{951}$	41 $2\frac{5}{8}$
42. $1\frac{3}{32}$	43 $\frac{21}{32}$	44 $\frac{2}{3}$	45 $2\frac{7}{16}$	46 $4\frac{1}{2}$	47. $\frac{16}{21}$
48 $1\frac{7}{20}$	49 $3\frac{3}{5}$	50 $5\frac{5}{8}$	51. $\frac{5}{18}$	52 $\frac{5}{96}$	53 $\frac{3}{35}$
54. $\frac{2}{45}$	55 $\frac{1}{24}$	56. $\frac{1}{33}$	57 $11\frac{1}{4}$	58 $25\frac{2}{3}$	59 $13\frac{1}{3}$
60. $49\frac{1}{2}$	61 $1\frac{1}{2}$	62 $\frac{7}{8}$	63 $\frac{13}{18}$	64 $\frac{5}{16}$	65. $5\frac{5}{11}$
66 12	67 $3\frac{11}{15}$	68 $2\frac{5}{8}$	69 $\frac{11}{12}$	70 $3\frac{1}{9}$	71. $3\frac{39}{112}$
72. $4\frac{2}{5}$	73. $\frac{7}{9}$	74 $\frac{2}{27}$	75. 75	76. $1\frac{46}{63}$	77 $7\frac{88}{107}$
78 $3\frac{1}{3}$	79 $\frac{1}{4}$	80 $\frac{28}{129}$	81 $\frac{11}{12}$	82 $3\frac{1}{2}$	83 $10\frac{163}{182}$
84 $14\frac{2}{15}$	85 $6\frac{5}{12}$	86 $\frac{2}{3}$	87. $\frac{56}{117}$	88. $10\frac{39}{50}$	89. 68
90. $8\frac{16}{21}$					

XXI.

	1. $7\frac{1}{2}$	2 120	3 $5\frac{11}{15}$	4 3	5. $\frac{1}{20}$.
6. $44\frac{4}{5}$	7. $7\frac{11}{19}$	8. $1\frac{45}{53}$	9. $\frac{28}{75}$	10. $\frac{1}{3}$	11. $9\frac{13}{20}$
12 12	13 1	14 $\frac{9}{25}$	15 $\frac{5}{6}$	16 $\frac{49}{72}$	17. $12\frac{13}{40}$
18 $15\frac{17}{81}$	19. $40\frac{31}{40}$	20. $\frac{14}{15}$	21 $19\frac{2}{3}$.	22. 11	23. $22\frac{2}{3}$

24 $9\frac{1}{8}$.	25. $55\frac{71}{128}$	26. $83\frac{27}{104}$	27 $3\frac{29}{30}$	28 $1\frac{5}{8}$	29 $2\frac{17}{120}$
30. $\frac{1}{11}$	31. $7\frac{1}{8}$	32 $6\frac{3}{4}$	33 $2\frac{15}{56}$	34. $231\frac{5}{8}$	35 $14\frac{11}{20}$
36. $2\frac{1}{7}$	37. $4\frac{5}{27}$	38. 4	39 9	40. $\frac{173}{882}$	41. $18\frac{1}{18}$
42. $5\frac{1}{3}$	43. $\frac{1}{4}$	44. $\frac{51}{76}$	45. 1	46 $\frac{58}{189}$	47. 11
48. $7\frac{41}{60}$	49 1	50. 0	51 $1\frac{1}{4}$	52. $10\frac{11}{70}$	53. $\frac{15}{16}$
54 $\frac{2}{15}$	55 $106\frac{1}{3}$	56. $\frac{64}{85}$	57. $\frac{5}{81}$	58. $\frac{1}{47}$	59. 10
60. $\frac{23}{25}$	61 $2\frac{13}{24}$	62. $\frac{1}{24}$.	63. $\frac{1}{18000}$	64 $\frac{9}{100}$	65 $\frac{527}{1225}$
66 $2\frac{2}{7}$	67 $\frac{13}{28}$.	68. $1\frac{140}{549}$.	69. 1	70. $6\frac{2}{3}$	71. $2\frac{6}{25}$
72. $1\frac{1}{5}$	73. $8\frac{2}{3}$	74. $1\frac{23}{105}$	75. $\frac{1}{5}$.	76. 4	77. 1
78. $\frac{53}{122}$	79. $\frac{273}{1315}$	80 $\frac{95}{737}$.	81 $1\frac{5}{22}$	82. 1	83. $\frac{1}{4}$
84 3	85. 1	86. 10	87. 2.	88. $\frac{1}{2}$	89. $\frac{1}{8}$.
90. 1.	91. 76	92 75	93. $\frac{1}{144}$	94. 4	95 $\frac{44}{105}$
96. $\frac{11}{17}$	97. $\frac{159}{3146}$	98. 9	99. 1	100. $\frac{3}{187}$	101. $\frac{3}{4}$
102. $\frac{4}{5}$	103. $\frac{13}{30}$	104 $\frac{16}{37}$	105. $\frac{16}{29}$	106. $\frac{32}{38}$	107. $\frac{5}{8}$
108. $\frac{58}{69}$	109. 2	110 $\frac{57}{115}$	111 $2\frac{13}{60}$	112. $\frac{2}{5}$	113. $\frac{37}{60}$
114. $\frac{43}{60}$	115 $1\frac{4}{15}$	116. $1\frac{3}{10}$	117 $\frac{43}{60}$	118. $\frac{1}{15}$	119. $\frac{5}{8}$
120. $1\frac{1}{9}$	121 $1\frac{29}{48}$	122. $1\frac{31}{45}$	123. $1\frac{1}{30}$	124 $1\frac{2}{15}$	125. $1\frac{37}{48}$
126. $\frac{40}{93}$	127 $\frac{32}{45}$	128. $1\frac{1}{9}$	129. $\frac{47}{48}$	130 $\frac{10}{27}$	131 17
132. 2	133. $\frac{11}{221}$	134 $6\frac{1}{4}$.	135. $1\frac{53}{121}$.	136. $1\frac{1}{2}$	137. $15\frac{1}{2}$
138. $1\frac{37}{72}$	139. 1	140 5	141. 2	142. $1\frac{7}{87}$	143. $\frac{1}{3}$
144. $\frac{54}{175}$	145. $4\frac{23}{74}$	146. $2\frac{19}{30}$	147. $1\frac{8}{195}$.	148. $\frac{348}{365}$	149. $1\frac{445}{1612}$.
150. $1\frac{2}{3}$	151. $\frac{288}{289}$	152. $4\frac{17}{47}$.	153. $\frac{286}{333}$	154. $96\frac{1}{4}$	155. 10
156 0	157. 0	158. $5\frac{2}{9}$	159. 2	160. $\frac{1}{9}$.	161. 8
162. $1\frac{5}{16}$	163. $15\frac{1}{2}$	164. $61\frac{10}{27}$.	165. $7\frac{25}{32}$	166. $129\frac{37}{42}$	167. $1\frac{8}{17}$
168. 1	169. $1\frac{11}{139}$	170. 1	171 1.	172 $3\frac{81}{125}$	173 $1\frac{289}{3600}$
174. $2\frac{24}{317}$	175. $2\frac{1}{6}$	176. $4\frac{1}{6}$.	177. $\frac{5}{22}$	178. $2\frac{7}{11}$	179. 1
180. $2\frac{5}{7}$	181. $1\frac{40}{441}$.	182. 6	183. 1.	184. $\frac{9}{25}$	185. 1
186. $\frac{15}{28}$	187. $\frac{223}{360}$	188 $1\frac{7}{18}$	189. $11\frac{50}{111}$	190. 1.	

XXII.

1 6*d* ; 3*d* ; $1\frac{1}{2}$*d* , $\frac{3}{4}$*d* 2 4*d* , 2*d* , 1*d* ; $\frac{1}{2}$*d* , $\frac{1}{4}$*d*.
3. $4\frac{1}{2}$*d* ; $7\frac{1}{2}$*d* ; $10\frac{1}{2}$*d* , $2\frac{1}{4}$*d* , $3\frac{3}{4}$*d* 4 8*d* , 10*d* ; $2\frac{1}{4}$*d* ; $5\frac{1}{2}$*d* ; $1\frac{1}{4}$*d*
5 10*s* , 5*s* , 2*s* 6*d* , 1*s*. 3*d* 6 6*s* 8*d* , 13*s* 4*d* , 3*s* 4*d* , 16*s* 8*d*., 1*s* 8*d* , 18*s* 4*d* 7. 15*s* , 7*s* 6*d* ; 12*s* 6*d* , 17*s* 6*d*., 3*s* 9*d*
8. 4*s* , 8*s* , 12*s* , 16*s* ; 1*s* 4*d* , 9*s* 6*d* 9. 1*d* , 1*s* 5*d* , 2*d* ; 4*d* ; 2*s* 4*d*., 3*d* , 2*s* 3*d* ; $\frac{1}{4}$*d* , $4\frac{1}{4}$*d* , $\frac{1}{2}$*d* 10. 1*s* 11*d* , 3*s* 1*d* , 4*s* 5*d* , 1*s*. 10*d*., 5*s* 2*d* , 1*s* 9*d* , $3\frac{1}{2}$*d* , $3\frac{1}{4}$*d* , $5\frac{3}{4}$*d* , $9\frac{1}{4}$*d* 11 £1, 18*s* $1\frac{1}{4}$*d*.
12. £2, 7*s* $4\frac{1}{5}$*d* 13 9*s* $5\frac{2}{5}$*d* 14. £1, 11*s* $8\frac{5}{8}$*d*. 15 3*s* $7\frac{2}{7}$*d*
16 6*s* $11\frac{4}{5}$*d* 17. £1, 14*s* $6\frac{1}{4}$*d* 18 £2, 3*s* $8\frac{13}{24}$*d*. 19 8*s* 4*d*
20 15*s* 21. £2, 16*s* 22. £2, 5*s* 23. £4, 5*s* $8\frac{4}{7}$*d*.
24. £5, 9*s* $1\frac{1}{11}$*d* 25 8*s* $3\frac{1}{9}$*d*. 26. 5*s* $7\frac{1}{12}$*d*. 27 17 hrs. 36 min.
28. 2 hrs 21 min. 40 secs 29. 3 ro 30 sq po. 30 480 yds (*or* 2 fur 7 po $1\frac{1}{2}$ yd) 31 3 yds 2 ft 8 in. 32 13 cwts 3 qrs
33. 8 yds 1 ft $8\frac{4}{7}$ in 34. 1 sq yd 8 sq ft $26\frac{2}{11}$ sq. in 35 £10, 3*s* $8\frac{55}{96}$*d*
36 £34, 6*s* $5\frac{11}{72}$*d*. 37 £4, 15*s* $2\frac{6}{7}$*d* 38 £11, 13*s* $10\frac{9}{11}$*d*
39 £3, 3*s* $8\frac{3}{4}$*d* 40. £1, 16*s* 10*d* 41 £2191, 3*s* 8*d*.
42. £117, 6*s* $5\frac{4}{5}$*d*. 43. 9 yds 0 ft $8\frac{1}{3}$ in 44. 2 tons 5 cwts 3 qrs 21 lbs
45. 1 ton 16 cwts 0 qrs $9\frac{4}{5}$ lbs 46 8 sq po $24\frac{5}{8}$ sq yds 47 $11\frac{1}{4}$*d*
48. 8*s*. 9*d*. 49. 11*s*. $9\frac{3}{4}$*d* 50. 3*s* $9\frac{1}{3}$*d* 51. 1*s* $6\frac{37}{84}$*d* 52. 2*s* $0\frac{21}{76}$*d*.

53 £7, 10*s* 54 £2, 11*s* 55 7 tons 8 cwts 1 qr $15\frac{11}{31}$ lbs
56 9 tons 19 cwts 1 qr $6\frac{22}{25}$ lbs 57 1 ton 2 cwts 2 qrs
58 9 hrs 28 min 45 secs 59 £11, 17*s* $5\frac{1}{18}d$ 60 £12, 10*s* $10\frac{1}{2}d$.
61 £4042, 17*s* $6\frac{3}{4}d$ 62 £2025, 5*s* $2\frac{2}{3}d$ 63 £65, 6*s* $1\frac{5}{7}d$
64. £83, 19*s* $2\frac{50}{63}d$ 65 16*s* $10\frac{3}{5}d$ 66 £1, 7*s* 9*d*
67 £1, 0*s* $10\frac{113}{122}d$ 68 £1, 6*s* $8\frac{13}{71}d$ 69 £17, 17*s* $3\frac{1}{2}d$
70 £9, 17*s* $1\frac{7}{25}d$ 71 7*s* 72 £5 73 £1 74 £3
75. £5 76 £7, 9*s* $7\frac{1}{2}d$ 77 5 yds 0 ft 9 in 78 3 ro 10 sq po
79 $4\frac{1}{4}d$ 80. 6*s* $3\frac{1}{4}d$ 81 2*s* $0\frac{1}{2}d$ 82 1*s* $0\frac{3}{4}d$ 83 £2, 16*s* 11*d*
84. 1*d* 85 1 ton 3 cwts 0 qrs $21\frac{1}{3}$ lbs 86 1 cwt 0 qrs 3 lbs
87 £7, 7*s* 7*d* 88. £14, 7*s* 89. 1 cwt 0 qrs 16 lbs
90. 9 tons 6 cwts.

XXIII. 1 $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ 2 $\frac{3}{8}$; $\frac{5}{8}$, $\frac{7}{8}$ 3 $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{24}$, $\frac{1}{48}$
4 $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{5}{24}$, $\frac{5}{48}$ 5 $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ 6 $\frac{3}{4}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$ 7 $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{1}{12}$
8. $\frac{1}{5}$, $\frac{2}{5}$; $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{15}$ 9 $\frac{1}{240}$, $\frac{1}{120}$, $\frac{1}{60}$, $\frac{1}{80}$, $\frac{1}{480}$, $\frac{1}{960}$ 10 $\frac{7}{240}$, $\frac{11}{240}$, $\frac{7}{480}$, $\frac{17}{960}$, $\frac{31}{960}$ 11. $\frac{5}{18}$ 12 $\frac{1}{5}$ 13 $\frac{5}{16}$ 14 $\frac{3}{26}$ 15 $\frac{1}{5}$
16 $\frac{11}{37}$ 17 $\frac{101}{276}$ 18 $\frac{19}{741}$ 19 $\frac{11}{32}$ 20 $\frac{11}{14}$
21. $\frac{35}{104}$ 22 $\frac{20}{99}$ 23 $\frac{47}{290}$ 24 $\frac{125}{228}$ 25 $\frac{77}{160}$
26 $\frac{31}{320}$ 27. $\frac{1}{10}$ 28 $\frac{1}{100}$ 29 $\frac{5}{51}$ 30 $\frac{112}{285}$
31 $\frac{9}{16}$ 32 $\frac{39}{224}$ 33 $\frac{9}{160}$ 34 $\frac{237}{320}$ 35 $\frac{7}{352}$
36 $\frac{109}{160}$. 37 $\frac{21}{320}$ 38 $\frac{1}{36}$ 39 $\frac{139}{200}$ 40 $\frac{7}{8}$
41 $48\frac{3}{4}s$ 42 $111\frac{5}{8}s$ 43 $63\frac{5}{16}s$ 44 $57\frac{3}{16}s$ 45 £$14\frac{13}{15}$
46 £$21\frac{67}{160}$ 47 £$7\frac{7}{16}$ 48 £$7\frac{3}{32}$ 49 $31\frac{3}{4}$ qrs 50 $33\frac{3}{4}$ cwts
51 $192\frac{1}{7}$ cwts 52 $13\frac{15}{56}$ qrs 53 $17\frac{11}{12}$ yds 54 $127\frac{1}{2}$ po
55 $12\frac{7}{20}$ ac 56 $7\frac{9}{10}$ ac 57 46 times 58 62 times
59 $\frac{1}{8}$ 60 $\frac{1}{12}$ 61 $\frac{4}{9}$ 62 $\frac{1}{39}$ 63 $\frac{6}{5}$
64. $\frac{6}{11}$ 65 $\frac{9}{28}$ 66 $\frac{1}{8}$ 67 $\frac{1}{6}$ 68 $\frac{1}{924}$
69 $\frac{1}{880}$ 70 $\frac{1}{160}$ 71 $\frac{3}{7040}$ 72 $\frac{5}{192}$ 73 $\frac{1}{11}$
74 $\frac{24}{25}$ 75 $\frac{7}{11}$ 76 $\frac{30}{49}$ 77 $\frac{3}{4}$ 78 $\frac{4}{33}$
79. $\frac{13}{125}$ 80. $\frac{73}{252}$

XXIV. 1. Nineteen *ninety-eighths* 2 $\frac{91}{149}$ 3 $\frac{126}{17}$ 4. $4\frac{9}{17}$
5 $\frac{147}{777}$ 6 $\frac{12}{19}$ 7. $\frac{64}{616}$, $\frac{49}{616}$ 8 $\frac{137}{140}$ 9 $8\frac{2}{315}$ 10. 2*s*
11 Ninety-one *ninetieths* 12 $700\frac{19}{91}$ 13 $\frac{1890}{19}$. 14 $52\frac{11}{19}$
15 $\frac{63}{153}$ 16 $\frac{2}{9}$ 17. $\frac{17}{38}$ 18 $17\frac{133}{162}$ 19 $46\frac{17}{10}$. 20 5*s*. 3*d*
21. 101 *tenths* 22 34 *fifty-oneths* 23 $\frac{25}{83}$ 24 $7\frac{161}{250}$ 25 $3\frac{11}{185}$
26 $\frac{3}{20}$ 27 $1\frac{1}{1460}$ 28 0 29. $\frac{11}{210}$ 30 $\frac{1}{3}$ of it 31 49 *units*
32 4 *ninths* 33 $\frac{9}{10}$ 34 $1\frac{11}{45}$ 35 $20\frac{501}{1001}$ 36. $8\frac{7}{16}$ 37. $14\frac{2}{3}$
38 $1\frac{3}{4}$ 39 $3\frac{4}{57}$ 40 30 acres 41 $\frac{442}{33}$ 42. $\frac{19}{88}$ 43 $\frac{19}{81}$
44 $10\frac{23}{102}$ 45. $3\frac{431}{448}$ 46 $1\frac{7}{9}$ 47 $\frac{10}{11}$ 48 $\frac{5}{19}$ 49 2
50 $\frac{1}{60}$ of the field 51 $19\frac{99}{101}$ 52 $\frac{123}{231}$ 53 $\frac{19}{32}$ 54 $14\frac{97}{195}$
55. $3\frac{268}{285}$ 56 6 57 $1\frac{61}{65}$ 58 $\frac{9}{10}$ 59 30 bits 60. 3*s* $1\frac{1}{2}d$
61 500 *seventieths* 62. $\frac{12}{89}$ 63 $8\frac{13}{44}$ 64 $13708\frac{43}{54}$. 65 5
66 1 67 $1\frac{1}{27}$ 68 33 69 The latter. 70 3 hrs 54 m
71 820 *ninetieths* 72 $\frac{94}{129}$ 73 $\frac{1}{15}$ 74 $\frac{1}{180}$ 75 0
76 $6\frac{2}{9}$ 77 $163\frac{26}{45}$ 78 240 79 £3, 11*s* 3*d* 80. 9 hrs 24 m

ANSWERS TO QUESTIONS 61*–100*

XXIV.

61*. 12
62*. 9*d*
63*. 91
64* None
65*. 2*s* 1*d*
66*. 6*d.*
67* 2*s.* 6*d.*
68* 2*s.* 8*d.*
69*. 1*s*
70*. 1*s*
71*. £60
72*. £292, 10*s*
73* 1*d*
74*. 2*s* 6*d.*
75*. 12
76*. 36 miles
77*. 24 feet
78* 24
79*. 305
80* 3*s* 6*d*
81*. £360.
82* 640
83*. 24.
84*. $\frac{2}{15}$.
85*. $\frac{3}{640}$
86* 128
87* 15 bits, 1½ in left
88* 2*s.* 7*d*
89* 6*s* 0½*d*
90* 48
91*. 4 ft 6 in
92*. 8 st. 8 lbs.
93*. $\frac{9}{16}$
94*. 6 A.M
95* Bat, 5*s* 6*d.*, ball, wickets, 2*s* 6*d.*
96*. 15*s*
97*. 29.
98*. 120.
99*. £9600
100* £22, 4*s.*

81 $\frac{999}{1147}$ 82 $\frac{6}{7}$ is greatest, $\frac{5}{8}$ is least 83. $9\frac{4}{7}$ 84. 81884
85 42 86 0 87 1 88 £2, 10*s* 9*d* 89. $\frac{1}{512}$.
90 $\frac{6}{7}$ of the book 91 $\frac{359}{777}$ 92 $\frac{11-9}{12-10}$ 93 $\frac{23}{120}$ 94. 7.
95. $82\frac{85}{126}$ 96 $\frac{8}{27}$ 97. £1, 11*s* $4\frac{7}{8}d$ 98 2*s* 99. $\frac{7}{192}$
100. $\frac{1}{100}$ of the pole 101 19 102 $\frac{1}{5}$ 103. 36 104. $\frac{2}{17}$
105. $\frac{1}{4}$ 106 2 107 $1\frac{281}{1000}$ 108 £7, 6*s* $0\frac{3}{4}d$ 109 $\frac{1}{308}$
110. A, £2, 12*s* 4*d*, B, £1, 19*s* 3*d* 111 $\frac{14904}{17112}$ 112 $13\frac{1}{3}$ 113 $58\frac{155}{396}$
114 0 115 7 116 $7\frac{8}{15}$ 117 15 118 £5, 11*s* $8\frac{1}{2}d$ 119 $\frac{1}{27}$
120. A, £11, 19*s* $7\frac{1}{2}d$, B, £7, 19*s* 9*d* 121 £4, 3*s* $1\frac{1}{2}d$ 122. The latter.
123 $1\frac{1}{20}$ 124 15 times 125. 16 times, rem^r $\frac{1}{35}$ 126. $2\frac{15}{49}$
127. $2\frac{13}{33}$ 128 $\frac{63}{5115}d$ 129 8 marbles 130 $\frac{7}{10}$ of the work
131 £5381, 11*s* 3*d* 132 B 133 $34\frac{79}{312}$ 134 $5\frac{91}{102}$ 135 $\frac{3}{4}$
136 $158\frac{351}{2450}$ 137. $\frac{1}{9}$ 138. £40, 19*s* 139 $\frac{9}{20}$ of the property
140 $\frac{6}{35}$ of the field 141. £281, 0*s* $11\frac{3}{7}d$ 142 G C M. $\frac{1}{24}$, L C M $26\frac{1}{4}$
143. $\frac{7}{37}$ 144 $\frac{157}{1728}$ 145 $\frac{72}{175}$ 146 21 dys 3 hrs 30 m
147. 3 dwts $8\frac{19}{40}$ grs 148 2 ft 6 in. 149 $\frac{4}{57}$ of his journey
150 £6, 14*s* $4\frac{1}{2}d$ for one, 2*s* 1*d* for each of the others 151 £4
152 G C M $\frac{12}{35}$, L C M 24 153. $\frac{1}{12}$ 154. $1\frac{28}{155}$. 155 3
156. $\frac{9}{806}$ 157. 17 cwts 3 qrs 10 lbs 158 67584 stamps 159. $\frac{1}{10}$.
160. 1 hr 12 min 161. $\frac{1}{1\frac{12}{23}}$. 162 233 tons 10 cwts 2 qrs $10\frac{2}{3}$ lbs.
163 77 times 164 $\frac{7+4}{11+4}$ is greatest, $\frac{7-4}{11-4}$ is least. 165. $286999\frac{72}{359}$
166 $1\frac{217}{221}$ 167. $\frac{42}{43}$ 168 $\frac{11}{15}$ 169. 4 inches. 170 $\frac{69}{160}$
171. $\frac{47619}{333333}$ 172. 144 tons 3 cwts 2 qrs. $4\frac{26}{55}$ lbs. 173. 52560 times
174 $\frac{31}{91}$, $\frac{41}{119}$, $\frac{51}{143}$ 175. $16699\frac{107}{274}$ 176. 5 177. $1\frac{30}{127}$ 178 $1\frac{114113}{1000000}$
179 955 feet 180 $\frac{91}{240}$ of the day 181. 1219 182 7245
183. $\frac{10}{31}$, $\frac{9}{28}$, $\frac{8}{25}$, $\frac{7}{22}$ 184. $1\frac{3}{17}$ 185 1510 186. 1126 lengths, 4 inches over 187. 1 188. $\frac{11}{18}$ 189. $\frac{17}{367}$ 190 $1\frac{51}{145}$.
191. 3213 192 11088 193. $\frac{2}{17}$, $\frac{3}{25}$, $\frac{4}{33}$, $\frac{5}{41}$ 194 $2\frac{113}{204}$ 195. $12\frac{47}{90}$
196. 9 times, $1\frac{3}{40}$ rem. 197 £49 198. $\frac{7}{8}$ 199. 999000 grains.
200 $\frac{7}{8}$

XXV.

1. 6*d*, 4*d*, 3*d*, 2*d*, $1\frac{1}{2}d$, 1*d*, $\frac{3}{4}d$ 2. 10*s*, 6*s* 8*d*; 5*s*; 4*s*; 3*s* 4*d*, 2*s* 6*d*, 2*s*, 1*s* 8*d*, 1*s* 4*d*, 1*s*. 3*d*, 1*s*, 8*d*, 6*d*, 4*d*, 3*d*
3. $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{16}$. 4. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$, $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{12}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$.
5. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{10}$; $\frac{1}{20}$, $\frac{1}{12}$, $\frac{1}{15}$; $\frac{1}{30}$; $\frac{1}{60}$. 6. $\frac{1}{2}$; $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{5}$; $\frac{1}{10}$, $\frac{1}{20}$; $\frac{1}{3}$, $\frac{1}{6}$; $\frac{1}{12}$ 7. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{6}$; $\frac{1}{12}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$ 8 $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$; $\frac{1}{3}$; $\frac{1}{6}$, $\frac{1}{12}$ 9. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{5}$; $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{40}$; $\frac{1}{80}$ 10. $\frac{1}{2}$; $\frac{1}{4}$, $\frac{1}{8}$; $\frac{1}{16}$; $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$; $\frac{1}{40}$ 11 £115, 10*s* 12. £96, 15*s*

13. £148, 8*s*	14 £127	15. £142, 6*s*
16 £197, 16*s*		
17. £68, 2*s* 6*d*	18. £84, 2*s* 6*d*	19 £218, 17*s* 6*d*
20 £237, 17*s* 6*d*.	21. £143, 6*s*. 8*d*	22. £177, 6*s* 8*d*
23. £513, 13*s* 4*d*	24. £417, 13*s* 4*d*	25 £14, 16*s*. 8*d*
26 £16, 3*s* 4*d*	27. £32, 1*s* 8*d*	28. £64, 1*s* 8*d*
29 £42, 12*s* 6*d*.	30. £59, 5*s* 6*d*	31. £2, 18*s* 6*d*
32. £2, 4*s* 9*d*.	33. £14, 10*s* 4*d*	34. £7, 17*s* 8*d*.
35 £10, 11*s* 2*d*	36. £21, 9*s*. 8*d*.	37. £8, 19*s*
38. £15, 6*s* $7\frac{1}{2}d$	39. £6, 2*s*. 3*d*.	40. £6, 7*s* $10\frac{1}{2}d$

41 £430, 2s   42 £557, 3s   43 £296, 2s
44 £528, 6s   45. £55, 16s   46 £64, 4s.
47 £303, 7s 6d   48 £451, 17s 6d.   49 £2666, 5s.
50 £3569, 7s 6d   51. £16, 6s 3d   52 £35, 16s 3d.
53 £245, 18s 8d   54 £206, 1s 3d   55 £34, 8s 9d.
56 £43, 10s 5d   57. £6, 9s 4½d   58 £10, 3s 7½d
59. £31, 4s 4½d   60. £34, 14s 4½d   61 £2, 14s 5¾d
62 £3, 11s 3d.   63 £1, 4s 11¼d   64 £2, 10s 0¾d.
65 £65, 16s 9½d   66. £116, 17s 3½d   67. £3, 14s 0¾d.
68 £8, 11s. 6¾d   69. £22, 15s 7½d   70 £31, 0s 5¼d.
71 £933   72 £1128, 15s   73. £879, 12s
74 £298, 2s.   75. £475, 17s. 6d   76. £615, 7s 6d
77 £125, 6s 8d.   78. £96, 16s 8d   79. £164, 13s 4d
80 £222, 1s 8d.   81 £152, 15s   82 £149, 12s
83 £799, 10s   84 £1902, 7s 6d   85 £2442, 8s
86 £1865, 10s.   87 £3805, 13s 4d   88. £9791, 13s 4d
89. £551, 9s 6d   90 £540, 11s 3d   91 £382, 2s 8d
92 £270, 15s 8d   93 £44, 2s.   94 £45, 10s 1½d
95. £2213, 8s 3d   96 £3238, 13s 7d   97 £45
98 £76, 16s   •99 £53, 8s   100. £198, 9s 4½d   101 £102, 15s
102 £161, 5s   103. £62, 8s   104. £73, 16s   105. £386, 2s
106 £597, 12s 6d   107 £449, 15s   108. £1402   109. £1430, 12s 6d
110 £692, 10s   111 £63, 15s   112 £245, 2s   113 £119, 3s 4d
114. £139, 18s 4d.   115 £517, 10s   116 £887, 5s.   117. £284, 4s
118 £435   119 £53, 15s 6d   120. £68, 18s   121 £22, 6s 8d
122 £21, 6s 1½d   123 £7, 4s 10d   124 £8, 6s 9d
125 £9, 3s 9d   126 £21, 10s 10d   127 £61, 4s 4½d.
128 £75, 16s 10½d   129 £86, 19s   130 £77, 7s
131 £42, 12s   132 £37, 11s 4d   133 £29, 18s 6d
134 £38, 1s 3d   135 £609, 5s 9d.   136 £957, 4s 1½d
137 £362, 14s   138 £336, 7s   139 £1340, 12s 6d
140 £2025, 7s 6d   141 £2479, 19s   142 £3729, 13s 9d
143 £273, 14s 1d   144 £302, 18s 7d   145. £348, 3s
146 £792, 11s 6d   147 £818, 0s 8d   148 £838, 15s
149 £8746, 6s 3d.   150 £7623, 8s 4d   151. £24581, 2s 10d.
152. £38162, 11s.   153 £1657, 18s 1½d   154 £1766, 7s 6d
155 £19410, 4s. 4d   156 £19295, 7s 6d   157 £90, 17s 6¼d.
158 £46, 1s 9¾d   159. £23, 13s 7½d   160 £22, 18s 1d
161 £13, 11s 11½d   162 £13, 8s 4d   163 £10, 19s. 9¼d
164 £12, 4s 3¼d   165. £13, 0s 9d   166 £24, 18s
167 £23, 0s 7½d   168. £19, 15s 3d   169. £46, 19s 10½d.
170 £85, 18s 9½d   171 £54, 18s 10d   172 £48, 9s 4½d
173. £312, 6s 7½d   174 £342, 3s 0½d   175 £41227, 5s 3¾d
176 £48103, 11s 9¼d   177 £8469, 10s   178 £10057, 19s
179 £36039, 6s 9½d   180 £39467, 11s 0¾d   181. £10416, 14s 2d.
182 £5211, 0s 8¼d   183. £903, 9s   184 £941, 14s
185 £801, 13s 4d   186 £783, 5s 1d   187 £564, 15s 1½d.
188 £458, 13s 9d   189 £2901, 19s. 11¾d.   190 £8885, 8s 1½d.
191. £1017, 6s 8¼d.   192. £1289, 9s   193. £1350, 1s. 0¾d.

194 £1226, 9s 7$\frac{1}{2}d$ 195 £4424, 3s 196. £4108, 13s
197 £855, 11s 198 £962, 7s 8d. 199 £1586, 16s 3d
200 £1615, 9s 8d. 201 £2186, 4s 5$\frac{3}{4}d$ 202 £2968, 9s 0$\frac{3}{4}d$.
203 £663, 8s 1d 204. £570 205. £1999, 12s 2d
206. £2816, 9s 0$\frac{3}{4}d$ 207. £79, 14s 8$\frac{1}{4}d$ 208 £69, 8s 3$\frac{1}{2}d$
209 £512, 15s 6d 210. £869, 18s 9d 211. £347, 11s 9d
212 £373, 1s 0$\frac{1}{2}d$. 213. £155, 0s 11d 214. £293, 7s. 6$\frac{3}{4}d$
215 £5588, 5s 5$\frac{1}{4}d$. 216. £4240, 9s 3$\frac{1}{4}d$ 217 £44, 14s 5$\frac{3}{4}d$.
218. £65, 6s 6$\frac{3}{4}d$. 219 £323, 18s 4d 220 £297, 14s 1d
221 £49, 4s 10$\frac{3}{4}d$ 222. £188, 17s 4$\frac{1}{4}d$. 223 £413, 18s 10$\frac{1}{2}d$.
224 £1105, 6s 8$\frac{3}{4}d$ 225 £1142, 12s 3d. 226. £1455, 17s 10$\frac{1}{2}d$.
227 £378, 10s 0$\frac{3}{4}d$ 228. £502, 1s 8$\frac{3}{4}d$ 229. £8, 11s 2$\frac{1}{4}d$
230. £8, 10s 11d 231. £25, 11s 8$\frac{3}{4}d$. 232 £42, 6s 4$\frac{1}{2}d$.
233. £32, 1s 9$\frac{3}{4}d$. 234 £49, 19s 1$\frac{1}{4}d$. 235 £362, 5s 0$\frac{3}{4}d$
236. £363, 16s 2$\frac{1}{2}d$. 237 £140, 0s 8$\frac{1}{4}d$. 238 £96, 1s 4$\frac{1}{4}d$.
239. £6330, 12s 2$\frac{1}{4}d$ 240. £7193, 6s 241 £17, 9s 7$\frac{5}{16}d$
242 £102, 1s. 0$\frac{7}{8}d$ 243 £146, 13s 8$\frac{5}{8}d$ 244 £312, 4s. 7$\frac{5}{16}d$
245 £104, 19s 11$\frac{1}{3}d$ 246 £337, 13s 6$\frac{39}{40}d$ 247. £128, 16s 0$\frac{7}{8}d$
248. £656, 15s 10$\frac{13}{20}d$ 249 £876, 19s 2$\frac{5}{8}d$ 250. £784, 18s 8$\frac{5}{8}d$
251. £87, 13s 2$\frac{2}{5}d$ 252. £54, 12s. 3$\frac{1}{13}d$ 253. £1311, 19s 7d
254. £3735, 15s 255 £631, 10s 256. £153, 16s 9$\frac{1}{2}d$.
257. £584, 18s 8d. 258 £2695, 14s 0$\frac{1}{2}d$ 259 £197, 9s 2d
260. £624, 17s 9$\frac{1}{2}d$ 261. £353, 17s 3$\frac{3}{5}d$ 262 £410, 17s 11$\frac{5}{8}d$
263. £11, 12s. 9d 264. £12, 7s 2$\frac{1}{2}d$ 265 £383, 5s
266. £389, 17s 10$\frac{1}{4}d$. 267 £526, 1s 5$\frac{5}{8}d$ 268 £4327, 7s 5$\frac{15}{32}d$
269. £9, 3s 1$\frac{1}{8}d$ 270 £38, 5s 11$\frac{1}{4}d$ 271. £5353, 1s 6d.
272. £2732, 11s 7d 273 5 tons 11 cwts 0 qrs 1 lb 6 ozs. 11 drs
274. 37 mi 1516 yds 1 ft 6 in *or* 37 mi 6 fur 35 po 4 yds 275. £397
276. £464, 18s 3$\frac{1}{4}d$ 277 £408, 16s 1$\frac{1}{8}d$. 278 £2671, 11s 2$\frac{1}{5}d$
279. 83 tons 10 cwts 0 qrs 4 lbs 280 561 tons 18 cwts 0 qrs 24 lbs.

XXVI. 1 10 cwts , 5 cwts ; 4 cwts. ; 2 cwts , 2 qrs , 1 qr , 16 lbs., 14 lbs. 2. 14 lbs , 7 lbs ; 4 lbs , 8 ozs , 4 ozs , 2 ozs , 1 oz.
3. 2 ro , 1 ro , 20 sq po , 10 sq po.; 20 sq po , 10 sq po , 8 sq po.; 5 sq po. 4. 1 ft 6 in ; 1 ft ; 9 in , 6 in.; 4 in , 3 in., 6 in ; 4 in.; 3 in ; 2 in ; 1 in 5. $\frac{1}{2}$, $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{10}$ $\frac{1}{2}$; $\frac{1}{4}$, $\frac{1}{8}$; $\frac{1}{7}$.
6. $\frac{1}{2}$; $\frac{1}{10}$. $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{7}$, $\frac{1}{14}$ 7. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{5}$; $\frac{1}{10}$ $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$; $\frac{1}{10}$.
8. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$, $\frac{1}{10}$. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{8}$, $\frac{1}{12}$ 9 $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$; $\frac{1}{6}$, $\frac{1}{9}$; $\frac{1}{12}$ $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{12}$. 10. $\frac{1}{2}$; $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$. $\frac{1}{8}$; $\frac{1}{10}$, $\frac{1}{2}$, $\frac{1}{11}$ 11 £6, 15s 11$\frac{1}{4}d$
12. £5, 4s. 6$\frac{1}{4}d$ 13. £161, 7s 3d. 14 £173, 12s 10$\frac{1}{2}d$
15. £40, 4s 2d. 16. £46, 8s 1$\frac{3}{4}d$. 17 £4, 17s 7$\frac{1}{2}d$
18. £4, 4s 8$\frac{3}{4}d$ 19. £24, 10s. 5$\frac{1}{2}d$. 20 £24, 15s 10d
21 £564, 13s 9d. 22 £468, 15s 23. £1350, 14s 0$\frac{3}{4}d$
24 £1183, 18s 3d 25. £14, 11s. 10d 26 £19, 10s
27 £31, 18s. 7d. 28 £51, 18s 6$\frac{3}{4}d$ 29 £33, 9s
30. £95, 17s 1d 31 £3, 14s 5$\frac{3}{4}d$ 32 £4, 6s 4$\frac{3}{4}d$
33. £63, 5s. 2$\frac{1}{4}d$. 34. £60, 6s 5d 35 £7, 6s. 3d.
36. £6, 9s. 11$\frac{3}{4}d$. 37. £12, 2s. 1$\frac{1}{2}d$. 38. £10, 11s 6d.

39 £7, 18s $0\frac{1}{4}d$ 40 £8, 10s $11\frac{1}{2}d$ 41 £104, 3s 1d
42 £71, 18s $10\frac{1}{4}d$ 43 £122, 6s $4\frac{1}{4}d$ 44 £87, 16s $1\frac{1}{4}d$
45 14s $9\frac{3}{4}d$ 46 3s $2\frac{1}{4}d$ 47 £18, 3s
48 £14, 16s $3\frac{3}{4}d$ 49 £160, 6s $5\frac{1}{2}d$. 50 £250, 11s $3\frac{1}{4}d$
51 £11, 11s $6\frac{1}{4}d$ 52. £12, 8s $2\frac{1}{4}d$ 53 £185, 12s $10\frac{3}{4}d$
54 £340, 12s $9\frac{3}{4}d$ 55 £204, 14s $11\frac{5}{8}d$ 56 £38, 2s $5\frac{11}{14}d$
57 £76, 12s $11\frac{7}{16}d$ 58 £12, 4s $2\frac{41}{56}d$ 59. £3, 0s $11\frac{1}{12}d$
60. £7, 19s $8\frac{2}{3}d$ 61 £54, 15s $1\frac{2}{3}d$ 62 £5, 19s $0\frac{37}{72}d$
63 £519, 18s $6\frac{27}{32}d$ 64 £2493, 4s $3\frac{27}{64}d$ 65 £11, 6s $3\frac{5}{8}d$
66 £59, 11s $10\frac{2}{7}d$ 67 £50, 12s $6\frac{7}{16}d$ 68 £45, 2s $10\frac{103}{224}d$
69 £4, 2s $1\frac{5}{7}d$ 70 14s $8\frac{11}{14}d$ 71. £179, 16s $7\frac{2}{7}d$
72 £155, 12s $3\frac{6}{7}d$ 73 £134, 5s $1\frac{4}{5}d$ 74 £128, 6s $5\frac{13}{16}d$
75 £24, 11s $6\frac{3}{4}d$ 76 £20, 14s $4\frac{10}{11}d$ 77 £145, 2s $3\frac{3}{11}d$.
78 £451, 3s $2\frac{47}{110}d$ 79 £4, 14s $9\frac{23}{64}d$ 80. £3, 11s $9\frac{13}{24}d$
81 £1704, 3s $6\frac{3}{4}d$ 82 £2033, 15s $3\frac{3}{4}d$ 83 £139, 17s $3\frac{3}{4}d$
84 £250, 15s $10\frac{7}{8}d$ 85 £257, 6s $11\frac{11}{32}d$ 86 £587, 7s $6\frac{3}{8}d$
87 £15527, 1s 8d 88 £19126, 16s 6d 89. £7117, 18s $8\frac{3}{4}d$
90. £2979, 0s $7\frac{1}{2}d$ 91. £896, 12s $6\frac{1}{2}d$ 92 £2085, 9s $10\frac{1}{2}d$
93 £353, 17s $9\frac{23}{48}d$ 94. £44, 14s $3\frac{3}{80}d$ 95 £146, 10s 10d
96 £6672, 5s $1\frac{1}{2}d$ 97 £734, 7s $8\frac{13}{16}d$ 98. £3262, 15s $9\frac{4}{5}d$
99. £2507, 11s $3\frac{21}{80}d$ 100. £23498, 3s $1\frac{1}{2}d$

XXVII.

1. 12s 1d 2 12s $3\frac{1}{4}d$ 3. 10s $11\frac{1}{2}d$ 4 14s 3d
5 13s 10d 6. 13s $8\frac{1}{4}d$ 7 10s 11d 8 18s $10\frac{1}{2}d$ 9. 19s $5\frac{3}{4}d$
10 14s $9\frac{1}{2}d$ 11 15s $0\frac{1}{2}d$ 12 9s 2d 13. 8s $3\frac{3}{4}d$ 14. 9s $11\frac{1}{4}d$
15 8s $8\frac{1}{2}d$ 16 16s $9\frac{1}{4}d$ 17 £6, 4s 6d 18 £7, 7s 19 13s $6\frac{1}{2}d$
20 £1, 14s $4\frac{1}{2}d$ 21 £1, 9s $9\frac{3}{4}d$ 22 £1, 3s 23. £1, 16s $1\frac{3}{4}d$
24 £2, 2s 1d 25 £2, 7s 10d 26 £1, 11s $1\frac{3}{4}d$ 27 £9, 8s $10\frac{1}{4}d$
28 £3, 2s $9\frac{1}{2}d$ 29 £2, 12s 7d 30 £2, 6s $3\frac{1}{4}d$ 31. $8\frac{3}{4}d$
32. $4\frac{3}{4}d$ 33. $7\frac{3}{4}d$ 34 3d 35 3s $0\frac{1}{4}d$ 36. 2s. $0\frac{1}{2}d$
37 1s $11\frac{1}{4}d$. 38 4s $4\frac{1}{2}d$ 39 11s 4d 40 2s $1\frac{1}{2}d$ 41 3s. $8\frac{1}{4}d$
42 £1, 15s $5\frac{1}{2}d$ 43 14s $3\frac{1}{2}d$ 44 12s 5d 45. £1, 0s. $9\frac{3}{4}d$
46 18s $9\frac{1}{2}d$

47.

	£	s	d
		18	0
		2	$2\frac{1}{2}$
		9	$9\frac{1}{2}$
	1	10	0
Discount,			9
	£1	9	3 *Ans*

48.

	£	s	d
		19	$10\frac{1}{2}$
		18	$9\frac{3}{4}$
	3	19	$7\frac{1}{4}$
	5	18	$3\frac{1}{2}$
Discount,		2	$10\frac{1}{2}$
	£5	15	5 *Ans*

49

	£	s	d
		11	5
	3	13	0
	2	12	1
	6	16	6
Discount,	1	0	4
	£5	16	2 *Ans.*

50.

	£	s	d
	3	9	9
	2	6	0
	2	18	2
	8	13	11
Discount,	1	13	6
	£7	0	5 *Ans.*

XXVIII. **1.** Seven *tenths*, Seven *hundredths*, Seven *thousandths*; Seven *ten-thousandths*, Seven *hundred-thousandths*, Seven *millionths*; Seven *ten-millionths*.

2. Eight *hundredths*, Eight *ten-thousandths*, Eight *tenths*, Eight *hundred-thousandths*, Eight *thousandths*, Eight *ten-millionths*; Eight *millionths*

3. Twenty-three *hundredths*, Forty-five *hundredths*, Four-hundred-and-eighty-six *thousandths*, Two-hundred-and-seventy-one *thousandths*; Eighty-three *thousandths*, Eight-hundred-and-three *thousandths*; Forty-one *ten-thousandths*, Six-hundred-and-thirty-two *ten-thousandths*

4. Two, and three *tenths*, Six-hundred-and-one *thousandths*, Five, and three *hundredths*, Twenty-four, and five *hundredths*, Seventeen, and seventeen *hundredths*; Eighty-nine *thousandths*, Sixty-six, and sixty-six *hundredths*, Six-thousand-six-hundred-and-sixty-six *ten-thousandths*

5. ·3, ·7, ·08, ·004 **6.** ·1, ·06, ·09, ·005
7. ·002, ·0007, ·000006 **8.** ·0008, ·000004, 009 **9.** 2·3, 12·07
10. 70·007, 10·0000001 **11** ·21, ·17, 1·1 **12.** ·12; ·99, 3·3
13. ·202; ·000014 **14.** 1·09, ·0087 **15** ·2·3 **16.** 5·7
17. 45 **18.** 72 **19.** ·8 **20.** ·06 **21.** 7·01 **22.** 92·1
23. 53·4 **24.** 724·5. **25** 123·4 **26** 500·3 **27.** 421
28. 503 **29** 1210 **30** 205·7 **31** 2315 **32** 41030
33. 5200 **34** 8 **35** ·23 **36** ·51 **37** ·05 **38.** ·07.
39 ·063 **40.** 6·1 **41** 2·9 **42** 6·03 **43** ·802
44. ·0001. **45.** 1·531 **46.** 4·076 **47.** ·437 **48** ·802
49. ·023 **50.** ·2361 **51.** ·4009 **52.** ·0213 **53.** ·0805
54. ·0012 **55** 472·8, 30·45, ·847, 2306·7, ·00703. **56.** 4728 304·5, 8·47, 23067, ·0703 **57.** 47280, 3045, 84·7, 230670, ·703
58. 234·07, 7030·057, 40·01, ·003, ·4367 **59.** 23·407, 703·0057, 4·001 ·0003, ·04367 **60** ·23407, 7·030057, ·04001, ·000003, ·0004367

XXIX. **1.** 20·92 **2.** 5·385. **3.** 17·81. **4.** 20·8
5. 39·638. **6.** 68·684. **7.** 934·4794 **8.** 467·1889.
9 207·5191. **10.** 3889·9611 **11.** 4·53 **12.** 3·072
13. 31·8 **14.** 42·7 **15.** 10·5969 **16.** 3·512
17. 1·545 **18.** ·632 **19** 15·3525 **20** 22·3022.
21. 4475·105045 **22.** 23·4397464 **23** ·45. **24** ·063
25. ·0044. **26.** ·001 **27.** 0 **28.** ·01 **29** ·3897.
30. 39·483. **31.** ·04202 **32.** 59·0522 **33.** ·88428
34. ·41925 **35.** 1·0129. **36** 3·5044 **37.** 9·99
38. 169·983 **39.** 45·45 **40** 77·922 **41.** 17·7082
42 2·61807 **43.** 2·2111. **44.** 313·95804 **45** 243·97578.
46 362·60703. **47.** 3·996 **48.** 11·27263 **49** 2·662
50. 1·111

XXX. **1.** 16·8 **2.** 5·64 **3** 24·31 **4.** ·805
5. 569·24. **6.** 2832·09. **7.** 3·48 **8.** ·371. **9.** 959·1.

10 48360 11. 305·3 12 32·4 13 3·65 14 1·68.
15 0657 16 ·02781 17. ·08815 18. 1 1055 19 ·1281
20 1·8139 21 4 03 22 17 907 23 ·12879
24 ·0009108 25. 03003 26. 337264 27 ·0690606
28. 36·94581 29 45 3 30 509·427 31 ·000274855
32 000185745 33 ·000105 34. ·000048 35 2·023
36 2·9791 37 ·00000072 38 00016807. 39. ·194481
40. 27 9841

XXXI. 1. 12·41 2 23·014 3 ·5509 4 ·00221.
5. 138·5 6 ·0121 7 ·2126 8. 57·825 9 5 2014
10 4 275 11 ·0425 12 ·1125. 13 23 218. 14 ·1539
15 ·0513 16 ·0868 17. ·88125 18 ·00548 19 ·115625
20 ·0002265625 21 ·021875 22. ·004375 23 ·0118
24 ·129875 25 ·72. 26 ·033 27 ·0084 28 ·00016
29. 9·6 30. ·175 31. 004 32 03008 33 ·012
34 10·3 35 ·002272 36 ·1326171875 37 5·976
38. ·013 39. 00305 40. 001702

XXXII. 1. 116·94 2 8393 3 8·815 4. 53·934
5. ·0121 6 28·8 7 193·19 8 3636 4 9 13648
10 5860 11 10500 12 6·40125 13 2·5 14. 20
15 3750 16 17500000 17 15·07 18. 1·728 19. ·9268
20. 242·156 21 13 846875 22 17·840625 23 ·0046875
24 152·546875 25 8·75 26 125 27 2440 28 1988
29 234·375 30 21 875 31 ·022 32 1·3 33 072
34 ·0009 35. 210·3 36 83 3 37 603 38 6·073
39 ·32 40 ·525 41 ·275 42 2 25 43 ·002215
44. 9·236 45 ·40105 46 13 756 47. 605000 48. 7500
49 500 50. 640000 51 2000 52 22500 53 600600
54. 21432 55 ·1857 56 ·0037 57 0530 58 ·5952
59 ·1263 60 ·0092 61 11·3207 62. 344·8275
63. 0049 64 6·59, 6590 65. 501000, 00501 66 3·25, 32500
67 1 2, 12, 12000, ·012 68. 1 3, ·13, 130000, 13, ·00013
69 Quotient 32; remainder ·153 70 518 times, and 14 over

XXXIII. 1 $\frac{3}{10}$, $\frac{7}{10}$, $\frac{9}{10}$, $\frac{1}{100}$ 2 $\frac{1}{10}$, $\frac{7}{100}$, $\frac{1}{100}$, $\frac{9}{1000}$
3 $\frac{3}{100}$, $\frac{1}{10000}$, $\frac{9}{100000}$ 4 $\frac{7}{1000}$, $\frac{3}{100000}$, $\frac{7}{10000000}$ 5 $\frac{13}{100}$, $\frac{23}{100}$, $\frac{17}{1000}$
6 $\frac{59}{100}$, $\frac{509}{1000}$, $\frac{5009}{10000}$ 7 $\frac{71}{100}$, $\frac{29}{1000}$, $\frac{413}{1000}$ 8 $\frac{17}{1000}$, $\frac{113}{1000}$, $\frac{1003}{10000}$
9. $\frac{871}{1000}$, $\frac{701}{10000}$, $\frac{4359}{10000}$ 10. $\frac{6789}{10000}$, $\frac{813}{100000}$, $\frac{1793}{1000000}$ 11 $3\frac{1}{10}$, $21\frac{7}{10}$, $1\frac{9}{1000}$
12. $5\frac{3}{10}$, $50\frac{3}{100}$, $5\frac{3}{10000}$ 13 $22\frac{7}{1000}$, $220\frac{7}{100}$, $2020\frac{7}{10}$
14. $170\frac{3}{10}$, $17\frac{3}{100}$, $1\frac{7}{100000}$ 15 $1\frac{23}{100}$, $12\frac{57}{100}$, $103\frac{31}{1000}$
16 $8\frac{47}{100}$, $19\frac{351}{1000}$, $104\frac{401}{1000}$ 17. $11\frac{11}{100}$, $1\frac{111}{1000}$, $11\frac{1011}{100000}$
18 $70\frac{77}{100}$, $77\frac{7}{100}$, $7\frac{7007}{100000}$ 19 $50\frac{571}{1000}$, $17\frac{8001}{10000}$, $1\frac{7598}{10000}$
20 $41\frac{209}{10000}$, $8\frac{30459}{1000000}$, $1\frac{405039}{1000000}$ 21 $\frac{1}{5}$ 22 $\frac{1}{2}$ 23 $\frac{3}{5}$
24 $\frac{4}{5}$ 25 $\frac{1}{50}$ 26 $\frac{1}{20}$ 27. $\frac{2}{25}$. 28 $\frac{1}{250}$ 29. $\frac{3}{20}$
30. $\frac{1}{4}$ 31 $\frac{2}{125}$ 32. $\frac{7}{250}$ 33. $1\frac{7}{90}$ 34. $11\frac{9}{20}$ 35. $2\frac{1}{8}$

36. $10\frac{9}{200}$. 37. $6\frac{13}{40}$ 38 $71\frac{5}{8}$ 39. $\frac{3}{625}$ 40. $\frac{3}{80}$
41. $1\frac{29}{40}$ 42 $3\frac{7}{80}$ 43 $17\frac{117}{500}$ 44 $50\frac{9}{125}$ 45. $1\frac{39}{40}$
46. $1\frac{21}{80}$. 47. $\frac{3}{4000}$ 48. $\frac{1}{15625}$ 49. $\frac{5}{32}$ 50. $1\frac{9}{16}$
51. ·1, ·7, ·01. 52 ·3, ·07, ·09 53. ·08, ·002, ·0007
54 ·009, ·0005, ·000001 55 ·17, ·027, ·0127 56. ·43, ·0174, ·02715
57 1·1, 20·3, 5·16 58. 2·03, 96·01 59 57·1, ·571, ·00571
60 9·9, ·099, ·000099 61. 4·03, 7·5, 21·023
62. 100·01, 7 009, 12·0014 63. 1·0001, 231·9, 4·073
64. 51·03, 8·0008, 1000·019 65. 38·9, ·0047, 80·03
66. ·023, 5·0715, 735 61 67. 43·256, 10·007, ·00153
68. ·1753, 1·001, 1457·03 69 10·1, 10·001, ·000101.
70. 7·0001, ·0071, 70·0001 71. ·2 72. ·8 73 ·5 74. ·25.
75. ·75 76. ·14 77. ·26 78 ·12 79 1·45 80. ·85.
81. ·56 82. ·125. 83 ·625 84. ·4375 85. ·824 86 1·056
87. 2·075 88. 7·462 89 10·1875 90 2·1375 91. ·05375
92. ·764 93. ·0016 94 ·00048 95 1·3224 96. ·09375
97. ·084375. 98. ·015625 99. 5859375. 100. ·17376.

XXXIV.

1. $1\cdot 7\dot{3}$ 2. $2\cdot 2\dot{8}\dot{1}$ 3. $\cdot 37\dot{9}\dot{1}$ 4. $5\cdot 451\dot{6}$.
5. $\cdot 041\dot{6}$ 6. $\cdot 0278\dot{3}$ 7. $\cdot 023\dot{2}$ 8. $\cdot 00358\dot{3}$ 9. $\cdot 042857\dot{1}$
10. $\cdot 0357142\dot{8}$ 11. ·5 12 $93\cdot\dot{3}$. 13 $36\cdot\dot{3}\dot{6}$ 14. $2583\cdot\dot{3}$
15. $1571\ \dot{4}28571$ 16. $5714\cdot\dot{2}8571\dot{4}$ 17. $\cdot 02\dot{7}$ 18 $\cdot 188\dot{6}\dot{3}$
19. ·009583 20 $\cdot 480\dot{5}$ 21. $\cdot 0357142\dot{8}$ 22. $\cdot 00\dot{5}0617283\dot{9}$
23 $\cdot 005\dot{6}\dot{3}$ 24. $\cdot 00486\dot{1}$. 25 $7\cdot\dot{4}0\dot{7}$ 26. $60\cdot\dot{6}\dot{0}$ 27. ·0138.
28. $\cdot 00\dot{9}25$ 29 $14090\cdot\dot{9}\dot{0}$. 30 $20\cdot 8\dot{3}$ 31. $1785\cdot\dot{7}1428\dot{5}$
32 $9\cdot\dot{8}7654320\dot{9}$ 33. $\cdot\dot{0}69\dot{3}$ 34. $\cdot 1\dot{4}8\dot{6}$ 35 $\cdot\dot{0}2439$
36. $\cdot\dot{0}7692\dot{3}$ 37. $\cdot 0\dot{4}05$. 38. $\cdot 00\dot{6}097\dot{5}$ 39. $\cdot 0544\dot{5}$. 40. $\cdot 2\dot{2}82051$.
41. ·00164 42. ·0108 43. $2\cdot\dot{4}\dot{2}$ 44. $47\cdot 6\dot{3}51$ 45 $\cdot 3652\dot{0}27$.
46. $1\cdot 37\dot{1}951\dot{2}$ 47. $\cdot\dot{1}32867$. 48. $\cdot 017\dot{2}07792$ 49. $\cdot 32258064516129\dot{0}$
50. $\cdot 0\dot{2}352941176470588$.

XXXV.

1. $\cdot\dot{3}$ 2. $\cdot\dot{4}$ 3. $\cdot\dot{2}\dot{7}$. 4 $\cdot\dot{6}\dot{3}$ 5. $\cdot\dot{1}42857$
6 $\dot{7}1428\dot{5}$ 7 ·07 8. $\cdot 07\dot{6}$. 9. $\cdot 4\dot{2}\dot{7}$. 10. $\cdot 511\dot{1}$
11. $\cdot 0\dot{4}2857\dot{1}$ 12 ·04428571. 13. $\cdot 8\dot{3}$ 14. $\cdot 91\dot{6}$ 15 $\cdot 1\dot{3}$
16. $\cdot 791\dot{6}$ 17. $\cdot 1\dot{7}1428\dot{5}$. 18. $\cdot 17\dot{8}5714\dot{2}$ 19. $1\cdot 0208\dot{3}$
20. $3\cdot 00\dot{3}\dot{6}$ 21. $4\cdot 018\dot{6}$ 22. $11\cdot 007291\dot{6}$ 23 $\cdot\dot{7}8\dot{3}$.
24. $\cdot\dot{1}951\dot{2}$ 25. $\cdot\dot{1}5384\dot{6}$ 26. $\cdot\dot{0}1234567\dot{9}$ 27. $1\cdot 0\dot{1}219\dot{5}$
28 $3\cdot 0\dot{6}7\dot{5}$ 29. $\cdot\dot{0}1587\dot{3}$ 30. $\cdot 2\dot{9}2307\dot{6}$ 31. $4\cdot 00\dot{2}47\dot{5}$.
32. $4\cdot 00\dot{2}2\dot{5}$ 33. ·0364583 34. $\cdot 02556\dot{8}\dot{1}$ 35. $\cdot 0266\dot{1}\dot{8}$
36 $\cdot 03912\dot{8}$ 37. $5\ 19\dot{8}7179\dot{4}$. 38. $2\cdot\dot{7}9894\dot{1}$
39. $\cdot\dot{3}52941176470588\dot{2}$ 40 $\cdot\dot{0}52631578947368421\dot{}$ 41 $\frac{3}{8}, \frac{7}{20}, \frac{1}{40}, \frac{19}{80}$.
42 $\frac{3}{16}, \frac{8}{25}, \frac{9}{32}, \frac{7}{64}$ 43. $\frac{1}{80}, \frac{1}{100}, \frac{1}{125}$ 44. $\frac{1}{160}, \frac{1}{200}, \frac{1}{250}$.
45. $\frac{7}{320}, \frac{13}{400}, \frac{17}{500}$ 46. $\frac{1}{625}, \frac{1}{640}$ 47. Two 48. Three.
49. Seven 50. Eight

XXXVI.

1. $\frac{2}{3}$. 2. $\frac{5}{9}$. 3 $\frac{2}{3}$ 4. $\frac{8}{9}$.
5. $4\frac{7}{9}$ 6. $11\frac{1}{9}$. 7. 1. 8. 8. 9. $\frac{8}{33}$ 10. $\frac{5}{11}$.

11 $1\frac{8}{11}$ 12. $\frac{432}{433}$ 13 $\frac{20}{111}$ 14. $\frac{10}{37}$ 15 $10\frac{35}{111}$ 16 $6\frac{31}{37}$
17 $\frac{31}{101}$ 18 $\frac{1355}{3333}$ 19. $\frac{44}{369}$ 20′ $\frac{7}{128}$ 21. $\frac{1}{7}$ 22 $\frac{3}{7}$
23 $1\frac{4}{7}$ 24. $3\frac{6}{7}$ 25. $\frac{75}{91}$ 26. $\frac{69}{91}$ 27. $\frac{10}{21}$ 28 $\frac{12}{13}$
29 $\frac{7}{81}$ 30 $\frac{5}{451}$ 31 $\frac{5}{6}$ 32 $\frac{11}{30}$ 33 $2\frac{26}{45}$ 34 $1\frac{1}{90}$
35 $\frac{116}{495}$ 36. $\frac{23}{66}$. 37 $\frac{1}{66}$ 38. $\frac{35}{36}$ 39. $\frac{3}{275}$ 40 $\frac{1}{44}$
41 $1\frac{29}{60}$ 42 $2\frac{17}{24}$ 43 $\frac{53}{222}$ 44. $\frac{71}{185}$ 45 $1\frac{17}{72}$ 46 $3\frac{1}{375}$
47 $\frac{301}{2775}$ 48 $\frac{51}{148}$ 49. $\frac{168}{205}$ 50 $\frac{104}{205}$ 51. $\frac{1}{9375}$ 52 $\frac{11}{96}$
53 $\frac{32083}{495000}$ 54 $\frac{297}{2960}$ 55. $\frac{13}{14}$ 56 $\frac{23}{35}$ 57 $\frac{6}{175}$
58 $\frac{113}{375}$ 59. $\frac{47}{56}$ 60 $\frac{3}{52}$

XXXVII. 1. ·6183 2 915005 3 $\cdot\dot{2}18933$ 4 ·561288
5 ·984 6 7247 7 2·421 8 $6\cdot156\dot{3}$ 9 ·513
10 1·257 11 2923 12. ·7481 13 $4\cdot00\dot{2}5$ 14 ·14382
15 12485 16. $2\cdot325\dot{6}$ 17 1·00002471 18 ·5189765
19 16·857142 20 4·923076 21 3·73450177 22 203337069
23 $149788770\dot{6}$ 24. ·426211089577 25 ·9, *i e* 1 26. 1 9, *i e* 2
27 1·19, *i e* 1·2 28 1·49, *i e* 1·5 29 ·89, *i e* ·9 30 0
31 $228\cdot037471\dot{3}$ 32. $778\cdot164391\dot{6}$ 33 12 003994358125
34 ·9, *i e* 1 35. $4\dot{9}$, *i e* ·5 36. 6·346285714 37 2·2473824
38 $2\cdot343176994546301\dot{4}$ 39 1000 40. 1·09, *i e* 1 1.

XXXVIII. 1. 1 $2\dot{8}$ 2. 10 $\dot{3}6$ 3 $36\cdot2\dot{1}$ 4. 5·477
5 32 681 6 $7\cdot2\dot{4}8$ 7 $1\cdot6218\dot{7}$ 8 ·005705 9 $00713\dot{6}$
10 $07\dot{6}95$ 11 $1\cdot8\dot{3}\dot{6}$ 12 ·02325883 13 02940065 14 036
15 ·35501 16 1173·26 17. 73·650793 18 $138\cdot0952\dot{3}8$
19 17135 $41\dot{6}$ 20 23 164983 21 343 22 $\cdot01\dot{0}9$ 23 ·25
24 7 12 25 $6\dot{6}3$ 26 ·37592 27 ·0150018 28. ·2780065
29. ·5 30 ·309375 31 19 $\dot{0}9$ 32. ·6 33 ·1629
34 $\cdot33\dot{8}58$ 35. 2·646 36 25 37 33·33 38 ·6249375
39 $3\cdot\dot{6}3$ 40. 3648 41 ·69 42 16·56 43 11 083
44 $6\cdot\dot{6}$ 45 3·99 46 3·3 47. 5·2428571 48 11·68428571
49 ·24 50 4 51 3 63 52 016 53 $9\dot{8}$ 54 $\cdot\dot{6}$
55. 1 56 2·88 57. 1 58. 2 59 ·5 60 ·01

XXXIX. 1 $4\frac{1}{2}d$ 2. 4*s* $7\frac{1}{2}d$ 3 17*s* 4 £2, 3*s* 6*d*
5. 2*s* 9*d* 6 18*s* 9*d* 7 $2\frac{1}{4}d$ 8 $11\frac{1}{4}d$ 9 7*s* $4\frac{1}{2}d$
10 5*s* $3\frac{3}{4}d$ 11 3*s* $1\frac{1}{2}d$ 12. 2*s* $7\frac{1}{2}d$ 13 £3, 7*s* 6*d*
14. £15, 3*s* 3*d* 15 11*s* $11\frac{1}{4}d$ 16 15*s* $6\frac{3}{4}d$ 17 £4, 15*s* $9\frac{3}{4}d$
18 2*s* $1\frac{1}{2}d$ 19 2*s* $3\frac{1}{2}d$ 20 3*s* $6\frac{1}{2}d$ 21 2*s* $9\frac{3}{4}d$ 22 16*s* $5\frac{3}{4}d$
23 £1, 2*s* $3\frac{3}{4}d$ 24 £3, 12*s* $7\frac{1}{2}d$ 25 £1, 9*s* $0\frac{3}{4}d$ 26 15*s* 6*d*
27 3*s* $10\frac{3}{4}d$ 28 4*d* 29 3 qrs 7 lbs 30 21 lbs
31. 1 cwt 2 qrs 8 lbs 12 ozs 32. 13 cwts 1 qr 12 lbs 4 ozs
33 2 mi 2 fur 30 po 34. 24 cu ft 810 in 35 1 ton 15 cwts 3 qrs
36 1 ton 3 cwts 2 qrs 1 lb 37 $15\frac{3}{4}$ poles, or 15 po 4 yds $4\frac{1}{2}$ in
38 2 ac 2 ro 39. £5, 15*s* 6*d* 40 £1, 0*s* $8\frac{3}{4}d$
41. 3 tons 7 cwts 3 qrs 22 lbs 42. 5 yds 1 ft $7\frac{1}{4}$ in. 43. 10 poles.

44. 1 lb 45 2*s* 1*d* 46. $3\frac{1}{2}d$ 47. 16*s* $7\frac{1}{2}d$ 48 1*s* $7\frac{1}{4}d$
49. 1 ft. 10 in 50. 3 in 51. 110 yds 0 ft 8 in 52 2 mi 622 yds 8 in
53. 6*d* 54. 7*s* 6*d* 55. $4\frac{1}{4}d$ 56. $3\frac{3}{4}d$ 57 13*s* 58. 6*d*
59. 1 lb 60. 1 ton 13 cwt 14 lbs.

XL. 1. ·4 2. ·6 3. ·15 4. ·35 5. ·25
6. ·75. 7 ·125 8. ·225 9 ·1125 10. ·0875 11. ·75
12. ·125 13. ·625 14. ·4375 15 ·0625 16 ·9375 17. ·155
18. ·175 19. ·021875 20. ·14625 21 ·03125 22. ·034375
23. ·115625 24. ·23125 25 ·01171875 26. ·001953125
27. ·0125 28. ·00875 29. ·109375 30. ·06525 31. ·015625
32. ·078125 33. ·0588125 34 ·38390625. 35. 5·16875
36. 14·54375 37 ·00063 38. ·000096 39. ·1390625
40. ·03671875 41 ·005625 42. ·00204 43. 642·4995 pence
44. 108·864275 pence 45. 1·19 ozs. 46. 724·57875 dwts
47. 20·52 pints 48. 17214·912 sq yds 49. 892·8 secs
50. 162·504 mins 51. $\cdot\dot{6}$ 52. $\cdot 41\dot{6}$ 53. $\cdot 08\dot{3}$ 54. $\cdot 1458\dot{3}$
55. $\cdot 8958\dot{3}$ 56. ·9583 57. $\cdot\dot{6}$ 58. $\cdot 8\dot{3}$ 59 $\cdot 358\dot{3}$
60. $\cdot 2541\dot{6}$ 61. $\cdot\dot{3}75$ 62. $\cdot 215\dot{9}\dot{0}$ 63. $\cdot 3\dot{5}069\dot{4}$ 64. ·14305
65. $\cdot 4\dot{3}2926\dot{8}$ 66. $\cdot 300\dot{6}75$ 67. $\cdot 803\dot{5}71428\dot{8}$ 68 $\cdot 8\dot{3}07692$
69. $\cdot 1306\dot{8}\dot{1}$ 70. $\cdot 06\dot{8}\dot{1}$ 71. $\cdot 0\dot{0}5$ 72. $\cdot 0011\dot{3}\dot{6}$ 73. ·003
74. $\cdot 7\dot{3}$ 75. $\cdot 0710\dot{2}\dot{2}7$ 76. $\cdot 23\dot{4}28571\dot{1}$ 77 $\cdot 82886\dot{9}04761$
78 $\cdot 00948660\dot{7}14285$ 79. ·056465 80. $\cdot 4097\dot{2}$.

XLI. 1. Three *hundredths.* 2 3·003. 3. Seven *millionths*
4. ·0123 5. 345·26579 6. 6·21004 7. 6 9557137 8. ·0462
9. 8·0881 10 24000 11. Neither, they are equal 12. ·000037
13. $\frac{47}{100000}$ 14. 619·62482 15. 12·499897 16. 364·406442
17 4·240671875 18. 470·34 19. 2·3681 20. ·8075
21 470·31 22. 2 in, 1 *tenth* in, and 6 *thousandths* in 23 ·703
24. $\frac{7}{625}$ 25. 9·293295 26. ·00047089 27. 49·0734375.
28 2665 29 ·012 30. 249*d* 31 ·02935 32 $3\frac{8713}{1000000}$.
33. 68·08792 34 33750 35. ·0314574 36. 506·9896925.
37 5·5546875 38. 1200 39 1245*d* 40. 0625. 41. 10·0121.
42. ·4783 43. ·00065 44. ·000012167 45 ·0705 46. 3·152.
47. ·00064 48. 6 fur 12 po 49. 7 times 50 ·61125.
51. 100·593 52 57830 53. 133·6336 54. ·04775 55 4670.
56. $\frac{3\frac{1}{3}}{3\frac{1}{2}}$ 57 ·00096875 58 8 ozs. 15 dwts 5 grs 59. ·065.
60 ·00275 61. ·35 62. ·04123 63. ·1875 64 304·3069436
65 ·02025 66 1 67. 562·1 68. 111. 69 3600 times
70. £1, 3*s* 6*d.* 71. Nothing; they are equal 72. ·0721 73 ·6512
74. 4·79 75. 1·45. 76 5616·1 77 2070000 78 31·79
79 7000 80 ·0484375 81. 1920 919 82. ·0065 83. ·24992
84. ·02919189 85. ·0575 86. 0 87. 1 ton 3 cwts. 2 qrs 1 lb
88. 308·7 inches 89 £4, 3*s* 4*d.* 90. A, £6, 1*s.* 3*d*, B, 12*s* $1\frac{1}{2}d.$,
C, £3, 8*s.* $8\frac{1}{2}d$ 91. $7\frac{14}{25}$ 92. 93. 8·91. 94 199·991.

95 ·000125 96 94 mi 7 fur 36 po . 97 670·68 mins 98 ·0625
99 18*s* 8*d* 100 £106, 8*s* 101 4 102 ·854 103 $\dot{2}59$
104 $\frac{27}{37}$ 105 7·2510 106 2 552084 107 2 108 1·108
109 3*d* 110 $\dot{1}4285\dot{7}$ 111 ·17 112 ·221951 113 2916
114 $\frac{73}{75}$ 115 3·4560 116. ·000535 117 ·345 118 495
119. 110 sq yds 120. ·357142$\dot{8}$ 121 6299·37 122 3753 2, ·0427
123 $4\frac{13}{55}$ 124. ·014583 125. ·357638 126. 4·2 127 5 6
128 $11\frac{1}{3}d$ 129 21421 62 mins 130 1785 pieces, and 001 of an inch over. 131 ·083 132 $\frac{2}{125}$, $\frac{1}{60}$ 133 ·04375, 22·85714$\dot{2}$
134 ·000002355 135 2 88 136 6199 137. 199 81$\dot{4}$
138. £16, 0*s* $7\frac{1}{4}d$ 139 132 09 perches 140. 23 bits, and 12 of an inch over 141 67 067 142 ·17 is the greater
143. $\frac{2321}{2331}$ 144 4·8 145 $\dot{2}44626264428$ 146 ·186420
147 ·0052083 of an inch 148 £123, 19*s* 6*d* 149 $1\frac{1}{2}d$ 150 ·1681
151 ·03$\dot{7}$ is greatest, ·037 is least 152. ·0030507 153 90
154. $\frac{19}{5400}$ 155 ·005733 156 13 157 3906250 times
158 2559·318 mins 159 3*s* $2\frac{1}{2}d$ 160 497 acres 161 $\frac{233}{67800}$
162. 6477·$\dot{7}$, 14·28571$\dot{4}$ 163 3·378 164 600 165 80, 79·92 166 ·$\dot{6}$3 167 4 cwts 3 qrs 14 lbs 14 oz
168 $\frac{1}{4}d$ 169. 2381 0625 ozs 170 A 171 ·351 172 714285
173 760, 18·5 174 ·00167 175 $\frac{5}{7}$ 176 6 177. ·00001, ·017745 178 179 ·1$\dot{4}$ 180 A 181 Four places
182. 3·72$\dot{6}$9 183. $\frac{219}{11000}$ 184 153 002 185 186. $\frac{75}{91}$ or ·824175 187. 19*s* $7\frac{1}{4}d$ 188 14375 189. 1171428 times, and 6·$\dot{8}$5714$\dot{2}$ pence over 190. 10·30 A M 191 Six places
192. $1\frac{2}{17}$ 193 ·06 194 ·0588235294 195 ·000625
196 ·0009027 and 9·027 197 11336 69204729001 198. 3·45
199. ·0625 200 Tuesday

XLII. 1. 15*s* 2 £1, 8*s* 3 £20 4 £27 5. £2, 15*s*
6 £4, 14*s* 6*d* 7 3*s* 4*d* 8 5*s* 3*d* 9. £3, 15*s* 10 £5, 12*s*
11. 2*s* 8*d* 12 12*s* 13. 10 lbs 14 21 lbs 15. 21 tons
16. 27 tons 17 12 lbs 18. 8 lbs 19. 20 yds 20 6 yds
21. $5\frac{1}{3}$ miles 22 $13\frac{1}{3}$ miles 23. 45 seconds 24 $17\frac{1}{2}$ seconds
25. 10 hours. 26 $8\frac{2}{3}$ hours 27 39 miles 28 92 miles
29. 35 weeks 30. 15 weeks 31. 4 days 32 1 day 33 5 days
34. 18 days 35. 2 days 36. 2 days 37. 9 days 38 56 days
39. 4 days 40 18 days 41 10 men 42 1107 men
43 21 men 44. 108 men 45 14 days 46 12 days
47. 9 horses. 48. 25 horses. 49 2 months 50 52 weeks
51. 1*s* $5\frac{1}{2}d$ 52. 13*s* 4*d* 53 11*s* 1*d* 54. 8*s* 8*d*. 55 £1, 5*s*
56 7*s* 1*d* 57 £143 58 £2800 59 3*s* 9*d* 60. 4*s* $11\frac{1}{2}d$
61. £1, 11*s* 6*d* 62 £8, 15*s* 63. £3 64 14*s* 2*d* 65 £31, 7*s*
66 £111, 16*s* 67 £1, 5*s* $10\frac{1}{2}d$ 68 1*s* $1\frac{1}{2}d$ 69 £2075
70. £1560 71 9*s* $4\frac{1}{2}d$ 72 13*s* $10\frac{1}{2}d$ 73 6*s* 6*d* 74 12*s*
75 £4, 13*s* 76 £5, 12*s* 6*d* 77. £57, 10*s* 6*d* 78 £284, 15*s*
79 £11, 0*s* 6*d* 80 £203, 10*s* 81 11 weeks 82. 5 weeks
83. 15 sheep 84. 19 sheep 85. 315 eggs. 86. 755 eggs.

87. 12 mins. **88** 40 mins **89.** 14 miles. **90.** $17\frac{1}{2}$ miles.
91. 15 days **92** 16 days **93.** 15 miles **94.** 90 miles
95. 28 days **96** $27\frac{1}{2}$ weeks **97.** 108 miles **98** 171 miles
99. £1, 0*s* 4*d* **100.** £1, 8*s* 4*d* **101.** £31, 5*s*. **102.** £42, 11*s* 8*d*
103 $5\frac{1}{4}d$ **104.** 1*s* $7\frac{1}{4}d$ **105.** £10, 17*s* **106.** 15*s* 6*d* **107.** 2*s* 4*d*
108. 2*s* $0\frac{3}{4}d$ **109.** £66 **110.** £22, 15*s* 10*d* **111.** $2\frac{1}{2}d$ **112.** $3\frac{3}{4}d$
113. 9*s* $2\frac{1}{4}d$ **114** £2, 5*s* 9*d* **115.** £4, 19*s* $1\frac{1}{2}d$ **116.** £5, 11*s* 7*d*.
117. £7, 11*s*. **118.** £6, 18*s* 10*d*. **119.** £810, 12*s* 6*d* **120.** £332, 12*s* $10\frac{1}{2}d$.
121. 6 ozs **122** 4 ozs **123** 324 bushels **124** 138 bottles
125. 45 weeks **126.** 62 qrs 4 bush **127.** 15 yards **128.** 96 yards
129. $16\frac{3}{5}$ inches **130.** $62\frac{1}{2}$ inches **131** 63 sheep **132.** 28 horses
133. £835, 13*s* $1\frac{1}{2}d$ **134.** £15213, 10*s* 10*d* **135.** £107, 8*s*. 9*d*
136. 34 qrs 3 bus 2 pks **137.** £70, 12*s* 11*d* **138** £109, 4*s*
139. £71, 8*s* $11\frac{1}{4}d$. **140.** £95, 4*s* $5\frac{1}{4}d$ **141** 121 loads **142.** $56\frac{1}{4}$ miles
143. £1, 6*s* $9\frac{3}{4}d$. **144** $9\frac{11}{16}$ hours (*or* 9 hrs. 41 min 15 secs)
145. 90 miles **146.** 11 inches **147** £182, 15*s*. **148.** £222, 1*s*. $1\frac{3}{4}d$
149. 96 feet **150** 93 feet 4 in. **151.** 4 ozs **152.** £63
153. £4, 5*s* $8\frac{1}{2}d$ **154.** £56, 18*s* 8*d* **155.** 694 A 3 R 23 P
156. £73073, 2*s* **157.** 6 P M on Oct 19 **158.** 4 hrs 27 min. 18 secs.
159. 5 min 1 sec fast **160.** 19 min 24 secs. past seven

XLIII. **1.** £25 **2.** £63 **3** £39, 4*s* **4.** £41, 5*s* **5.** £27
6. £44, 16*s* **7** 42 acres **8.** 63 acres **9.** 81 acres **10.** $69\frac{3}{10}$ acres
11. £8. **12.** £3 **13** 12 acres **14.** 16 acres **15.** £13, 15*s*
16. £26, 5*s* **17.** £12 **18.** £7, 8*s* 6*d* **19.** $22\frac{1}{2}$ acres. **20.** 55 acres
21 5 men. **22.** 8 men. **23.** 12 men **24.** 36 men **25.** 24 men
26 30 men. **27.** 4 men **28.** 35 men **29.** 16 men **30.** 21 men
31 4 days **32.** 1 day. **33.** 8 days **34.** 4 days **35.** 32 miles
36. 6*s*. **37.** 1 day. **38.** 6 days **39.** £8 **40** £54, 16*s* 6*d*.
41. 60 days **42.** 36 days **43** £6, 18*s* **44.** £109, 7*s* 6*d*.
45. £106, 17*s* 6*d* **46.** £3, 4*s* $5\frac{1}{2}d$ **47** 60 days **48** 28 horses
49. 17 days **50** $20\frac{1}{4}$ miles **51.** 9*d* **52** 2 lbs 10 ozs
53. $10\frac{2}{3}$ hours **54.** 24 men. **55.** 6 days. **56.** 10 hours.
57. 8 days **58.** 171 men **59.** 81 men. **60.** 16 days
61. 30 days **62.** 14 days **63** £198, 18*s*. **64** 21 days.
65. 432000 gallons **66.** $4\frac{1}{2}$ days **67.** 20 men **68.** 4 men.
69. 36 days **70.** $10\frac{1}{2}$ hours. **71** 9 days **72.** 20 men
73. 18 days **74.** 20 days. **75.** £2, 12*s* 3*d* **76** £13, 15*s* $7\frac{1}{2}d$
77. 2 men. **78.** 66 days. **79.** 750 men. **80.** 50 men

ANSWERS

TO THE EXERCISES OF PART II.

XLIV. **A.**—**1.** 12 days **2** 32 days **3.** 4 days **4.** 2 days **5.** £5, 7*s* 4*d* **6** 11 days. **7.** 5 mo **8.** 26 boys **9.** $1\frac{1}{4}$ hrs. **10.** 4 days **11.** 15 days. **12.** 8 days. **13.** 11 days **14.** 21 men. **15.** 45 days **16.** 15 men **17.** 2400 men. **18.** 177 days **19.** 1 hour. **20.** $9\frac{3}{4}$ weeks; £341, 5*s*.

B.—**21.** $7\frac{1}{4}$*d* **22.** 14*s* 9*d* **23** £239, 7*s* $7\frac{1}{2}$*d* **24.** £101, 10*s*. **25.** £722, 13*s*. 4*d* **26.** £545, 4*s* 5*d* **27.** 7*s* $7\frac{1}{4}$*d* **28.** £3840. **29** 4*s* 8*d* **30.** £23, 16*s* 8*d*

C.—**31.** £19, 0*s* 11*d* **32.** £17, 9*s* 3*d* **33.** £666 **34.** £1456, 8*s*. **35.** £17, 0*s*. 6*d* **36** £34, 0*s* 4*d* **37.** £8, 18*s* $1\frac{1}{2}$*d*. **38.** £5676, 13*s* 4*d*. **39.** £65. **40.** £934, 10*s*. **41.** £496, 10*s* 1*d*. **42** £4979, 6*s*. 8*d*. **43.** £840. **44.** £650, 3*s*. 3*d*. **45.** £1725.

D.—**46.** 72. **47.** 91. **48.** 55 **49.** 35 **50.** 105 **51.** 315 **52.** 60. **53.** 504 **54.** £23, 12*s* 6*d*. **55** £1, 0*s*. $0\frac{3}{4}$*d* **56.** £15, 16*s*. $6\frac{3}{4}$*d* **57.** 39 **58** 12*s* 3*d*. **59.** 114 miles **60** 24 ft **61.** 2*s* **62** £1. **63.** 2*s*. 8*d* **64.** 63 **65.** 24. **66.** $1\frac{1}{2}$ days **67.** $\frac{7}{10}$ day

E.—**68.** 6 days **69.** 4 days. **70.** 12 hrs. **71.** $13\frac{1}{3}$ min **72.** $5\frac{1}{7}$ min. **73.** $2\frac{2}{3}$ days. **74.** 4 min. **75** $1\frac{1}{2}$ days **76.** $1\frac{1}{29}$ days **77.** 12 days **78** 12 days **79.** 9*s* 9*d*. **80.** $37\frac{1}{2}$ min. **81.** $17\frac{1}{7}$ min **82.** $14\frac{2}{5}$ days **83** $17\frac{1}{2}$ days. **84.** 30 days. **85.** $9\frac{1}{11}$ days **86.** $5\frac{5}{8}$ hrs **87.** 30 days. **88.** A, 6, B, 12, days. **89.** 20 days **90.** 120 days

F.—**91** 88 ft. **92.** 66 ft **93.** $15\frac{2}{5}$ ft **94.** $5\frac{2}{15}$ ft **95.** $117\frac{1}{3}$ **96.** 152 **97.** $7\frac{1}{2}$ mi **98.** $18\frac{3}{4}$ mi. **99.** $21\frac{9}{11}$ mi. **100.** $426\frac{2}{27}$ mi **101.** $22\frac{8}{11}$ mi. **102** $4\frac{1}{2}$ mi **103.** 36 mi **104.** $22\frac{1}{2}$ mi. **105.** $16\frac{1}{2}$ secs **106** $26\frac{1}{4}$ mi **107** 228 yds. **108** 8.49 A.M **109.** 90 mi **110.** $3\frac{1}{2}$ hrs. **111** $8\frac{1}{4}$ mi **112.** $2\frac{2}{3}$ mi per hr. **113.** 3 hrs. **114.** $3\frac{1}{3}$ hrs **115.** $104\frac{4}{11}$ mi **116** 9 mi per hr; $3\frac{3}{4}$ secs **117.** (i) $3\frac{15}{16}$ secs; (ii) $31\frac{1}{2}$ secs. **118.** 88 yds, 132 yds **119** No. **120.** 5 mi per hr. **121.** (i) 3 hrs, (ii) $4\frac{1}{5}$ hrs **122.** $1\frac{2}{7}$ hrs **123.** $16\frac{4}{11}$ min past 3 **124** $38\frac{2}{11}$ min past 7 **125.** $27\frac{3}{11}$ min past 5 **126.** $21\frac{9}{11}$ min past 4. **127** $43\frac{7}{11}$ min past 2 **128.** $21\frac{9}{11}$ min past 10 **129.** $10\frac{10}{11}$ min past 8 **130** $49\frac{1}{11}$ min past 3 **131.** $16\frac{4}{11}$ and $49\frac{1}{11}$ min past 6. **132.** $21\frac{9}{11}$ and $54\frac{6}{11}$ min past 1 **133.** $16\frac{4}{11}$ and $49\frac{1}{11}$ min. past 12 **134** At 9, and at $32\frac{8}{11}$ min past 9. **135.** $15\frac{3}{11}$ min. **136.** $57\frac{9}{11}$ min past 9. **137.** $32\frac{8}{11}$ min past 4. **138.** $54\frac{6}{11}$ min past 11.

G.—139 $8\frac{4}{5}$ yds 140. 38 yds 141 30 points 142. 271 points
143. 1 min 144. $10\frac{1}{3}$ secs

H.—145 10*s* 146 595 147. 15. 148 99 149 816
150. 3*s* $4\frac{1}{2}$*d*

XLV. 1. 2 3 2. 11 1. 3. 14 11 4 2 21
5 47 57 6 8 13 7. 6 13 8. 5 16 9 14 1
10. 37 57 11 The latter 12 The former 13 — 14 —
15 £1, 2*s* 11*d* 16 560 17. Decreased 18 Increased
19 $4\frac{2}{5}$ 20 $2\frac{1}{3}$ 21. $5\frac{2}{5}$ 22 $10\frac{22}{65}$ 23 —
24 —. 25. — 26. 98 27 $6\frac{8}{15}$ 28 £46, 13*s* 4*d*
29 1 cwt 0 qr 14 lbs 30 $1\frac{3}{5}$ 31 10*d* 32 4*s* 8*d* 33 $3\frac{1}{3}$
34 54 35 $2\frac{1}{3}$ qts 36. $3\frac{1}{3}$ 37. 12 tons 38 28
39. 80 40. $11\frac{1}{3}$ 41 — 42 7·65 43 — 44 Yes
45 No 46 54 ft 47 15 yds 48 88 ft 49. $11\frac{1}{4}$ and $13\frac{1}{2}$ chs
50. 28 chs 50 lks, 25 chs 65 lks, 39 chs 90 lks

XLVI. 1. 99, 209 2 315, 1287 3. 69, 161, 253
4 3125, 12500, 12500; 18750 5. $12\frac{2}{7}$, $9\frac{5}{7}$ 6. $7\frac{1}{4}$, $9\frac{1}{4}$, $11\frac{1}{2}$
7 70, 63 8 240, 476 9 3125, 6250, 12500, 25000
10 30, 48, 72 11 1440, 144, 1017 12. 135, 36 13 17. 19·5
14 £1700; £5100, £8500, £13600
15 60, 100, 260, 340, 420, 580, yds 16 £1, 7*s*. 9*d*, £3, 4*s* 9*d*
17. £115, 15*s*, £135, 0*s* 10*d*, £154, 6*s* 8*d*
18 £23, 12*s* $9\frac{1}{2}$*d*, £30, 7*s* $10\frac{1}{2}$*d*, £43, 18*s* $0\frac{1}{2}$*d*
19 £5, 10*s* 6*d*, £26, 19*s* 6*d* 20 £2, 8*s* 7*d*, £1, 19*s* 5*d*
21 £9, 4*s*, £10, £10, 16*s* 22 £60, £40, £30, £24
23 £400, £450, £480. 24. 88, 120, 130, ft
25 A, £666, 13*s* 4*d*, B, £1333, 6*s* 8*d*; C, £1000
26 A, £312, 10*s*; B, £187, 10*s*, C, £125 27. £306, £336; £366
28. 152; 171, 361 29 A, £202, 10*s*, B, £225, C, £112, 10*s*, D, £75
30. £760, 18*s*, £384, 6*s*, £959, 18*s* 8*d*, £584, 5*s* 4*d*, respectively.
31. £9736, 3*s* $2\frac{3}{4}$*d*, £6954, 8*s* $0\frac{1}{4}$*d* 32. 186, 83, 231
33 A, £42, B, £48, C, £60
34 A, £28, 9*s* $5\frac{1}{4}$*d*, B, £18, 19*s* $7\frac{1}{2}$*d*, C, £12, 13*s* 1*d*
35 A, £16, 13*s* 4*d*, B, £33, 6*s* 8*d*, C, £100
36 A, £1, 6*s* 8*d*, B, £1, 3*s* 4*d*, C, £1, 8*s* 4*d*
37 £208, £104, £52; £156 38. £212, 2*s*, £353, 10*s*., £388, 17*s*.
39 A, £700; B, £600, C, £480, D, £320
40 $15\frac{3}{4}$, $26\frac{1}{4}$, $33\frac{3}{4}$, $38\frac{1}{4}$, gal
41 £6, 8*s* for each man; £3, 14*s* 8*d* for each woman, £2, 6*s* 8*d* for each child
42. £26, 2*s* 6*d* for each man, £16, 10*s* for each woman, £12 for each boy
43 A, £13, 10*s*, B, £14, 8*s*, C, £14, 14*s*
44 A, £126, B, £112, C, £504 45. A, £105, B, £225, C, £240

46 A, £300, B, £192. 47. A, £309, 2s 5d; B, £144, 5s 1d.
48. £2266, 13s 4d 49. A, £160, B, £110 50. £89, 9s 9d
51. 7 mo after A 52. Equally, B, 2 mo before the division of profits
53. A, £2, 5s 5d, B, £2, 14s 7d 54. £870. 55 A, £8750, B, £6250
56. A, £146, 5s., B, £232, 17s 6d 57. A, £94, 10s; B, £168; C, £396
58. £2018, 3s 4d nearly 59 6 shillings worth
60. 4 cwt 1 qr 14 lbs of 1st, 4 cwt. 0 qr 14 lbs. of 2nd; 3 cwt 1 qr of 3rd

---

XLVII. 1. 56 sq ft. 2. 143 sq. ft 3. 7 sq ft. 126 sq in
4 24 sq ft. 128 sq in. 5. 65 sq ft 38 sq in
6. 9 sq yds 1 sq ft 72 sq in 7 $13\frac{1}{30}$ sq in 8 $3\frac{7}{16}$ sq in
9. 47 sq ft $15\frac{3}{4}$ sq in 10. 7 sq ft $8\frac{5}{8}$ sq in 11. 4 ac 12 24 ac
13. 3 ac 3 ro. 14. 3 ac 0 ro 33 po. 15. 11 ac 0 ro 32 po
16. 20 ac 1 ro 24 po 17. 1 sq ft 25 sq in 18 $54\frac{3}{8}\frac{5}{4}$ sq in
19 8 sq yds 1 sq ft 97 sq in 20. 10 ac 21 1 ac 0 ro. 36 po
22. 26 ac 3 ro 4 po 9 sq yds 23 8 ac. 0 ro 16 po.
24. 5 ac 1 ro 1 po 25 3 ac 1 ro 9 po 26 301 sq ft 18 sq in.
27. $39\frac{3}{8}$ sq in 28. 5 sq yds 2 sq ft $76\frac{1}{2}$ sq in 29. 2 sq ft 36 sq in
30. 7 sq yds 31 680 ac 32 1 ac. $13\frac{1}{3}$ po. 33. 5 ac 1 ro.
34 56 ac 0 ro 31 po. 35. 17 ft 36. 15 yds. 37. 63 ft.
38 $3\frac{3}{5}$ in. 39. 10 ft 6 in. 40. 26 ft 3 in 41. 110 yds.
42. 572 yds **43. 1 ft. 6 in.** 44. 1 yd 1 ft 45. $10\frac{2}{7}$ in.
46. $\frac{3}{16}$ in 47 5 chains. 48. 8 chains 49. 18 po 1 yd; or 100 yds.
50. 50 yds. 51 18 ft 6 in 52. 8 ft 53. 76 yds 54 4 ft.
55 3 ft $4\frac{1}{2}$ in. 56 $5\frac{1}{2}$ yds 57 $7\frac{1}{2}$ in 58. 14 yds.
59 1 ft 9 in. 60. $7\frac{1}{4}$ in each 61 198 62 288 63. 56
64. 1584 65. 36 66. 432 67 2304 68. 118800 69 1152
70. 36 71 1024. 72 $74\frac{1}{3}$ 73. $32\frac{2}{3}$ 74. $61\frac{3}{4}$ 75 81
76. $52\frac{1}{2}$. 77. 64 78. $149\frac{1}{3}$ 79 48 80 $24\frac{3}{8}$ 81. 32
82 $43\frac{11}{18}$ 83. 33 84. $80\frac{2}{3}$ 85 £3, 15s. $6\frac{1}{2}d$ 86 £9.
87. £10. 88. £23, 9s 4d 89. £4, 13s 6d 90 £8, 10s $7\frac{1}{2}d$
91. £26, 1s 4d **92. £39,** 2s. 93. 9s 7d 94 6s 10d.
95 £7, 16s. 9d; £1, 10s. 96. £1, 3s; 64 yds, £12. 97 £29, 6s 8d.
98 £30, 1s 11d 99. £20, 9s 6d, £1, 16s. $11\frac{1}{2}d$ 100. £75, 7s 4d.
101 £49, 2s 6d 102 £2, 9s $4\frac{1}{2}d$ 103. 165 sq ft 104. 18 ft.
105 $3\frac{3}{4}$ ft 106 3s 6d 107. 7s. 108. 23 ft 109. 9d.
110 88 ft 111. 63 ft 112. $22\frac{1}{2}$ ft; 45 ft 113 (i) 64 ft, (ii) 704 sq ft.
114. (i) 60 ft, (ii) 600 sq. ft 115. (i) 74 ft; (ii) 666 sq ft
116. (i) 74 ft., (ii) 777 sq. ft 117 (i) 70 ft, (ii) 1225 sq ft.
118 (i) 81 ft.; (ii) $992\frac{1}{4}$ sq ft 119 (i) 82 ft; (ii) 943 sq. ft.
120 (i) 147 ft, (ii) $2719\frac{1}{2}$ sq ft 121 (i) 79 ft, (ii) $941\frac{5}{12}$ sq ft
122 (i) 72 ft, (ii) 711 sq ft 123. 2 sq ft 62 sq in
124 3 sq ft 27 sq in 125. $731\frac{1}{2}$ sq ft 126. 38 127 8.
128 $23\frac{1}{2}$ sq. ft 129. $43\frac{5}{16}$ sq ft 130. 160 131. 88 132. 138
133. 108. 134. 11 135. 902 136. 420 137. £2, 12s 1d.
138. £1, 16s 3d 139. £5, 5s. 140. 17s 6d 141 £1, 7s.
142. £8, 7s 1d 143. £1, 0s 9d. 144. 978. 145. 1442 146 $1211\frac{1}{3}$.

147 £2, 10*s*. **148.** £2, 3*s* $1\frac{1}{2}d$. **149** £2, 6*s* 9*d* **150** £3, 18*s*
151 12 ft **152** 13 ft 7 in **153.** $6\frac{1}{2}$ in **154** 5 ft
155 6 ft 6 in. **156.** 7 ft. 6 in **157.** $11\frac{1}{2}$ ft **158** 37 in. **159** 1*d*
160 15 ft wide; 11 ft high

XLVIII.

1 108 cub ft **2** 595 cub ft. **3** 154 cub ft
4 525 cub ft **5.** 7 cub ft 504 cub in **6** 91 cub ft 936 cub in
7 16 cub ft 864 cub in **8.** 1228 cub ft 432 cub in
9. $5\frac{23}{64}$ cub in. **10** 12 cub ft 1216 cub in **11** 1 cub ft 469 cub in
12 29·791 cub. in **13** 13 ft **14** 13 ft 4 in **15** $1\frac{1}{2}$ in
16. 3 yds 2 ft **17** 1 yd. **18** 10 in **19.** 3111 **20** 8
21. 30 **22.** 11 ft **23** $15\frac{1}{4}$ ft **24** $4\frac{4}{5}$ ft **25** 108 ft
26 $\frac{3}{4}$ in **27** 162. **28** 24 ft **29** 30 ft wide, 16 ft 8 in high
30 0625 **31** £1, 3*s* **32** 14*s* $10\frac{1}{2}d$ **33.** $1633\frac{1}{2}$ **34** $1633\frac{1}{2}$
35 $47\frac{197}{224}$ **36.** 117 tons 3 cwts 3 qrs **37** $6\frac{1}{3}$ **38** £1, 17*s* 4*d*
39 £2, 14*s* 2*d*. **40.** 317 yds **41** 18432 **42** 440 **43** 1120
44 16 **45** 51200 **46.** 44352 **47.** 60 **48.** 4 **49** 26800
50 11680 **51.** 350 **52** 1000 **53.** 459, and $\frac{3}{4}$ pint over
54 6 **55** 8 **56.** 16 **57** 2376 **58** 2700 **59.** 15770
60 $5376\frac{3}{8}$. **61** 8680 **62.** $1568\frac{1}{2}$ **63.** 37 ft **64** $6\frac{3}{16}$
65 $136\frac{1}{3}$ **66** 13 cwt 59 lbs **67.** 1 ton 1 lb **68.** 1 grain
69. £37, 10*s*. **70.** 3 ft 2 in

XLIX.

1. 5 ft. 10′ 6″ **2** 6 ft 7′ 6″ **3** 4 ft 3′ 3″
4. 11′ 9″ **5** 8′ 7″ 6‴ **6** 2′ 1″ 6‴ **7.** 6 ft 4′ 6″
8 14 ft 9′ **9.** 16 ft. 2′ 3″ **10.** 3 sq ft 6′ **11** 2 sq ft 3′
12. 4 sq ft 7′. 4″ **13** 11′ 4″ **14** 11′ 11″ **15** 7′ 5″ 6‴
16 5 sq ft 9′ **17** 6 sq ft 7′ 6″ **18.** 46 sq ft 8′ 3″
19 10 cub ft 9′. **20** 17 cub ft 4′ 6″ **21** 3 cub ft 9′. 9″
22. 3‴ 6^{iv} **23.** 7″ 2‴ 9^{iv} **24** 9″ 9‴. 1^{iv} 6^{v}
25. 6 cub ft 3′ 0″ 4‴ **26.** 11 cub ft 6′ **27** 2 cub ft 10′ 7″
28 2 ft 5 in **29** 1 ft $7\frac{1}{2}$ in **30** 5 ft $4\frac{3}{4}$ in **31** 11 ft $10\frac{2}{3}$ in
32. 7 ft $0\frac{1}{3}$ in **33** 16 ft $2\frac{5}{8}$ in **34** 5 sq ft 108 sq in
35 14 sq ft 44 sq in **36** 6 sq ft 141 sq in
37. 1 sq. ft $103\frac{1}{2}$ sq in **38.** $131\frac{2}{3}$ sq in **39** $112\frac{11}{24}$ sq in
40 12 cub ft 288 cub in **41** 7 cub ft 60 cub in
42 3 cub ft 807 cub in **43** 1547 cub in **44.** $1246\frac{1}{2}$ cub in
45 $840\frac{25}{48}$ cub in **46.** 22 ft. 3′ 1″ 8‴ **47** 17 sq ft 9′ 4″ 1‴ 6^{iv}
48 17 sq ft 3′ 10″ 4‴ **49** 4 cub ft 5′ 4″ 9‴ 6^{iv}
50 (i) 31 ft 8′, (ii) 5 ft 3′ 4″, (iii) 34 ft 3′ 8″
51 (i) 66 sq ft 9′ 4″; (ii) 400 sq ft 8′; (iii) 261 sq ft 6′ 6″ 8‴
52 (i) 1 ft 4′. 3″ 5‴; (ii) 1 ft 7′ 6″ 6‴, (iii) 8′ 1″ 8‴ 6^{iv}
53 £1, 17*s* $1\frac{1}{4}d$. **54.** £12, 0*s* $1\frac{3}{4}d$ **55** 10 sq ft 1′ 6″.
56 1 ft 1′ 5″ 3‴ **57** 1′ 1″ 8‴ 3^{iv} **58** 5″ 6‴ 8^{iv}
59. 7 sq ft 8′ 3″ **60** 11 sq ft 10′ 6″ **61.** 44 sq ft 4′ 8″
62 99 sq ft 3′ 11″ 8‴. **63** 277 sq ft 11′ 2″ 3‴.
64 10 sq. ft. 7′ 2″ 7‴ 6^{iv} **65** 104 sq ft. 5′. 0″ 9‴. 11^{iv}

66. 339 sq ft. 1′ 6″. 7‴. 6^{iv} 67. 21 sq ft. 0′. 1″. 68. 2′. 5″. 4‴. 1^{iv}.
69 2 sq ft 5′. 0″ 5‴. 4^{iv} 70. 190 sq ft 2′ 6″ 3‴.
71. •12 cub ft. 4′ 6″ 72. 3 cub ft. 1′ 6″. 73. 42 cub ft 0′ 2″
74. 3 cub ft 4′ 6″ 75. 57 cub. ft. 3′ 6″. 76. 6 cub ft. 9′ 10″ 6‴.
77. 7 cub ft. 11′ 10″ 6‴ 78. 56 cub ft 11′. 1″ 9‴.
79 382 cub ft 11′ 5″ 3‴ 6^{iv} 6^{v}. 80. 216 cub. ft 7′. 5″ 11‴ 9^{iv} 4^{v}
81. 2315 cub. ft. 7′ 11″ 2‴. 6^{iv}. 82. 6185 cub ft 2′ 10″ 5‴ 9^{iv}.
83. 12 cub. ft 8′ 5″ 4‴ 84. 49 cub ft 3′ 6″ 8‴
85 4′ 6″ 2‴ 11^{iv}. 6^{v} 8^{vi}. 86. 444 cub ft 6′ 4″ 11‴. 10^{iv} 1^{v} 7^{vi}.
87 $136\frac{1}{8}$ sq ft. 88. $292\frac{1}{3}\frac{7}{2}$ sq ft. 89. £57, 1*s* $10\frac{1}{2}d$
90. £92, 15*s* 91. £9, 8*s* $3\frac{7}{12}d$ 92. 62 sq. yd. 0 sq ft $58\frac{2}{3}$ sq. in.
93. 4231 sq ft. $26\frac{2}{3}$ sq in 94. £2622, 12*s* 95. 3375 cub in.
96. £38, 8*s* 8*d* 97. 5 ft. 7′. 98. 16 ft 8′. 6″. 99. 2 ft 4′ 3″.
100. 13 ft. 9′ 6″.

L. 1. (i) 4·563; (ii) 4·56328 2. (i) ·033, (ii) ·03265
3 (i) 15·248, (ii) 15·24763. 4. (i) ·279, (ii) ·27893
5. (i) 6·150; (ii) 6·14993 6. (i) ·423; (ii) ·42337
7. (i) 2·068, (ii) 2·06833. 8. (i) 473·667, (ii) 473·66667
9. (i) ·059, (ii) ·05859 10. 47·67 11. 346·0 12. 49·642
13. 99·783 14. 11·3092 15. 6·48007 16 1327. 17. 40,000000
18. 170·9 19. 91·8 20 152462·4 21. 59·91 22. 400·7397.
23 116·510 24. 28·989 25 5·228. 26 12·454
27 5296·627 28. 10·603 29. ·178 30 13·627.
31. ·012 32. ·615 33. ·544 34. ·014 35. 5·014
36. 9·376 37 ·774 38. 2·545 39. 2033·941 40 30·906
41. 144·383 42. 15·461 43. ·949 44. 12·294 45. 2·486
46 18·991 47. £2·821. 48 £13 908 49. 1·829
50. £43·654 51. £4·779 52. £·487, or £·488 53. £63·554.
54. £10·183. 55. £3·175 56 £75·992 57 £2·096
58 £·885 59. £1·49167 60. £16·62917 61 £·69583.
62 £·28333 63. £453·80833 64. £197·67083 65. £4·93542.
66. £87·16562 67. £142·08021. 68. £4, 16*s*. 4*d* 69. £1, 11*s* 8*d*.
70. £17, 18*s* 2*d* 71. £13, 14*s* 1*d* 72. 12*s* 9*d*. 73. £38, 7*s*. 2*d*.
74. £123, 17*s* 8*d* 75. £1, 11*s*. 11*d*. 76. £11, 2*s*. 77. £47, 10*s* $11\frac{1}{4}d$.
78 13*s* $8\frac{1}{2}d$ 79 £1, 10*s* $4\frac{1}{2}d$. 80. £11, 0*s* $11\frac{1}{4}d$.
81. £546, 9*s* $2\frac{1}{2}d$ 82. £3, 5*s* $11\frac{1}{2}d$ 83. £33, 10*s* 5*d*
84. £347, 9*s*. 4*d* 85. £45, 18*s* 10*d* 86. £7, 18*s* 10*d*
87. £5989, 12*s* 10*d*. 88 £1672, 1*s* 1*d*. 89. £1255, 16*s* 4*d*
90 £3223, 15*s* 11*d*. 91. £3788, 10*s* 2*d* 92. £411, 14*s* 3*d*.
93 £517, 19*s* 11*d* 94 £4, 8*s* 8*d* 95. £50, 7*s* 6*d*.
96 £4, 13*s*. 3*d* 97. £6, 15*s* 98 £42, 3*s* 2*d*. 99. £7, 4*s* 6*d*
100 £11, 7*s* 2*d*. 101. 727·23 102 15·24 103. 7·79
104. 29163·99 105. ·126 106. 27·201 107. 3·50. 108. 230·29.
109. 46·916 110. 33·61 111 1435·283 112. ·168 113 24·94
114 ·3573. 115. ·0004. 116. ·656 117 ·05 118. ·088808
119. 6·7630 120. 65·261905 121. 17·228. 122. 9·870 123. ·33.
124. 145·3833. 125. ·000457. 126. 10·22. 127. 31·01 128. ·07.

129 ·00002. 130. ·135 131. 1·137 132 1·267 133. 1·13141.
134 1·42331. 135 1·710 136. ·16 137 12·077 138. 315·0677
139. 281·524. 140 1368 141. 1713 142 1700 143. 6984000
144. 13·275 145 9244 146. 8·297 147. ·351 148 27426 54
149. 3802 150 ·0002588 151. £1, 9*s* 1*d* 152 £10, 3*s* 11*d*
153 £3032, 9*s* 7*d* 154 £43741, 11*s* 3*d* 155 £348019, 10*s* 10*d*
156 £168023, 19*s* 3*d* 157 £9250, 4*s* 1*d* 158. £713, 1*s*
159. £1363, 5*s* 5*d* 160 £9286, 10*s* 10*d*. 161. ·519 162 17·16
163 ·0219 164. ·629 165 ·180 166. ·237 167 305·46
168. 701·94 169. ·0014 170 0028 171 ·12947 172 14·9491
173. ·050193 174. 44·814217 175. 25·107 176 9·9804
177. ·02467 178. ·079091. 179 1·45 180 15·94 181 12·056
182 ·375 183. ·192. 184 6·48 185 ·0041. 186 26
187. 2525 188. ·4617 189 216·0 190 464·3642 191 £1, 8*s*
192 6*s* 6*d* 193. £2, 10*s* 9*d* 194 £107, 16*s* 6*d* 195. £5, 3*s* 11*d*
196 £15, 8*s*. 1*d* 197 ·71828 198. ·36788 199 ·41069
200 ·27522 201 £31, 4*s* 2*d* 202 £256, 8*s* 7*d* 203 6*s* 11*d*
204 14*s* 205 £318, 1*s* 11*d*, £155, 16*s* 4*d* 206 11*s* 6*d*
207 (i) £26, 12*s* 5*d*; £18, 13*s* 6*d*; £11, 10*s* 5*d*, (ii) £40, 8*s* 5*d*, £48, 3*s* 10*d*, £59, 1*s* 6*d*, (iii) £338, 14*s* 10*d*, £501, 12*s*, £573, 5*s* 2*d*, (iv) 6 tons 12 cwt 2 qr, 16 tons 1 cwt 2 qr, 34 tons 19 cwt. 3 qr, (v) 153 ac 2 ro 23 po, 162 ac 0 ro 28 po, 157 ac 3 ro 26 po
208 916·3 sq ft. 209. 2 sq in. 210 47·94 mi

LI. 1 50. 2 25 3 20 4 5 5 4 6 2
7 75 8 40 9 70 10. 45 11. 44 12 $3\frac{3}{4}$
13 $33\frac{1}{3}$ 14 $12\frac{1}{2}$ 15 $8\frac{1}{3}$ 16 $2\frac{1}{2}$ 17 $7\frac{1}{2}$ 18. $3\frac{1}{2}$
19 $62\frac{1}{2}$ 20 $12\frac{3}{4}$ 21. $\frac{2}{25}$ 22 $\frac{6}{25}$ 23 $\frac{3}{20}$ 24. $\frac{7}{20}$.
25 $\frac{39}{50}$ 26 $\frac{3}{50}$ 27 $\frac{9}{200}$ 28 $\frac{1}{30}$ 29 $\frac{1}{16}$ 30. $\frac{2}{3}$
31. $\frac{1}{12}$ 32. $\frac{1}{3}$ 33 $18\frac{3}{4}$ 34. $26\frac{6}{19}$ 35. $14\frac{1}{6}$ 36. $41\frac{3}{17}$.
37 $3\frac{37}{81}$ 38 $37\frac{1}{2}$ 39. $\frac{1}{32}$ 40. $\frac{2}{45}$ 41 $\frac{11}{160}$ 42 $\frac{7}{320}$
43 $\frac{17}{40}$ 44 $\frac{23}{125}$ 45. 270 46. 453 47 177 48 82
49 40·5 50 3551 51 4 52 $31\frac{1}{3}$ 53 $37\frac{1}{2}$ 54 $81\frac{9}{11}$
55 5 56 1·5 57 920 58 576 59. 915 60. £4, 14*s*
61 £5, 16*s* 62 £13, 15*s* 63 £17, 7*s* 64 12*s* 6*d*
65 £1, 3*s* 3*d* 66 8*s* 9*d* 67. 17*s* 6*d* 68 £1, 16*s* $7\frac{1}{2}$*d*
69 4*s* 4*d* 70 9*s* 8*d* 71. £2, 11*s* $2\frac{1}{2}$*d* 72 £4, 6*s* 5*d*
73 £6, 12*s* 10*d* 74 £9, 9*s* 7*d* 75 £26, 17*s* 11*d* 76 10*s* 1*d*
77. 3*s* 7*d* 78 £1, 6*s* 11*d* 79 14*s* 8*d* 80. £2, 3*s* 9*d*
81 £3, 2*s* 7*d* 82 £5, 7*s*. 4*d* 83 £9, 1*s* 5*d* 84 £23, 7*s* 8*d*
85 17*s* 86 £1, 8*s* 6*d* 87. 6*s* 3*d* 88 3*s* 9*d* 89 7*s* 3*d*
90 14*s* 2*d* 91. 5*s* 5*d* 92 2*s* 2*d* 93 £1, 9*s* 10*d* 94 6*s*
95 £2, 17*s* 96 3 cwts 3 qrs 97 16 ozs 3 dwt
98 29 025 grs 99 £1, 15*s* 7*d* 100 12*s* 101 £9, 1*s* 3*d*
102 £9, 12*s* $4\frac{1}{2}$*d* 103 £23, 16*s* $7\frac{1}{2}$*d* 104 $16\frac{2}{3}$ 105 $12\frac{1}{2}$
106 $\frac{5}{8}$ 107 $97\frac{1}{2}$ 108. $6\frac{2}{3}$ 109 $20\frac{5}{6}$ 110 5 111 $42\frac{6}{7}$
112 $4\frac{1}{6}$ 113. $18\frac{3}{4}$ 114. $3\frac{1}{3}$ 115 2 116 14*s* 2*d*.
117 £2, 9*s*. 5*d*. 118. £1, 7*s* 1*d* 119 £3, 19*s*. 7*d* 120. £6, 9*s*. 10*d*.

121. x4, 19*s*. 9*d* 122. £10, 16*s* 5*d* 123. £21, 11*s*. 10*d*
124. £16, 15*s*. 8*d* 125. £4, 15*s* 2*d*. 126. £3, 4*s* 3*d*
127. £2, 10*s* 3*d* 128. £7, 4*s* 9*d* 129. £18, 1*s*. 6*d*
130. £24, 19*s* 6*d*. 131. £5, 11*s*. 132. £1, 18*s*. 2*d*
133 £28, 7*s*. 134. 459 135 295 136. £15, 4*s* 6*d*
137. £35, 2*s* 138. £48, 19*s* 6*d* 139. £109, 4*s* $3\frac{1}{2}$*d*.
140 £49, 8*s* 141 £53, 4*s* 4*d* 142. £19, 8*s* 1*d*
143. £93, 13*s*. 6*d* 144 £163, 19*s* 11*d* 145. £295, 0*s*. 1*d*
146 1701 147. 3927 148. £50, 4*s* 3*d* 149. £91, 15*s* 6*d*
150 10 ton 3 cwt. 2 qr 151. 8 ton 11 cwt 2 qr 152. £35, 16*s*. 7*d*
153. £15, 5*s* 154. £71, 7*s* 8*d* 155. £102, 16*s* 7*d*.
156. £260, 10*s* 7*d* 157. £506, 9*s* 11*d* 158. £85, 10*s*
159 139195 160. 5 161. $8\frac{1}{3}$. 162. 11120 163. £644, 3*s* 4*d*
164. 1435 165. 3·06 166. $37\frac{1}{2}$ 167. £5, 1*s* 4*d* 168 165200.
169. $5\frac{1}{2}$ 170. 1683 171. 22 172 700 173. 88.
174. 20200 175 20% increase 176 (i) $3\frac{1}{3}$; (ii) $3\frac{1}{23}$.
177 £1500 178. 8600 179. 3 180. £5, 4*s* 2*d*.
181 4*s* in the pound 182. 60000 183. £551 184. £520.
185 £733, 19*s* 186. 44% increase 187. $2\frac{1}{4}$% decrease
188 4947·014 cub ft nitrogen, 1520·4456 cub ft. oxygen, 80·5404 cub ft carbonic acid. 189. 1671 ozs 190. $62\frac{1}{2}$
191 800 192. £7400 193. $535\frac{1}{2}$, $484\frac{1}{2}$ 194. £468,750000
195. 34 196. E, 72·9, W, 4·0; S, 10·7, I, 12·5
197 1st class, 3·5, 2nd, 6·6, 3rd, 2·2; total, ·8.
198 1st class, 2·7 decrease, 2nd, 12·4 dec; 3rd, 2·0 inc; total, 1·1 inc
199 1st class, 3·8 decrease, 2nd, 10·0 dec; 3rd, 2·3 inc.; total, ·8 inc
200 1st class, ·8 dec, 2nd, ·3 inc.; 3rd, 4·9 inc., S T., 24·8 dec; total, 4·3 inc

---

LII. 1 20% gain 2. $12\frac{1}{2}$% gain 3. $16\frac{2}{3}$% loss 4. $9\frac{3}{8}$% loss
5. 8% gain 6. $66\frac{2}{3}$% loss. 7. 10% gain 8. 18% gain.
9. 2% loss 10. 20% gain. 11. £9, 18*s*. 12. 8*s* 3*d*
13 £2, 0*s* 3*d* 14. £6, 6*s*. 15. 2*s*. $11\frac{3}{4}$*d*. 16. 11*s*
17 £8, 15*s* 18. £1369 19 £5, 8*s* 20 £1, 4*s* 10*d*
21. 13*s* 4*d* 22. £1166, 13*s* 4*d* 23. 8*s*. 4*d* 24. £1, 3*s* 4*d*.
25. £44 26. 8*s* 4*d*. 27. £560. 28. 1*s*. $0\frac{1}{2}$*d*
29. £5, 5*s* 30. £60 31. £33, 17*s* 3*d* 32. $5\frac{7}{8}$ 33. $12\frac{1}{2}$
34. £37, 10*s* 35. £90, 8*s*. 4*d*. 36. 16*s* 8*d*. 37. £15. 38 £75
39. $2\frac{3}{4}$*d* 40. $2\frac{1}{2}$*d* 41. $29\frac{17}{27}$ 42. $16\frac{2}{3}$ 43 £25
44. $12\frac{1}{2}$ 45 150 46. 44. 47. $36\frac{1}{3}$. 48. 10. 49. 100
50 3*s* $4\frac{1}{2}$*d* 51. 2*s* $10\frac{1}{2}$*d* 52. $31\frac{1}{4}$ 53. £1, 8*s* 9*d*.
54 £8 55 £1642, 13*s*. 4*d* 56 25 57. £59, 17*s*.
58. $7\frac{1}{2}$% gain 59. 2% loss 60. £21, 5*s*. 61. 3*s*. $7\frac{1}{2}$*d*
62. Bought at 12*s*. 6*d*.; sold at 12*s*. per yd 63. 3456. 64. 1% loss
65. £1, 4*s* loss 66. £12, 10*s*. 67. £2, 4*s* gain.
68. $4\frac{1}{8}$% gain. 69. 80% gain 70. 48 71. £225
72 14% gain. 73. £39, 6*s* 74. 2*s* 6*d* 75. $15\frac{9}{27}$.
76. 1*s*. 10*d* 77. $16\frac{2}{3}$ 78. 2*s*. 79. $21\frac{31}{35}$. 80. At 2*s* 6*d*. per lb.

81. 20% gain. 82. 40. 83 1⅝ pints 84. 9s per gal, 18 1/17 %.
85 12½ 86 8 gals 87 156 gals 88 4 gals 89 £60
90 10s 91. £133, 6s 8d 92 9¼ 93 13s 9d 94 22½% gain
95 £60 96. 8s 4d 97. £1, 13s 4d 98 £8, 6s 8d.
99. 2s 7½d. 100. £808, £606, £404

LIII. 1 £73, 12s 9d 2 £25, 12s 3 £3, 4s 1d 4. £48, 11s 5d
5 £83, 13s. 7d. 6 £1, 7s 6d 7 £4, 6s 3d 8 £2, 8s 5d
9. £9, 9s 9d 10. £12, 14s 6d 11 £24, 12s 8d 12. £117, 11s 11d.
13. £22, 2s. 9d 14 £1757, 16s 2d 15. £12, 12s 9d.
16. £57, 18s 4d. 17. £1, 7s 18 £2, 12s 6d 19. £3, 1s 4d
20 £30, 10s 8d. 21 £117, 16s 6d 22. £326, 5s 23 £71, 5s
24 £733, 6s 8d. 25. £4, 3s 4d 26. £976, 1s 3d 27. £7625
28 £455, 5s 29. £775 30. £61, 19s 9d

LIV. 1 £21, 13s 2 £12, 10s 6d. 3. £19, 2s 6d. 4 £93, 15s
5 £28, 11s 6d 6. £2, 16s 8d 7 £72, 5s 8. £35, 2s 9d
9 £34, 9s 5d 10 £561, 15s 7d 11. £179, 4s 12 £78
13 £649, 13s 9d 14 £26, 3s 6d 15 £28, 19s 9d. 16 £123, 7s 6d
17. £369, 15s 18 £96, 5s 8d 19. £1065, 17s 3d 20. £4249, 0s 9d
21 £4, 1s 8d 22. £50, 10s 3d 23 £19, 3s 3d 24. 15s 10d
25 £12, 15s 6d 26. 13s 6d 27. £43, 4s 3d 28 £2, 16s 6d
29 £6, 11s 3d 30 £784, 4s 31. £7, 7s 7d 32 £4, 4s 4d
33 £1, 1s 1d 34. £138, 18s 8d 35 £239, 15s 6d.
36 £8, 4s 9d, £282, 16s 5d 37 £1, 15s 11d, £91, 14s 1d.
38. £10, 12s 10d., £1713, 18s 8d 39 £1, 12s, £28, 5s 7d
40 £96, 14s 10d, £813, 7s 3d. 41 18s 6d, £70, 8s 4d
42 £23, 9s 11d; £1048, 17s 11d 43 £90, 4s 9d, £3474, 3s 9d
44. 3s 9d, £10, 14s 7d 45 £2, 1s 11d, £109, 15s 4d
46. £16, 5s 47. £6, 8s 48 £3, 11s 6d 49 £3, 12s 9d.
50. 19s 10d 51 7s 1d 52 10s 7d 53 £6, 13s 6d.
54. £31, 10s 6d 55 £4, 10s 6d 56. 5s 11d. 57. 6d
58 £1, 0s 5d 59 8s 9d 60 15s 8d 61. £2, 18s 6d
62. £7, 19s 5d. 63 £213, 11s 64 £14, 11s 6d 65. 6s 8d.
66 £17, 12s. 7d. 67 £13, 0s 8d 68. 1s 6d 69 2s 5d
70. £2, 12s 9d 71 3 72 3¼ 73 4⅖ 74. 2⅛ 75 4½
76 3⅓ 77. 4 78. 4½ 79 3½ 80 3 81 4¼ 82. 6
83. 4 84. 3 85 2½ 86. 2½ 87. 4 88 4 89 18
90. 12½ 91 4 yrs 92. 3½ yrs 93 25 yrs 94. 6 mo.
95. 16 yrs 96 89 yrs 97 5 mo 98. 25 days 99 10 mo.
100. 219 days 101 3 yrs 102 2 yrs 103 5 yrs 104. 8½ yrs
105 1 yr 106 5⅜ yrs 107. 4½ yrs 108 8 mo 109. 5 mo
110 50 days 111. £425 112 £2375 113. £3720 114. £2325
115. £525 116. £1575 117 £291, 17s 6d 118. £20,000
119 £6666, 13s 4d 120. £164, 5s 121 £750 122 £225
123 £425 124. £555 125. £375 126. £314, 10s. 127 £710
128. £365. 129. £148, 7s. 11d, 130. £1036. 131 £173, 5s.

132. £2, 1*s* $9\frac{3}{8}$*d*. 133. £1, 6*s* $3\frac{3}{4}$*d*. 134. £297, 1*s*. 8*d*. 135. £5, 8*s*. 9*d*.
136. £9, 2*s* 1*d* 137. £1, 12*s*. 6*d*. 138. £3, 12*s* 2*d*. 139 £125, 2*s* 6*d*.
140. £178, 14*s* $1\frac{1}{2}$*d* 141. 2 yrs 7 mo 142. 3 yrs. 143. £576, 18*s*. 9*d*.
144. $4\frac{1}{2}$% 145. £375 146 $2\frac{3}{8}$% 147. $6\frac{1}{4}$ yrs. 148. $26\frac{2}{3}$ yrs
149. Mar. 20, 1898 150. $3\frac{1}{3}$% 151. £4250 152. £87, 10*s*.
153. 90% 154. £30. 155. 5*s* 3*d* 156. £48. 157 44 45
158. A, £84; B, £21, C, £105 159 £104932, 6*s* 8*d*. 160. 4%; $3\frac{1}{2}$%.
161. $2\frac{1}{2}$ yrs 162 12 yrs 163. £1500; £1800 164 £2037, £1746
165. $4\frac{3}{20}$%. 166. £63, 3*s*. 8*d*. 167. £8732, 18*s* 9*d*. 168. £2, 8*s*. 6*d*.
169. £200, 17*s* 5*d* 170. £2, 13*s*. 4*d*

LV.

1. £2756, 5*s*. 2. £4596, 16*s*. 3. £694, 11*s* 6*d*.
4 £336, 4*s* 5. £1098, 1*s* 6*d*. 6. £38, 1*s* 3*d*
7. £289, 15*s* $5\frac{1}{4}$*d* 8. £615, 2*s*. 6*d* 9. £106, 16*s* 9*d*
10. £409, 13*s* $11\frac{1}{4}$*d*. 11. £572, 17*s* 8*d*. 12. £1687, 5*s* 11*d*.
13. £2677, 3*s* 7*d*. 14 £188, 1*s* 3*d* 15 £106, 2*s* 1*d*.
16. £3021, 8*s* 17. £1129, 11*s*. 18 £476, 9*s* 2*d*
19. £1773, 14*s* 8*d* 20. £2672, 2*s* 5*d*. 21 £16, 11*s* 6*d*
22. £631, 10*s* 3*d* 23 £29, 9*s* 7*d* 24. £51, 12*s* 4*d*
25 £25, 13*s*. 10*d* 26. £144, 10*s* 5*d* 27. £56, 7*s* 3*d*
28 £54, 6*s* 2*d* 29 £43, 11*s* 2*d* 30. £339, 15*s* 10*d*
31. £875, 15*s* 6*d* 32. £468, 7*s*. 7*d* 33. £810, 6*s* 9*d*
34. £4993, 10*s* 3*d*. 35. £179, 11*s* 6*d* 36. £1187, 13*s* 9*d*
37. £5321, 8*s* 4*d* 38. £876, 13*s* 2*d* 39. £1345, 9*s*. 11*d*.
40. £258, 16*s* 11*d* 41. £463, 5*s* 7*d* 42. £811, 8*s* 9*d*
43. £2806, 0*s* 6*d* 44. £121, 16*s* 3*d* 45. £3864, 18*s* 3*d*
46. £1029, 14*s* 4*d* 47. £27, 12*s* 3*d* 48. £108, 10*s* 3*d*
49. £11, 14*s* 9*d* 50. £26, 17*s* 11*d* 51. £1711.
52. £1303 53. £1363 54. £1426 55. £802 56 £18573
57. £873 58. £3404 59. £2, 1*s* 4*d*. 60. £20, 13*s* 6*d*
61 £9, 7*s* 5*d*. 62 £1, 3*s* 3*d* 63 £7, 2*s* 11*d* 64. 3 yrs
65 3 yrs. 66 3 yrs. 67 $3\frac{1}{2}$ yrs 68 $2\frac{3}{4}$ yrs. 69. $2\frac{1}{3}$ yrs
70 $1\frac{1}{2}$ yrs 71. £1250 72 £4000 73. £6400. 74. £2430.
75 £64000 76 £64, 15*s* 77 £10, 3*s* 4*d* 78. £6, 13*s* $7\frac{119}{250}$*d*.
79. £11, 2*s* 80. £84 81. 35909 82 £875 83. £7000.
84. £2000. 85. £563, 5*s* 11*d*. 86. 7 yrs 87. £441.
88. £901, 6*s* 8*d* 89. £3225, 16*s* 90. £9331, 4*s*

LVI.

1. £4, 1*s*. 3*d*. 2. £2, 13*s* 7*d* 3. £6, 8*s* 9*d*
4. £12, 13*s* 10*d* 5. £3, 17*s* 9*d* 6. £3, 12*s* 5*d*.
7. £3. 8. £2, 2*s*. 3*d* 9. £276, 10*s*
10. £407, 3*s*. 9*d* 11. £322, 19*s* 5*d* 12. £250, 8*s* 6*d*
13. £4, 13*s*. 3*d*. 14 £1, 9*s* 11*d* 15. £10, 10*s* 2*d*.
16. £8, 16*s* 1*d* 17. £6, 19*s* 4*d* 18. £296, 11*s* 9*d*.
19. £271, 16*s* 8*d*. 20. £1374, 13*s* 8*d*. 21. £664, 12*s* 6*d*.
22. £373, 8*s* 6*d*. 23. £1111, 12*s* 4*d* 24. £560, 6*s* 9*d*.
25. £2399, 9*s*. 10*d*. 26. £582, 10*s* 27. £346, 19*s* 7*d*.

28 £991, 4s. 7d   29. £296, 10s 11d   30. £2100
31. £515   32. £750.   33. 5%.   34 £561, 3s 9d   35 May 30
36 (i) £30,   (ii) £750   37 (i) £200,   (ii) £4000
38. (i) £24,   (ii) £640   39. (i) £7,   (ii) £400
40. (i) £30,   (ii) £1000   41. (i) £4, 18s,   (ii) £280
42 (i) £10, 8s 4d, (ii) £833, 6s 8d   43. (i) £4, 14s.,   (ii) £470
44 (i) £1, 8s 7d., (ii) £149, 1s. 5d. nearly
45 (i) £5, 3s. 3d, (ii) £498, 6s 9d nearly   46 £1, 2s $8\frac{1}{4}$d
47 5s $2\frac{2}{3}$d   48. $2\frac{1}{2}$.   49 $3\frac{1}{3}$ 50 5 mo   51. 45 days
52 £264, 17s 3d.   53. £312   54 £826, 0s 10d   55 £1300
56 £1717   57. £10.   58 £50   59. £195, 16%   60 —
61 £770, 19s 6d   62. £222, 5s   63 £596, 17s 5d
64 £3525, 17s 4d   65 £1351, 2s 7d   66. £863, 16s 9d
67. £354, 11s 11d.   68. £1048, 8s. 6d   69. £293, 15s nearly.
70. £449, 5s. nearly

LVII. 1. £696   2 £2300   3. £507   4 £567   5 £1386
6 £1281   7 £769, 10s   8 £2252, 10s   9. £433, 2s 6d
10. £7329   11 £1250 stock.   12 £875 stock.   13 £875 stock.
14 £832 stock   15 £1333, 6s 8d stock   16 £1600 stock
17. £2000 stock   18 £533, 6s 8d stock   19 £800 stock
20. £3200 stock   21. £102   22 £30   23 £72, 10s.
24 £21, 5s   25. £21, 15s   26 £391, 10s.   27 £112, 9s.
28. £17, 12s   29 £46, 13s 4d   30. £56, 10s
31. £46, 17s 6d   32. £70   33 £21, 17s 6d   34 £30
35 £34, 7s 6d   36. £17, 7s 6d   37 £37, 2s 6d
38. £26, 13s 4d   39. £103, 10s   40 £10, 7s 6d.   41 £33, 8s.
42 £5, 5s   43 £10, 19s   44 £15   45 £10, 13s 4d
46 £3, 17s   47. £4, 17s 6d   48 £20, 12s 6d   49 £39, 1s 3d.
50 £9, 3s 4d   51. £2392   52. £581, 17s 6d   53 £2609, 5s
54 £679.   55 £400, 12s 6d   56 £342   57. £1401, 15s.
58 £980   59 £1889, 16s   60. £1834, 1s 9d   61. £560 stock.
62 £1000 stock.   63 £875 stock   64 £1500 stock
65 £800 stock   66 £3200 stock   67. £140   68 £22, 10s
69 £11, 5s   70 £43, 15s.   71. £7, 3s 9d   72 £4, 2s 6d
73 £40, 19s   74 £39   75 £48, 15s   76 £20, 9s 6d
77 £65   78 £23, 8s 9d   79 £13, 1s   80 £21, 5s 4d
81 £12, 1s 8d   82 £33, 16s 8d   83 £58   84 £12, 13s 9d
85 £19, 5s   86 £52, 18s 9d   87 £4, 2s 6d   88 £2, 17s 9d
89 £6, 17s 6d   90. £10, 11s 9d   91 $2\frac{13}{56}$ 92 $4\frac{4}{39}$
93 $3\frac{19}{27}$ 94 $3\frac{11}{23}$ 95. 4   96 $5\frac{5}{7}$ 97. The former
98 The latter   99 The former   100. The latter   101 The latter
102. The former   103 £20 increase   104. £34, 7s 6d increase
105 £25 decrease   106 £33, 15s increase   107 £75 increase
108 £190 increase   109 £25 increase   110 £20, 11s 3d increase
111 £3350, 0s 3d   112. £34, 12s 5d   113 £1852, 10s 7d stock.
114 £2110, 16s 4d stock   115 £691, 4s   116. £2606, 3s 6d
117 £18, 3s 8d.   118 £16, 1s 2d.   119 £7, 1s 3d.

120 £5, 10*s* 8*d* 121. £156. 122. £245. 123. £62, 10*s*.

124 £952 125 £3675 126. £75 127. £543, 15*s*. 128. £18.

129 £45 130 £185, 18*s* 9*d*. 131. £28, 14*s* 132. £43, 4*s*.

133. £1, 11*s* 6*d* 134 £1125 135. £24, 7*s* 6*d* 136. 70.

137 400 138. 60. 139. 9 140. 70 141. 140 142. 54.

143. 80 144 5 145 7. 146. £138, 10*s* 147. £847, 10*s*.

148 £315 149 £75, 12*s*. 150 £73, 10*s* 151 £109, 16*s*.

152 £74, 8*s* 153 £1261, 10*s* 154. £416. 155. £171, 5*s*.

156 $4\frac{1}{3}$ 157 $3\frac{5}{7}$ 158. $5\frac{1}{3}$. 159 The former 160 The latter.

161 £1945 stock 162. £4000, £4250, £12000 stock. 163. £3500.

164 £5950 165. £3480 166. £4875 167 £3600.

168 £3678, 6*s* 169. £436, 2*s* 11*d* 170 £9000 171. £8700.

172. £22080 173. £21840 174 £14520 175. £148, 4*s*.

176 £6, 17*s* 6*d* 177. No difference 178. £7750 stock

179 £215. 180. £139, 19*s* 181. $80\frac{2}{3}$ 182. $93\frac{3}{4}$ 183. $98\frac{3}{4}$

184 £3240 185. At 90 186. £90 187 £196, 13*s* 4*d*

188. £1430 189. £54 increase 190. The former 191. 4

192. $5\frac{1}{19}$ 193. $3\frac{67}{191}$. 194. 4 195. $4\frac{4}{29}$ 196. $133\frac{1}{3}$

197. 85 198 $96\frac{1}{4}$ 199 £72, 10*s* 200. £$120\frac{3}{16}$ 201. 6%

202. $19\frac{9}{19}$% decrease 203. 96 204 £2000 stock 205 120

206. £10489, 10*s* 207. £19, 16*s* 8*d* less 208 4*s*. less.

209 £1260 210 £3820 211. The latter 212. 128.

213 $114\frac{7}{8}$ 214. $166\frac{2}{3}$ 215. £5290 stock 216 £1600; £1706, stock.

217. $133\frac{1}{3}$ 218 $121\frac{3}{4}$ 219. £14, 1*s* 220 £100.

221 £10000 stock. 222. £525 before, £625 after.

223. $18\frac{74}{377}$% decrease 224 B 225. 84. 226. 80.

227 £2859, 10*s* 228 £14 229. At 98. 230 £174735

231 $4\frac{1}{4}$% 232 £24, 5*s* 4*d* 233. £144, 10*s*

234 The former is better 235 £7765, 6*s* 8*d* 236 £40000

237 £81577 238. £$133\frac{1}{3}$ 239. £52, 16*s*. 240 10%.

241 £37, 10*s* 242. £1300 in $4\frac{1}{2}$ p c 's, £1350 in 4 p c 's.

243 £4207, 10*s* in 5 p c 's, £5409 in $2\frac{3}{4}$ p.c 's 244. £12967, 10*s*.

245 £3880 in 3 p c 's; £3120 in 4 p c 's.

246. £3894 in $2\frac{1}{2}$ p c 's; £3700 in $2\frac{3}{4}$ p c 's

247 £958, 18*s* 8*d* 248 £20416 249. £105 decrease

250. The latter. 251 £7500 252. 3%. 253. £1728

254. 80 255. £3990 256 £3900 stock at 93; £1100 stock at 88

257 £2000 stock at $110\frac{1}{2}$, £3333, 6*s* 8*d* stock at 113

258 £2000 stock; $114\frac{1}{2}$. 259. 13 · 31. 260. $8\frac{1}{2}$%

261. £3, 6*s* 9*d* increase. 262. £264, 12*s* 263. £10, 9*s* $9\frac{3}{4}$*d*.

264. £936, 10*s* 8*d*. 265 £16, 3*s*. 7*d*. increase.

266. £4733, 6*s* 8*d* stock, £63, 11*s* 8*d* gain. 267 £11, 18*s*. increase

268 —. 269. £236, 12*s* 6*d* 270 £104, 19*s* 2*d*

LVIII. 1. 103 *fr* 5 *c* 2. 1587 *fr* 50 *c* 3 236 *M*. 70 *pf*.

4 76 *M* $5\frac{1}{2}$ *pf* 5 846 *l* 75 *c* 6. 48 *fl* 25 *kr* 7. 93 *R* 45 *kop*

8 $7 45 *c* 9. $154 70 *c* 10. $6 $5\frac{3}{4}$ *c* 11. 74·25 *fr* 12. 8·05 *fr*.

13. 105·5 *M*. 14. 16·055 *M*. 15. 24·05 *l*. 16. 6·25 *fl*. 17. 62·05 *fl*.

18 150·19 *R* 19. $17·35 20 $175·05 21. 2297 *fr* 88 *c*.
22. 1982 *M* 60 *pf* 23. $155 65 *c*. 24 213 *fr* 15 *c*. 25 1053 *fr*
26 415 *M* 80 *pf* 27 $698 25 *c* 28 7 *fr* 7 *c* 29 $13·85.
30 67 times 31. £16, 18*s*. 2½*d* 32. £98, 13*s* 2*d*. 33 £3, 2*s* 2*d*
34 £18, 1*s* 7*d* 35. 4*s* 9*d* 36 £3, 2*s* 2*d* 37. £2, 5*s* 3*d*
38 £8, 13*s* 9*d* 39. 14*s* 8*d* 40 £54, 13*s* 9*d* 41 £7, 9*s* 4*d*
42. £2, 13*s* 11*d* 43 £908, 13*s* 4*d* 44 £892, 12*s* 1*d*
45 £8, 18*s* 6*d* 46 £9, 1*s* 9*d* 47. 192 *fr* 63 *c* 48 953 *fr*. 26 *c*
49. 534 *fr* 84 *c* 50 10246 *fr* 73 *c* 51 311 *M* 23 *pf*. 52. 46 *M* 12 *pf*.
53. 1915 *M* 40 *pf* 54. $17 60 *c* 55 $1039 20 *c* 56 1291 *fl* 65 *kr*.
57. 380 *fl* 15 *kr*. 58. 65 *l* 47 *c* 59 340 *R* 48 *kop* 60 676 *R* 72 *kop*.

LIX. 1. 4 *Km* 5 *Hm* 7 *Dm* 3 *m* 2 4 *Mm* 5 *Km* 7 *Hm* 3 *Dm*.
3. 4 *m*. 5 *dm* 7 *cm* 3 *mm* 4. 4 *Hm* 5 *Dm* 7 *m* 3 *dm*
5 4 *m* 5 *dm* 7 *cm* 3 *mm*. 6 4 *Km* 573 *m* 7 45 *Km* 730 *m*.
8. 457 *Km* 300 *m* 9 20 *Km* 534 *m* 10 2 *Km* 53·4 *m*
11. 3 *m* 65 *cm* 12. 36 *m* 50 *cm* 13 1 *m* 37 5 *cm*
14 8 *m* 46 *cm* 15 7 *m* 30·5 *cm* 16 2·347 *Km*
17. 51·0702 *Km* 18 90·125 *Km* 19. 13·025 *Km*
20 7·5 *Km* 21 4·005 *Km* 22 8·75 *Km* 23 ·0437 *Km*
24 04 *Km* 25 ·0004 *Km* 26. ·0002 *Km* 27 6000 *m*
28 13500 *m* 29 70 *m* 30 ·7 *m* 31 ·045 *m*
32 300 *cm* 33 750 *cm* 34 60 *cm* 35. ·3 *cm*
36. 300,000 *cm* 37 60·57 *Km* 38 718·1 *m* 39 1120 85 *cm*.
40 9·775 *Km* 41 87·3 *m* 42. 18 5 *cm* 43 50 *m*
44 24·54 *Km* 45 58·65 *Km* 46 8100 *m*. 47 939 4 *m*
48 763·8 *Km* 49 4·535 *Km* 50 2·05 *Km*
51 59 *cm*. + 13 *cm* over 52 64 *cm* + 1 *cm* over
53. 5 *m* + 500 *cm* over 54 3 04 *m* + 32 *cm* over 55 47 times
56 18 times 57 702 times 58. 305 times
59 242 times + 11·3 *m*. over 60 43 times + 2·5 *mm* over
61. 102 *fr* 34 *c* 62 68 *fr* 59 *c* 63 46143 *fr* 75 *c*
64 195 *fr* 91 *c* 65. 133 *M* 21 *pf* 66. 16380 *M*
67 50 *fr* 37½ *c* 68 32 *fl* 62½ *c* 69 1880 70 167.
71 18 *fr* 24 *c* 72 61 *m*. 50 *cm* 73 51 *Km* 562½ *m*
74 26 times + 16 *mm* over 75 1006 *fr* 75 *c* 76 1655 *fr*
77 54 *fr* 90 *c* 78. 17 *M* 85 *pf* 79. 23 *M* 65 *pf*
80 237 *fl*. 60 *c* 81. 8 *fr* 10 *c* 82 34 *fr* 20 *c*
83 48 *fr* 79 *c*. 84. 56 *fr* 5 *c* 85 282 *M* 51 *pf* 86 21 *M* 39 *pf*
87. 561 *Kg*. 88. 16 *cg* 89 241 times + 1525 *g* over
90 73 *fr* 6 *c* 91. 20 *M* 52 *pf* 92 9 *fr* 60 *c* 93 6 *M* 5 *pf*
94 18 *fr* 50 *c* 95 178 *fr* 10 *c* 96 65 *c* 97 5 *fr* 25 *c*
98. 175 *fr*. 50 *c* 99 48 *c* 100 20886 *fr* 25 *c*. 101 300 *sq m*
102. ·03 *sq. m* 103 3000000 *sq m* 104 2300 *sq m*. 105 230000 *sq m*
106. 23 *sq m* 107. 700 *sq cm* 108. 70000 *sq cm*. 109 4·85 *sq cm*.
110 3400000 *sq cm* 111. 5·8 *a* 112. 43 *a* 113 1·98 *a*
114. 630 *a* 115 166,304 *sq cm* 116 21·4496 *Ha* 117. 6·375 *Da*
118 23 times 119 5131 *fr* 87½ *c* 120. 61200 *fr* 121 24·15 *sq. m*.

122 54·81 *a*. 123. 136 *M* 80 *pf* 124. 145 *fr*. 20 *c*. 125. 333 *fr*. 35 *c*.
126. 28 *fr*. 40 *c* 127. 146 *M* 75, or 80, *pf* 128. 3000000 *cub cm*
129. 4·85 *cub. m*. 130. 150 *cub m*. 131 ·015 *steres*. 132. 168 *cub m*
133. 48037·5 *cub dm* 134. 57 *fr*. 9 *c* 135. 30000 *Kg*. 136. 6·25 *mm*
137. 105 *Hl*, 10500 *Kg* 138. 55225 *tonn* 139. 1·1811 *in*.
140. 1093·6 *yds* 141. 4·9710 *ms* 142. 134·52 *ft*. 143 1550·1 *sq in*.
144. 5·5802 *sq m* 145. 2·4711 *ac* 146. 61027 *cub in*
147. 1·3080 *cub yds*. 148. 1·7608 *pints* 149. 15·432 *grs*
150. 2·2046 *lbs* 151. 2·5400 *cm* 152. 15·240 *dm*. 153. ·91438 *m*
154. 4·8279 *Km* 155. 6·4512 *sq cm* 156. 2·0234 *Ha* 157. 36·420 *a*.
158. 16·386 *cub cm* 159. 5·3517 *cub m* 160. 4·5435 *l*.
161. 31·103 *gram* 162. 11·340 *Kg* 163. 4000 *sq. m*.
164. 51 yds. nearly 165. 175 *cub dm* 166. 1609 *m*
167. 160·9308 *Km* 168 12 sq yds 169. 784·54 *m*
170. More expensive 171. 12193 *sq cm* nearly. 172. 78 *fr*.
173. 5*s* $4\frac{1}{2}$*d* nearly 174. 1*d* 175. 25515 *Kg* nearly.
176 144·7 *Kg* nearly. 177. 215·55 *M* 178. 18·7% nearly
179. 6·4 *c* nearly 180. 1226 *g* nearly

LX. 1. 13356 *fr* 2. 5799·75 *M*. 3. 9882·60 *l*. 4. 19774·80 *kron*.
5. 7696·20 *gul* 6. 2250 *fl* 12 *st*. 7. $1185·30. 8. 11691·09 *R*.
9. 3008 *dol* 10. 2109·554 *mlr*. 11 £340, 12*s* 5*d*. 12. £159, 12*s* 7*d*
13. £61, 9*s* 6*d* 14. £54, 5*s* 2*d* 15. £181, 8*s*. 11*d*
16 £308, 19*s* 4*d* 17 £345, 13*s* 7*d* 18 £3816, 13*s*. 10*d*
19. £72, 12*s* 6*d*. 20 £59, 18*s*. 6*d*. 21 12197·44 *fr*
22. 3627·88 *M* 23 19300·12 *kron* 24 3051 *fl* 4 *st*
25 14097 *Rs* 7 *a* 26. 3238·4 *mlr* 27. £824, 12*s* 7*d*
28 £190, 2*s* 9*d*. 29 £590, 2*s* 1*d* 30 £80, 9*s* 7*d*
31. £169, 0*s* 3*d*. 32 £640, 11*s* 5*d* 33. £198, 2*s*.
34. £719, 9*s* 3*d*. 35. 25·07 36. 25·12 37. 49 38 20·46.
39 94·64. 40. 1*s* $2\frac{1}{2}$*d* 41. 25 $40\frac{1}{2}$ 42. 12 $2\frac{3}{8}$
43. 12·19 44 $40\frac{1}{2}$*d* 45. $2\frac{1}{2}$% 46. 4% 47. £157, 12*s* 1*d*
48 £243, 6*s* 10*d*. 49. £140, 11*s* 10*d* 50. £1730, 8*s* 1*d*.
51 10667·09 *fr*. 52. 8097·22 *R* 53. 6457·15 *l* 54. 3274 *Rs*
55. 24983·75 *fr* 56. Better to *remit* 57. Better to *draw*
58. 12·06 *fl*. for £1. 59. 122·35 *fr* for 100 *M*. 60. $4·81½ for £1.
61 Better through London, $151·5 62 25·23 *fr* for £1
63. £556, 15*s* 3*d* 64. 25·25 *fr* = £1 65. £1 = 16·145 rupees

---

LXI. 1. 1094·29. 2. 2961·02 3 6018·595. 4 17894·86
5 231·732. 6. 1491·77475 7 18·4629 8. ·7902 9 ·1521.
10. ·1464 11. £6, 9*s* $4\frac{1}{2}$*d* 12. £25, 3*s* 3*d* 13. £35, 9*s*. 6*d*
14. £63, 10*s* 6*d*. 15. £407, 19*s* 3*d* 16. 9*s* 7*d* 17. 11*s* 7*d*
18. 8*s* 6*d* 19. £1, 4*s*. 7*d* 20. £1, 16*s* 3*d* 21. £3, 13*s* 10*d*
22. £1, 17*s* 3*d*. 23. £5, 2*s* 4*d*. 24. £19, 11*s*. 25 93 yds
26. 269 yds 27. 734·9 yds 28 1862·5 yds. 29. 103·7 yds
30 335·2 yds. 31. 95·7 yds 32. 649·2 yds. 33. 190·19 *m*

34 474 15 m 35. 69·04 m 36 1503 02 m 37 383 lbs 10 ozs
38 7045 lbs 10 ozs 39 866 lbs 40 35 lbs 7 ozs 41 4590 96 fr
42 27750 80 fr 43 42276 71 fr 44 8811 43 fr 45 775·48 fr
46 13185 70 fr 47 £400, 8s 11d 48 £2114 0s 1d
49 £17, 11s 5d 50 £78 9 11d

— —

LXII.

1 30 2 50 3 80 4 100 5 600
6 1100 7 15 8 28 9 99 10 72 11 16
12 63 13 7 14 9 15 35 16 96 17 132
18 230 19 91 20 170 21 290 22 132 23 18
24 570 25 24 26 32 27 36 28 63 29 66
30 81 31 96 32 330 33 128 34 243 35 325
36 625 37 735 38 6400 39 1111 40 9009 41 43
42 58 43 86 44 167 45 209 46 237 47 277
48 291 49 321 50 327 51 409 52 417 53 527
54 745 55 777 56 1079 57 1209 58 1763 59 1858
60 2307 61 3313 62 3526 63 4379 64 4722
65 5243 66 6015 67 9008 68 9709 69 13765
70 15215 71 20103 72 29075 73 33104 74 34002
75 37196 76 92829 77 93139 78 104196 79 3000071
80 7077077 81 $\frac{1}{100}$ 82 [illegible] 83 $\frac{7}{26}$ 84 $\frac{9}{11}$ 85 $\frac{9}{26}$
86 [illegible] 87 $\frac{1}{15}$ 88 [illegible] 89 $\frac{8}{9}$ 90 [illegible] 91 [illegible]
92 [illegible] 93 $1\frac{1}{4}$ 94 $1\frac{1}{15}$ 95 $1\frac{2}{15}$ 96 $1\frac{6}{7}$ 97 $5\frac{4}{5}$
98 $1\frac{1}{12}$ 99 $1\frac{5}{14}$ 100 $1\frac{6}{17}$ 101 [illegible] 102 [illegible] 103 $5\frac{4}{5}$
104 $8\frac{1}{6}$ 105 [illegible] 106 $140\frac{1}{2}$ 107 $268\frac{1}{3}$ 108 $549\frac{1}{4}$
109 [illegible] 110 [illegible] 111 $4\frac{5}{7}$ 112 192 113 [illegible]
114 $18\frac{3}{5}$ 115 49 116 $\frac{2}{7}$ 117 $\frac{10}{19}$ 118 1
119 [illegible] 120 [illegible] 121 $\frac{5}{6}$ 122 [illegible] 123 1 1
124 1 2 125 2 126 3 127 8 128 9
129 01 130 ·06 131 07 132 002 133 005
134 012 135 2 1 136 17 137 73 138 00022
139 14 6 140 3 13 141 303 142 35 7 143 4 57
144 5 05 145 1·001 146 11 01 147 140 5 148 1 683
149 30 03 150 265 9 151 1263 152 0905 153 35 04
154 35 79 155 4 312 156 562 1 157 65 43 158 2907
159 79 05 160 8 008 161 11 211 162 120 98 163 1353 1
164 1807 165 1 8827 166 241 03 167 5 5555 168 655 36
169 7790 5 170 7 9901 171 490 304 172 65 4321
173 60010 06 174 1 41421 175 1 73205 176 2 23606
177 8 36660 178 1 58113 179 6 95701 180 714142
181 225831 182 547722 183 632455 184 1 77245
185 0236643 186 00316227 187 63 4997 188 8 13236
189 1 291 190 577 191 2·061 192 852
193 065 194 4 518 195 120 196 804
197 [illegible] 198 [illegible] 199 [illegible] 200 [illegible] 201 $1\frac{5}{6}$ 202 [illegible]
203 [illegible] 204 [illegible] 205 882 206 559 207 1·058
208 2 035 209 775 210 302 211 645
212 1 061 213 1·472 214 1 099 215 1 773

216 ·848 **217** ·594 **218** 3·545 **219** ·101

220 ·083 **221** 9·8995 **222** 69·2820 **223** 4 0557

224 2 1928 **225** 9·7980 **226** 4 4721 **227** 1 9319

228. 3·4912 **229** 2·2361 **230** 1·7321 **231** 4 8990

232 2 6390 **233** 5 0599 **234** 1·2890 **235** ·4142

236 ·8090 **237** ·6899 **238** ·2743 **239** 15 9373.

240 15 2942 **241.** 5·2779 **242** ·3482 **243** ·2679

244 33 9705 **245.** ·0142 **246** ·0455 **247** 45685

248 1 80278 **249.** ·33538 **250.** 1·875 **251** 7 yds 1 ft.

252 16 ft 7 in **253** 32 ft 10 in. **254** 86 ft **255** 110 yds.

256 219 yds **257** 71 yds. **258** 880 yds. **259** 31 po.

260. 91 ft. **261.** $\frac{1}{4}$ hr. **262** $5\frac{9}{10}$ min **263.** £198.

264 £10, 5*s* 6*d* **265** 137 ft. **266** £96, 6*s*. **267.** £196, 10*s*.

268 £3, 11*s* **269** $61\frac{1}{2}$ yds.; 5*s* $1\frac{1}{2}$*d* **270** 48 days **271.** 31

272 186. **273** 7 in **274** 10 in **275** 7 in, 1 ft 9 in, 3 ft 6 in

276 16 9 **277** ·1367 **278** 6 140033 **279** ·0707106

280 — **281** 14 ft **282** $120\frac{1}{2}$ yds **283** 498 15 yds

284 14 **285** (i) 438, (ii) 1 **286** 2 **287** 12, $2\frac{1}{5}$

288 103 **289** 2 7 **290** 2·0305 **291** 1302 **292** 96 8115.

293. 80 ft. **294** 24 ft **295** 8 ch 50 lks **296.** 4 950 in

297 40 ft **298** 4%. **299** $3\frac{1}{3}$% **300** $6\frac{1}{4}$%

LXIII.

1. 20 **2** 30 **3** 40 **4** 50 **5** 6 **6** 12

7 24 **8** 63 **9** 77 **10.** 26 **11** 70 **12** 88 **13** 9

14 15 **15** 24 **16** 28 **17** 33 **18** 96 **19** 125

20 567 **21** 17 **22** 23 **23** 34 **24** 37 **25** 48

26 65 **27** 79 **28** 83 **29** 131 **30** 203 **31** 950.

32 1060 **33** 234 **34** 298 **35** 658 **36** 735 **37** 1101

38 2901 **39** 4372 **40** 7436 **41** 10090 **42.** $\frac{3}{20}$ **43** $\frac{5}{12}$

44 $\frac{4}{7}$ **45** $1\frac{1}{6}$ **46** $1\frac{1}{8}$ **47** $2\frac{3}{4}$ **48** $3\frac{5}{11}$ **49** $13\frac{2}{7}$

50 $75\frac{1}{8}$ **51** $1\frac{1}{3}$ **52** $\frac{1}{15}$ **53** $\frac{1}{8}$ **54** ·2 **55** ·3

56 1·2 **57** ·01 **58** 1·3 **59** 3·1 **60** ·76 **61** 18·6

62 3 27 **63** 91 6 **64** 31·07 **65** 513·5 **66** 127 65

67 1 26 **68.** 1·44 **69** 1·91 **70** ·86 **71** ·53 **72** 1 05

73 1 52 **74** ·86 **75** ·43 **76.** ·55 **77** 69 **78** ·77.

79 1·46 **80.** ·89. **81** 1 82 **82** 198 **83** 10 ft 11 in

84 171·61 sq ft **85** $121\frac{1}{2}$ sq ft **86** — **87** 11 ft 11 in.

88 3·2809 ft **89** $\frac{3}{4}$ in **90** 88 ft, 176 ft **91** 18 ft

92 14, 21; 28 **93** £8, 9*s* **94** 6 **95** 2604 **96** 11

97. 16 **98** 18 **99** 25 **100** 44 **101** 59 **102** 83

103 97 **104** 44 **105.** $3\frac{1}{2}$ **106** 1 122 **107** 11

108 5·4 ft **109** 4% **110** 2%

LXIV.

1. 11420 **2.** 43657 **3** 163125 **4** 20331. **5** 38160

6 5514*t* **7.** 12114 **8** 101 **9** 111515 **10** 333478

11 34*t*539 **12.** 913*e*3. **13** 1312304 **14.** 552716 **15** 1425400

16 20452443 **17.** 1*t*3*t*5166 **18** 58*e*164851. **19** 803845

20. $362t07+6$. 21 1023 22 $104032+14$ 23. 14173.
24 $20t39$ 25. 22602 26 122202220 27 44111. 28 $39e9e$
29 496 30 1331 31. 102332 32. 10172 33. 10020101222
34 762 35 427011 36 5752 37 1010101111. 38 823
39. 11211123. 40 16806 41 $214{\cdot}043$ 42 $2t9{\cdot}39$ 43 $46{\cdot}\dot{5}$
44 $40{\cdot}85714\dot{2}$ 45. $375{\cdot}\dot{3}48\dot{t}$ 46 11011100 001 47 134
48. 577 49. $3t7$ 50 1613 51 $e1$ 52 1295, 216
53. 65 54 42 55. Scale 7. 56 $\cdot 69$ 57. $1te{\cdot}e3$
58 $143{\cdot}69\dot{4}$ 59 $7t{\cdot}t88$ 60 Scale 8 61. 2, 2, 3, 10, 14, 16
62 $\cdot 1\dot{3}$ 63 43 34 64 31 $28e$
65. When the den[r] contains no factors except 2's and 3's, a *terminating* radix fraction 66 $\frac{61}{400}$ 67. $\frac{251}{550}$ 68 $\frac{51e}{ee0}$
69 1, 2^3, 2^5, 2^6 and 2^7, lbs 70 (i) 3^5, 3^4, 3^2, and 1, lbs, (ii) 3^6, 3^4, 3^2, and 1, lbs. in one scale-pan, and 3^5 and 3, lbs in the other

---

LXV. A 1. 65 2 8*s* $1\frac{1}{2}d$ 3. $1\frac{59}{112}$. 4 (i) $12{\cdot}3$, (ii) 12300, (iii) $\cdot 123$, (iv) $\cdot 0123$ 5. 970 pieces, 90 cm over
6. 48 mi per hour 7 114 acres 8 Either 112 out, or 72 not out 9 $93\frac{1}{3}$ hours 10 A, £48, 12*s*, B, £24, 8*s*

B 1.

47	47	35	35	47	47
35	35	47	47	5	7
235	141	245	140	235	329
141	235	140	245	7	5
1645	1645	1645	1645	1645	1645

2 £1354, 14*s*, find 276 times (£5 − 22*d*) by Practice 3 12, 18, 36
4 1*s* 7*d* 5. $1{\cdot}89$ 6 £20,550 7 Five times
8 1 hour 59 min 9 5 yards 10. French

C. 1.

1958	
1570	ten times
388	
157	once
231	
157	once
rem[r] 74,	quot 12 times

2 13*s* 2*d* 3. $26{\cdot}3$
4 141 fr 12 c 5 14 min
6 19, $14{\cdot}5$ years 7 £17, 2*s* 10*d*
8 11 20 A M 9 68 %
10. $\frac{9}{22}$

D 1. 1 yard 2. (i) £7, 3*s*, (ii) 143 times 3 $\cdot 44$ 4 10*s* 11*d*
5 (i) $\frac{193}{360}$, (ii) $\cdot 5361$, (iii) $53{\cdot}61$ 6 36 miles, 52 8 feet
7 988 31 francs 8 75 9 48 10 $8\frac{1}{2}$ miles

E 1 (i) 67, *i e.* 73 times $(5-4)-(3+2+1)$
(ii) 49700, *i e* $497 \times 100 \times 52 \div 52$
(iii) £6, 2*s* 7*d*, *i e* 73 times 1*s* 10*d*. − twice 5*s*. 7½*d*
2. Three 3 1 4. 28600, 9500 5. 8 times 6. 3¾%.
7 66 ft ; 63 ft 8. 17 per thousand 9. 6 sheets ; 8*s* 3*d*
10 58 lbs 6 ozs

F. 1. "One thousand and seventy-seven", "Ten hundred and seventy-seven"; "Ten, seventy-seven"; "One, nought, seven, seven"; "One, O, double seven"
2. (i) 75, rem^r 6, (ii) 117, rem^r 27; (iii) 150, rem^r 6 3. 3*d*.
4 (i) £151, 4*s* 2⅝*d*, *i e.* 3675 times $(10-\frac{1}{8})$ pence;
(ii) $\frac{11}{1160}$, *i e* the diff of $1-\frac{1}{40}$ and $1-\frac{1}{29}$;
(iii) $38032{\cdot}8\dot{3}$, *i e* $(500-5)$ times 76·834
5. £16,666, 8*s* 6. (i) 7½, (ii) 4½ 7. £1347, 7*s* 6*d*. 8 2 days
9. 30 10. 744.

G. 1.
$$\begin{array}{r} 279 \\ 279 \\ \hline 2790 \\ 2790 \\ 2790 \\ \hline 8928 \end{array}$$
2. 17½. 3. 1·44 4. (i) 10000 miles, (ii) a mile, (iii) a yard, (iv) an inch, (v) $\frac{1}{16}$ of an inch.
5 1 16 P M 6. £1, 15*s*. 11*d* 7. £6, 4*s*.
8. 5 ft 7½ in 9. 3 francs 10 centimes
10. Each man £9, 4*s*., each woman £3, 9*s*

H 1. 446 2 £204, 16*s*. 3. 97 4 £33, 13*s* 0¼*d* 5 400
6. 66·9 Rough check, the expression is roughly $35 \times \frac{56}{18} \times \frac{34\ 5}{56}$, so is rather less than *twice* 34 7 To within half of a millimetre.
8. 71 years 9. 4 min 10 1849

I. 1. April 1st, 1904. 2 1*s* 1*d* 3. 8 st. 3 lbs. 4. 90·10
5 12 marks 6 Scotland 2·0%, Ireland 4·6% 7. 142 feet.
8 Yes, about 3*s* more. 9 £5, 17*s*. 1*d* 10. 243⅓%.

J 1. 4662 2. (i) £37, 0*s*. 1*d*, *i e* 83 times (6*s* 7*d* + 2*s* 4*d*);
(ii) 37139⅞, *i e* $37610 - 3761 \times \frac{1}{8}$ 3. 1·285
4 32, 19, 6, miles 5. 28·3 6. A 7. 140 8. £1,765,000.
9 5½*d* 10. £4, 1*s*

K. 1. One hundred and fifty; by the figures 111. 2. 0
3. 7674 lbs ; 89 feet 4. 18 points 5 5564·16 francs.
6. 1408 yards 7. 19 ft 6 in 8. £1, 7*s*. 6*d* 9. (i) 1 year, (ii) 4 years, (iii) 10 years, (iv) 28 years 10. 250

L. 1. MCDLXII 2. C pays A 8s 11d and B 10s 4d 3. 4
4. ·066106 5. 16, 13, 11, 11 6. 14·1 years, 5 ft., 7 st 11$\frac{1}{2}$ lbs
7. 14 miles 8. \$2·47, \$15·57. 9. £148, 7s 3d
10. 998001, 100489

M. 1 DCCCXLVII. 2. 10584 3 £512, 2s 8d, by the method of Example vii, page 98 4. 253 5 79 6 3s 6$\frac{1}{2}d$ in the £. 7. 8$\frac{1}{2}$ mins., so as to indicate 3$\frac{1}{2}$ mins past 3
8. 14·4% 9. 2 miles 10. 400 sq in., 512 biscuits

N. 1. 9990; 1036 2. 40 cm 3 ·00126 4. £238 5 $\frac{4}{17}$.
6. £51, 12s 8d 7 4$\frac{1}{2}$ mi per hr 8 4 loads 9. 66$\frac{2}{3}$%
10. £11, 9s 1d

O. 1. 2704. 2. 72 cm 3. 2·958 4 1$\frac{1}{2}d$ 5. 63 miles.
6 60 7 25,654 8. —. 9. 218$\frac{3}{4}$ gals 10 20 min. past 12; 4$\frac{1}{2}$ miles

---

P. 1. Eight. 2. £30, 6s 7d 3. 40·7 4 12300. Roughly, about $\dfrac{300 \times 500}{12}$, or 12500. 5. 27$\frac{1}{2}$% 6. \$6·83
7. £25, 4s 8 £4, 1s. increase 9 (i) 153 sq ft, 2448 tiles, (ii) 805 sq ft, (iii) 1530 cub ft 10 3292·8 litres.

Q 1 5555. 2 38 thousand times 3 £720, 17s 6d 4 83$\frac{11}{16}$
5. 10·8 sq ft. 6. 43%. 7 D 8 (i) £16, 2s, (ii) £16, 6s., (iii) £15, 6s 8d. 9. 50 10. 297,000 tons

---

R. 1 222 2 31 3. 126 4 11·19 5 £42, 19s 11d
6 £11, 8s 7. 50, by 30, inches, 25 feet of string 8 £45,000
9 (i) £11, 7s 6d, (ii) £495,495 10. 600, £76, 4s

S 1. 225 × 231 2 £1, 16s 1d. 3 0. 4 1545.
5 157 651 6. 1 pint 7. 9 years 7 months.
8. 30 miles per hour. 9 6241. 10. 13·5, 5·4, 3 6, cm

T 1 (i) 54,656,375; (ii) 42,850,598 2 $\frac{5}{21}$, $\frac{6}{35}$ 3. 24 bolts; 2·4 inches 4. ·051936 5 2·9% 6 £170, 5s 10d
7. (i) 4 min 40 secs; (ii) 4 min 26 secs 8. 184 yards 9. 84.
10 £3800.

U. 1. $9 \times 2 + 8 \times 3 + 7 \times 4 + 6 \times 5 + 1 \times 0$ 2. 4*s* $0\frac{1}{2}$*d*.
3. $43\frac{2}{3}$ 4. Rough estimate, (i) 1000, *i.e* 20×50; (ii) 800, *i e* $30^2 - 10^2$; (iii) 625, *i e* $\frac{40 \times 125}{2 \times 4}$ Results to one dec. place, (i) 1001·6; (ii) 802·6; (iii) 624·2 5. $4\frac{3}{4}$ inches.
6 33 seconds 7 $16\frac{2}{3}$%. 8. 288. 9. £1360, 5*s*. 10*d*
10. $6\frac{1}{4}$ gallons.

V. 1 31, 58; 19 years 2 £28, 15*s*., £9, 1*s* 3*d*
3. (i) $2\frac{1}{4}$, *i e* $\dfrac{9\left(11 - \frac{13}{444}\right)}{4\left(11 - \frac{13}{444}\right)}$; (ii) 134·862, (iii) 8·5631 4. £300
5. 4*d*, $1\frac{1}{3}$*d*, 1*d* 6. 28, 76. 7. A, £2, 12*s*. 6*d*, B, £5, 5*s*; C, £4, 4*s* 8. 5·2 kilometres, 87·4 miles.
9 £26, 6*s*. 8*d* increase 10 87, 88, 89

W. 1 37 2. — 3. 217 and 341. 4. 4620. 5. 1343
6 £276 7. £1346, 5*s* 6*d* 8. 23% nearly. 9. $5\frac{5}{9}$ cub yds.
10 6 ft 8 in

X 1 (i) 3, (ii) 2, (iii) 8, (iv) 52. 2 17*s* 3. $\frac{4}{3}\frac{2}{3}\frac{3}{0}$. 4. —.
5. 5 centimes 6 £37, 10*s* 7. 44·2 miles; 54·8 miles per hour
8 $156\frac{1}{4}$ sq yds, 150 sq. yds 9. £213, 17*s* 8*d*. 10. —.

Y. 1. (i) 1971; (ii) 30710 2. £68, 5*s*. 3 1 4 About $2\frac{1}{2}$ miles
5. 2 47 acres 6 18 miles 7. £445 8 ·0025 inch, nearly
9 1215 acres 10. £47, 17*s* 9*d*; £128, 18*s* 5*d*, £151, 0*s* 6*d*

Z. 1 1061. 2 36,075,539, 1068 3 £4, 5*s* 3*d* 4. 4.
5. (i) ·000073, (ii) 58·4. 6. 36000 7 99 men 8. 3212·41.
9. $\frac{1}{4}$ mile, 60 seconds 10. $7\frac{1}{2}$ in, $6\frac{1}{2}$ in.

AA 1. 1521 2. 12*s* 8*d* 3 1887 million. 4. Correct to 5, but not to 6, places of decimals. 5. 10 ft 11 in
6 150 lbs 7 £350 8 3 ft 4 in. 9. As 14 to 9
10. £15, 2*s* 6*d*

BB. 1 428 2 More than 200 years 3. 800 4. £1098
5. About 530 Kg 6. The latter 7 (i) £·025 is 6*d*., so ·0248 is less than 6*d*; (ii) 3 is a factor of 132, but 3 is *not* a factor of 1148, (iii) The second rectangle has 4 times the area of the first 8. — 9. 20 gallons 10. 2 miles.

CC. 1. (i) £1, 5s 6d, (ii) 1 half-sov, 13s 6d in silver, 2s in copper 2. 2520, 2688, 2856 3 ·09 4 2·44 miles 5 7 30 A M on Friday, $41\frac{1}{4}$ mins past 7 6 60% 7. £106, 11s. 2d. 8. 17·65 9 21 minutes. 10 About 11,100 cub metres

---

DD. 1. 455,832 times 2 1513 and 712 3. 22 quarts 4 3·9% 5 £7, 17s 2d 6 265 black, 264 red 7. 9 inches 8. 18 9. 2808 kilogrammes 10 23·1%

EE 1. 5 times 2 (i) £30, 14s 3d, (ii) $2\frac{1}{4}d$, (iii) £10, 4s 9d; (iv) $33\frac{1}{3}$% 3 (i) 22, (ii) 99, (iii) 220 4 ·2231

5 $I = \frac{Prn}{100}$, $P = \frac{100I}{rn}$, $r = \frac{100I}{Pn}$, $n = \frac{100I}{Pr}$, $1\frac{2}{3}$%

6. (i) 41° F, (ii) 95° F, (iii) 176° F, (iv) 35° C, (v) 15° C, (vi) 10° below zero C 7 3 ft 8 in 8 132 posts, 396 rails, £13, 4s 9 70 miles 10 The circle by more than 683 sq ft

---

FF. 1 1,000,000,000 milligrams 2 Seven 3 Scotland, 6·4% increase, Ireland, 1 7% decrease 4 14 mm nearly 5. £5, 0s $2\frac{1}{2}d$ 6 1% decrease in revenue 7 A, 22s 10d, B, 21s 8d, C, 18s 6d 8. 97 to 15 9 93 10 $5\frac{1}{4}$ miles.

GG 1. 17s 5d 2. $3\frac{3}{4}d$, $6\frac{2}{33}$% 3 3100 miles 4. £2, 14s 5 82 sq in 6 23 girls 7 250 cub ft 8 (i) $4\frac{1}{2}$ mi; (ii) $31\frac{1}{2}$ mi; (iii) $22\frac{1}{2}$ mi 9 89,460 kilogrammes 10 819

HH 1. 347 and 253 2. £451, 0s 8d, 382·6 cub ft 3 (i) 1, (ii) 2s $3\frac{1}{2}d$, (iii) 218·8 sq cm 4 104° F = 40° C = 32° R; 85° C = 185° F, 48° R = 140° F 5. $37\frac{1}{2}$% 6. $9\frac{1}{2}d$ nearly 7 437 8. $3\frac{1}{3}$ miles per hour 9. 15s 11d 10. Panama more than twice as costly

---

II 1 2,012,174,541, 2,057,695,731 2 Just over 4 francs 37 centimes 3 1609 metres 4 69 grammes 5 323 girls 6. 62 7 1324 8 £2604, 3s 4d 9. $9\frac{5}{8}$ miles per hour 10 25 7 inches nearly

---

JJ. 1. 18,000,000,000,000 miles 2. 135 lbs 3 4,306,451 4. 1013 litres 5 72·9% 6. £3200 7 3680 metres 8 At half-past 12 9. $1\frac{11}{16}$ cub feet 10. One hundred and seventy-five and a half, 37·3

KK. 1. 7743, *i.e.* $(88+1)(88-1)$, or 88^2-1 2 (i) 0, (ii) $\frac{3}{8}$, (iii) £22, 7s 4d, (iv) 41,100 sq yds 3. £261, 4s 2d. 4 3·937 inches 5. £25, 6s 8d 6. $\frac{1}{2}$ cwt 7. 894,432 cub yds 8. 58,778 million cub ft The error arose from the omission of the word "thousand", 778 is about $\frac{3}{4}$ of a thousand, so 58,778 million is about $58\frac{3}{4}$ *thousand* million. 9. 40 years 10. £1836, 0s 10d

---

LL. 1 27 2. 44 feet 3 The sovereigns, by about ·05 lb 4. Government 305, Opposition 281, Independents 84 5 81·6% 6 £320 7. 578 sq yds 8. 128 ft per min nearly. 9. 3·7321 10. 27 feet

---

MM. 1. 189; 8 2. £280 3 23,513,768,964 4 ·02237, 45 miles an hour, nearly. 5. 581 million 6. £16, 6s 8d 7. 168 8. Yes, 2 ft 3 in by 1 ft. 9 in. by $8\frac{1}{2}$ in. externally, leaving 3 in. for waste in sawdust 9. 17 feet. 10. 402854, 20590

NN. 1. Monday. 2. 1840 kilogrammes 3 21 feet 4. 8 ft by 4 ft by 2 ft 5 $67\frac{1}{2}$ seconds, 135 yards 6 (i) 86 ft, (ii) 50 ft, (iii) $18\frac{1}{2}$ ft, approx 7 12000 gallons. 8. 40 cm 9 $48\frac{3}{4}$ minutes; 2 miles; 10d. 10. 35 and 51.

EXAMINATION PAPERS.

Camb. Local Exams, Dec, 1912

PRELIMINARY —1. 20,604,819 2. £17, 18s $11\frac{1}{4}$d 3. 26520 4. (i) $\frac{1}{2}$, (ii) $\frac{11}{30}$. 5 22·5, ·075213 6. (a) £10; (b) $\frac{3}{250}$ 7. £1, 19s $0\frac{1}{2}$d. 8 £37, 2s 6d. 9. 3s 10 2s

---

JUNIOR —A 1. 308, 93632 A 2. (i) 1, (ii) ·09. A 3 £30, 16s 5d A 4. 2481 francs 60 centimes A 5 £22, 19s 4d A 6. £16, 11s 9d A 7 By £25 B 1. 2 hours B 2. 14 days. B 3. The 1st by $5\frac{5}{8}$ miles B 4. 12 gallons B 5. $34\frac{2}{7}$%.

SENIOR —1. Three 2. (i) $\frac{17}{10}$, (ii) $9\frac{1}{8}$ 3 369000; 26·46 4. 34·47 miles per hour. 5. £15, 5s 6d. 6. £65, 9s 3d 7. ·3937 inch 8. £2, 5s 6d 9. 26% 10. The Bank stock.

Oxford Local Exams, March, 1913.

JUNIOR —1 17s. 6d. 2. 81; $\frac{3}{8}$ pint 3. 1s $0\frac{3}{4}$d. 4. £38, 16s $1\frac{1}{2}$d 5 1·853 6 £1, 19s $4\frac{1}{2}$d 7. £15187, 10s; £100 loss 8. 9801; 1024 9. 231; 24389

SENIOR —1. ·521875 2. £12, 0*s.* 6½*d* 3 17 4 1*s* 9*d*
5. 1200 dollars 40 cents, nearly 6 £2, 8*s* 6*d* 7. £140.
8. $43\frac{3}{16}$ sq ft. 9. 150 lbs

London University Matric. Exam, 1913.

1 53 8, ·000364. 2 107·52 feet; 1·22 miles, per hour, approx
3. £17500

Board of Education, Prelim Certificate Exam, 1912

1. 4497 fr 78 c 2 2·0873. 3. 361 equal tiles 4. 19·23
5. 21 6 97·0299% 7 4 lbs 8 $\frac{5}{81}$, $\frac{11}{34}$ 9. 1600 sq in, 13 sheets, 740 sq in 10. 39208

11. (i) £13, 18*s* 4*d*, *i e* $\frac{1}{12}$ of £(168 − 1), or £14 − 1*s* 8*d*
 (ii) 140, *i e.* $\sqrt{16 \times 7 \times 25 \times 7}$, or $4 \times 7 \times 5$
 (iii) The first, by inspection, for its numerator is greater, and its denominator is less, than those of the second fraction
 (iv) 70½ miles, *i e* 6 times 12 miles − ⅛ of 12 miles

---

1. 16*s* 4*d.* 2 ¾*d* 3. 314·2 mm, 309·4 mm.

Board of Education Science Exams., 1912

PRACTICAL MATHS., LOWER PAPER —1 ·00457 2. 4·9 inches
HIGHER PAPER —2·600

Civil Service Exams, 1912

BOY CLERKS, FIRST PAPER —1. 6506 2 15 3. 4180 yds
4. 88 mm 5. 114 mm 6 — 7 5*s* 1½*d* 8 Seven
9 £1, 1*s* 9*d.* 10 185 lbs 11. 94⅛% 12. £4, 0*s* 8*d* 13. 48 mi
14. 749 cub decimetres 15 20 inches

SECOND PAPER —1 A, £23, 14*s*, B, £15, 18*s* 2 £7, 19*s*.
3 160,000 cub ft; 237 cub yds 4. 9 min
5 27 blocks, each 17 7 by 7·7 by 5·7, cm approx; ·9 Kg

WOMAN AND GIRL CLERKS —1. £1130, 17*s* 9*d* 2. 2126¼ cub in; 65¼ lbs, nearly 3 46%, £42,000,000. 4 *8, 58 5 The second; 2·5 cub ft. 6. 15 chairs in a row; 28 rows; 840 persons